Modern Construction Materials
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Module No - 03
Lecture No - 09
Part 02 of 02
Fracture Mechanics – Part 02

This is the second part of the lecture on fracture mechanics until. Now, we have been looking at linear elastic fracture mechanics and where it can be applied. Now, will go on to see how fracture occurs in different materials that may or may not follow linear elastic fracture mechanics strictly, and then we look at the end on what we call probabilistic fracture, where the variability of the defects comes into play.

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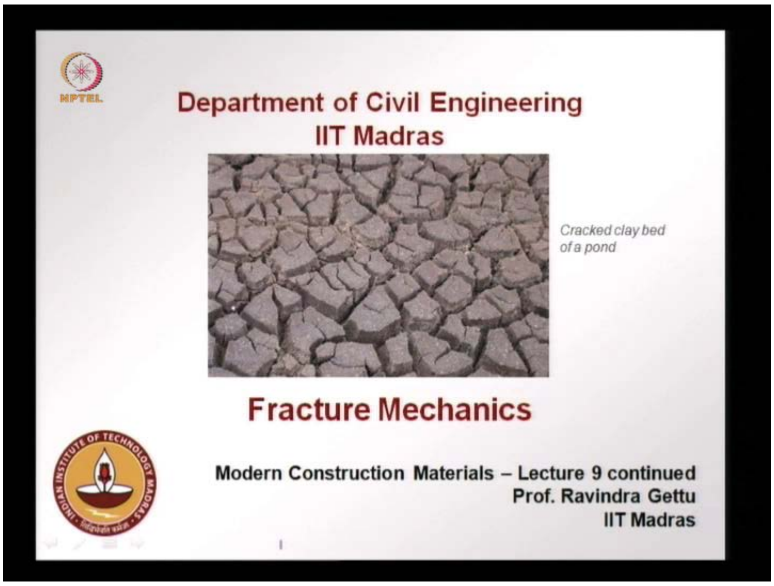

Here you see the picture of a cracked clay bed, where you see cracking occurring extensively when clay dries up. So, different materials crack in different ways and for that we have to understand what happens at a crack tip.

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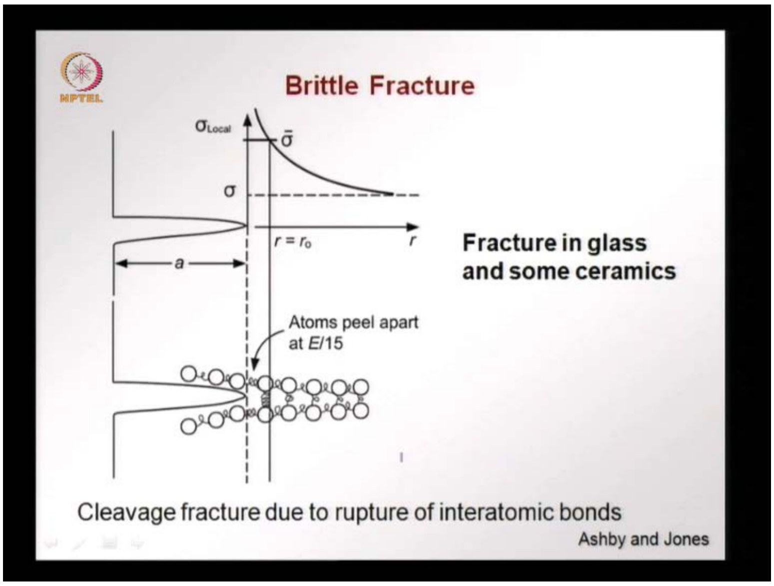

In the case of the fracture in glass and ceramics brittle fracture as the stress intensity goes up near that crack tip. This stress reaches in the vicinity of the crack tip a value of stress, which causes the rupture of the bonds in front of the crack tip.

So, thats what we see here we have the atomic bonds and if we go back to the ((Refer Slide Time: 01:31)) diagram. We know that there is a maximum attractive energy that is keeping the bonds together, and when we stretch the bonds what the crack is doing now is as it is opening its stretching these bonds. And they will be a time when the bond energy is surpassed and the atoms or particles peel apart break the bond breaks and the crack now starts to extend; this is called cleavage fracture, which occurs due to rupture of the inter atomic bonds themselves.

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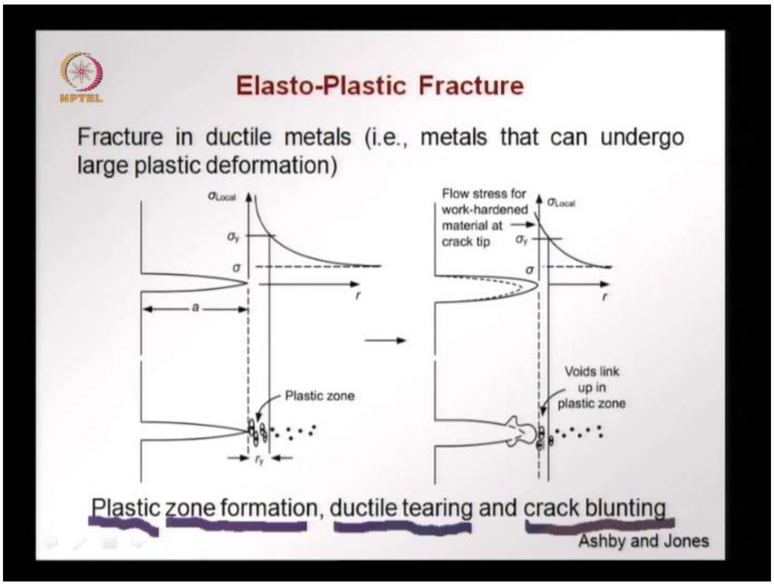

In metals especially in metals that can undergo large plastic deformation ductile metals. We have a different type of mechanism we have what is call is Elasto Plastic Fracture that is there is a plastic zone in front of the crack tip always, and this occurs for the following reasons. Now, we know that in front of the crack tip we have high stresses as we approach the crack tip now the stresses will increase, and there will be a point in time; where the stresses according to linear elastic fracture mechanics would be higher than the yield strength.

So, what happens is the region were this stress can become higher than the yield strength, now becomes plastic. So, the stress over this region is always equal to yield strength. So, this becomes the plastic zone and there can be cavitation forming, and plasticity occurring in this region of with r y; with further increase in stress what will happen is this crack now propagates. In the micro structure what happen is that the these cavities that form now link up with the crack or the crack extends to include the cavities.

This would result in the blunting of the crack, because you will not have a sharp crack any more here initially we had a sharp crack. Now, with the inclusion of the cavities the crack blunts that is it loses its sharp end. Since there is this blunting now the stress intensity at the crack tip decreases; the stresses drop slightly at the crack tip. So, this now requires further energy to propagate the crack and there is an extension of the plastic zone beyond the crack blunt.

So, what we see is first there is a zone, which becomes plastic with further increase in stress this plastic zone; joins up or the cavities that form in the plastic zone join up with the crack you have a crack extension, but a blunt crack forming. This crack blunting now requires additional energy to propagate the crack, because the blunting decreases the stress concentration. And then progressively there will be a plastic zone formed ahead of the crack and the crack will extend.

So, there is a toughenifng effect or a beneficial effect to arrest the crack due to the formation of the plastic zone, ductile tearing and crack blunting. In metals it is common even in practice to stop a crack from propagating by drilling a small hole at the edge of the crack. So, that the crack sharpness is lost and you have a blunt crack.

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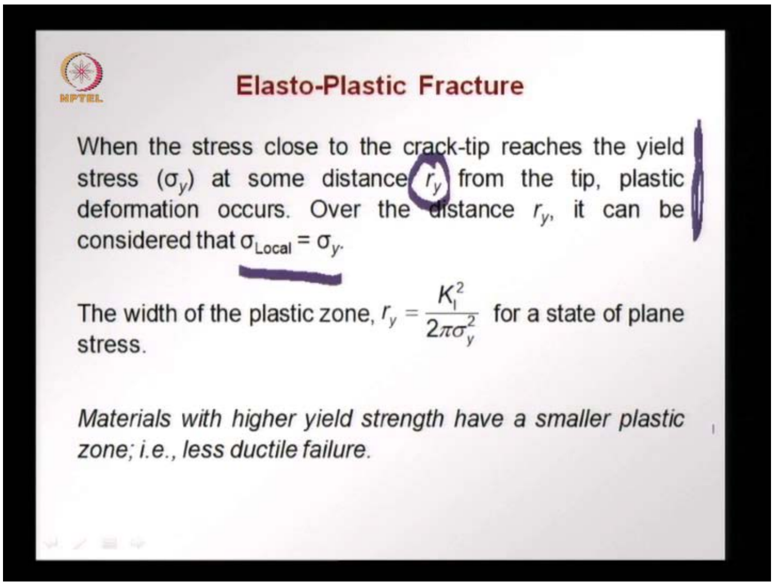

This plastic zone that forms in front of a ductile material that is cracking can be quantified. As we said when the stress close to the crack tip reaches the yield stress there will be a region where it is yielding over a distance r y; plastic deformation occurs in this region. And we can consider that the stress over that region is equal to yield strength, the value of r y, which is the width of the plastic zone is given by this k 1 square divided by 2 pi sigma square y when there is a state of plane stress.

So, what we see here is that when sigma y is larger, r y is smaller that is for a material, which has a higher yield strength the plastic zone is smaller; that means, the ductility of the failure is also less, larger the r y more ductile is the failure more energy is dissipated there is a larger zone were the toughening of occurs and energy is dissipated during failure. And this also explains why materials even ductile materials that are stronger behave in a more brittle manner and the origen is the size of the plastic zone that develops in front of the crack, which is smaller when the yield strength is higher.

