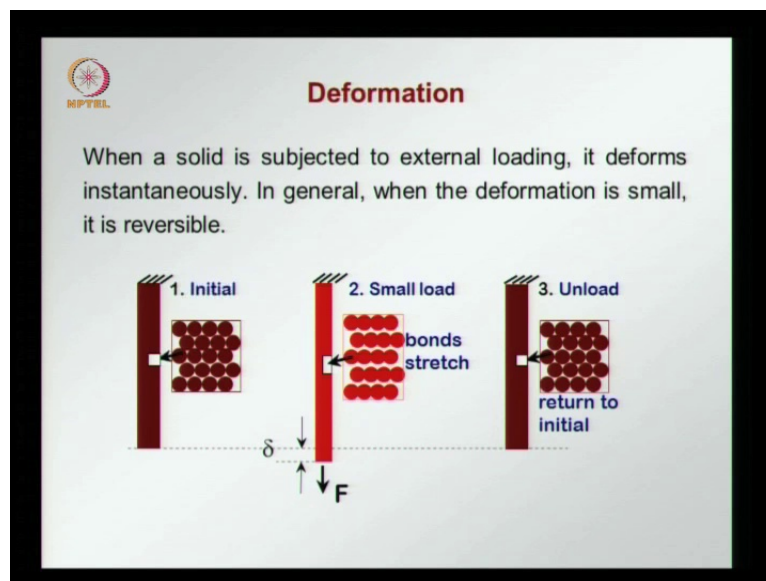


Modern Construction Materials
Prof. Ravindra Gettu
Department of Civil Engineering
Indian Institute of Technology, Madras

Module No - 3
Lecture No - 7
Part 01 of 03
Response to Stress

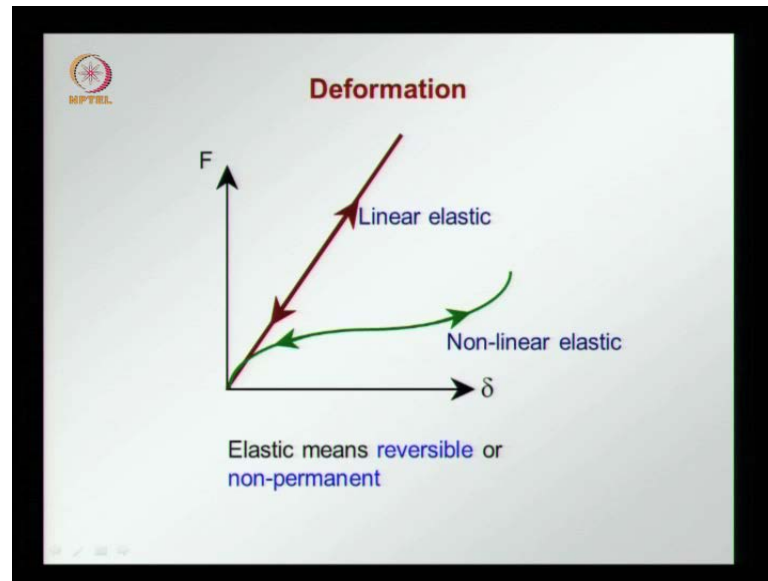
Welcome to lecture 7 of modern construction materials. Today, we are going to talk about the response of material to stress. And, until now we have looked at how the microstructure forms? And, now we will go on to look at the different properties and how the materials behave? When stress is imposed on them.

(Refer Slide Time: 00:34)



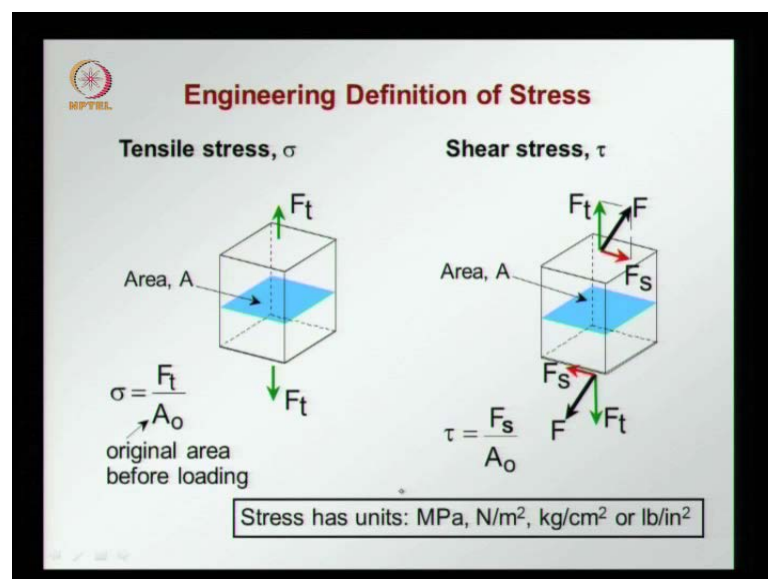
Now, we have already seen that when there is a small deformation in the material. It is in the elastic or reversible state that is when a small pull is given that is when we have an initial structure that is pull or deformed by a small distance or displacement. And, then released that is disposed is removed. the material deforms and then goes back to the original state. So, when we have the material subjected to a certain stress, there is a stretching of the bonds and there is an instantaneous deformation that the material experiences. When this force is removed and the displacement that has been applied was small the material returns to its original state when unloaded. So, that means the deformation when it is small is reversible or it is in the elastic state.

(Refer Slide Time: 01:47)



We can have the elastic state being linear or non-linear. In both these cases when the load is released the material goes back to the original state. So, when the load is applied the displacement increases and then when the load is released the material goes back to the original state. And, this could be linear the relation between the force and the displacement could be linear or non-linear. In this case the material is called linear elastic and in this case it is called non-linear. Elastic here now means that the deformation is reversible or non-permanent.

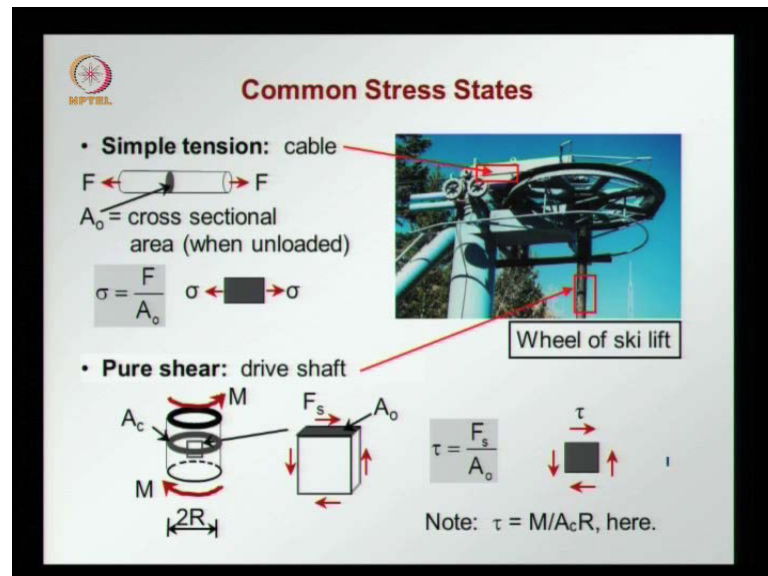
(Refer Slide Time: 02:38)



We can now define stress as follows this is something that you all studied before in mechanics and we can look at different types of stress briefly and quickly. Tensile stress is that which we define as the load divided by the area perpendicular to it. That is if we have the small piece of material subjected to this tensile load F sub t . And, this is taken by this area A then the tensile stress is the load applied divided by the original area before loading that is A sub 0 , so this is the tensile stress.

Shears stress is where we have a stress applied along the plane that we are concerned of, so, this happens when you have an inclined force or if you have a force that is along the direction of the plane. So, in this case let us have let us look at A inclined force being applied to the top of this cubical body, has F and this will have now two components. The vertical component acting perpendicular to this area F sub t , which will then be the tensile force. And, a horizontal component F sub s , which is the shears force. The shear stress across this plane, now is shear load divided by the original area this is called the shear stress. Stress has units of Mega Pascal's or Newton per meter square or kilogram per centimetre square or pounds per inch square.

