Modern Construction Materials
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Module - 3
Lecture - 8
Failure Theories

Welcome to the eight lecture of Modern Construction Materials. In the previous lecture, we had looked at how materials respond to stress and today we talk about failure theories we look at what kind of criteria.

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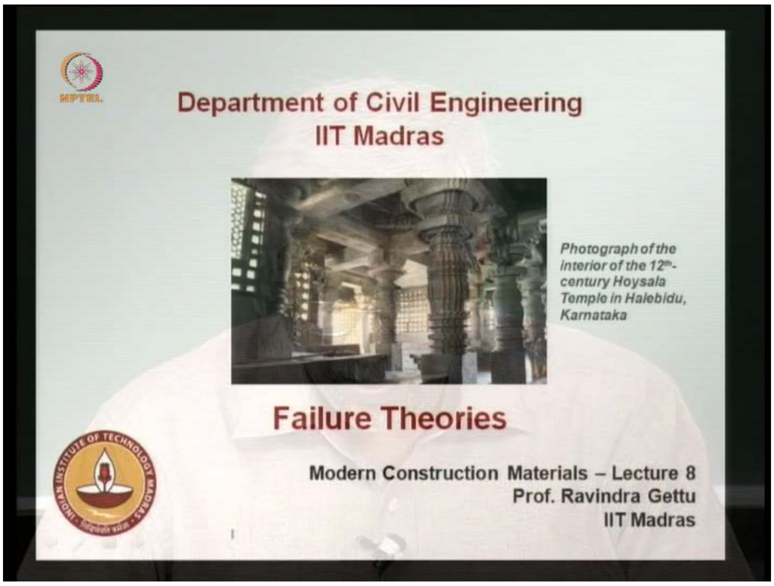

We have to use for determining when a material will fail and a multi-axial stresses I start with this picture of a temple in hale Bedu Karnataka which has survived, since in the twelfth century.

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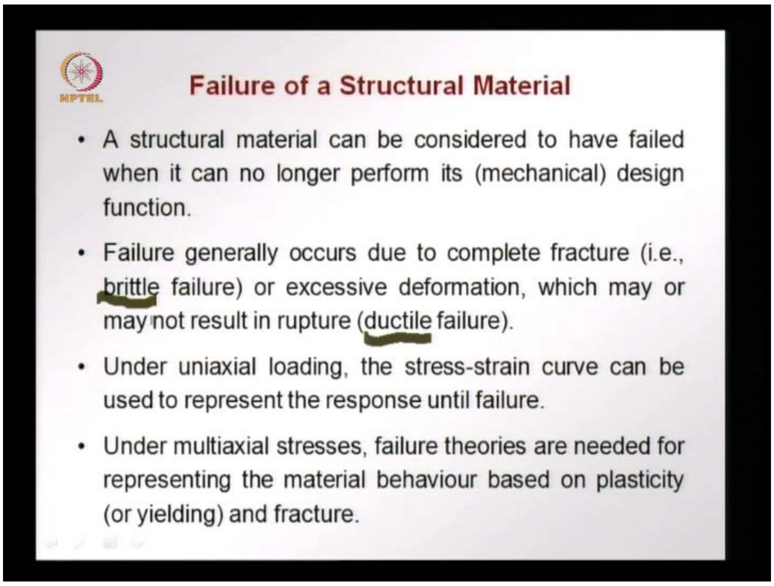

We see in the pictures lot of nice tone Collins which has survive taken the load for many, many hundreds of years and have not fail what do we consider as failure a structural material is consider to have failed when it can no longer perform its design function, and in this case we are talking about mechanical aspects failure my occur into ways complete fractures that is brittle failure or rupture when an element a structure break apart failure can also be considered as an excessive deformation that is the structural element deform. So, much that though it does not rupture or break the deformation is such that it cannot be use for what it has is designed form that is something that we can called ductile as supposed to brittle failure in the first case now when we have to study failure and determine when a material fails and a uni-axial loading it is pretty simple.

We look at the stress strain curve and this stress strain curve when drawn up to failure can tell us how a material well respond to stress and when the material fail and how it will fail and a multi-axial stresses it is more complicated to visualize and therefore, failure theories are needed a representing a material behavior, and this is usually done for metal's based on plasticity or yielding an in other material plasticity.

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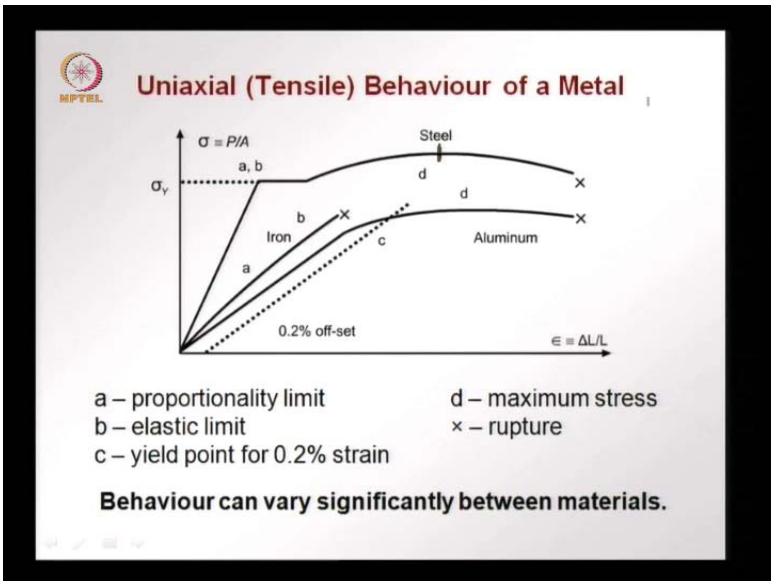

May be combine with fracture when we look at the uniaxial behavior of a metal say in tension we can get different types of behavior's when we compare different materials the top curve here is from I will still may be have a elastic response a linear elastic response culminating in what is call the proportionality limit for the elastic limit in this case both co-inside, then we have the a yield Plato then there is hardening and at this point d which is the maximum stress we have necking occurring. Now the next start to develop and finally, you have failure in the case of aluminum, we find that there is no definite point where we can identify the yielding for the proportionality yielding to start a proportionality to complete it is...

So, in order to define an yield point objectively what is done for materials such as this where we do not have a clear well defined yield point like we had in in my steel is that a 0.2 percent of set is taken that is a line is drawn parallel to the initial slope add a strain of 0.2 percent and wherever it intersects is taken as the yield point, this is classical of poly crystalline materials which have different grains yielding at different points in time and this we will discuss in the some other previous lectures we can also have some metals which do not undergo significant yielding or necking and failure like in the case of cast iron again we...

So, in the previous lecture cast iron was a brittle material and we have slightly curve behaviour instead of a linear elastic a behavior, we can say that the proportionality limit is somewhere here in this region the curves stops being linear and then again an elastic limit could be somewhere here and then failed in this case, we do not really defined the different reasons like we add in the case of mil still.

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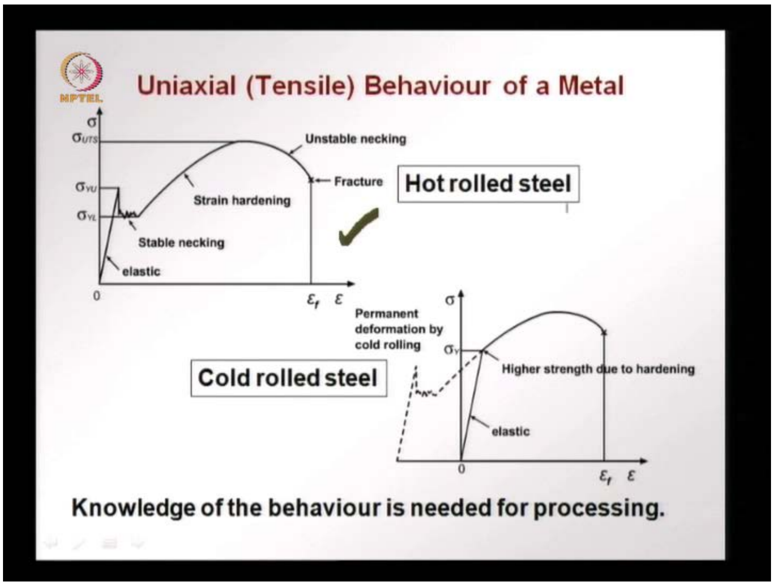

So, what is clear is that between metals itself the behaviour can vary significant the tensile behaviour is also use to determine how a material should be processed going back to the behaviour of mil steel say in the case of hot rolled steel, we have this type of behaviour that we just discussed we have a elastic behavior signified by a linear elastic range followed by the unsettled yielding the upper yield limit, then we have a lower yield limit a stable necking occurring hardening. We have the peak years and then we have failure complete structure occurring it is strain which can be given a name of epsilon which can be called epsilon f for failure.

