NPTEL ONLINE CERTIFICATION COURSES

EARTHQUAKE SEISMOLOGY

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Module 02 : Snell's law, Plane wave reflection and transmission
Lecture 01: Wavenumber vector, Slowness, P-and S-wave Polarization

CONCEPTS COVERED

- Introduction to wavenumber vector
- Slowness
- P-and-S-wave Polarisation
- Summary

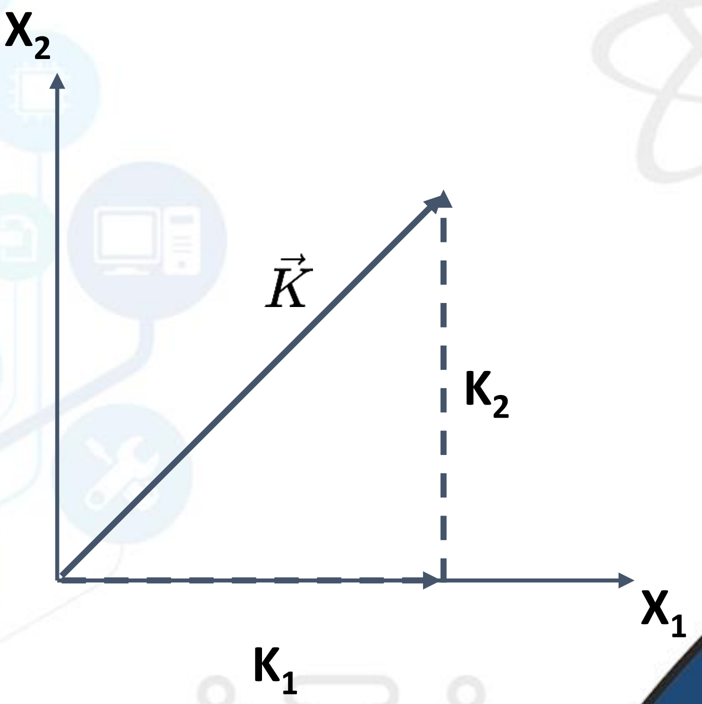
Wavenumber Vector

$$\vec{k} = k_1 \hat{e}_1 + k_2 \hat{e}_2$$

$$\left\| \vec{k} \right\| = \sqrt{k_1^2 + k_2^2}$$ In 2-D

$$\vec{k} = k_1 \hat{e}_1 + k_2 \hat{e}_2 + k_3 \hat{e}_3$$

$$\left\| \vec{k} \right\| = \sqrt{k_1^2 + k_2^2 + k_3^2}$$ In 3-D



Wavenumber Vector

$$\vec{k} = \left\|\vec{k}\right\|\hat{k}$$ This is wavenumber vector

Where

$$\left\|\vec{k}\right\| = \sqrt{k_1^2 + k_2^2 + k_3^2} = \frac{\omega}{\alpha}$$

$$\vec{k} = \left\|\vec{k}\right\|\hat{k} = \frac{\omega}{\alpha}\hat{k}$$

Wave number vector tell us about the path a wave is propagating through

We can define plane wave solution

$$\phi\left(\vec{x},t\right) = Ae^{-i\left(\omega t - \vec{k}.\vec{x}\right)}$$

This is scalar potential not the final displacement. If the amplitude depends on the frequency, then,

$$\phi\left(\vec{x},t\right) = A(\omega)e^{-i\left(\omega t - \vec{k}.\vec{x}\right)}$$

Slowness ?