(Refer Slide Time: 04:37)

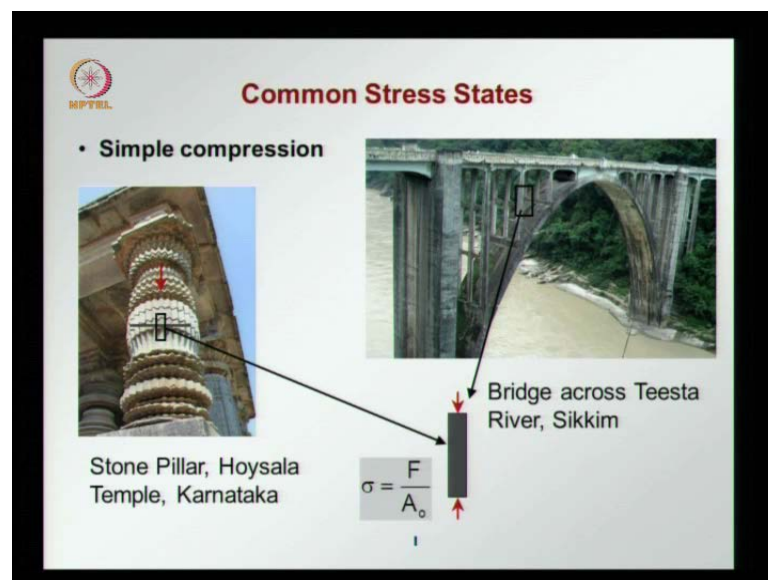


There are several common stress states that we can look at, in an example of this which is a wheel of a ski lift you have. Now, a cable which is around this wheel and this is what is moving this ski lift this wheel is rotated by a shaft. The cable is now in simple tension this cable is being subjected to a force such as what is given here we have the cable

which is being pulled by a force F . The cross-sectional area initially was A_0 . Now, that tensile force that this cable is having is F the tensile stress that this cable is subjected to is F divided by A_0 . If, you look at the shaft this shaft is being twisted.

So, what we have here is we have a piece of shaft here which has a moment being applied a cross check, so, this is the moment here. And, we have this cross section now being sheared if you look at a small piece of a material within the shaft you find that we are applying a shear force across this plane. So, the shear stress would then be the shear force divided by A_0 . And, in this case it is given as M divided by A_0 times R . So, here this is A_0 , and M is the moment that is occurring while this ski lift is moving and R is the radius of the shaft.

(Refer Slide Time: 06:45)

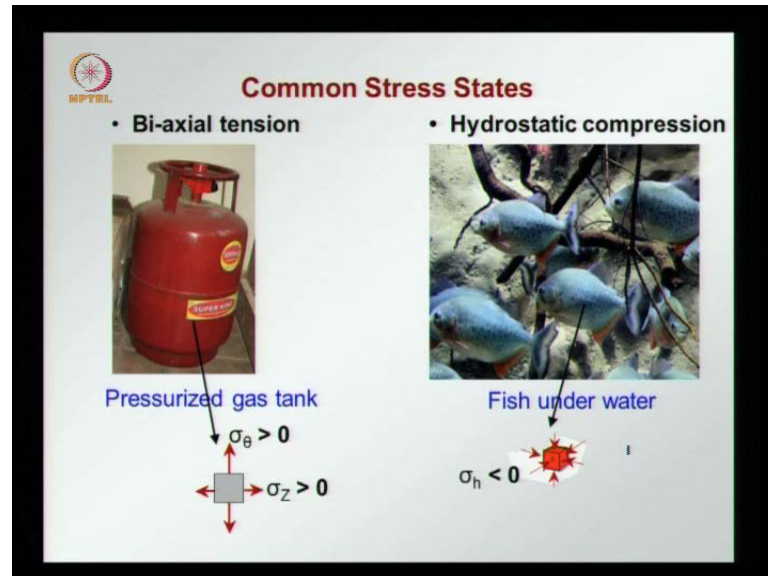


So, we have several cases, where we can identify common stress states which could be close to pure tension compression or shear. Now, in this case we see pure compression which you can have say in this example this is a stone pillar from a hoysala temple in Karnataka. Karnataka in the south of India, and we find that basically in the stone pillar you have a compressive load leading to simple compression or compressed stresses in the middle.

We see a similar behaviour in this struck this is a bridge across teesta river in Sikkim in the north of India. And, we find that now in this struck we will basically have a compressive force leading to a compressive stress in the element. In both these cases the

compressive stress will be calculated as the load divided by the initial area of the element.

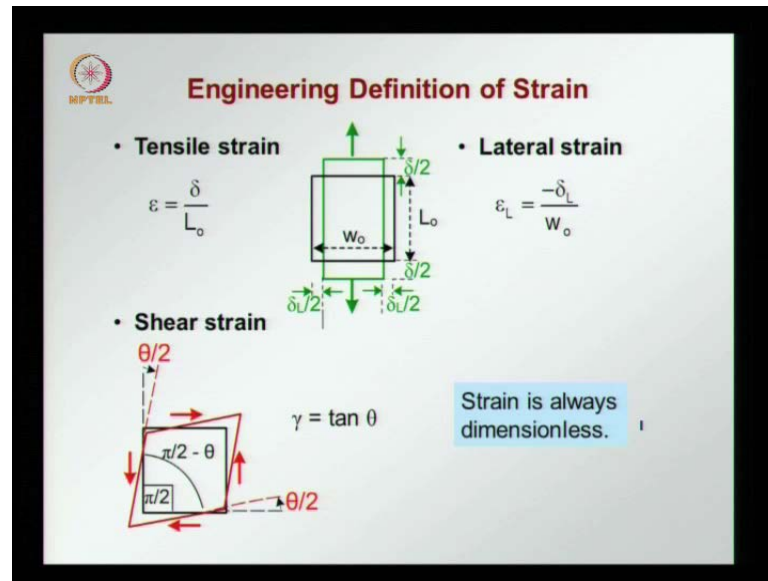
(Refer Slide Time: 17:41)



We can have other stress states which are not as simple as pure compression or pure tension. For example in the case of a pressurized gas tank or a gas cylinder, we find that the envelop of the cylinder will be under bi-axial tension. That is its being stretched in two directions they will be tension in two directions. So, you have a piece of the cylinder skin will be stretched along one direction as well as the other direction.

A case of hydrostatic compression occurs when the stresses are the same along all directions. Suppose, you have a small element with the same stress being applied in all directions we call it hydrostatic compression. That would occur say in a fish that is under water we do not have many civil engineering examples of hydrostatic compression. But, we can have some structures and a very high confinement by they will be a significant component of hydrostatic compression.

(Refer Slide Time: 08:58)



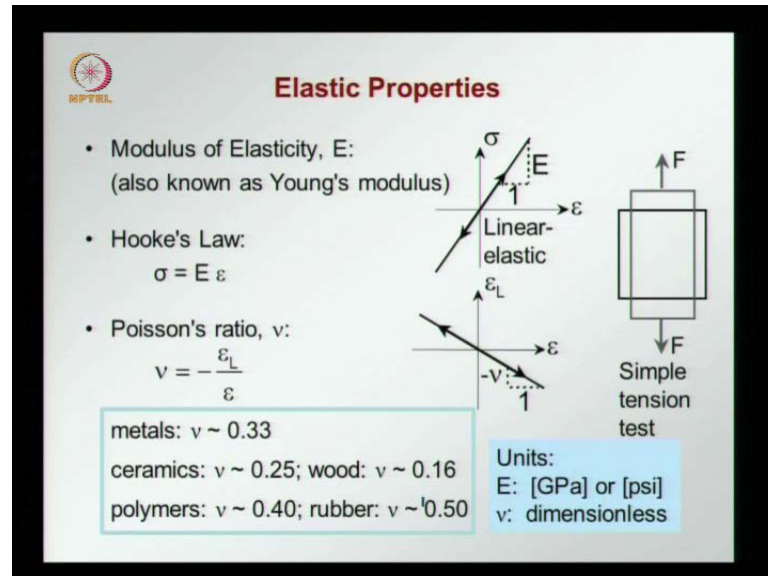
Now, we looked at stress the material when under stress under goes strain it has to respond or deform due to the applied stress and that is what we characterize in terms of strain. Tensile strain is where you have the element subjected to tensile forces tensile stresses and there is an elongation or a stretching of the material. The tensile strain is now defined as the stretch or the increase in length divided by the original L delta divided by L sub 0.