Now, instead of looking at the hot rolled steel behavior, if you have that a think of a cold rolled steel what has happened cold rolled steel is that this material that we have in the upper diagram was loaded to a certain point and then unload that what we see here. So, the hot rolled behaviour is this, but what has happened is that in cold rolling we have deformed the material certain extent unloaded and this new material. Now is the cold rolled steel and if we were to determining the tensile behaviour of the cold rolled steel you will now has an elastic part starting from the 0 reference stress strain and then here we will have yield then necking and rupture. So, we need this behaviour to determine how much of cold working has to be done. So, that we get the desire yield value an also have a sufficient amount of elongation before failed.

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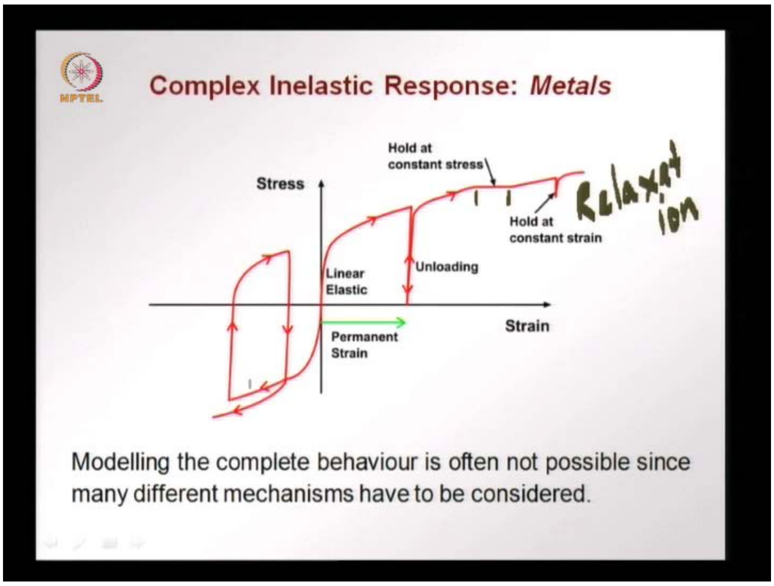

So, the knowledge of this behaviour is not only needed for design, but also for process the complete stress strain behaviour under all sorts of loading conditions is difficult to model is quite complex in this diagram that you have in the screen. We put together different types of loading conditions and how the material can respond this is to understand that the behaviour is quite complex a single simple model may not be sufficient to explain all the possible scenarios in this case we have linear elastic behaviour then if you continue an unload, you will have a certain loading unloading generally this will have the same slope of the initial part of the curve, and then if we go on save the whole stress say from this point this point you are holding stress constant now due to creep that we study before this strain will continue to increase.

So, here you need a model which looks at creep polls than if we continue and we come to a certain point a we hold this strain constant that is your holding strain constant we will see that the stress drops this is call relaxation this as mechanisms similar to creep, but here what we are doing is looking at the behaviour when strain is a l constant instead of stress being l constant in creep in relaxation we have strained being l constant and now this stress relaxes of the stress drops in along this part in any case that be unload behind the elastic region we will have a permanent strain. So, whichever model is needed for the plastic part as to correctly look at how permanent strain occurs then we can also have fatigue we looked at cyclic loading in the previous lecture and we have to see what sort of stress strain diagram we have when the stress is cyclic this is called a hysteresis loop. So, the behaviour in elastic region is quite complex main if phenomena can intervene.

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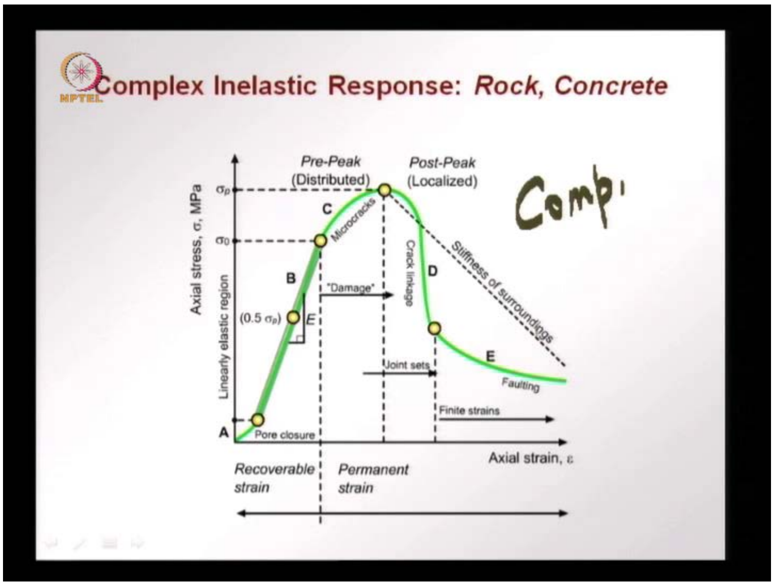

And therefore we need complex models and simple models are not sufficient, let us look at a material that does not have ductile fail says rock or concrete we would have stress strain diagram this time may be looking more at compression rather than tension because these material are generally I weak in tension cannot be use there more in compression. So, if we were to test a specimen of corncrake and the uniaxial compression, we will have initially a small non-linear a part which can be a attributed for the closer of polls to the settling of the boundaries under the load. And then we have a elastic part this is where we will expect the material to be under service conditions the slope of this is the young’s modelist then damages initiated defects which occur in the material slowly propagate there is micro-cracking and this is called the pre peek non-linear region have distributed microcracking occurring at a certain point of strain the micro cracks quails localized larger crack starts foaming the cracks link and the load carrying capacity comes down in a rock this is where the joins sets joins starts opening up then you have a major cracking or faulting large strains occur and then you have failure this part of the curve may also depend on the stiffness of the testing machine are of the surroundings engine getting a unique objective post peak part in these materials is sometimes very difficult this behaviour where the stress drop after a peak is called strain siphoning has supposed to what?

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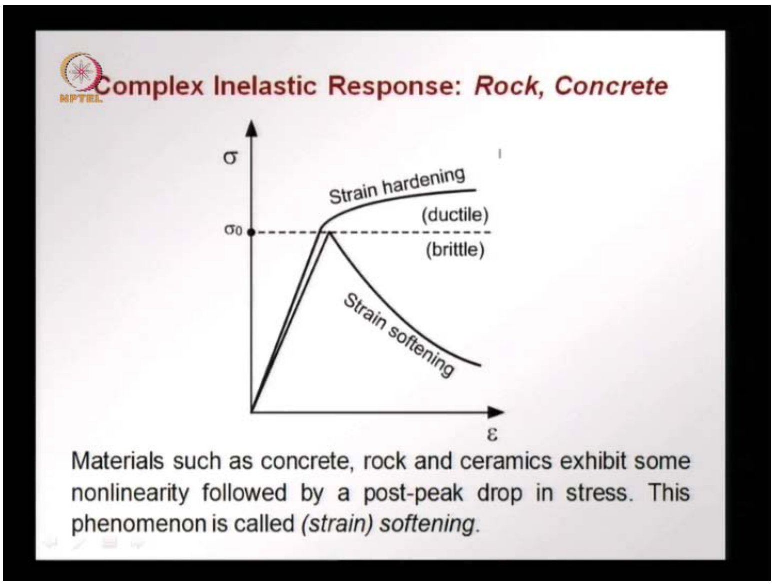

We saw in the metals when the curve goes up and it call its strain hardening this is called strain siphoning. So, any behaviour that is below a plastic Plato's brittle and is called the strain siphoning and what is above a plastic flat behaviour is called strain hardening an as I said material such as concrete rocks ceramics exhibits such type of nonlinearity there.