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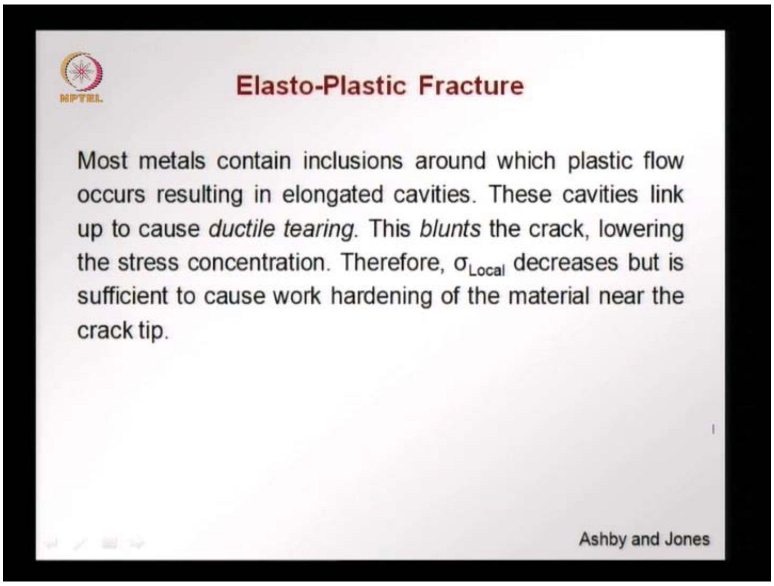

Metals that contain inclusions also end up having elongated cavities like we saw in the picture two slides back; and these cavities now link up and cause ductile tearing and this blunts the crack as I explained. And this blunting of the crack lowers the stress concentration after the lowering of the stress concentration now the stress decreases and to further propagate the crack a higher stress is needed.

However, the blunting of the crack also leads to work hardening this again strengthens the material in front of the crack and again leads to higher stress requirement for propagating the crack. So, all these mechanisms that occur in front of the crack tip in a ductile material tend to increase the energy requirement and therefore, arrest the crack and slow down the propagation of the crack.

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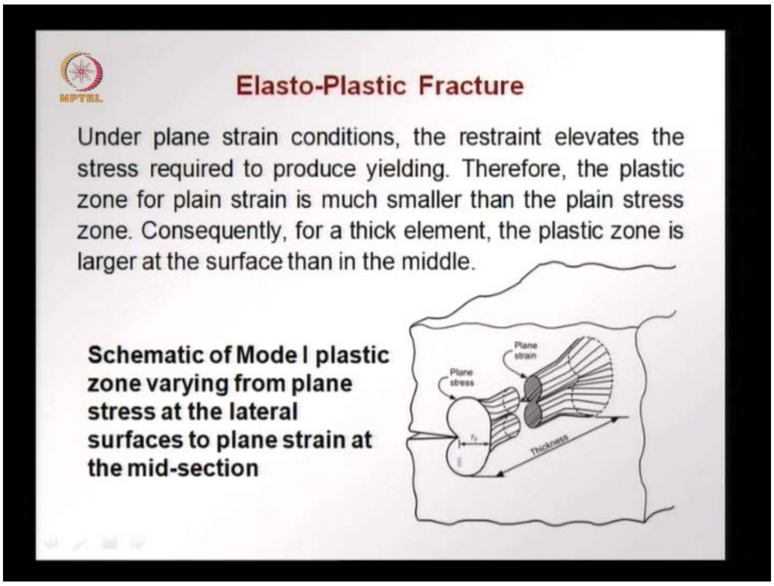

We see a difference in the plastic zone size depending on whether the conditions are close to plane stress or plane strain; in plane strain there is a confinement of the region there is a restraint and therefore, the zone is small compared to when you have plane strain when you have plane stress. The plastic zone in plane strain is smaller than the zone that we have in a plane stress; suppose we have a very thick element we will have plane strain in the middle, because of confinement and on the surfaces we have plane stress. And if you look at the development of the yield zone in front of the crack in a thick plate metal plate we will find that there will be a larger yield zone in front of the crack tip at the surfaces due to plane stress conditions, and inside it will be smaller.

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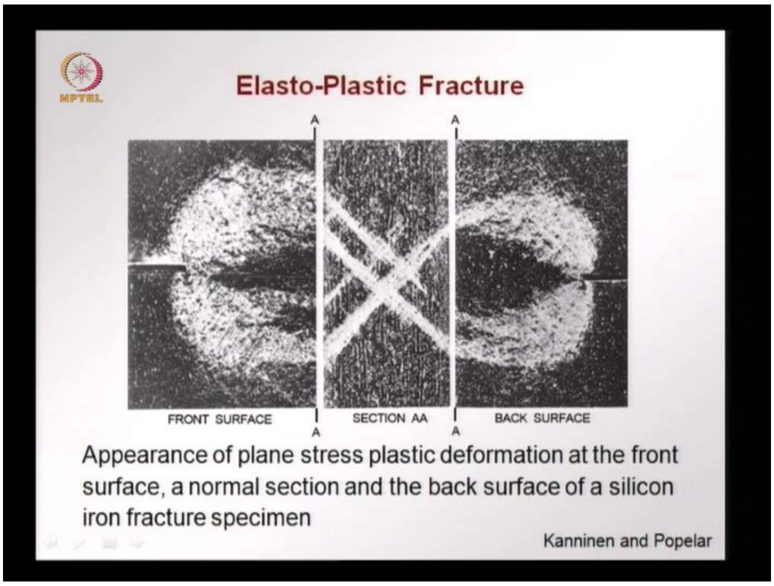

This is shown in this picture of fracture occurring in iron specimen on the left we have the front surface this is now the crack or the notch that was made, and this is the plastic zone that is occurring in front of the crack; on the other surface the back surface again now we see this is the notch, this is the crack tip and this is the plastic zone. Now, if we were to take a section across we will see that on the edges we have a large plastic zone corresponding to the regions of plane stress, but in the interior due to the confinement due to the constrain we have a smaller region of yielding, because of plane stain conditions.

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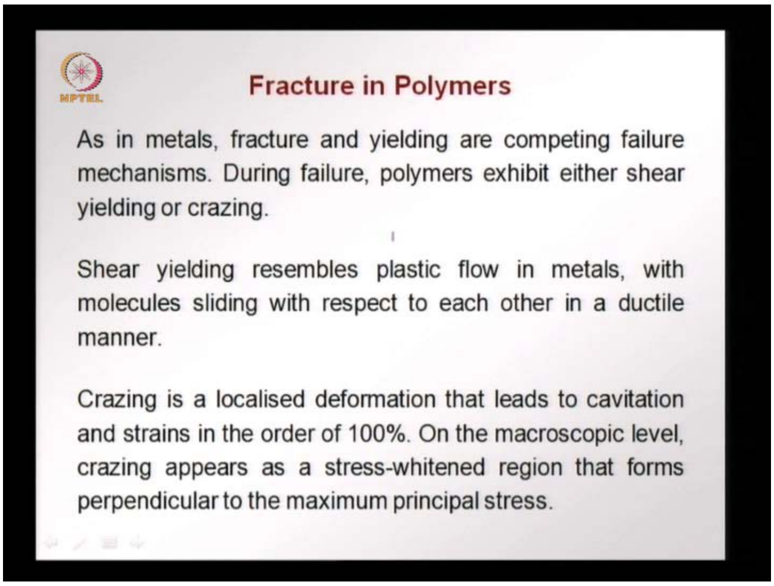

How do other materials like polymers fracture just like we see a competition of fracture in yielding in metals, polymers also have a competition between cracking and shear yielding or crazing. Shear yielding is similar to what we see in the yielding of metals due to plastic flow, but here in a polymer the molecules slide with respect to each other; the molecule change in the polymer slide with respect to each other, and we can have yielding occurring or something similar to yielding occurring. Crazing is more localized that can lead to cavitation and very high strains occurring. And if you see crazing occurring in a polymer you would see that there is a whitened zone; a zone becomes more opaque in front of the crack tip called the stress whitened zone, and this forms perpendicular to the maximum principal stress.

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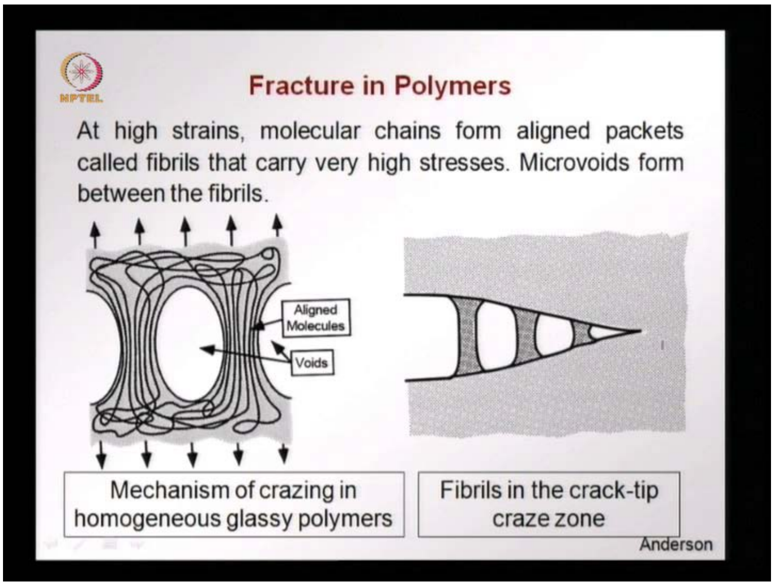

So, what happens in crazing is that we have very high strains. So, the polymer change now get aligned in the direction of the applied stress. So, this is the direction were the crack is propagating perpendicular to it these molecules start aligning; this now bridges the crack, but also leads to cavities forming or voids forming between these aligned molecules these aligned molecules are called fibrils. Little bit away from the crack this is what you will see or a little bit when you zoom out of the crack you will see that at the crack tip; you see these fibrils bridging the crack, and then you have the voids occurring. So, these are the fibrils at the craze zone near the crack tip.