So, if you take this case and your stretching this element you can imagine that there is a change of half delta on this side an half delta that is side and your total strain is now delta divided by L_0 . As, we are stretching this element there is also a lateral deformation instead of the width being W_0 , it decreases by a certain quantity say delta L . The lateral strain is now given as minus delta L divided by W_0 delta L being the total change in the lateral dimension. We saw that there are cases were a material has shear stress right there is stress is apply across a plane causing a slip type behaviour.

The corresponding strain or shear strain is now calculated in terms of this angle, suppose we have originally this element with an angle of 90 degrees π by 2. And, if we shear this element this square instead of being the shape of the black square becomes like this red rhombus. So, there is a deformation of the angle from 90 to something, else say 90 minus theta. And, the shear strain is now given by $\tan \theta$, you will notice that in all these

cases strain did not have any dimension. That it is because strain is always dimensionless, we are dividing one length maybe other or we are looking at an angle.

(Refer Slide Time: 11:34)



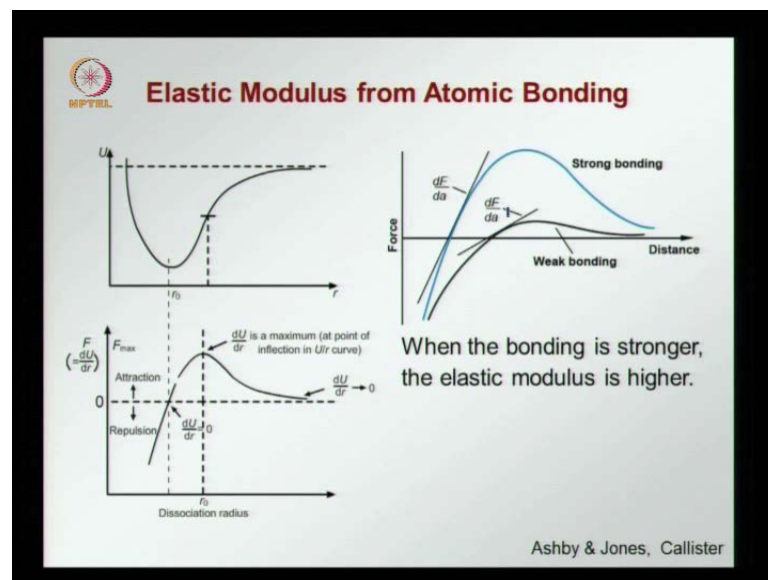
There are several elastic properties associated with the response that we have looked at the most fundamental is the modulus of elasticity which is also called the young's modulus. If we were going to do a simple tension test, so we take a bar and we stretch it. We will have a behaviour such as this when we draw the plot between this stress. And, this strain the axial stress and the axial strain over the linear elastic region. This linear behaviour as a slope which is called the young's modulus, and the relation between these two is governed by what is called the Hooke's law.

It says that stress is equal to the young's modulus times this strain. We also saw that there was a lateral deformation. And, if we were to plot the lateral strain versus the axial strain, we will also see that there is a linear behaviour and the slope is minus nu. So, if we were to divide the lateral strain by the axial strain. And, put a minus sign in front of it, we get the Poisson ratio represented by nu. We find that while the units of the young's modulus can be Giga Pascal mega Pascal or PSI. The Poisson ratio is dimensionless, since we are dividing to strains.

The value of the young's modulus where is a lot, we will come back to it later and look at typical value for different materials. However, the Poisson ratio does not vary much between different groups of materials, we find that the Poisson ratio is about 0.33 one

third for most metals for ceramics is slightly smaller about 0.25 wood will be still smaller 0.16. Whereas, more flexible polymers will have a higher Poisson ratio can be polymers will have Poisson ratio 0.4. Whereas, rubber is 0.5, rubber is called incompressible, because it has such a high Poisson ratio.

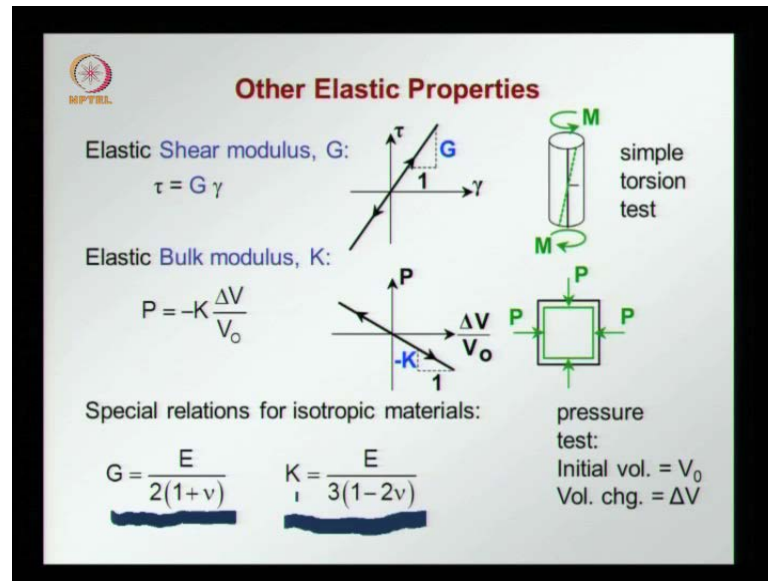
(Refer Slide Time: 14:10)



We looked at these graphs earlier. And, we saw that he can explain the elastic behaviour and a very small loads very small elongations through the condemn most diagram an the load inter atomic spacing diagram, where we saw that when we differentiate the condemn most diagram which has the energy with respect to inter particle distance or inter atomic spacing.

We saw that the diagram of load versus inter atomic or inter particle spacing had a small linear range in the atomic scale itself. And, this we saw that when the it was strongly bonded the material had a higher slope here and when it was weakly bonded the slope was less. So, we find that in strongly bonded materials we will have a higher young's modulus. That means a higher slope and here we will have a lesser slope when the material is weakly bond.

(Refer Slide Time: 15:23)

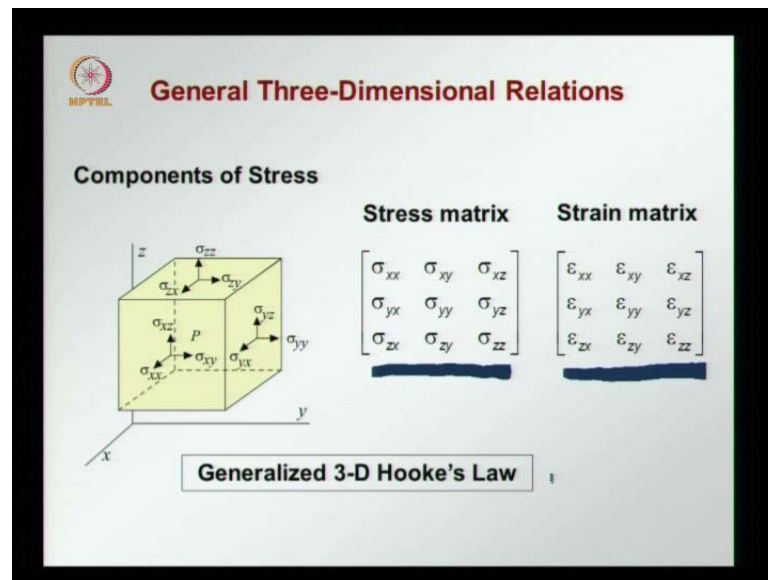


All other elastic properties are somehow related to the young's modulus and the Poisson ration, for example when we have a case of torsion, when we have a bar like a shaft that we saw earlier subject to a twist. And, if we draw a diagram between the shear stress and the shear strain, we will now have an elastic response or a linear response. And, the slope of it is call the shear modulus that is tau is equal to G the shear modulus times gamma which is the shears strain, however we find that this is not an independent parameter.

But, G is now related to E and u as given here E by 2 times 1 plus mu gives the shear modulus similarly when you have a hydrostatic condition that is we have a cube being subjected to uniform stresses all around. And, if we where to plot now the hydrostatic pressure applied versus the volume matrix strain. That is the change in volume divided by the initial volume again we find a linear or linear elasticity response.