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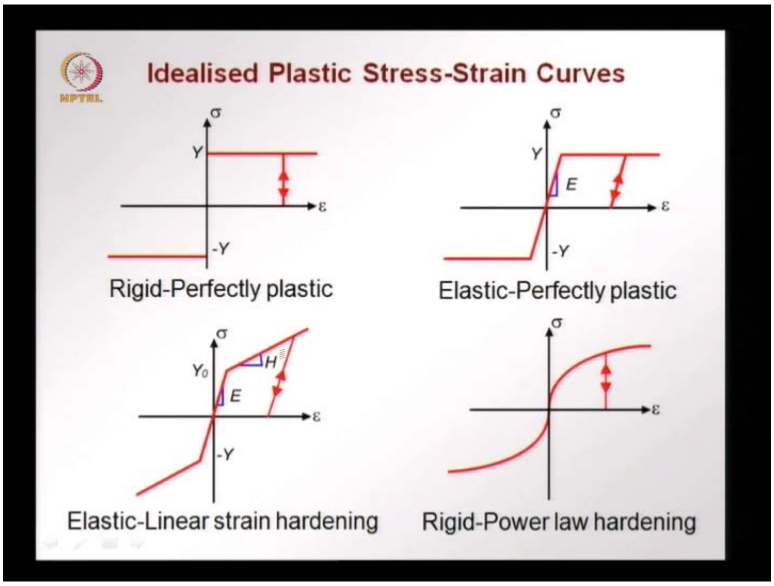

We have a post peak drop in stress as strain increases and this is called strain soponing now plastic stress strain curve can be idealize for modeling and these are the different model that often used on the left top, we have a rigid of perfectly plastic model where instead of a finite elastic mode list. We have rigid material the elastic mode list is infinite then we have the plastic part of an unloading reloading is now vertical and the plato start at the stress equal to the yield start this may be more realistic the curve top right where we have an elastic part both in compression and in tention then we have the plateau occurring and the yield strength and initial slope is given by the young's modelist.

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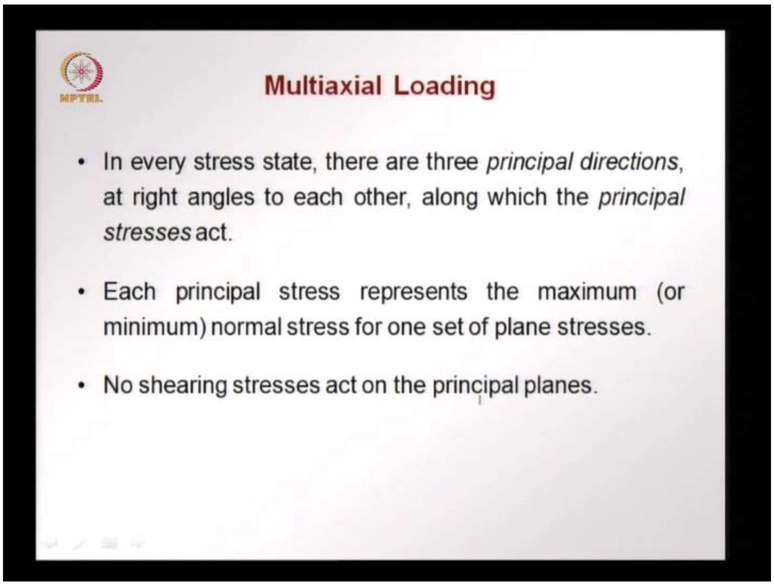

And unloading reloading follows just slope of the young's modelist hardening behaviour can be model by these two curves at the bottom with a bi linear behaviour signifying represented by two slopes e and h or you can have a power law type hardening where we have the curve behaviour when we look at multiaxial loading. We have to remember remind as well principal stresses and principal directions, and if you go back to strength of materials you would have studied that every stress state there are three principal directions orthogonal to each other that is right angle to each other along with the principal stresses act each principal stress the maximum or minimum normal stress for one set of plane stresses along those principal planes we do not have initial stresses.

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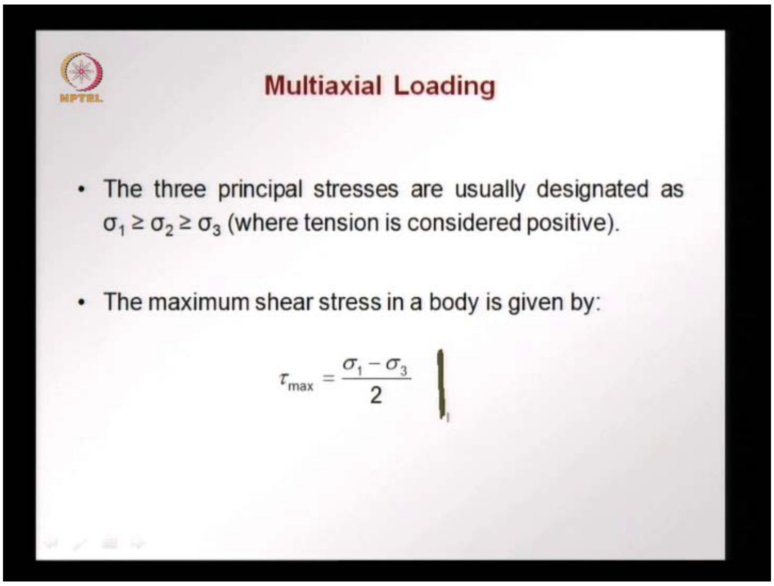

We have only axial stresses the principal stresses or usually designated sigma1 sigma 2 sigma 3 considering tension as positive sigma 1 would be the maximum principal stress sigma 3 would be the minimum principal stress the maximum shear stress in the body is given by the difference between the maximum and minimum principal stresses divided by 2.

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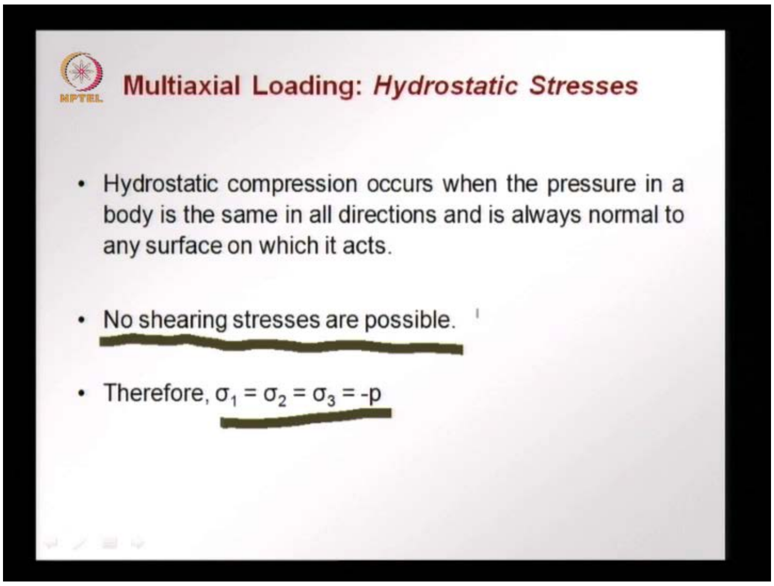

So, tau max is equal to sigma1 minus sigma 3 by 2 in the cases hydrostatic pressures when we have hydrostatic compression the pressure on the body is the same in all directions that is it is always normal to any surface an which it acts sigma 1 sigma 2 sigma 3 or all equal equal to minus p which is the pressure and therefore, no sharing is possible. So, we can we will not be able to have any shear stress when the material is a and the hydrostatic compression.

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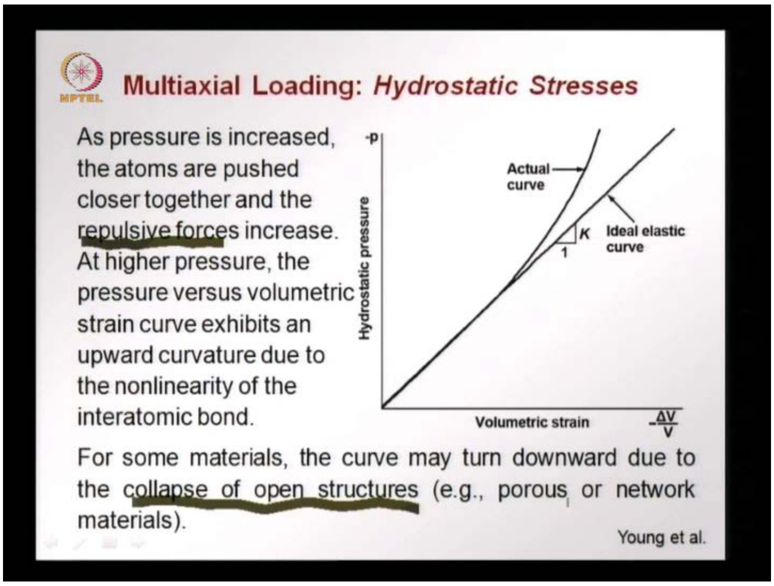

So, this means failure due to sharing will not occurs in under hydrostatic compression and this goes back to when we started looking at the condom most diagram we said that failures cannot occur under pure compression, because the atoms as they are pushed together have very high repulsive forces and we have a behaviour like what we see as the pressure is the increased as is the hydrostatic pressure is increased the atoms are pulled pushed together. So, that is the strain is increasing atoms are push together the repulsive is increased an at very high pressures we find that the curve between hydrostatic pressure and volumetric strain actually starts going up ward due to the nonlinearity that we saw in the inter atomic bond going back to the condom most diagram.