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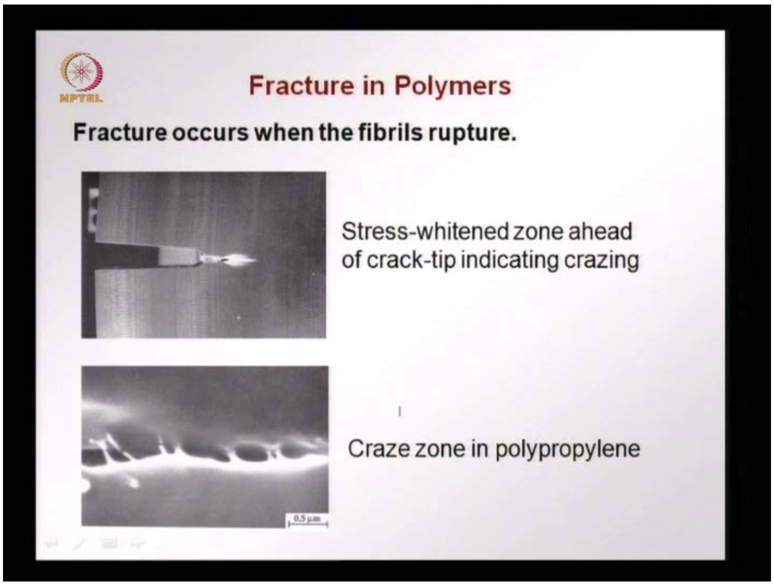

Some more pictures we see the on the top we see the stress whitened zone or the crazing zone near the crack tip and closer we see the craze zone in a polypropylene. So, you see the fibrils here the voids and this is now the craze zone.

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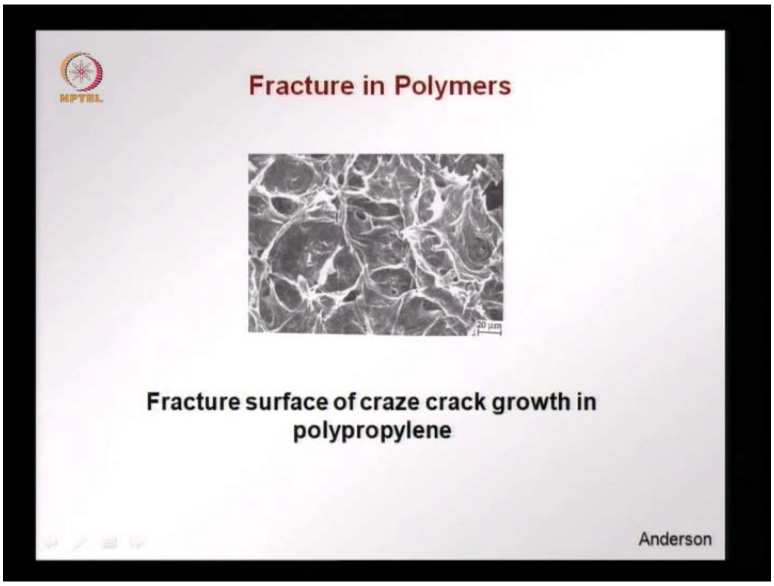

In the micro scope when we look across the craze zone we will see this in polypropylene this is now the section of the fibril all the bunches, the bundles of fibers, bundles of the polymer chains; and here between or outside of the fibrils we see the voids or the cavities occurring. So, this would be the cavity and bounded by the fibril.

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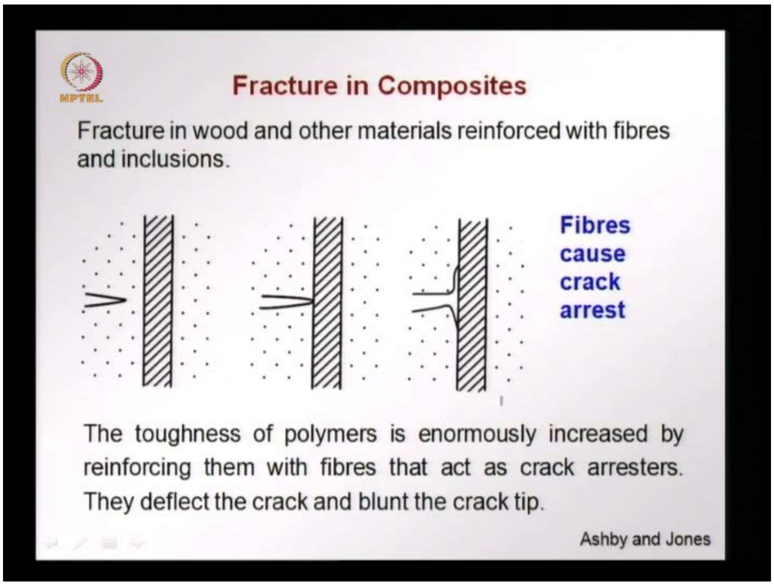

What happens in composites; suppose we have fracture occurring in wood or other materials reinforced with fibers or inclusions, the fibers will tend to arrest the crack they will stop the crack from propagation that is first you will have the crack occurring in the matrix; see in the polymer matrix the crack is propagating with higher stress; the crack propagates further and it reaches the fiber. Now, the fiber is stronger has a higher elongation capacity, so it will stop the crack from propagating it will not allow the crack to break the fiber and propagate.

So, what we see when the stress continues to increases now the crack envelopes the fiber; and it does not propagate, does not extend further until the fiber is eager broken or its slips and you have significant deformation in this region. So, this gives rise to two phenomena that help control the crack one is the crack is now blunted instead of a sharp crack we have now of blunt crack tip. So, this decreases the stress concentration and therefore, you need now more stress to keep the crack propagating.

Secondly, some of the stress now is transferred by the fiber. So, there is a less there is less of a driving force for continuing the crack, because some part of the stress now is transferred from one end of one side of the material to the other by the fiber itself.

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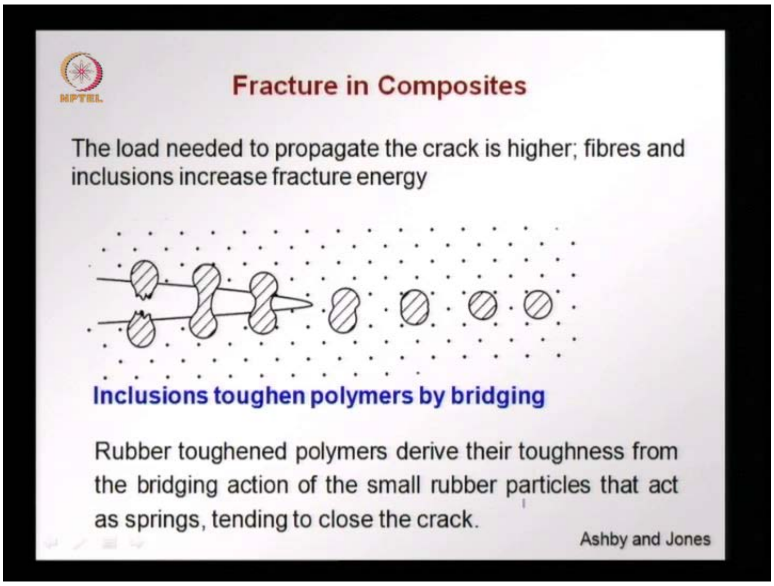

So, the toughness of the polymer is increased significantly when fibers reinforce polymers they act as crack arresters, they deflect the crack and they blunt the crack tip. When we have inclusions similar effect occurs; the load need to propagate the crack becomes higher as the inclusions as we saw in the case of fibers increase the fracture energy. The energy requirement to keep driving the crack what would happen when we have rubber inclusions in a polymer is that when the crack propagates we have these rubber inclusions acting like springs. The bond the faces of the crack the pull the faces of the crack together and until the rubber inclusion breaks the crack cannot propagate very much further.

So, this tends to close the crack and as the crack propagates it will encounter more inclusions, which will again arrest the crack more energy will be required to break these rubber particles and keep going. So, rubber toughened polymers derive their toughness from the bridging of the small rubber particles they act as springs and tend to close the crack.

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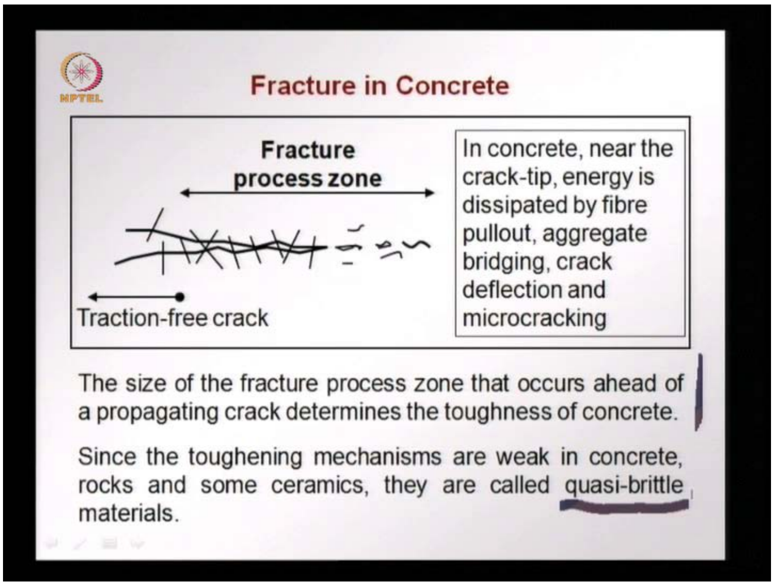

What happens in concrete? concrete as we know is a heterogeneous material it is a very brittle material and in front of a crack tip in concrete we have what is called a fracture process zone. And in this zone there are many mechanisms which occur, which prevent a very brittle fracture from occurring. Near the crack tip we have fibers in case this is a fiber reinforce concrete we would have fibers otherwise we have also aggregates, which are interlocking or aggregates that are bridging a crack they aggregate either to be broken or pulled out for the crack to propagate.