Reciprocal of velocity

$$\vec{s} = \frac{\hat{s}}{c} = \frac{\hat{s}}{\text{velocity}} = \frac{\hat{s}}{\alpha}$$

Here, “c” is the arbitrary velocity and “**α**” is the P-wave velocity

Since, $\vec{k} = \dfrac{\omega}{\alpha}\hat{k}$

so, $\vec{k} = \omega|s|\hat{k}$ $\quad as \quad |\hat{s}| = \dfrac{1}{\alpha}$

Direction of $\hat{s}$ and $\hat{k}$ will be same so,

$$\vec{k} = \omega\vec{s}$$

It will help us to rewrite scalar potential in terms of slowness (s).

$$\phi\left(\vec{x},t\right) = A(\omega)e^{-i\left(\omega t - \vec{k}.\vec{x}\right)} = A(\omega)e^{-i(\omega t - \omega\vec{s}.\vec{x})}$$

$$= A(w)e^{-i\omega(t-\vec{s}.\vec{x})}$$

Similarly, S-wave potential can be written as :

$$\vec{\psi}\left(\vec{x},t\right) = B(\omega)e^{-i(\omega t - \omega\vec{s}.\vec{x})}$$

$$= B(\omega)e^{-i\omega(t-\vec{s}.\vec{x})}$$

where, $\vec{K} = \dfrac{\omega}{\beta}\hat{k}$

$\&\,'\beta'$ is the S-wave *velocity*

P- and S-wave polarisation

How the particles of the earth moves when seismic waves passing by? Or what would be the displacement "u"?

To answer this question, we will have to find the particle motion (or displacement "u") from the potential function. All we need to do is reverse Helmholtz Decomposition.

$$\text{For P-wave} \qquad \vec{u_p} = \nabla \phi$$

$$\text{For S-wave} \qquad \vec{u_s} = \nabla \times \vec{\psi}$$

For a P-wave travelling in the X-direction , the P-wave potential equation may be written as

$$\nabla^2\phi - \frac{1}{\alpha^2}\left(\frac{\partial^2\phi}{\partial t^2}\right) = 0$$

Since, P-wave is not travelling in Y-and-Z-directions, so general solution is

$$\phi = \phi\left(t \pm \frac{x}{a}\right)$$

We can rewrite it as the sum of the +ve X-direction propagating wave and -ve X-direction propagating wave

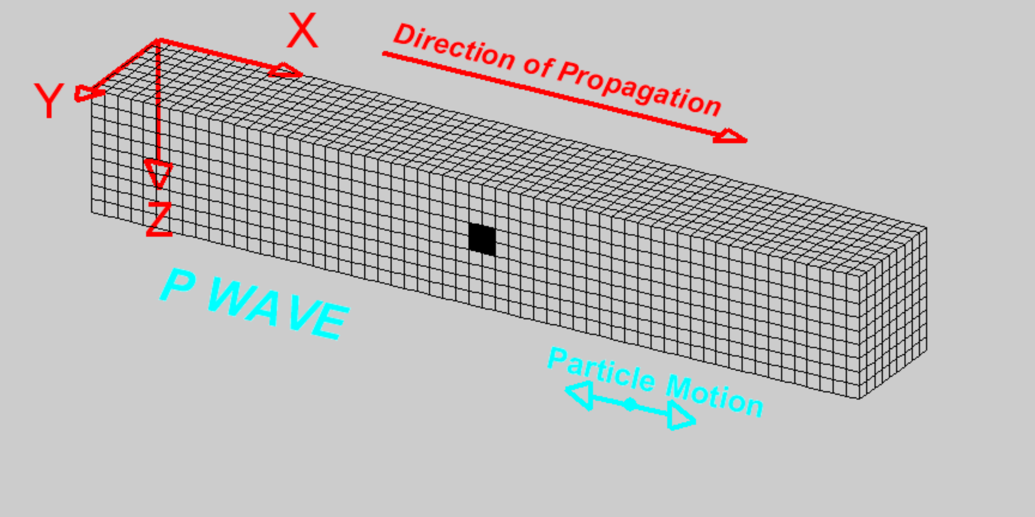
$$\phi = \phi_1\left(t - \frac{x}{a}\right) + \phi_2\left(t + \frac{x}{a}\right)$$