So, here there is a lot of repulsion and the we enter into the rigid way the bonds are behaving in a non-linear manner this slope of this line is the bulk modelist k in some cases; however, we can have this curve instead of going up going down were this that would indicate the collapse of the microstructure.

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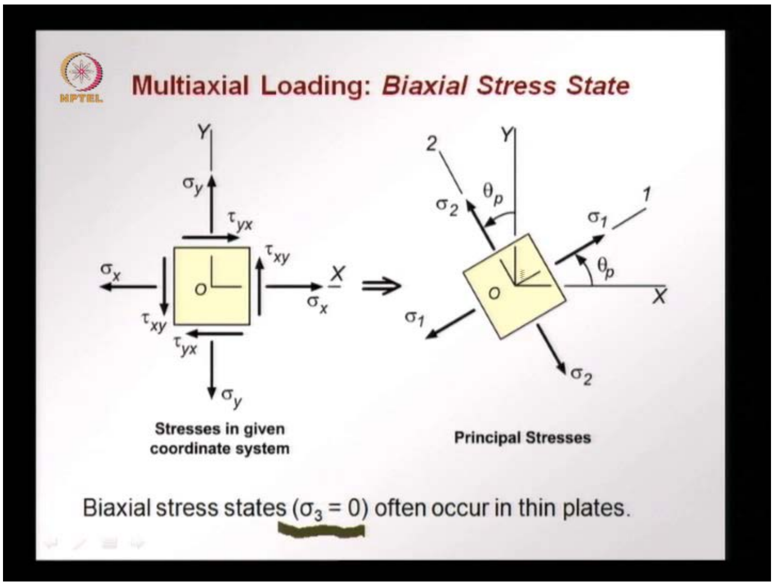

That is if you have a lot of porous in the material this porous can collapse due to the very high hydrostatic pressures that is being applied coming back to biaxial stress state, and principal stresses and principal planes suppose we have a point o around at which we are applying this is where in generic stress state given by the different normal stresses and the shear stresses we would find that they will be a set-off planes on which a principal stresses act sigma 1 and sigma 2 this is a case of planes stress. So, we are looking at something like a thin plate where sigma 3 is equal to 0 for simplistic, but the same ideas apply to all multiaxial behaviors. So, the principal stresses and a sigma 1 sigma 2.

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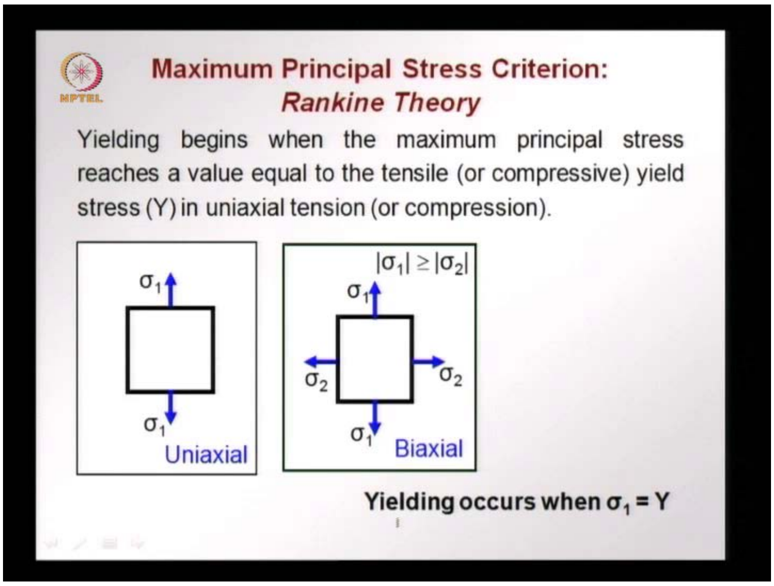

You see that on the principal planes is do not have any shear stresses now how do we used this concept to determine failure there are difference failure criteria the first one and most simple is the maximum principal stress criterion on the rankine theory according to the rankine theory yielding begins when the maximum principal stress reaches a value equal to the tensile or compressive yield stress under uniaxial tensile compression that is yielding in any state that occur when the principal stress reaches the value corresponding to yield in the uniaxial case I am in the uniaxial case. Now we know that yielding will occur when the stress reaches the yield stress. So, what we are saying is under a biaxial stress state as long has as soon as the maximum principal stress reaches the yield strength yielding will occur under yield stress. So, according to rankine theory yielding occurs.

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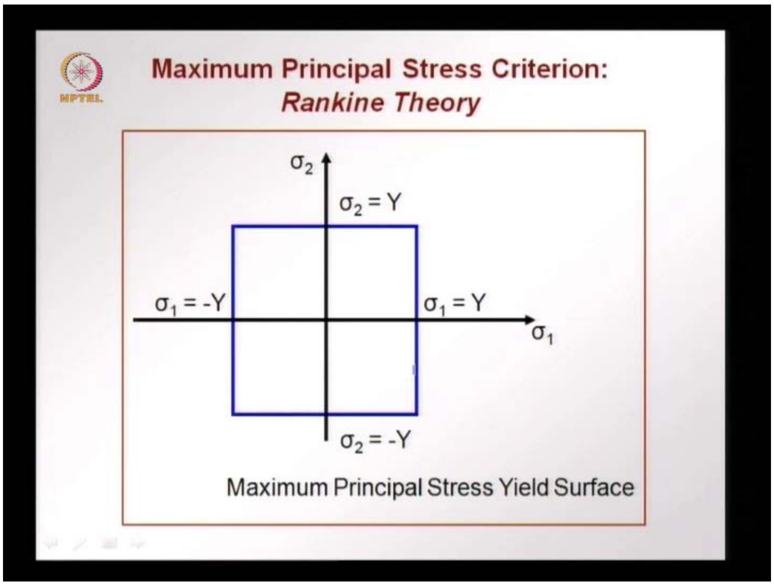

When the maximum principal stress reaches the value of the yield stress and this is represented by the yield surface which says that as long as the stress state his within the surface defined by sigma 1 equal to y sigma 2 equal to y and on the four sides when we have a stress state determined by sigma 1 sigma 2 inside this yield surface failure does not occur failure occurs as soon as the stress state reaches or touches this yield surface that is when yielding will start to occur.

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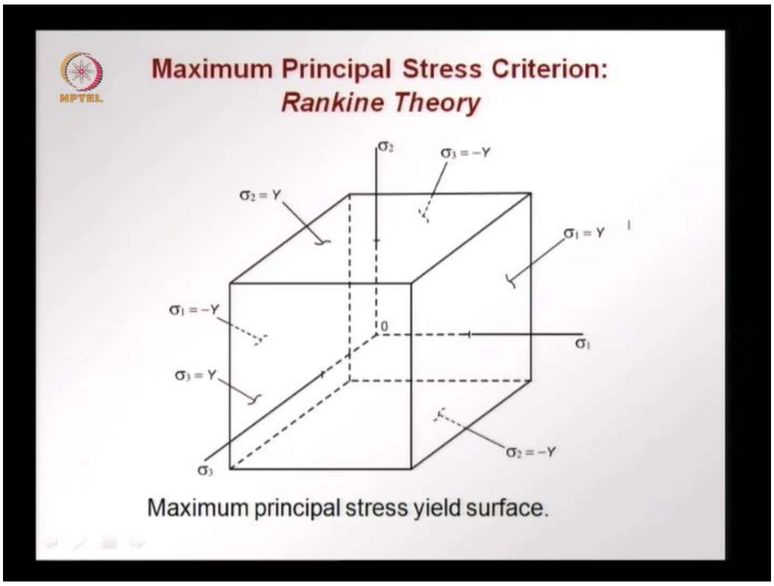

That is the meaning of the yield surface this is in two-dimensions with sigma3 equals to 0 in the case of sigma3 all. So, being nonzero in the case of a multiaxial stress state instead of a square yield surface we have a cubic yield surface again the surfaces or defined either yield strength this surface called example is sigma one equals to y and. So, now, again as long as the stress state is such that with sigma 1 sigma2 sigma3 known the point is inside this cube then failure is not occur yielding is set to occur as soon as the point as stress increases the reaches.