The aggregate also deflects the crack, if there is a very hard aggregate here the crack will rather go around if the interface is weaker rather than going through the crack. There is also some micro cracking due to the defects that occur. So, all this together dissipates energy during cracking and this is called the fracture process zone, which occurs in front of the traction free crack that is the crack. The part of the crack, where there is no bridging between the crack phases that is called a traction free crack.

The size of the process zone determines the toughness of concrete larger the process zone higher will be the toughness similar to what we saw in yielding of metals. The process zone will be larger when we have a material such as fiber reinforce concrete and the process zone will be much smaller when we have a brittle concrete like a high strength concrete; there the toughness is lower. Since toughening mechanisms in materials like concrete, rock and some ceramics are week they are called quasi brittle materials. There is not brittle as glass, but still closed enough and they are called quasi brittle materials that is almost brittle materials.

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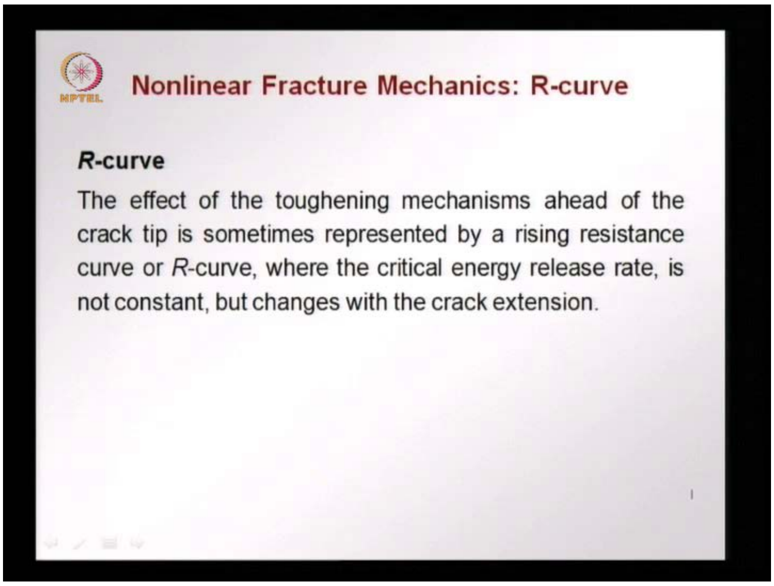

How do we model this mechanism; we understand that since there is a zone were energy is dissipated that occurs in front of the crack we cannot apply linear elastic fracture mechanics directly. So, there are different models the simplest of which is called the R-curve model as part of non-linear fracture mechanics. Here what we do is inside of a constant fracture energy the g c that we saw earlier in the fracture criteria according to linear elastic fracture mechanics we have a variable fracture energy called the R-curve.

The effect of the toughening that we saw in many of these cases in this lecture is represented by an increasing or a raising resistance curve or R-curve. Where the critical energy release rate earlier we called it g c when it was constant, is now changing with cracking extension that is it is increasing with crack extension.

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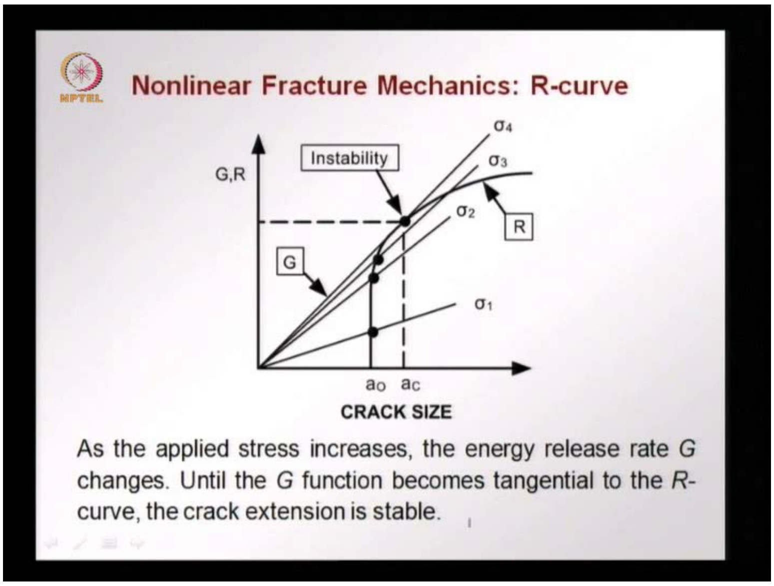

So, this curve will tell us how we apply this R-curve it looks complicated, but let us see how to understand how it works. On the x axis we have crack extension say initially we have a crack length or a defect length of a 0 as the crack propagates we will move from left to right. On the y axis we have both R, which is the resistance or the variable fracture energy instead of a constant fracture energy we are now having a fracture energy that is increasing with the crack size, G is the energy release rate that we discussed in the previous lecture.

The material property is this curve; is the R-curve. So, this is now a material property instead of a constant we have a curve fracture energy as a function of the crack size. So, what we are saying is as the crack is extending more and more of this toughening mechanisms come into play.

The process zone becomes more prominent and therefore, the resistance to crack propagation increases. These lines are the energy release rate as a function of the applied stress as we know the energy release rate now will depend on the load or stress applied the geometric of the element and the crack length just like in the case of k1. In this case the G-curve cuts the R-curve at this point so; that means, this is the stable crack length for this stress.

Now, we increase the stress the have a stress sigma 2 and the corresponding curve for G is this. So, the crack length is slightly longer the crack propagates a little bit, but still its stable it does not runaway and have catastrophic sudden cracking sigma 3 the crack extends little bit more, because it now encounters more resistance and it stops after propagating little bit more.

But beyond this at sigma 4 we find that we reach a point were any further increase in stress will create more G than R. So, at the point where we have the G and the R-curve meeting and the G-curve is tangential to the R-curve instability starts any further increase in stress will cause a sudden increase in crack length and we will have failure. Until this point the crack can propagate in a stable manner keep increasing, encountering more resistance and stop the growth does not increase suddenly.

So, as this applied stress increases the energy release rate G increases and when the G function becomes tangential is when the crack extension stops being stable; until that point the crack extension is stable that is it is slowly progressing, but it stops when the stress does not increase any further.

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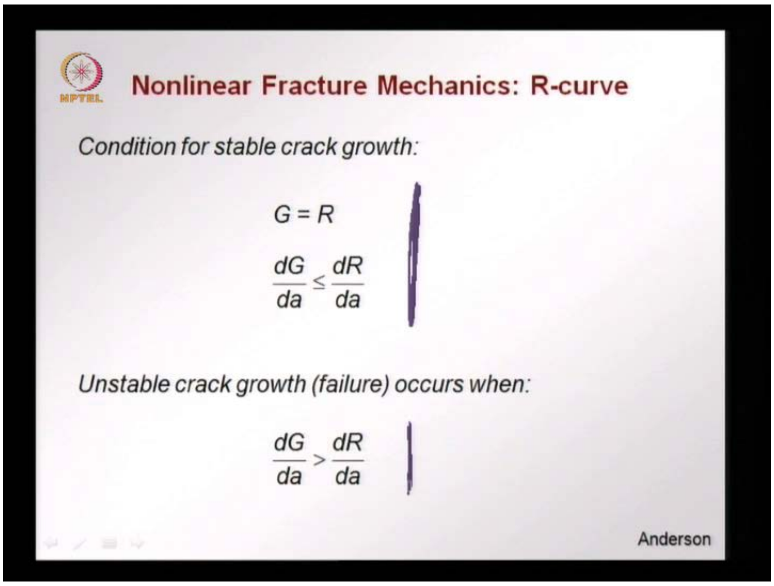

So, the criteria for stable and unstable crack growth for the R-curve are defined as follows for stable crack growth we have two conditions G equal to R, and the slope of G with respect to the crack length should be less than that of R. So, as long as these conditions are satisfied we have stable crack growth that is the crack grows, but it stops at that stress. If the crack growth continues due to higher stress this means that the slope has now increased for the G-curve higher than that of the R-curve, this is when failure is said to occur there unstable crack growth occurring.

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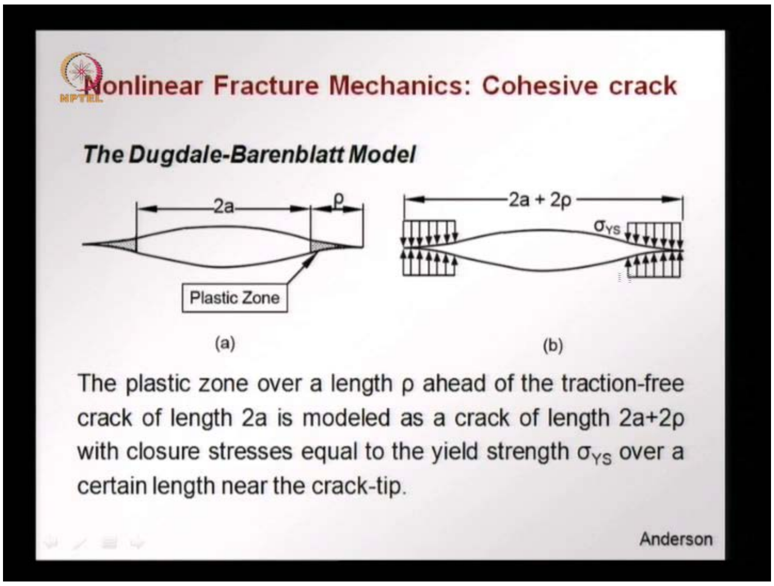

A more complicated and more comprehensive set of models or called the Cohesive crack models and this started when two people dugdale and barenblatt independently proposed models for looking at plastic fracture, where they considered the crack, which was traction free and with a plastic zone at the ends. And this plastic zone comes about, because of the mechanism that we discussed in the beginning of this lecture there is yielding there is cavitation and. So, on occurring here.