travelling in +ve x direction   travelling in -ve x direction

Since Φ is a function of x only then we can write as

$$\vec{u} = \nabla\phi = (\partial_x\phi,\, \partial_y\phi,\, \partial_z\phi) = \left(\frac{\partial\phi}{\partial x}, \frac{\partial\phi}{\partial y}, \frac{\partial\phi}{\partial z}\right)$$

$$= \left(\frac{\partial\phi}{\partial x}, 0, 0\right)$$



so, the P-wave moving along the X-direction is $\phi = e^{i(wt-k_x x)}$

If we take $$\frac{\partial\phi}{\partial x} = -ik_x e^{i(wt-k_x x)}$$

$$\implies \vec{u} = (-ik_x\phi,\, 0, 0)$$

which means that for P-wave , particle motion is in the direction of propagation.

For S-wave

$$\nabla^2\vec{\psi} - \frac{1}{\beta^2}\left(\frac{\partial^2\vec{\psi}}{\partial t^2}\right) = 0$$

Consider an arbitrary S-wave vector potential

$$\vec{\psi} = (\psi_x, \psi_y, \psi_z)$$

Let us assume it is travelling along X-axis then,

$$\vec{k} = (k, 0, 0)$$

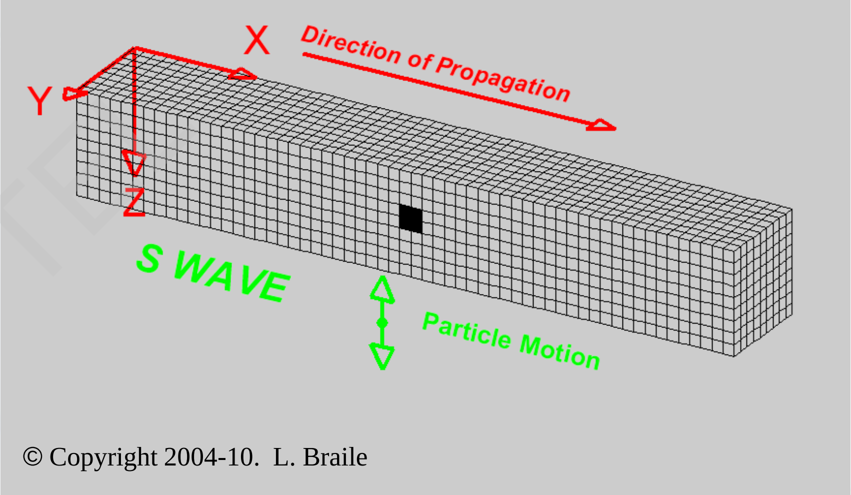
$$\vec{\psi}\left(\vec{x}, t\right) = B(\omega)e^{-i(\omega t - k_x x)}$$

Since, the S-wave is propagating in the X-direction, so potential will a function of X, then

$$\psi_x = \psi_x\left(t - \frac{x}{\beta}\right) \qquad \psi_y = \psi_y\left(t - \frac{x}{\beta}\right) \qquad \psi_z = \psi_z\left(t - \frac{x}{\beta}\right)$$

Displacement is $\vec{u} = \nabla \times \vec{\psi}$

$$\vec{u} = \begin{pmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \psi_x & \psi_y & \psi_z \end{pmatrix} = \hat{x}\{\partial_y \psi_z - \partial_z \psi_y\} + \hat{y}\{\partial_z \psi_x - \partial_x \psi_z\} + \hat{z}\{\partial_x \psi_y - \partial_y \psi_x\}$$



Since ψ_y, ψ_x and ψ_z are function of x only. So,

$$\partial_y \psi_z = \partial_z \psi_x = \partial_y \psi_x = \partial_z \psi_y = 0$$

$$\implies \vec{u} = \hat{x}\{0\} + \hat{y}\left\{-\frac{\partial \psi_z}{\partial x}\right\} + \hat{z}\left\{\frac{\partial \psi_y}{\partial x}\right\}$$

$$\vec{u} = \left(0, -\frac{\partial \psi_z}{\partial x}, \frac{\partial \psi_y}{\partial x}\right)$$