The slope is given by minus K, where K is now the bulk models. And, again we find that this bulk module can be related to the young's modules and Poisson ratios. Given as E divided by 3 times 1 minus 2 mu this is, now the bulk module. So, we find that all properties in the elastic regime of a homogeneous isotropic material or related somehow to the young's modules in the Poisson ratio.

(Refer Slide Time: 17:16)



In a three dimensional case now the linear hooke's law has to be expanded and put in terms of matrices. So, instead of having one stress now we have a stress matrix and we will have a corresponding strain matrix. The stress is given here are shown in the cube where we have this three access X Y Z. and we have sigma Y Y, now the normal stress along the Y direction. Sigma X X the stress along the direction x. sigma Z Z the stress along the Z direction and the other stress is an out the shear stresses is acting a along the different planes in different directions. This now gives as a stress matrix, and this is now the corresponding strain matrix along the similar directions. Now, let us see what is the hook's law that follows.

(Refer Slide Time: 18:28)

General Three-Dimensional Relations

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{zx} \\ \sigma_{xy} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{yz} \\ \varepsilon_{zx} \\ \varepsilon_{xy} \end{bmatrix}$$

Stiffness matrix

We have now the generalized hook's law. So, we need to find the equivalent of the young's modulus or the stiffness. And, we find that the components of stress can now be related to a stiffness matrix times the strains that we saw before. So, instead of just one value of stiffness or young's modulus, now we have to content with the stiffness matrix. But, what we will soon is that all these components can be defined in terms of just the young's modulus and Poisson ratio.

(Refer Slide Time: 19:08)

Hooke's Law for Isotropic Elastic Material

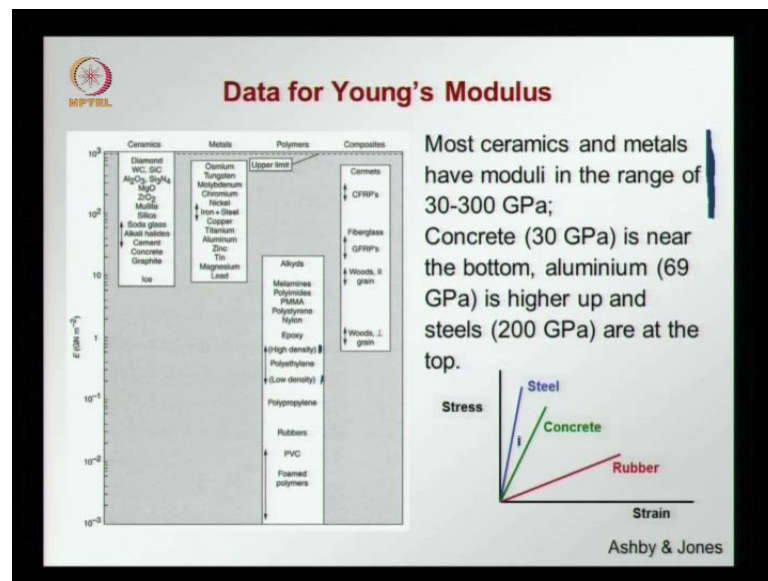
$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{zx} \\ \sigma_{xy} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & 1-2\nu & 0 & 0 \\ 0 & 0 & 0 & 0 & 1-2\nu & 0 \\ 0 & 0 & 0 & 0 & 0 & 1-2\nu \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{yz} \\ \varepsilon_{zx} \\ \varepsilon_{xy} \end{bmatrix}$$

This constitutive relation depends only on two material properties, E and ν.

So, this is how they equation becomes for a three dimensional case of an elastic behaviour this would be the hooke's law for an isotropic elastic material. Isotropic meaning that in all directions, we have the same properties and the same behaviour. And, we are talking about an elastic material or linear elastic material to be more specific. And, we find that all the stress that we will looked at in the three dimensional case can be related to this strains through this matrix.

Which is which has components all defines in terms of the young's modulus and Poisson ratio and many of these components as here, therefore we find that constitutive relation that is the fundamental relation between stress and strains in an isotropic elastic material depends only on 2 material properties. Young's modules and the Poisson ratio this is something that we should always remember.

(Refer Slide Time: 20:13)

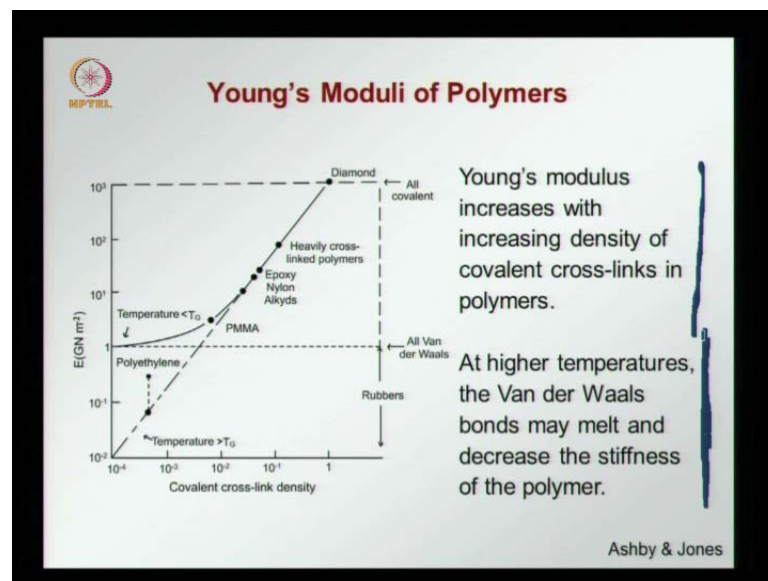


As I said before there are arrange of values for the young's modulus or modulus of elasticity this is a chart taken from as we enjoys, where we see on the Y axis different values of the young's modulus. We have different values of the young's modulus, and we find the stiffest materials are the once which have the highest young's moduli or materials like diamond stiff materials like silica alumina and so on. Metals also have very high stiffness followed by other composites like concrete and FRP's fibre reinforced polymers follow.

And, then at the bottom we have polymers which are relatively flexible polyethylene rubber PVC foam polymers and o on. Note that in this case of polyethylene will come back to it later, we have something called a high density polyethylene and a low density polyethylene. And, we find that the low density polyethylene has a lower young's modulus, then the higher density polyethylene which is stiff. So, what we see is that most ceramics and metals have moduli in the range of 30 to 300 Giga Pascal's.

The bottom of this range is maybe defined by concrete another similar materials aluminium is higher up and much higher at the top or metals like steel tungsten and so on. So, what this means is that if we were to do a tensile test of pieces of steel concrete, and rubber we will have different behaviours as shown in this plot rubber for a certain stress will deform much more. Then, concrete which will deform more than steel or for a certain strain, we find that rubber requires less stress to produce this strain concrete more and steel much more. So, higher the young's modulus higher is this stiffness or the stress required to produce a unit strain.

(Refer Slide Time: 22:49)



Let us, look at how young's moduli of polymers vary as a function of the micro structure, if you remember when we talked about the micro structure polymers. We said they were linear polymers and cross link to our branch polymer. And, we find that the young's modulus increases in the polymer with an increase in the density of the cross

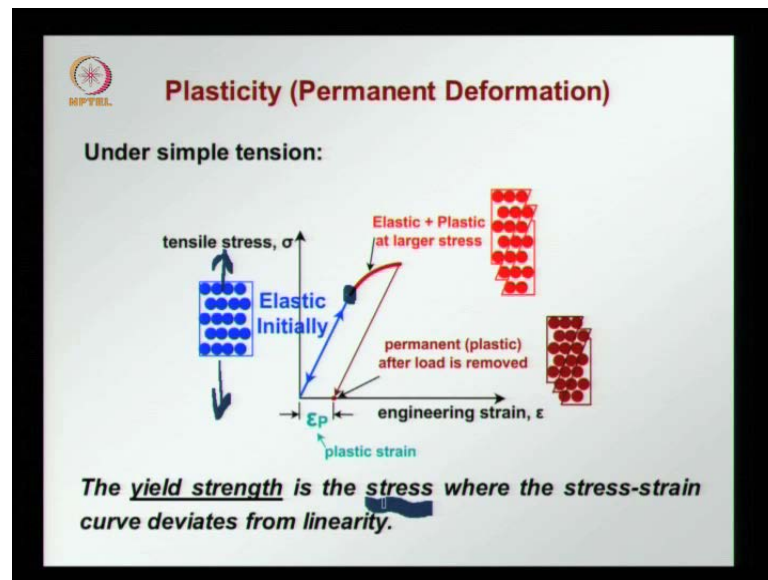
links which are covalent in the polymer. We remember that the cross links were covalent bonds that link the different branches of the polymer.