So, the stress here is equal to the yield strength that is what we saw. So, what dugdale and barenblatt did is that they said that to represent this we can take a traction free crack say a linear elastic crack, and add these closing stresses at the end equal to the yield strength. So, they super posed two solutions one a crack without any stresses like a linear elastic crack, and then they super imposed the stresses at the end to give the solution for this crack with a plastic zone. So, in these models the plastic zone over the length rho ahead of the traction free crack 2 a of length 2 a is modeled as a crack of length 2 a plus 2 rho. So, this is the whole crack with the closure stresses equal to the yield strength over this ends of the crack.

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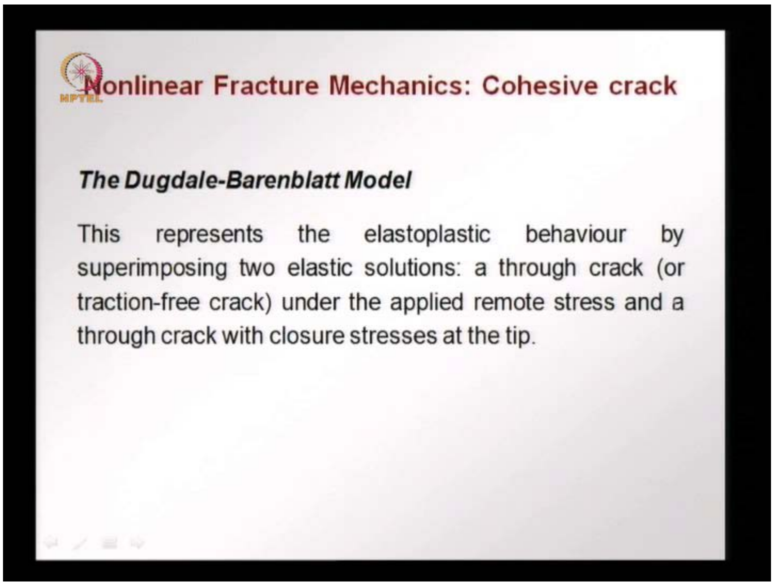

So, what they did is represent the Elasto Plastic Fracture by superimposing two elastic solutions a through crack, a traction free crack under the applied remote stress, and then the behavior of the crack with the closure stresses. The first part is a through crack under the applied stress, remote stress far away from the crack tip and the other solution, which is superimpose is the crack with the closure stresses at the tip.

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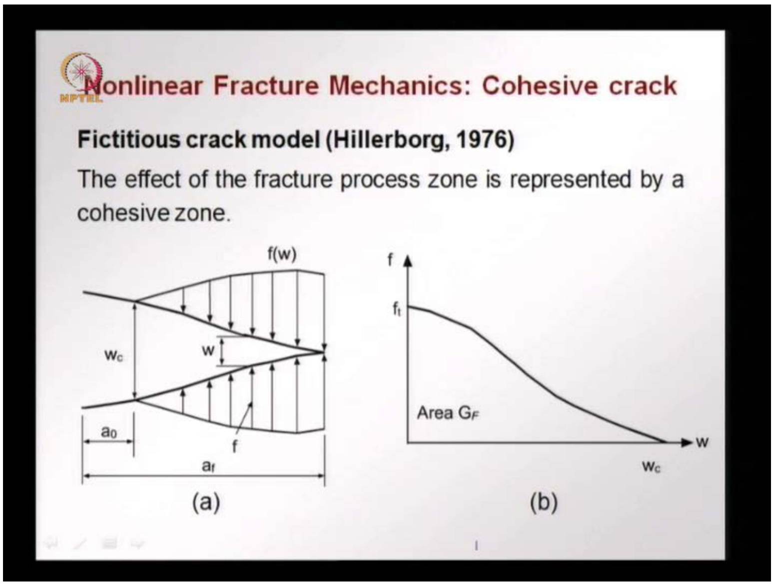

A further extension of this model called the Fictitious crack model was given by Hillerborg in 1976 with application to concrete and other brittle materials. Here instead of a plastic zone we saw that a fracture process zone forms this is what we saw in the case of concrete so; obviously, there we do not have a yield strength that is governing the stresses across the crack tip.

So, and a more flexible model a has been suggested, where we have a crack with ends having these cohesive or closing stresses that could be defined depending as a function of the material. And a possible function for this closing stresses is given here, where we have on the x- axis the crack width that is the opening of the crack W, and on the y-axis we have the stress that is trying to close the crack this is the closing stress or the cohesive stress. At the crack tip, where the crack is just starting to open the stress will be equal to the tensile strength, because when the strength is exceeded that is where cracking will start.

And far away from the crack tip we reach a point, where there is no closing stress at all that is the crack is significantly separated that all the mechanisms that cause toughening or nullified this is called the critical crack width W sub c. The area under this curve is called the fracture energy, G f is the fracture energy.

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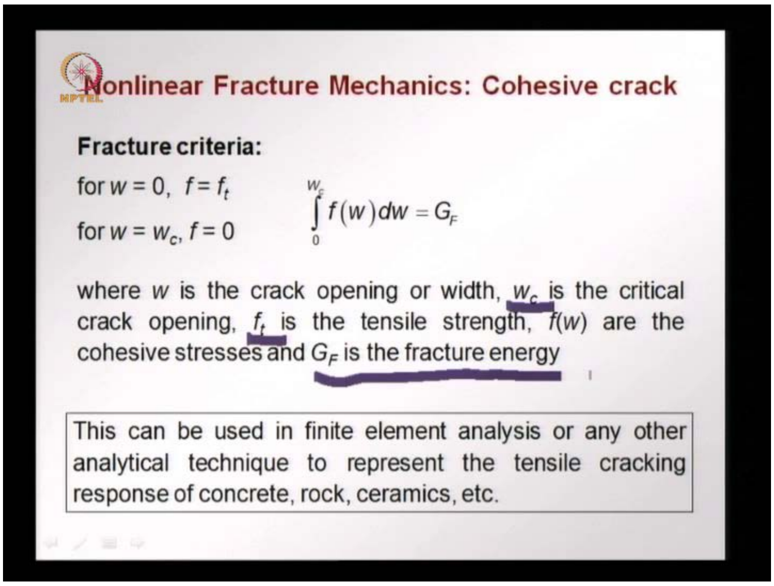

So, the fracture criteria according to the cohesive crack are a combination of these three at W equal to zero that is at the crack tip being zero crack is just about to start the stress across the crack is equal to the tensile strength. Far away from the crack tip at a distance, where we have a crack width equal to the critical crack opening W equal to W c there is no more closing stress or cohesive stress. And then we see that the integral of the closing stresses as a function of the crack width integrated between 0 to W c is called the fracture energy.

So, we have this as a parameter the tensile strength as a parameter and the critical crack opening now as a parameter. In addition to the shape of the curve; the shape of the curve also can vary from one material to the other. Now, this representation can be used in finite element analysis and any other analytical technique to represent the tensile cracking response of materials such as concrete, rock, ceramics even polymers.

When we have other modes of fracture this curve can be modified to include the effects of mode 2 and mode 3, but it has been very much used in the more critical failure that occurs in these materials that is in mode 1.

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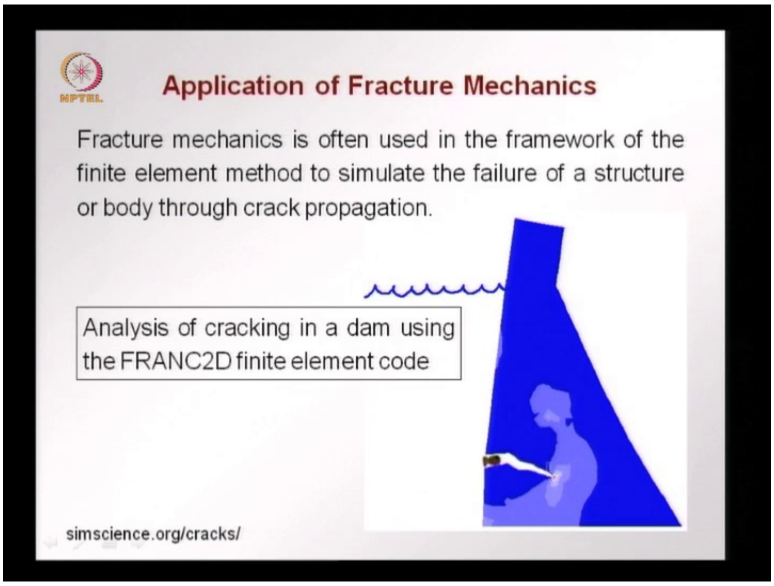

So, let us see some simulations this is from a sim science, where we see the simulation made by FRANC2D, which is a finite element code coming from the group of ((Refer Time: 30:19)) Toni Graphia in Cornell university. Where they have used fracture mechanics to simulate what happens in a dam that is subjected to a excessive height of water. The pressure of the water creates a crack in the dam and you see a crack; propagating the red zone is, where we have a very high stress concentration and we see how the model is able to show, how the crack will propagate and ultimately how the dam will collapse.