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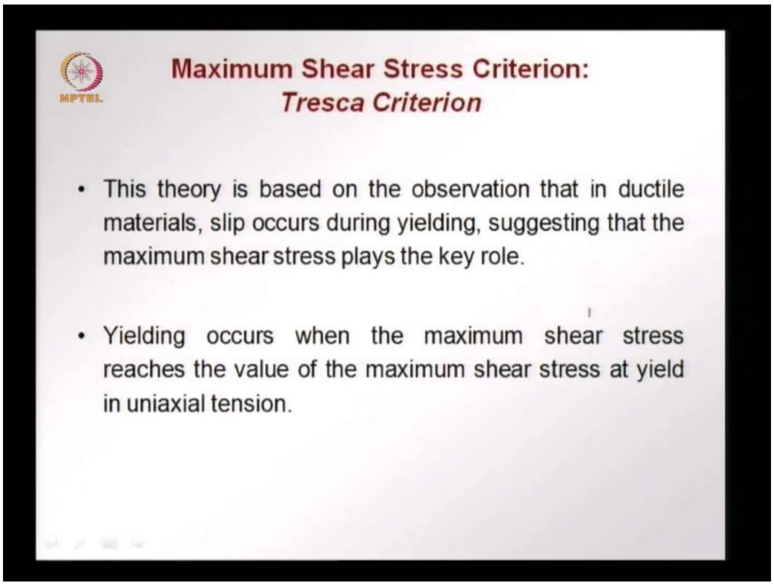

The yield surface then failure is to occur in the tresca criterion or what is called the maximum shear stress criteria we give more important to shear stress this theory is based on the observation and this we discuss extensively in previous lectures that in ductile materials slip occurs due to yielding.

This slip is provoke by sharing and therefore, the tresca criterion gives the maximum important to the maximum shear stress the criterion says the yielding occurs when the maximum shear stress under in arbitrary stress state reaches the value of the maximum shear stress at yield in uni-axial stress. So, we have to find out at what value the maximum shear stress will have at yielding and the uni-axial tention and when the same maximum shear stress occurs under.

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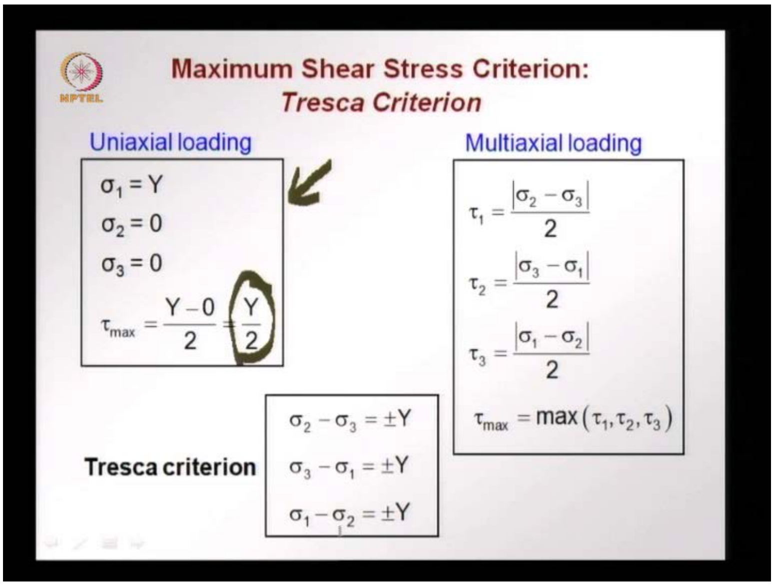

Any arbitrary condition yielding is set to occur and the uni-axial condition this will be the case sigma 1 is equals to y at yield sigma 1 sigma2 sigma 3 or 0 remember this is uni-axial loading. So, now, the shear stress are the maximum shear stress is sigma 1 minus sigma 3 divided by 2 that is half the yield stress. So, what we have said is when the maximum shear stress under any conditions reaches in this value then yielding will occur under multiaxial loading now there are three possible's s stresses which can occur given by the difference half the differences between in different principal stresses the maximum now will go an failed. So, linking these two we have the tresca criterion which says that failures will occur if anyone.

The these conditions are satisfied sigma 2 minus sigma create this is equals to plus minus y sigma 3sigma 3 minus sigma 1 is equal to plus minus y sigma 1 minus sigma 2 is equals to plus minus y.

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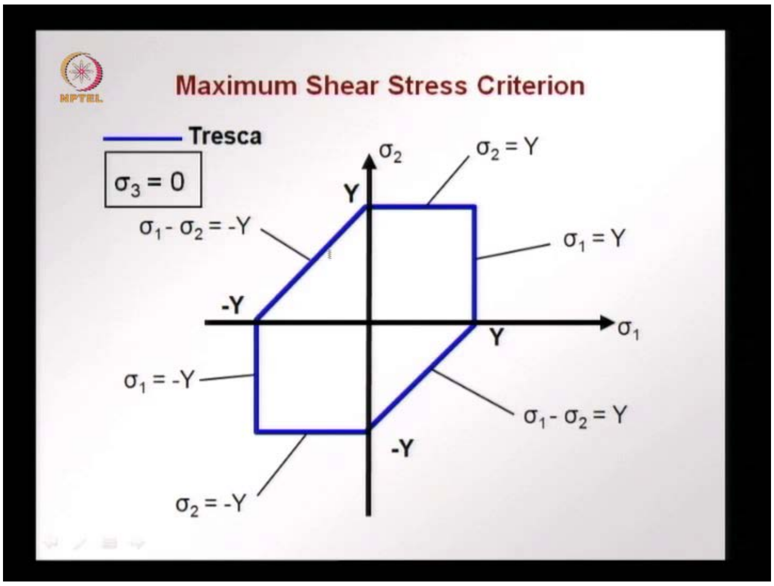

And this is represented by now by this yield surface for the tresca criterion this is a gain for sigma 3 is equals to 0 planes stress situation this surface comes from sigma 2 equals to y sigma 1 equal to y this sigma 1 minus sigma 2 equal to and so an. So, again as long as stress state is inside this yield surface failure has occur.

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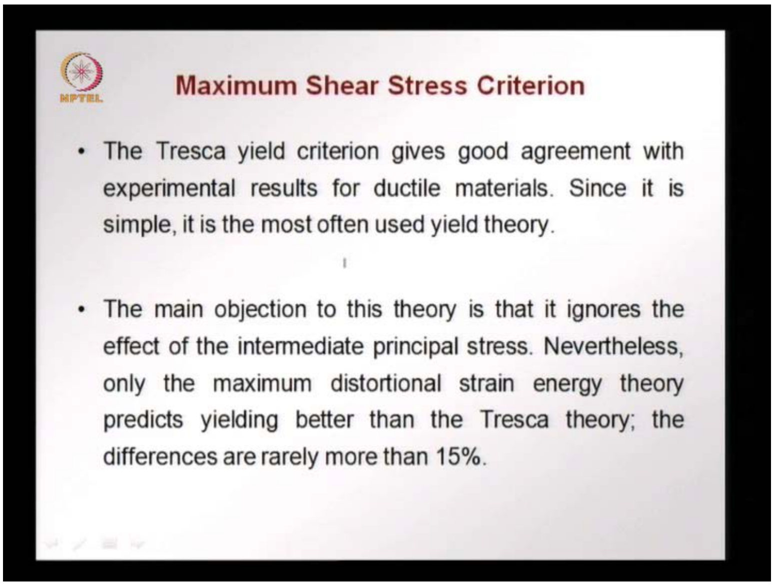

And then they stress state reaches in this yield surface yielding is set to start the tresca yield criterion gives good agreement with experimental results for most ductile materials and it is very simple therefore, it is the most often used yield theory the main objection are the limitation of this theory is that its ignores the effect of the intermediate principal stress if you remember the shear stress only depends on two of the principal stresses what is better is that will what we look at now is the maximum this distortional strain energy theory which predicts yielding slightly better than the tresca theory.