And, we find that the young's modulus, now increases with an increase in the density of the cross, so on the X axis in this plot we have an increase of the cross link density going from left to right. And, the corresponding young's modulus or modulus of elasticity in the logs scale on the Y axis, and we have what find is higher the number of cross links going from poly ethylene to poly methyl. Methacrylate epoxy is more cross link polymers, and diamond which is a covalent solid with a network structure.

We find that the young's modulus increases almost linearly when we do not consider the influence of temperature. This would be the range of rubbers then we have the polymers and then a material such as time. Beyond, this value that is below this dash line we have a larger influence of the vander Waals bonds. If you remember, we saw that in linear polymers the chains are held together by vander Waals forces which can be broken easily by temperature.

So, we find that if temperature is considered there is a softening of the material the material does not behave in a linear manner, but you have a non linear change in the young's modulus as the cross link density chains. Above, the glass transition temperature. We have this linear type behaviour which depends only on the cross links. But, when the temperature is less than the glass transition temperature, we have a non-linear behaviour that comes about because at higher temperatures. The vander Waals bonds melt and this decreases this stiffness of the point.

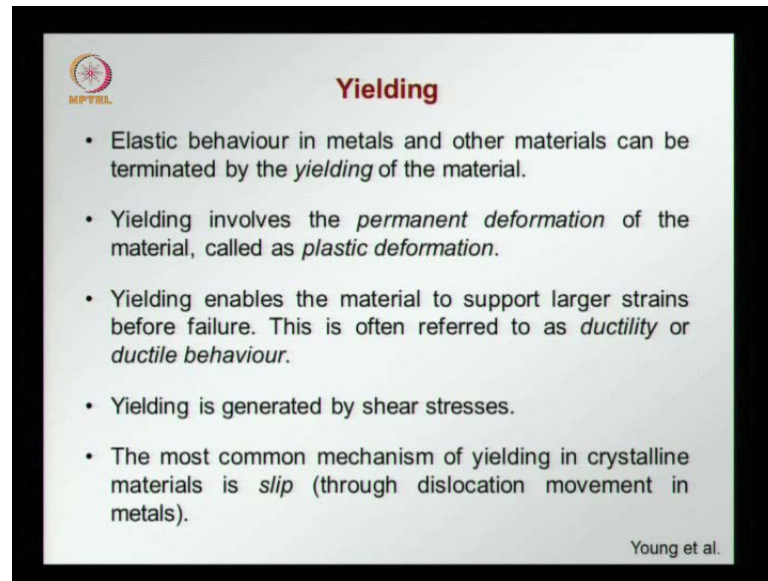
(Refer Slide Time: 25:42)



Now, beyond the elastic state governed by the young's modulus and the poisson ratio, we can have permanent or plastic deformation. So, if we look at the case of an element being pulled like a small element being pulled that we are considered earlier. So, we look at this and we pull this element. We apply a tensile force we find that there would be deformation in this structure inside this would be small deformation in the elastic range where if we unload it goes back to 0. But, if we keep pulling there are slip planes forming shearing of the micro structure and eventually you have a plastic or irreversible part of the deformation.

So, if we stretch it like this, and now unload remove the stress you do not go back to this initial point the 0 state. But, you end up here with a certain amount of permanent or plastic strain here given by epsilon. And this keeps increasing as we stretch it more and more, if we stretch more or apply more stress, then this increases more and more. The yield strength is this stress where the material queries from the linear. So, beyond this point there will be always some plastic deformation, if the material is unloaded. This point is called now the yield strength of the material this is this stress, where the stress strain curve deviates from a linear response.

(Refer Slide Time: 27:29)



The slide features a logo in the top left corner consisting of a red circle with a white star-like shape inside, and the text 'MPTEL' below it. The title 'Yielding' is centered at the top in a bold, dark red font. Below the title is a list of five bullet points. The text is in a standard black font. In the bottom right corner, the text 'Young et al.' is visible.

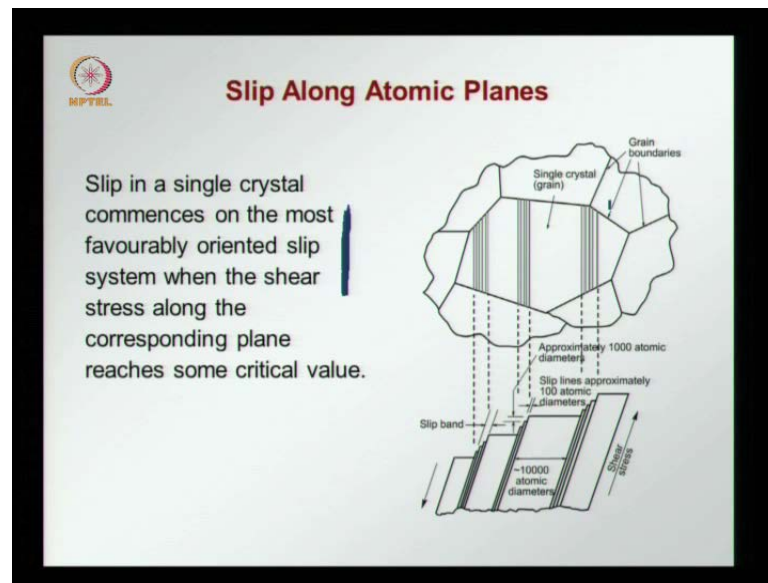
- Elastic behaviour in metals and other materials can be terminated by the *yielding* of the material.
- Yielding involves the *permanent deformation* of the material, called as *plastic deformation*.
- Yielding enables the material to support larger strains before failure. This is often referred to as *ductility* or *ductile behaviour*.
- Yielding is generated by shear stresses.
- The most common mechanism of yielding in crystalline materials is *slip* (through dislocation movement in metals).

Young et al.

So, in terms of yielding we can say that the elastic behaviour in metals and other material is terminated by the yielding of the material. beyond the elastic state the material starts to yield and not before. yielding involves permanent deformation which is also called plastic deformation. So, when we unload after some yielding, they will always be some permanent deformation. yielding enables the material to support larger strains, we saw that there is slip occurring and there is a strain which is causing shearing within the material.

This ability to support larger strains before failure happening is called ductility or there is a ductile response. Ductility is the ability to support large strains, and this as we saw in the previous slide is generated by shearing or shear stresses. And, this shearing occurs thorough slipping basically through this location moment in metals, and other mechanisms in other structures. So, in crystalline materials yielding occurs through slip and dislocation moment as in metals.


(Refer Slide Time: 29:05).



Coming back to this slide that we saw earlier, we find that in a single grain slipping and shearing will happen along the closely packed crystal planes within the crystal or the grain. And, in a polycrystalline material each grain now deforms slips along different directions along the direction that is most favourably oriented as far as that crystal or grain is concerned.

So, we find that when a single crystal has to undergo slip the most favourably oriented slip system is what is cost to slip by the shear stress? When the shear stress reaches its certain critical value corresponding to that slip plane, we have shearing occurring within the slip. Now, this has to propagate to other grain boundaries, and that requires more shear stress, if the crystal planes or its angle to each other across the different grains.

(Refer Slide Time: 30:19)

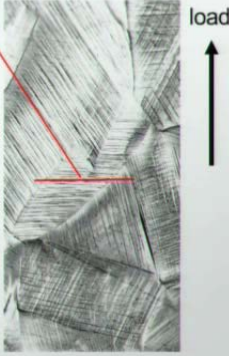
 **Slip in Polycrystalline Solids**

Slip planes and directions change from one crystal to another.

Materials with crystal structures having more slip systems undergo plastic deformation more easily.