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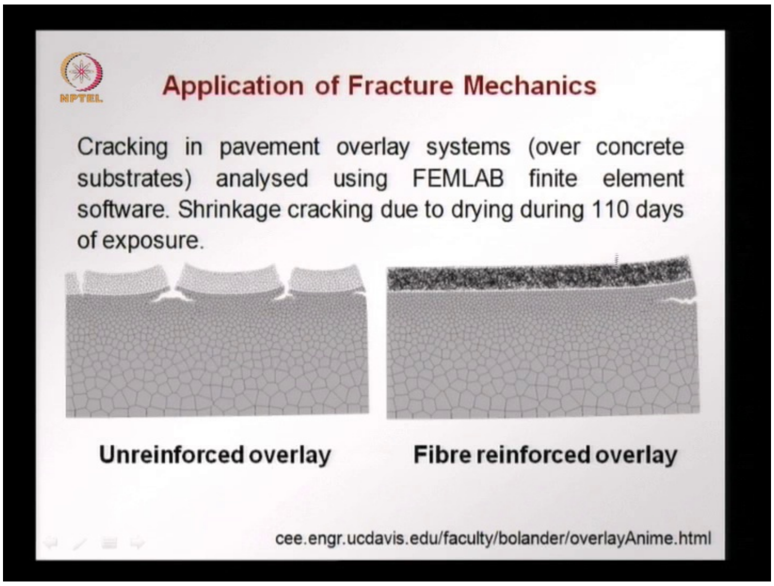

Another simulation is from the group of professor bollander from the university of California Davis, where we see a simulation of concrete overlays subjected to shrinkage; on the left we have an overly that is plane concrete; on the right we have an overly that is fiber reinforce concrete. And this is what is expect to happen due to the shrinkage over a period of hundred plus days and this was simulated using the software called FEMLAB; you see on the left that the shrinkage causes stresses in the overlay, which cannot be resisted by the plane concrete the fracture toughness is not that high.

So, we have a crack through the overlay at several places. In fiber reinforced concrete now the fibers cause toughening there is a higher fracture toughness of the overlay. So, even though there is shrinkage the overlay does not crack, but the deformation causes a delimitation running through the sub base instead of creating cracks in the concrete.

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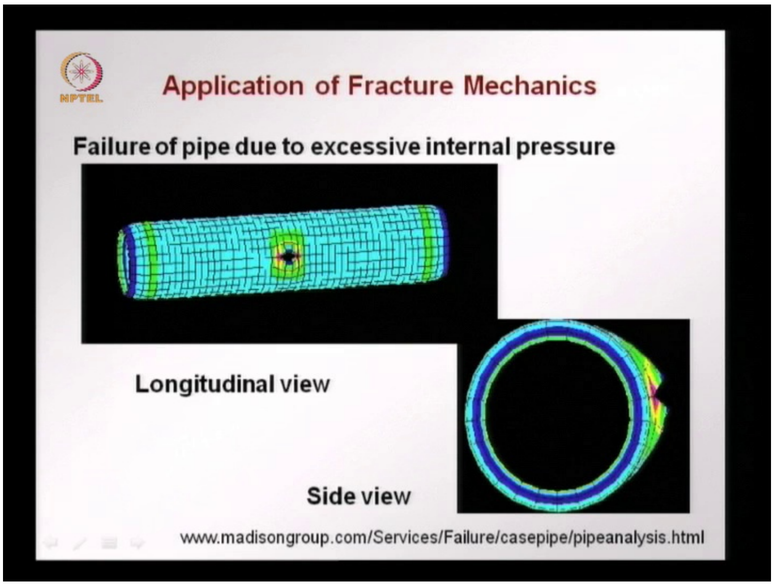

A third simulation that I am going to show you is from the medicine group, where we have the case of a PVC pipe and this is been simulated to show how the file the pipe will crack and fail when there is excessive internal pressure. So, we see here the crack opening this is the section, and here you see the longitudinal view how the crack propagates. So, such type of fracture analysis can be used to determine how materials will fail, how structural elements will fail and this we can use for design to prevent such failure from occurring, and also to have better materials developed that can resist such type of failures.

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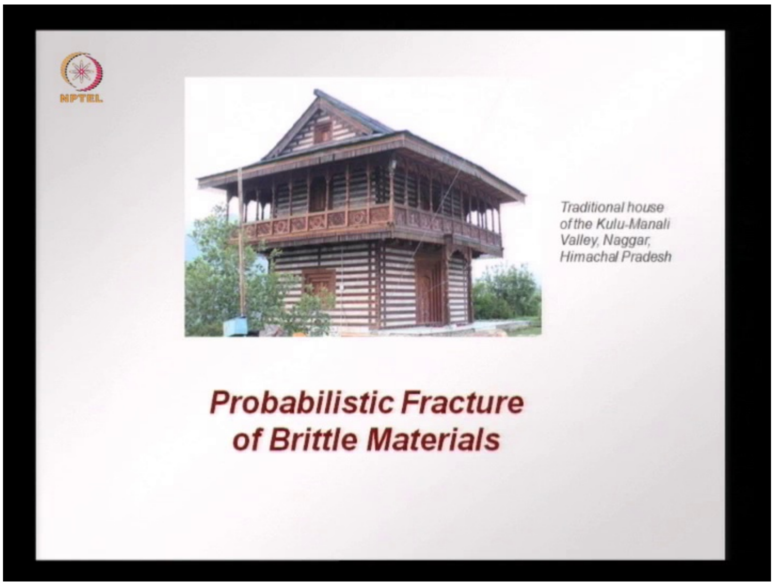

Now, we will go onto Probabilistic Fracture of Brittle Materials and for the transition I have included this photograph of a traditional house in the Kulu-Manali Valley, which has a lot of repeated elements look like a simple house, and it is made out of wood and stone. And we have a lot of repeated elements, similar looking elements like these pillars on the balcony and this walls are all made of blocks of stone. Though all these materials look similar each of them will have a different set of defects.

So, when we look at the failure of a brittle material these defects and the variation of this defects from one element to the other; even though the material is all the same can affect how fracture occurs.

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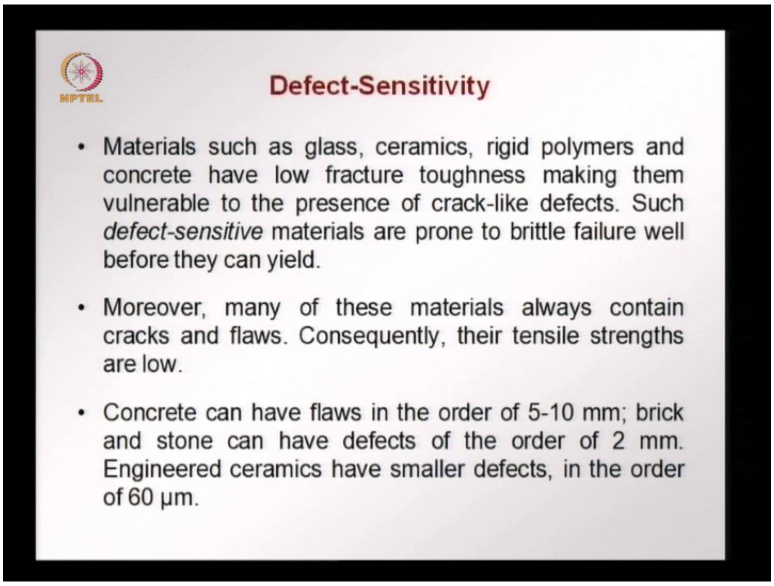

So, materials like glass, ceramics, rigid polymers, concrete to some extent would have low fracture toughness this we saw in the previous lecture and this makes them vulnerable to the presence of defects that can cause cracks or crack like defects. These materials are therefore, called defects sensitive materials and they are prone to brittle failure before they can have yielding type of failure. So, they crack and fail rather than yield and fail also what we see is most of these materials also have cracks and flaws in them they are not without defects.

This combined with the defect sensitivity decreases their tensile strength significantly concrete can have inherently flaws in the order of 5 to 10 millimeters normally you would expect flaws to be at up to the size of the aggregate. Bricks and stones have smaller defects in the order of few millimeters then we have engineered ceramics; ceramics that are made in the factory under very well defined conditions control conditions we can have smaller defects say in the order of a few microns.

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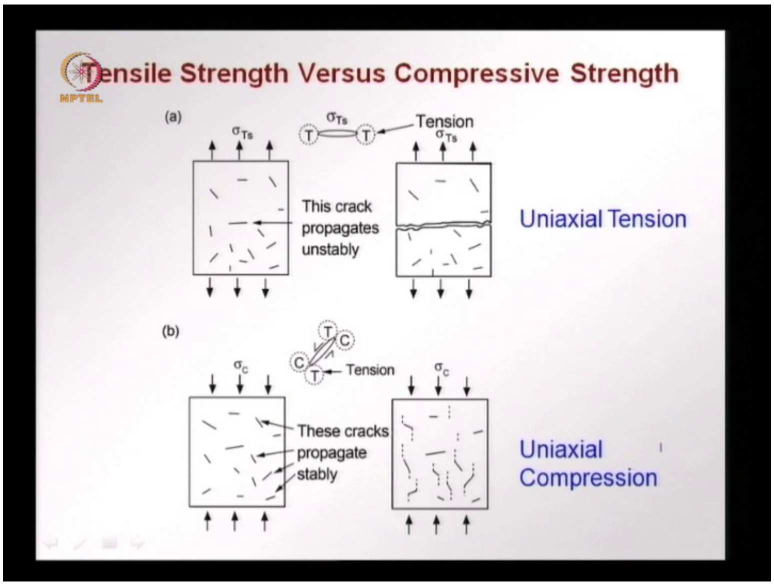

So, let us look at how a piece of material subjected to tension and compression differ in their behavior; on the top we have a panel subjected uni-axial tension and this panel now we will have a lot of defects this the inherent defects occurring in the material. And now, when tension is applied we will have the most significant crack that is the longest most favorably oriented crack; we know that under tension cracking will occur perpendicular to the direction of the applied stress.