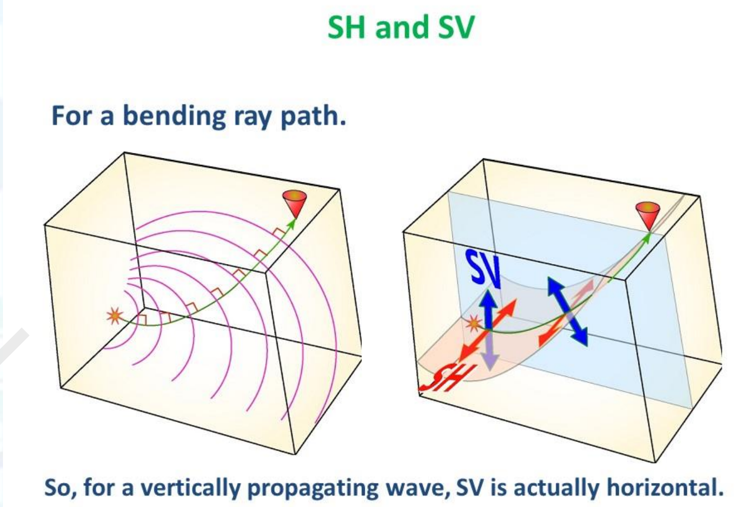
So, for the S-wave particle motion is purely in the direction perpendicular to the wave propagation

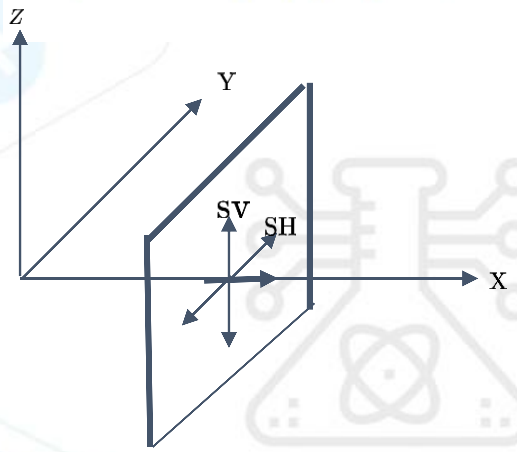
SH- and SV- wave

S-wave field is composed of SH and SV waves

- ➔ **SH - Shear wave polarised in horizontal direction**
- ➔ **SV - Shear wave polarised in station receiver plane**

Note: SV wave is always perpendicular to the propagation direction & in the vertical plane containing ray path





Summary

wave number vector shows the direction of wave propagation $\overrightarrow{K} = \left\|\overrightarrow{K}\right\|\hat{k} = \frac{\omega}{\alpha}k\,\hat{}$

Slowness vector has direction same as wavenumber vector $\vec{s} = \frac{\hat{s}}{c} = \frac{\hat{s}}{\text{velocity}} = \frac{\hat{s}}{\alpha}$

and incorporated slowness vector into wavenumber vector $\vec{k} = \omega|s|\hat{k}$

Particle motion of P-wave is parallel to wavenumber vector $\overrightarrow{u} = (-ik_x\phi, 0, 0)$

Particle motion of S-wave is perpendicular to wavenumber vector $\vec{u} = \left(0, -\frac{\partial\psi_z}{\partial x}, \frac{\partial\psi_y}{\partial x}\right)$

REFERENCES

- Stein, Seth, and Michael Wysession. An introduction to seismology, earthquakes, and earth structure. John Wiley & Sons, 2009.
- Lowrie, William, and Andreas Fichtner. Fundamentals of geophysics. Cambridge university press, 2020.
- Kearey, Philip, Michael Brooks, and Ian Hill. An introduction to geophysical exploration. Vol. 4. John Wiley & Sons, 2002.
- https://geologyscience.com/geology-branches/structural-geology/stress-and-strain/
- https://www.wikipedia.org/
- Seismology course, Professor Derek Schutt, Colorado State Univ., USA.

THANK YOU !