For example, Aluminium and Copper, with the FCC structure (12 planes) are more malleable and ductile than zinc, which has a HCP structure (3 parallel planes).

Slip lines on surface of polycrystalline Cu

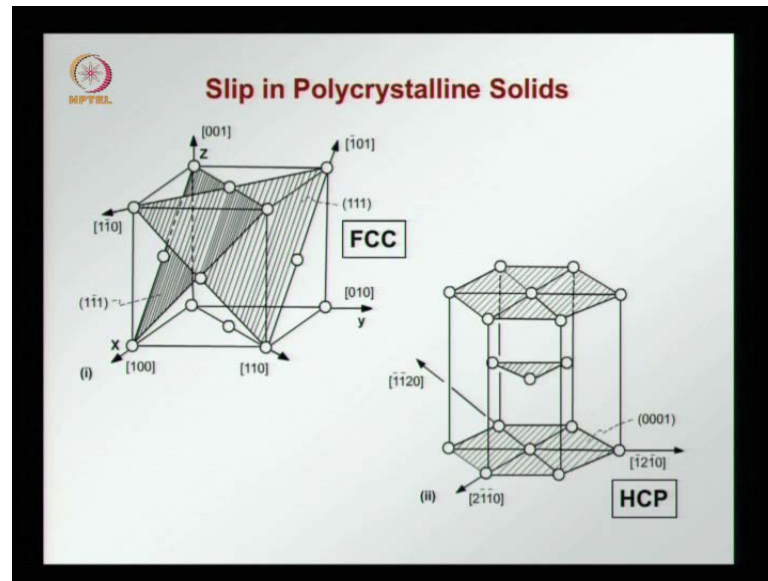


That is what we see here? In the case of say a polycrystalline. Copper given here, we have the different grains. Each of these grains has different orientation of the crystal planes. So, slipping has to change direction as even though it is loaded along one direction, we have slip changing from one crystal to the other following the crystal planes. So, slip planes and directions change from one crystal to the other.

And, materials with crystal structures having more slip planes will undergo plastic deformation more easily. That is they will be lower requirement of load lower stress that will cause slip to occur when you have a structure with more slip planes, and this we saw already that if we have an FCC structure for example in aluminium and copper. The material is more malleable and ductile, that means it can deform more easily.

We can make them make these materials into different shapes more easily, because of the multiple slip planes or closely pack crystal planes in the FCC structure. Whereas, in a material with a HCP structure like in zinc. There are only three planes and those are parallel planes, therefore zinc which has an HCP structure is less malleable less ductile than the others.

(Refer Slide Time: 31:55)



And, this is what is shown here on the left we have a material with an FCC structure with different possible slip planes covering different orientations, whereas in HCP material we have three crystal planes and all of them are along the same direction. So, a HCP material will need more stress in the poly crystalline state to be deformed, and also we find that an HCP material will be less ductile less malleable than an FCC.

(Refer Slide Time: 32:31)

Yielding in Polycrystalline Materials

- In polycrystalline materials, slip will first begin in grains that are most favourably oriented with respect to the maximum shear stress, and then will progress successively to the less favourably oriented crystals. In addition, there will be movement along the grain boundaries.
- Since yielding does not occur throughout the material at the same time, the yield point is not sharply defined.

In poly crystalline materials, what we have seen? Is that slip now will occur in some grains. The grains that are most favourably oriented in terms of the stress that is the

maximum shears stress slip will if I start there. And, then it will be progressing to other crystals where the slip planes are less favourably oriented. In addition there will also be some moment along the grain boundaries. You remember that we consider grain boundaries as surface defects, but there is a lower packing of the atoms and we thought of it as defects in the crystal structure.

So, we find that yielding now will occur in some grains those grains that are favourably oriented and then it will progress to other grains. Therefore, in a poly crystalline material there is no definite yield point. Since, yielding does not occur throughout the material at the same time we saw here that yielding occurs in some grains. And, then it will go on to other grains. So, yield point is not very sharply defined, we have the curve transition between the elastic and the plastic rigid.

(Refer Slide Time: 33:58)

A presentation slide with a black border. In the top left corner is a logo for MPTEL. The title 'Yielding in Polycrystalline Materials' is centered at the top in a dark red font. Below the title are two bullet points in black text. The first bullet point states that the yield strength of polycrystalline materials is higher than that of single crystals because many slip systems must be active. The second bullet point states that grain boundaries act as barriers to dislocation movement, delaying yielding. The citation 'Young et al.' is in the bottom right corner.

Yielding in Polycrystalline Materials

- The yield strength is considerably higher than that of a single crystal of the same material since many slip systems must become active before generalised yielding can occur.
- The grain boundaries act as barriers to dislocation movement, which further delays the onset of yield.

Young et al.

Also we find that in poly crystalline materials the yield strength is higher. since we saw that in some crystals which are not favourably oriented to make this systems active there should be more force and more stress applied, and therefore for generalize yielding to occur across the material. We need a higher stress this leads to a higher yield strength in a poly crystalline material than that of a single crystal of the same material. If there is only a single crystal than that crystal starts yielding at a certain stress along the most favourably oriented slip plane.

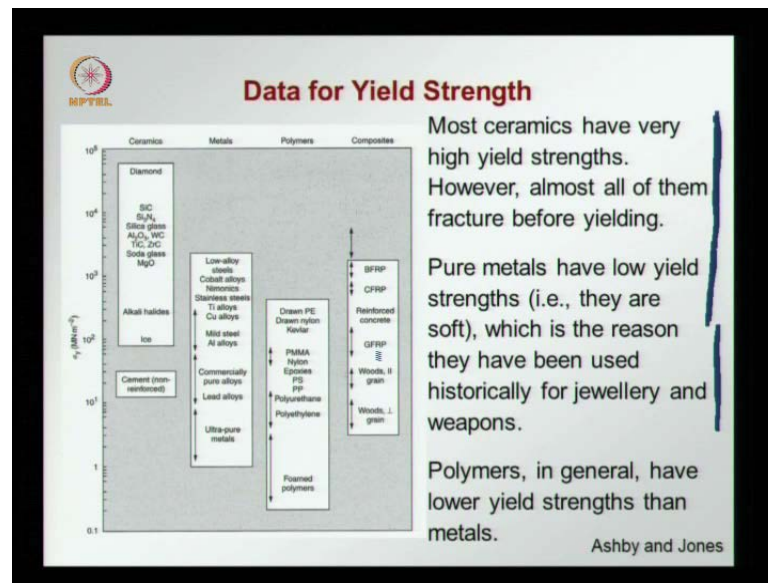
But, when you have a poly crystalline material some crystal will start yielding followed by the others and so. This raises the yield strength also we saw that grain boundaries act as barriers to dislocation movement that is the dislocations have a higher energy requirement to cross the boundaries. And, this further delays yielding or delays the onset of yielding and increases the strength. So, we saw earlier that material with smaller grains that is more grain boundaries will have a higher yield strength, because there are more barriers to dislocation.

(Refer Slide Time: 35:25)

A presentation slide with a black border. In the top left corner is a circular logo with a star and the text 'NPTEL' below it. The title 'Yielding in Amorphous Materials' is centered at the top in a dark red font. Below the title are two bullet points in black text. The first bullet point states that in amorphous materials, several different mechanisms are responsible for yielding. The second bullet point states that in thermoplastic polymers, yielding is due to the slip of long chain molecules past each other, and this process is highly time-dependent, so the response depends on the rate of loading. In the bottom right corner, the text 'Young et al.' is visible.

In amorphous materials there are several different mechanisms, which create an impression of yielding or we have something similar to yielding occurring not in the same respect as in a metal. But, we have yielding type behaviour occurring in amorphous materials also. For example in thermo plastic polymers yielding occurs due to the slipping of the chain molecules. The long chain molecules slip past one and other giving a yielding type behaviour. This is how ever time dependent, and therefore the response of these materials depends on the rate of loading. If we load very slowly, we find that there is more slip more deformation rather than when the material is loaded very fast.

(Refer Slide Time: 36:21)



From Ashby and Jones we have taken here this diagram giving the yield strength for different types of materials. And, on the y axis the log scale we have the yield strength and different types of materials are denoted here. We find the yield strength is very high for materials such as diamond, silicon, carbide, glass and so on. Ceramics we find generally have high yield strength given by the first box.