So, this being a largest most favorably oriented crack will start to propagate tension occurs at the ends of the crack and you will have propagation, and ultimately failure. So, what we see is under tension a single crack dominates failure it is the largest crack or largest flaw that will become a crack and fail. Now, let us see what happens if the same specimen instead of subjecting it to tension had been subjected to compression uni-axial compression; what we find is this crack, which dominated the failure and tension does not do anything.

It is just going to close, where as the more inclined cracks will have tension and compression at the ends if the inclination is such and you have a vertical loading; you will have tension and compression occurring, which will tend to cross some short of splitting stresses at the ends; you see here that at the end of the crack we have crack extension occurring almost vertically due to splitting.

And just one crack does not cause completely failure now many of these cracks have to open up some of these crack have to join before finally, you have failure under uni-axial compression. So, we see a big difference in how failure occurs and how the flaws will lead to failure in tension and compression.

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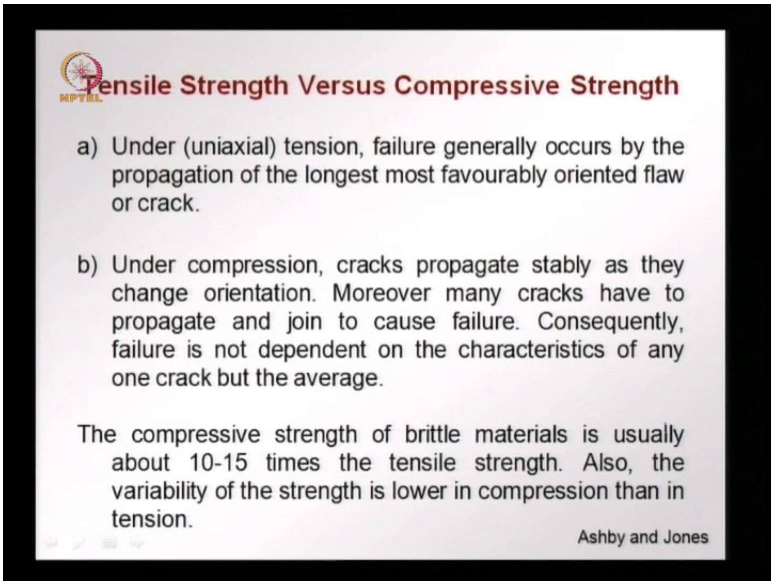

Under tension, failure occurs by the propagation of the longest most favorably oriented flaw that is one flaw can control the failure; and it is obvious to understand that in different specimens you will not have the same flaw size and same orientation of the flaw. And therefore, there will be a large variation when you test many specimens that is why we find that the variability of the tensile strength is much higher than in compression.

Also now, when we look at compression we noted that cracks will propagate in a stable manner change orientation you saw the inclined crack becoming vertical at the ends. Many cracks have to join propagate and join to cause failure; and consequently the failure is not depended on the characteristics of any single crack, but an average many cracks contribute; many cracks have to propagate stably joined together and crack.

So, this has two important consequences when we compare compression and tensile behavior the compressive strength of brittle materials like concrete, rocks, ceramics and. So, on is about 10 to 15 times that of the tensile strength, because you have many cracks propagating the cracks have to change orientation and join together and cause failure. The other thing is the variability of the strength is much lower in compression than in tension, because here you are averaging the behavior; even though the crack distribution between specimen and specimen change a lot since many cracks have to contribute we get an averaging of the behavior and therefore, the variability decreases.

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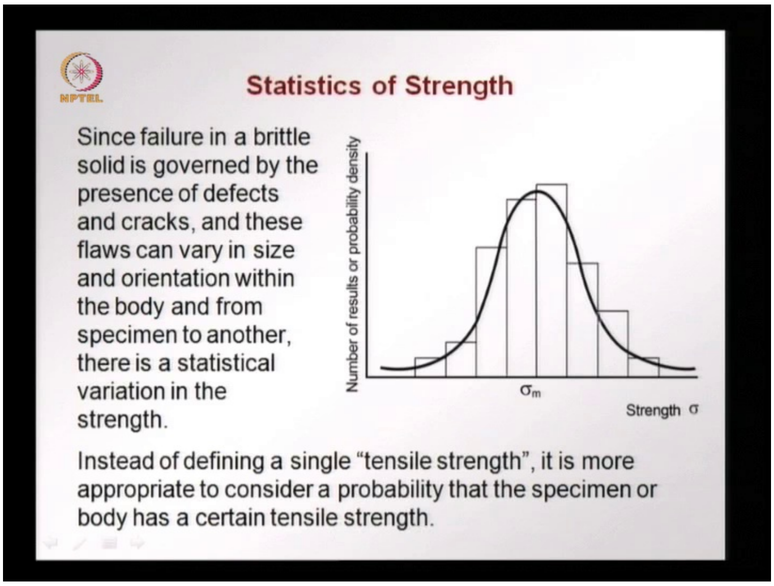

So, we to do it correctly we have to look at the Statistics of Strength. These flaws can vary in size and orientation within the specimen itself; and from one specimen to the other there is a statistical variation in the strength given like this. This could be test conducted over several identical specimens and you will see that there is a variation of the strength not all specimens will fail at a certainly value depending on the flaws. If there is a large flaw by chance; you will have a less strength if you have a specimen with very few flaws then the strength will be higher.

So, instead of defining at single tensile strength it is probably more appropriate to look at a probability that the specimen or body has a certain tensile strength; that is we say that there is a probability that the strength is at least this much or there is a probability that the strength is at least this much.

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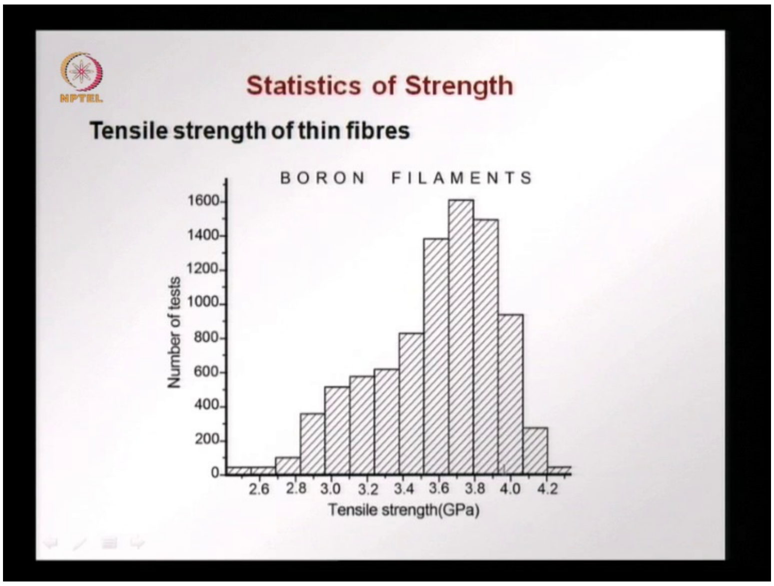

And this can be looked at when we see the experiments on different materials; this is how we have test data, this is the test data from B O R O N F I L A M E N T S under tension, and we find that there is a variation of the strength that we get; this is the number of test and we find that lot of test fail in this region, but they are lot of specimen, which fail with lower tensile strengths some fail at higher tensile strength.

So, there is a certain probability that the strength is at least 3 Giga Pascal and this probability is quite high. If we take 4 Giga Pascal the probability is quite low that the strength will be at least 4 Giga Pascal or more, because we see that lot of specimens fail at lesser values.

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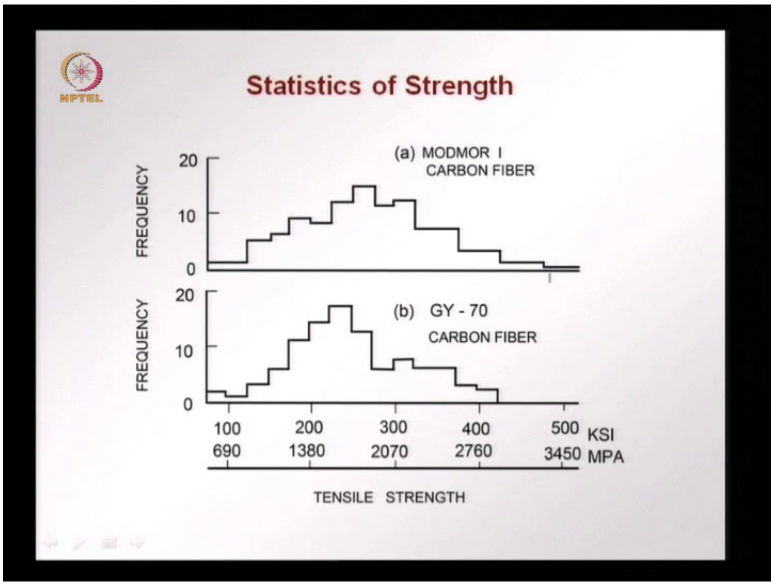

Similarly by the other materials like carbon fibers we have a similar distribution and we see a variation of the strength from specimen to specimen.