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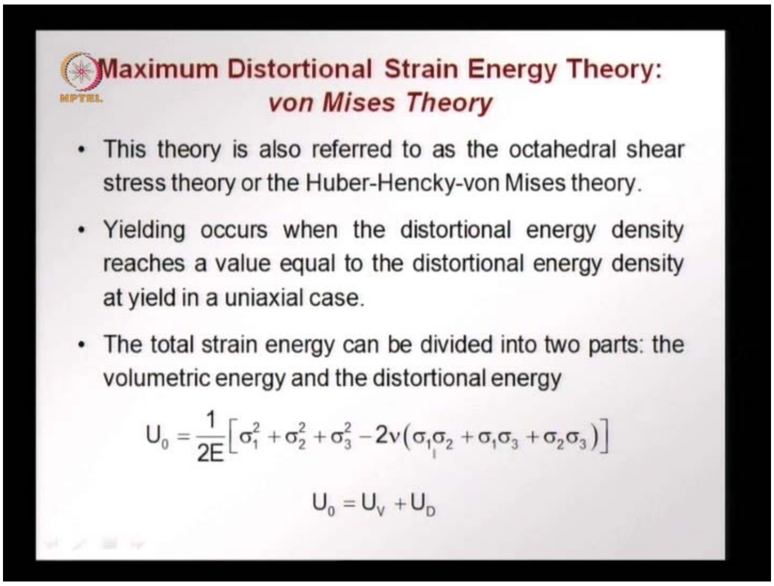

But the differences are not generally more than fifteen percent tresca is more easy to apply a more exact theory would be that of the maximum distortional strain energy theory this maximum distortional strain energy theory is called also the von mises theory other names are the octahedral shear stress theory or the Huber-Hencky-von mises theory in all these cases we consider the yielding occurs when the distortional energy density reaches a value equal to the distortional energy density at yield in a uni-axial case. So, we find out what is the distortional energy density at yielding in the uni-axial tension and when that same distortional energy density occurs in any other arbitrary state we be considered yielding occurs strain energy the total strain energy can be divided always into two parts a volumetric part on a distortional part u 0 is the total strain energy given by in this equation where sigma 1 sigma 2 sigma 3 are the principal stresses he’s the young modelist and you as the u 0 can be divided into the volumetric energy and the distortional energy the volumetric energy is now…

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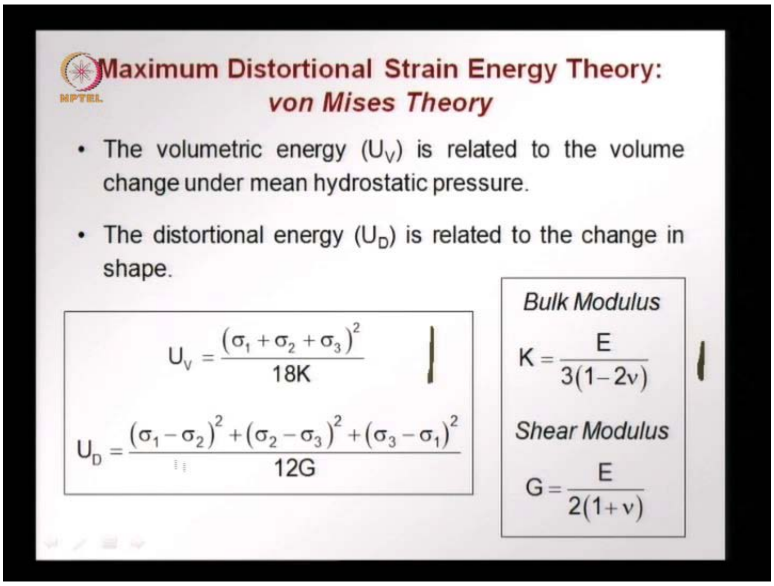

What is given a hydrostatic situation distortional energy that corresponds to the change on a distortion in the shear u v as I said this is the volumetric energy related to volume change under mean hydrostatic pressure under any arbitrary stress state they by always be a component that can be attributed to hydrostatic pressure the remaining the other part is the distortional energy is related to the change in shape.

U v u sub v is equal to the sum of sigma 1 sigma 2 sigma 3 square divided by a 18 k the k again bulk mode list given by this equation the distortional energy is given by this equation the some of the squares of the differences between the different principal stresses divided by two well g where g is the shear mode list. So, we can see that this clearly is from the hydrostatic part and we know that hydrostatic pressures will not cause failure it only causes volumetric compression as the stress increases and we saw that sharing leads to slip and yielding and that is what is behind the distortional energy.

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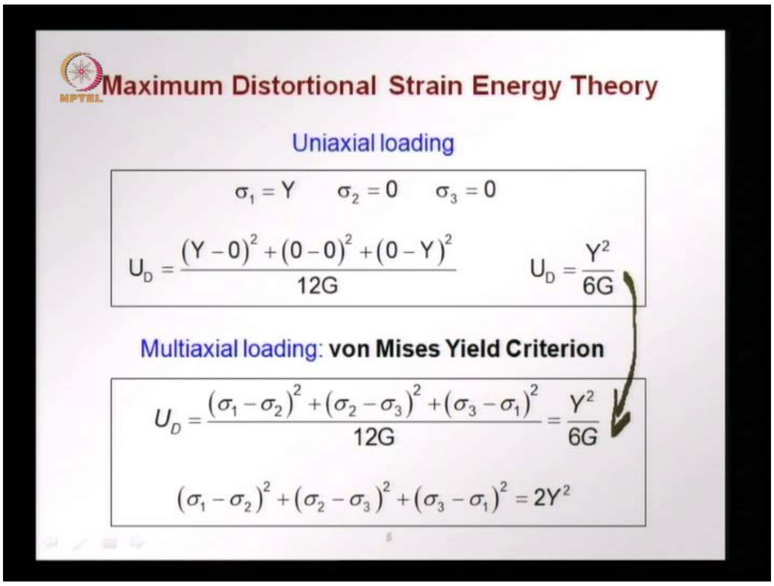

So, let us see how to apply this criteria to failure. So, what we said is the distortional energy density at uni-axial loading must first be determine. So, under uni-axial tension we have sigma 2 sigma 3 has 0 at yielding sigma 1 equal to y. So, using the equation of the previous slide the distortional energy is given by y square divided by 6 g. So, what we as said is the distortional energy and any arbitrary stress state when it reaches this value yielding will occur. So, in a multi-axial loading we say that this would be the equation this is set equal to this value and we get finally, in this equation.

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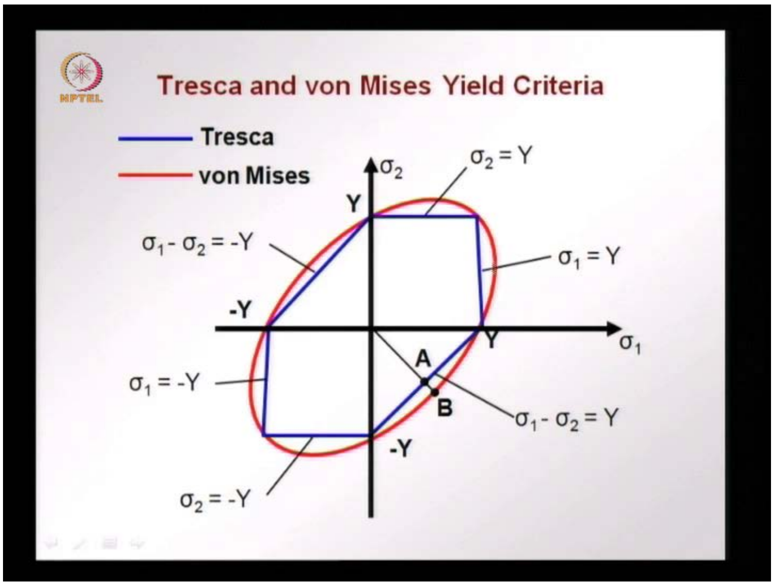

So, this is the von mises yield criteria failure through yielding when we draw the failure surface we get these red line which circum yield surface that we add in the tresca condition. So, an as the blue line, so we are find that that very similar except that there is a slightly higher stress that can be taken before failure occurs given by the difference between the red line and blue line this because we have not taken in to consideration the volumetric part and we are only looking at the shear part.