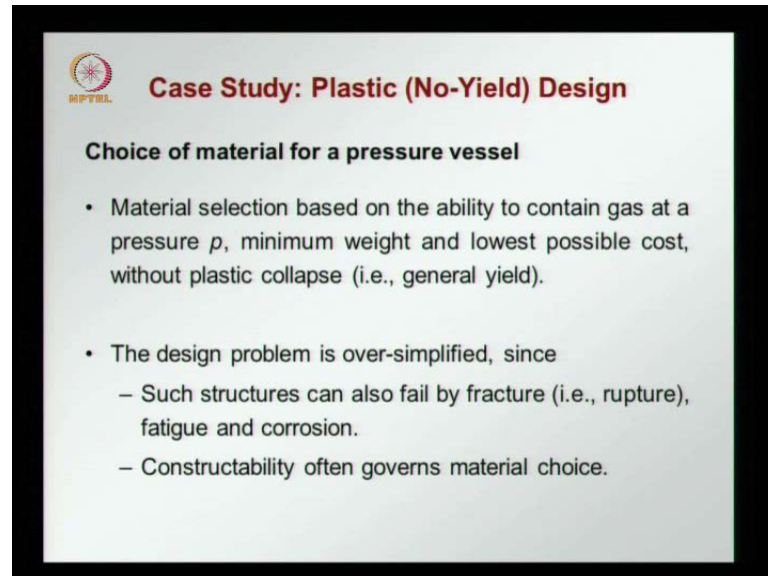
So, ceramics have very high yield strength and therefore, what happens is these materials never reach yielding, they crack or rupture before yield. So, we find that ceramics, since they have very high yield strength never reach this strength in practice. Most of them will fracture or crack before yielding. Pure metals, however we have all the pure metals here. Pure metals, we find that will have lower yield strength than the alloys. We saw discuss this behaviour earlier when we looked at point defects.

And, we saw that when a defect is introduced in the crystal, that is there is a distortion of the crystal planes leading to an increase in the yield strength because now the slip has to occur across a plane that is not smooth. So, we find that pure metals have lower yield strength than alloys they are softer. So, we find here that alloys are at the top and we have lower yield strength in the purer metals.

And this is the reason why pure metals like gold and silver have been use historically for making metals by hands say in jewellery and weapons. Polymers on the other hand in general have always lower yield strength than in metals. They are softer and deform

more easily composites are somewhere in the middle at the top. We have composites such as fibre in force polymers we in force concrete and at the bottom we have wood.

(Refer Slide Time: 38:45)




The slide features a logo in the top left corner with the text 'MPTEL' below it. The main title is 'Case Study: Plastic (No-Yield) Design'. Below the title is the sub-heading 'Choice of material for a pressure vessel'. The content consists of two main bullet points, with the second one having two sub-bullets.

- Material selection based on the ability to contain gas at a pressure p , minimum weight and lowest possible cost, without plastic collapse (i.e., general yield).
- The design problem is over-simplified, since
 - Such structures can also fail by fracture (i.e., rupture), fatigue and corrosion.
 - Constructability often governs material choice.

Let us, see how we can use the concept of the yield strength in design. Let us, take the case of a design of a pressure vessel which we are designing for the know yield or plastic condition. The material selection can be made based on its ability to contain gas at a pressure p . We want the lowest weight for the pressure vessel or the lowest cost possible without any plastic collapse occurring due to the pressure.

We obviously over simplified the design problem, because in such structures can also fail by rupturing or fracture fatigue corrosion and so on, which is ignored here just to make the point how we go about using the yield strength in the design? Also we find that in many cases constructability the ability to make the pressure vessel with a certain material also governs the choice of the material.

(Refer Slide Time: 39:48)

 **Case Study: Plastic (No-Yield) Design**

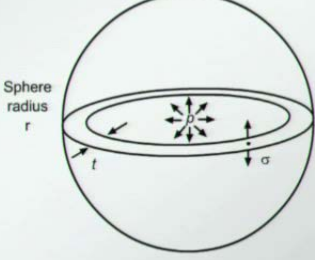
Stress in the vessel wall is:
 $\sigma = pr / 2t$

The radius of the pressure vessel, r , is fixed by the design.

For safety, $\sigma \leq \sigma_y / S$, where S is the safety factor.

The vessel mass is $M = 4 \pi r^2 t \rho$, where ρ is the density of the material.

So, $t = \frac{M}{4 \pi r^2 \rho}$




Ashby and Jones

However, in under this condition let us, see how we can go about with this case study of a pressure vessel which should not yield. The stress in the pressure vessel is given by the equation σ equals p times r divided by $2t$, where p is the pressure inside the vessel, r is the radius of this sphere. We are going to consider for simplicity a case of a sphere. So, this sphere has a radius r and the thickness t given here. And, for safety we consider a design stress that is lower than the yield strength. We will divide the yield strength by a safety factor S .

So, that given by accident this yield strength is not reached. So, we take a safe design strength σ , which is less than or equal to σ_y which is the yield strength divided by S . So, we are saying that the stress in the pressure vessel lining should never exceed σ_y divided by S . The mass of the vessel is given by the surface area times the thickness times the density M is equal to $4 \pi r^2 t \rho$ and from this. We can get the thickness as M divided by $4 \pi R^2$ divided by ρ , where ρ is the density of the material.

(Refer Slide Time: 41:27)

 **Case Study: Material Selection in Plastic Design of Pressure Vessel**

Candidate materials for pressure vessels and their properties

Material	σ_y (MPa)	ρ (kg/m ³)	Cost, C (Rs/kg)	ρ / σ_y	$C \cdot \rho / \sigma_y$
Reinforced concrete	200	2500	5	13	65
Alloy steel (pressure vessel steel)	1000	7800	20	8	160
Mild steel	220	7800	10	36	360
Aluminium alloy	400	2700	40	7	280
Fiber glass	200	1800	100	9	900
CFRP	600	1500	2,000	3	6,000

σ_y = yield strength; ρ = unit weight; C = cost per kg

Ashby and Jones

We can have different materials that can be used. Let us, say the in force concrete alloys steel or pressure vessel steel, mild steel, aluminium alloy, fibre glass and carbon fibre in force polymer. So, in the in these columns, we now see the yield strength, the density and a cost again. We should caution you that this cost is only for indicated purposes. And, it does not mean that these costs are absolutely correct for all location and time and so on. This just shows what would be the relative cost of the different materials that we have considered in this table rho divided by sigma y gives as this ratio the cost times rho divided by sigma y is this. We will use these parameters in the following slides. So, sigma y here is the yield strength rho is the unit weight and C is the cost per kilo of the material.

(Refer Slide Time: 42:39)

Case Study: Material Selection in Plastic Design of Pressure Vessel

Material selection based on minimum weight:
(Relevant in the design of an aircraft body, spacecraft hull, rocket fuel tank)

$$\frac{\sigma_y}{S} \geq \frac{2\pi p r^3 \rho}{M} \quad \text{or, the minimum mass} \quad M = 2S\pi p r^3 \left(\frac{\rho}{\sigma_y} \right)$$

Therefore, for minimum weight, we require the smallest $\left(\frac{\rho}{\sigma_y} \right)$

From the previous table, the best choice of the lightest vessel is CFRP. Aluminium alloy and pressure-vessel steel are next.