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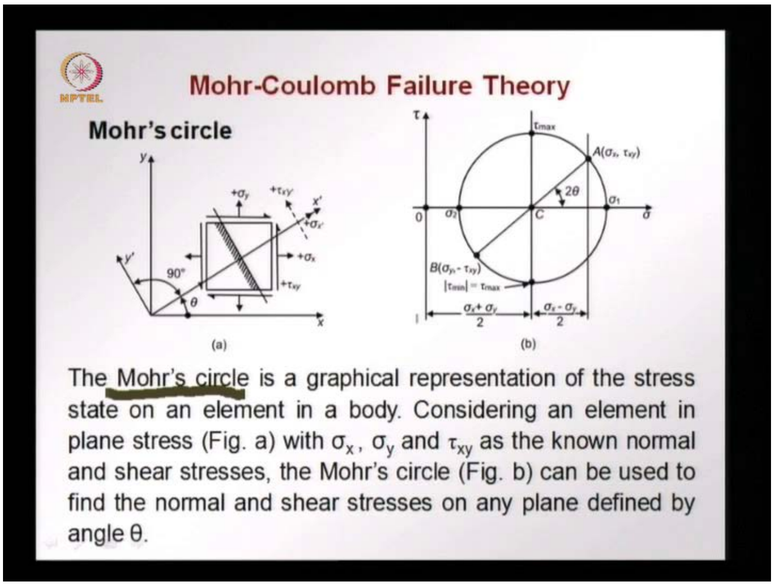

So, actually the stress that can be slightly higher in some cases than what is given by the tresca criteria we can also have failure that is occurring it by others mechanisms such as fracture or cracking rather than just by yield for this we have look at a failure theory call the Mohr-coulomb theory and for this again you have to revise the Mohr's circles which your studied in the strength of materials, and we know that Mohr's circle is a graphical representation of stress state if we consider a element given in figure a with sigma x sigma y and tow x y as the known normal and shear stresses along a certain plane we can now draw Mohr's circle fixing point a and b from known values of sigma x sigma y and tow x y we can now drawn this circle where this circle cuts x axis the normal stress axis are the principal stresses sigma 1would be the maximum principal stress sigma 2 would be the minimum principal stress, and the value given by the top of the Mohr's circle is the maximum shear stress y axis is gives shear stress the x-axis gives the normal stresses.

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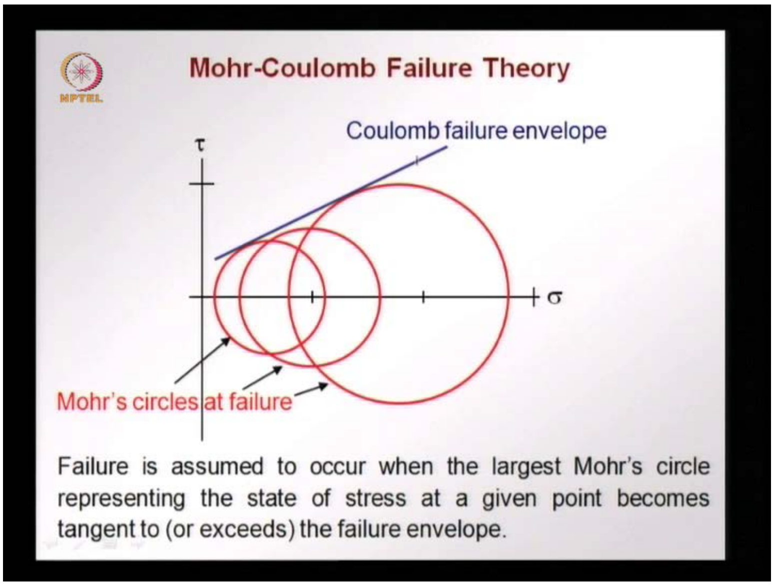

So, this would be the Mohr's circle corresponding to that element in this body failure is now taken to occur when the look at the Mohr's circle at failure at a certain stress state when we draw all the different Mohr's circle at failure for this material and we take the envelope this is called the coulomb failure envelope. We now assume that failure will occur when any Mohr's circle touches this failure envelope when the circle is smaller failure is not occur stress state is such that failure is for from occurring and as the stress is increases this Mohr's circle will become larger and larger and finally, it will touch the failure envelope the coulomb envelope which is taken linear and failure is then set to occurs.

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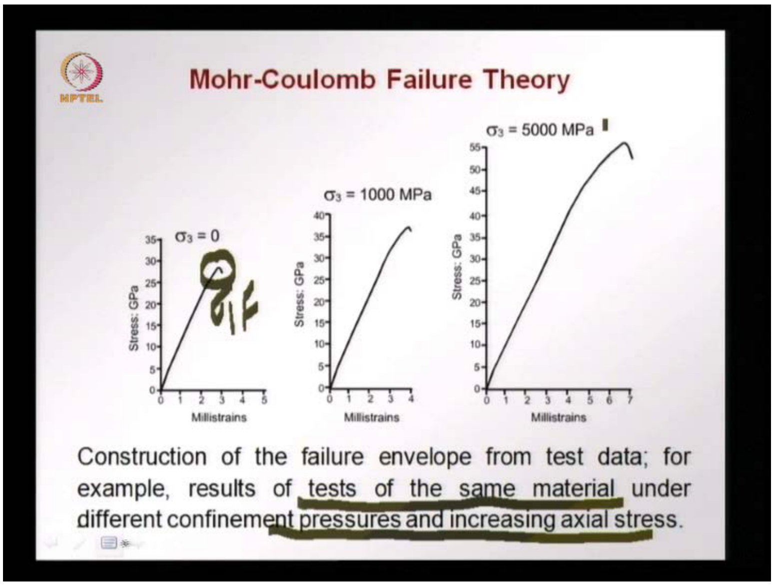

We can determining this failure envelope through test for example, we have a material that is subjected to different confinement pressures say we take a cylinder or a core some material and subjected to different confinement pressures the sigma three this is the case very have no confinement pressure, you have1000 mega fascicle confinement pressure and this is 5000 and when we increases the sigma 1 the axial pressure say sigma 3 is not confinement pressures and now we have increasing under that confinement pressure the axial stress in the case of 0 confinement.

We have in this particular case a linear part and then the peak occurring at around 27 mega fascicle and then failure and this would be the sigma 1 that failure under this value of sigma 3. So, we repeat an a identical specimen at test with sigma 3 equal to 1000 mega fascicle and we again load axially slipping sigma 3 constantly increasing sigma 1 and at failure we observe that the sigma 1 is around 36 mega fascicle again another test is done we want to get a at list 3 points to come form failure envelope the now apply a confinement pressure of 500 mega fascicle and we keep increasing sigma 1 the axial part and the find now that with higher confinement the failure now occurs at sigma one equal to 55 mega fascicle.

So, we have now 3 sets of sigma 1 sigma 3 values at failure for this material we also seen that have confinement increases we have a higher failures stress failure axial failure stress and this is the concept that we use a lot in civil engineering to confine a material to provide effectively for a higher load carrying capacity a simple example a is concrete coulomb with hopes at ductile which confined the concrete and therefore, increase its effective load carrying capacity.

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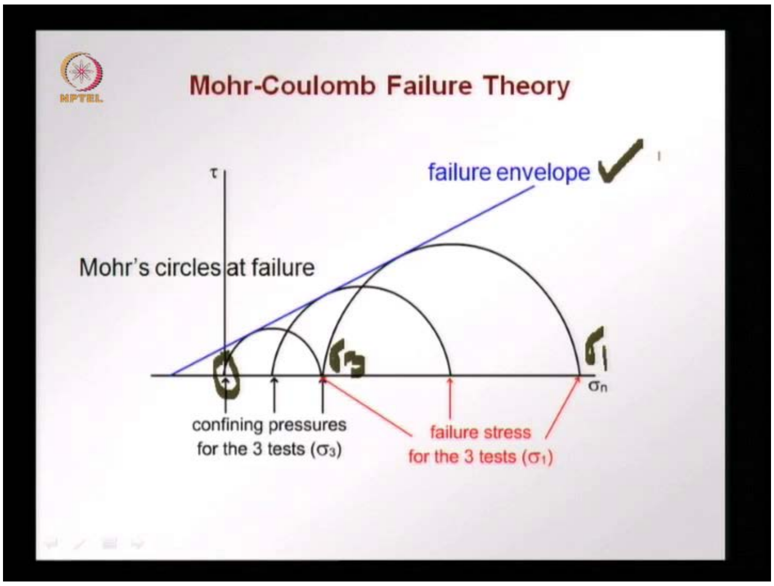

So, test like this construct the failure envelope and what We have done is huge tests of the same material under different confinement pressures and axial stresses we now draw the Mohr's circle for the cases represented by a tests that failures stress is sigma one. So, this is sigma one and this would be sigma three for the last test for the test that we did sigma three equal to thousand mega fascicle this would be the Mohr's circle and this would be the case were that is 0 confinement sigma three is equal to 0 now we draw at envelope tangent to the is Mohr's circle and this is now the failure envelope for the material that we have tested.