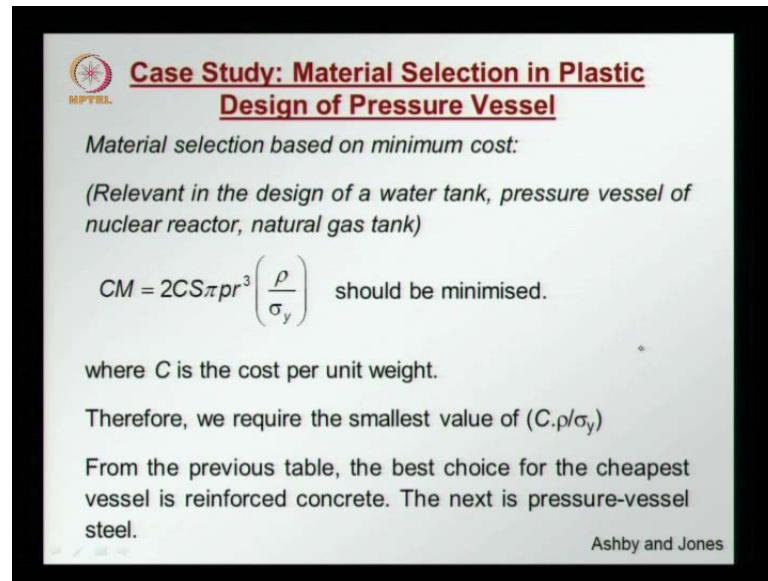
Ashby and Jones

So, suppose we want to design just based on minimum weight. That means cost is not a relevant issue this could be in the case of a design of an aircraft body, space craft hull, a rocket fuel tank and so on, where safety is more of a concern minimum weight is more of concern than cost. And, we find now this equation substituting for the value of strength. We find that the equation that has to be satisfied is sigma y divided by S should be greater than or equal to 2 pi p r cube rho divided by M.

We find that the minimum mass M is equal to 2 S, S is the safety factor pi p, which is the pressure r cubed times rho, which is the unit weight divided by sigma y, which is the yield strength. So, to minimize M we have to minimize this parameter. that is rho divided by sigma y. So, for minimum weight we need the smallest value of rho divided by sigma y and from the previous table.

We see that if we go to this table we see the minimum values of rho divided by sigma y or for composites alloy steel, aluminium alloy, fibre glass and so on. Something like pile steel and reinforced concrete have much higher values. So, we find that the best choice in terms of just a minimum weight is CFRP followed by aluminium alloy and pressure-vessel steel.

(Refer Slide Time: 44:29)



Case Study: Material Selection in Plastic Design of Pressure Vessel

Material selection based on minimum cost:

(Relevant in the design of a water tank, pressure vessel of nuclear reactor, natural gas tank)

$$CM = 2CS\pi pr^3 \left(\frac{\rho}{\sigma_y} \right) \text{ should be minimised.}$$

where C is the cost per unit weight.

Therefore, we require the smallest value of $(C \cdot \rho / \sigma_y)$

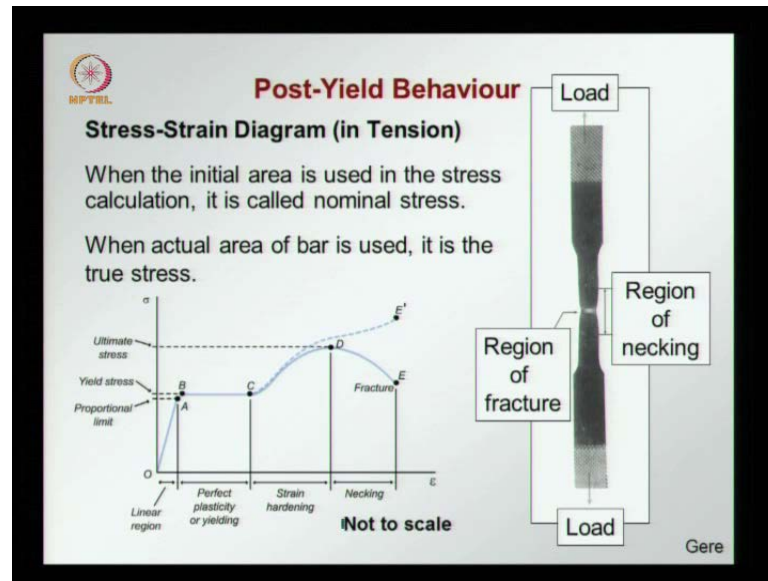
From the previous table, the best choice for the cheapest vessel is reinforced concrete. The next is pressure-vessel steel.

Ashby and Jones

However, if we want to look at cost, this would be relevant in the design of a water tank pressure vessel of a nuclear reactor or a natural gas tank. We find that the product C times M. C is the cost per unit weight. M is the mass of the gas tank given by two times C S pi p r cubed multiplied by rho times sigma y should be minimized. So, we find that for the smallest cost, we should minimize this quantity C times rho divided by sigma y. And, if we go back to this table, we find in the last column that the material which would give us the minimum cost through the combination of weight and cost criteria would be reinforced concrete followed by alloy steel and aluminium alloy.

These are the materials which are normally used in the construction of tanks which seems to indicate why these materials are so called. So, this was an example, where we saw how yield strength can be used in design to choose between different materials. And, we can consider different candidate materials, and then reach the material which gives us the minimum weight or the minimum cost or any other criterion that we might choose.

(Refer Slide Time: 46:01)



Let us see what happens beyond yield? Until now, we were looking the elastic regime and then we saw what happens when the material yielded. But, the behaviour of the material does not stopped on yielding we have something called the post-yield behaviour. So, we look at the stress-strain diagram in tension, we see that initially we might have a linear elastic behaviour then yielding occurs.

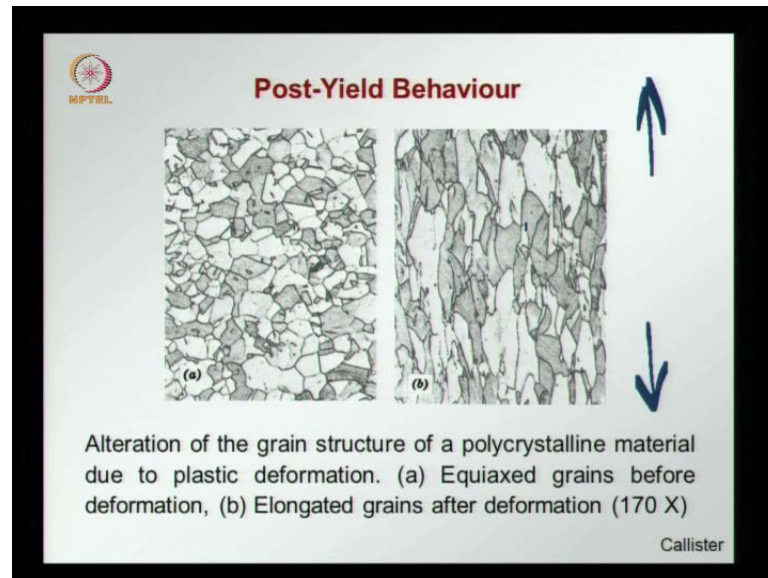
And, then we have a plastic regime that is a flat sigma epsilon curve or stress-strain curve, and then there is an increase in the stiffness this is called strain hardening. And, it reaches a peak and the neck drops. Now, if we look at the specimen that has given us this type of behaviour say, we take what is called a dog bone shaped element or a coupon. We find that when this is stretched initially, they will be a reversible deformation or a elastic deformation, then there is yielding.

Then, there is necking occurring that is they will be a neck form or a decrease in the cross sectional area. When we use the initial area in the calculations we get the nominal stress that is this blue line. If we use the actual area for the calculations of stress that is when the actual area of the bar is used as it is deforming it is called true stress. And, we have what we find? is we get the behaviour according to this dashed line. That means the stress is actually increasing it does not reach a peak and drop.

But, it continues to increase until fracture occurs in. this region of the neck the stress continues to increase and we have failure. This regime is called strain hardening that is

the material seems to harden or increases stiffness until a certain point. Now, here necking dominates, and we have a decrease in the cross section area and finally. We have failure note that this is not to scale, but indicates the different regimes of the failures.

(Refer Slide Time: 48:30)



And, during this classification or yielding or strain hardening, we find that the microstructure also changes. This is one of the reasons why when we release the load it the material does not go back to its original state. In this case of a polycrystalline material, we find that on the left we have equiaxed grains before any deformation. Now, there is stretching, so if we stretch this material we are pulling it along this direction. We will find that the grains are now elongated or stretched after the deformation.

And, now if we release we find that there is a plastic deformation and a permanent deformation. In this material we just change its structure from equiaxed to elongated grains. So, until now we looked at the elastic regime plastic response and we also looked at how we can differentiate between brittle and ductile failure? Now, in the second part of the lecture, we will continue to look at failure of materials. We will see how brittle failure occurs? And, we will go on to see how they can be even a transition of the material behaviour from ductile to brittle under some conditions.

Thank you.