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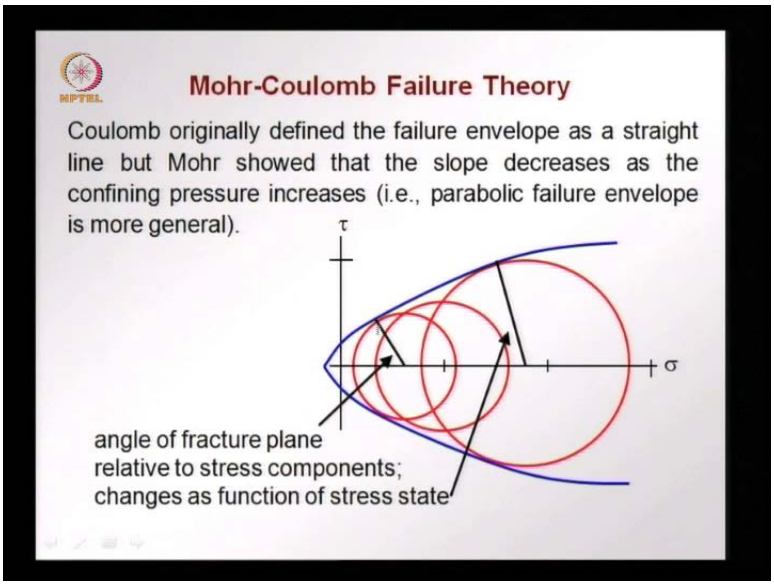

So, this is the way that the failure can be constructed for the different materials what we generally see is that instead of a straight line the failure envelope coulomb originally defined the failure envelope is a straight line, but later more showed that the slope decreases as the confinement pressure increases there is the flattening out of this curve and. So, you get a pinched response initially an at parabolic failure envelope and this is now called the Mohr's coulomb failure envelope what we also observed from this diagram is that the angle of the fracture plane remember that this angle, now tell us their how the failure will occur and has the confinement changers this angle indicating the failure plane also changes you see that when we look at the angle of line joining the point which coincides with the failure envelope and the centre of mohr's circle.

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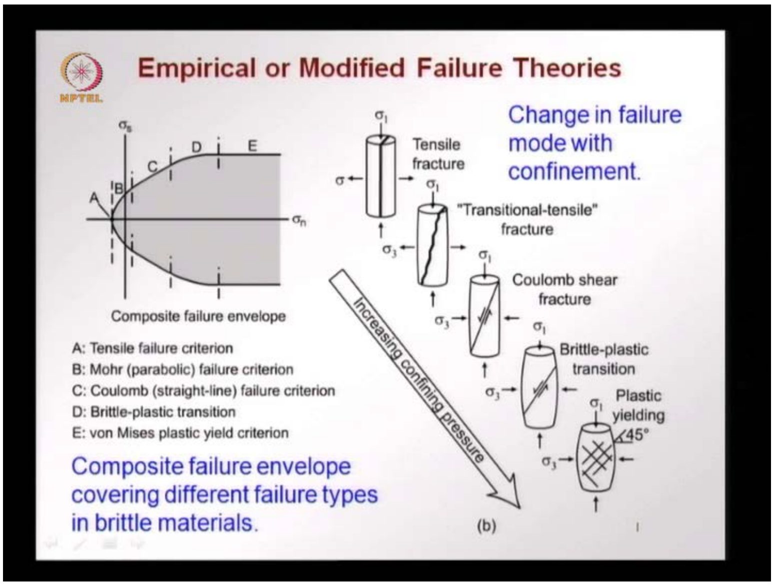

We find the this angle changes this indicates that the failure plane changes its direction have this stress state increases lastly we should understand that there are different materials which have a varying failure mode as the stress state increases or changes when this stress state changes say in a brittle material like concrete the rock the have to employ lot of varying failure theories sometimes these are empirical based or modified failure theories, and one such failure theory or a combination of failure theory is shown on the figure on the left top which is a composite failure envelope starting of with tensile failure criteria.

So, we could have a material failing under tension under vary confinement seen this case we have a specimen which is subjected to compression little bit of lateral tension and you can splitting type failure and tensile fractures occur, then we can little bit more confinement you can have the behaviour that will discussed earlier the more parabolic failure criteria this is the spark where we have a change from the vertical splitting to shear type failure this is called the transitional tensile fracture. You have the crack that is sliding and opening due to this stress state some amount of compression or confinement then we have a area which is corresponding to the coulomb a straight line failure criteria this is where you should have purely shear dominating the failure there is some amount of confinement here until in the cases the confinement was very low here you have most significant confinement.

And you have sharing occurring when we have a brittle transfers brittle plastic transition going towards the von mises we will criteria given by this diagram may be have sharing and some amount of a plasticity indicated by the bulk then finally, we will have very high confinement leading to plastic type behaviour where we can apply the von mises plastic yield criteria. So, you have here the shear banns forming at 45 degrees and failure occurring with plastic yield. So, this; obviously, is very complicated complex failure envelope which will require lot of para meters to develop and something like this is needed when we want to cover different types of failure in brittle material and other materials we also see the the mode of failure in many material such as concrete rocks ceramics would change that type of failure.

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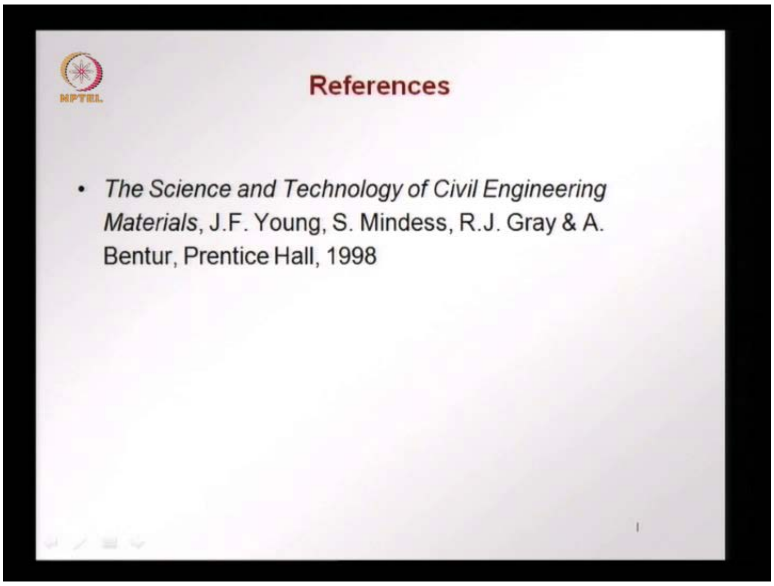

The failure mode changes with the confinement show to conclude we have we have looked in the previous lectures on how materials respond to stress and today we will look at what sort of criteria we need to explain failure to design for failure and to understand failure and with this will look that the different mechanical properties of continuous media. So, in all these cases we are looking at materials which do not have any discontinuity what we will do in the next lecture is look at fracture of materials with we look at this continuities forming defects becoming cracks and then the concept of stresses strain sort of breakdown if a you remember from strengthen materials and mechanics and what we look at in previous lecture all. So, stress and strain hard define at a point strain was the change in length divided by the original length and from that definition, and we can understand why the concept of stress and strain breakdown when there is a crack if you can a imagine a point which where you suddenly have a crack the original length is 0 between the plane defined by palms now when you have a crack you suddenly have a displacement between them. So, there is a change in length whereas, the original length was almost 0. So, if you divide the finite change in length divided by a 0 original length you have a length infinite strain. So, whenever there is a discontinuity or a crack you have infinite strains and what you will also see in next lecture is you sometime have infinite stresses. So, therefore, the concepts that we have a discussed in the previous lecture in this, where we look that continuous media breakdown and we have to bring in the concepts of fracture mechanics that will deal with in the next lecture.

Thank you very much.