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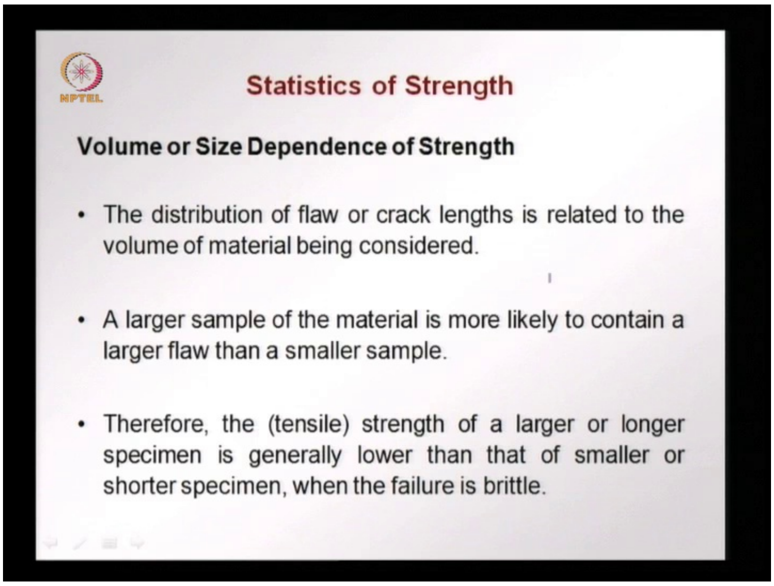

We also see that the volume of the specimen will dictate how failure occurs, because the distribution of flows the occurrence of large flaws will varying depending on the size of the specimen. So, the distribution of flaws and crack length are related to the volume of the material being considered. A large specimen or large sample of the material is more likely to have a larger flaw than a smaller one this is very important.

A larger specimen has a higher probability of having a large flaw preferably oriented than a small specimen. Therefore, generally we find that the average tensile strength of a larger or longer specimen is lower than that of a smaller or shorter specimen when we have brittle failure. These two are related we said that a larger specimen or a longer specimen can have a higher probability of a large flaw.

So, on the average what happens is when we test a lot of these larger or long specimens we get an average tensile strength that is lower than if we test lot of small or short specimens.

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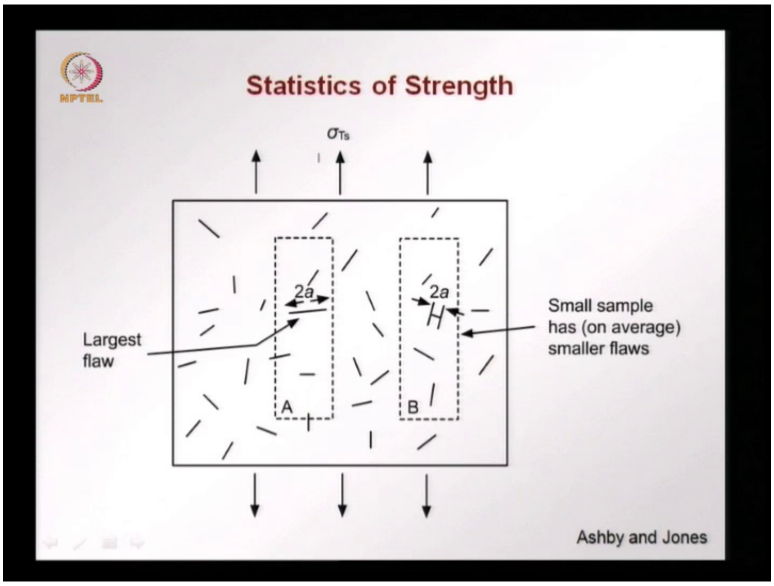

And this is further illustrated by this diagram from Ashby and Jones, where you see a panel with many flaws and in this case the most critical flaw is this; when this flaw with is the largest flaw propagates you will have failure. Now, instead of this large specimen suppose we had made small specimens from the same piece of material. The probability that we got exactly this piece the small piece is very low; we could have made specimens from here and so on.

Suppose we had made a small specimen such has this denoted as B, instead of this specimen denoted as a we would have a small flaw instead of large flaw this will give a higher tensile strength than this one. So, what we see is in a large specimen there is always a higher probability of a larger flaw than in a small specimen.

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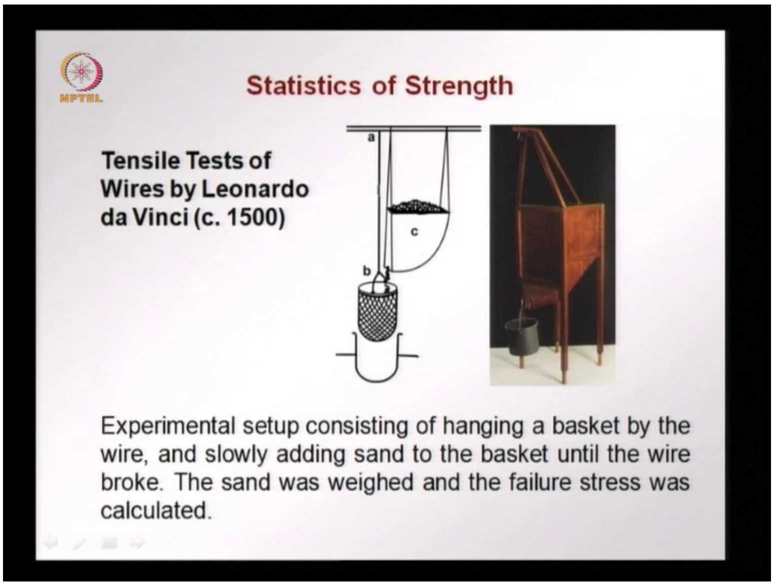

And this is something that goes back to the test of Leonardo da Vinci on wires he did tensile test of wires with this experimental set up that is shown here, where the wire was tied to a bar and on the other end to a wire basket and sand was added to this basket until the wire broke. Then the quantity of material in the wire would be measured or weighed and the failure stress was calculated.

So, what he found was when we had different lengths of wires all having the same cross section; the crass section is the same. So, theoretically the stress is the same if the load was the same, because load by area is the stress.

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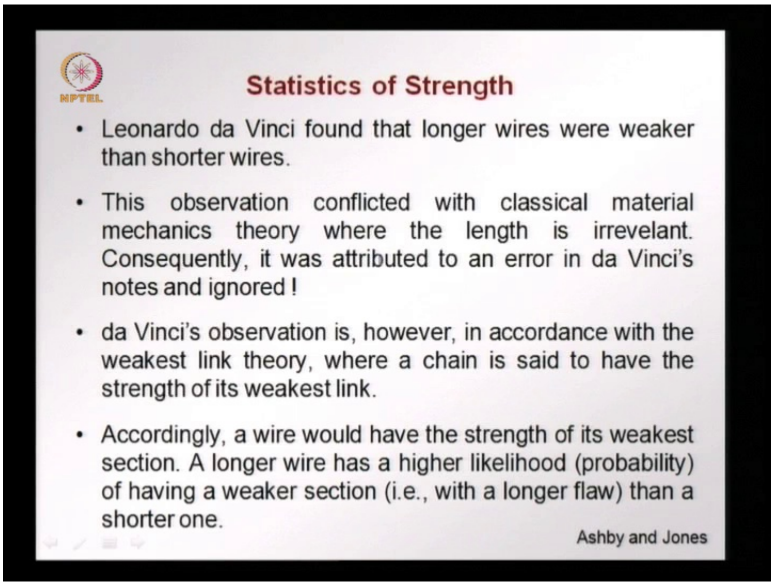

What he found was different lengths of wires failed in different ways what Leonardo da Vinci found was that longer wires were weaker than the shorter wires remember all are of the same cross-section. So, this conflict with classical materials mechanics theories, where strength does not come into play remember in the previous two lectures when we looked at the material response we always talked about a strength, we talked about a stress. But we never brought in the size or the shape of the element to determine what the strength would be.

So, this was considered to be a mistake in da Vinci's notes ignored for a very long time, but now we know that da Vinci's observation is in agreement with what is called the weakest link theory. The weakest link theory says a chain is a only a strong as its weakest link and the longer the chain there is a higher probability of a very weak link. So, the same argument for a wire would say that a wire would have the strength of its weakest section. A longer wire has a higher probability or likelihood of having a weaker section than a stronger one; a weaker section means that with a longer flaw.

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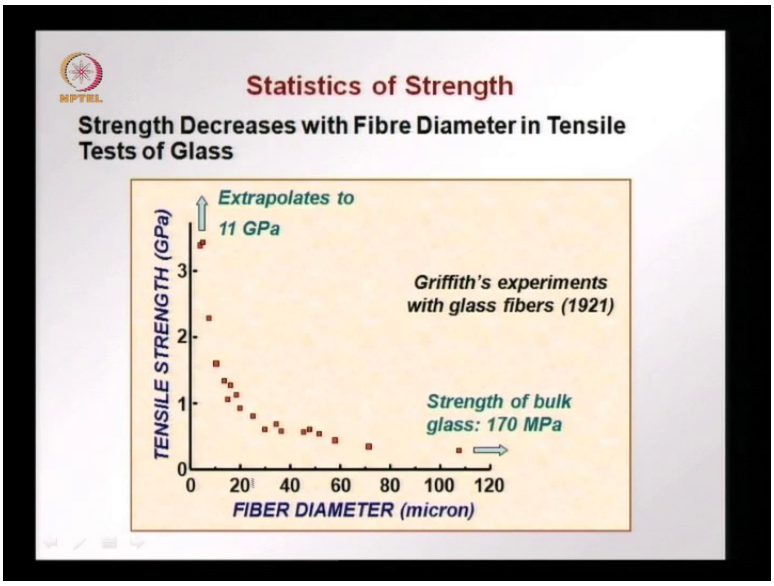

This was shown in extensive test by Griffith in the 1920 and these results are from tests of glass fibers instead of different lengths what he did is different diameters; and what he found was as the glass diameter decreased then we went to very thin glass fibers. The flaws were eliminated and therefore, the strength went up strength even went up 11 Giga Pascal in a glass with a bulk strength of 170 mega Pascal; a large specimen would fail at a strength of 170 mega Pascal, but very thin fibers could take even 11 Giga Pascal of stress. And this strength decreases as the diameter increased as we diameter; there is a higher probability of defects and flaws.

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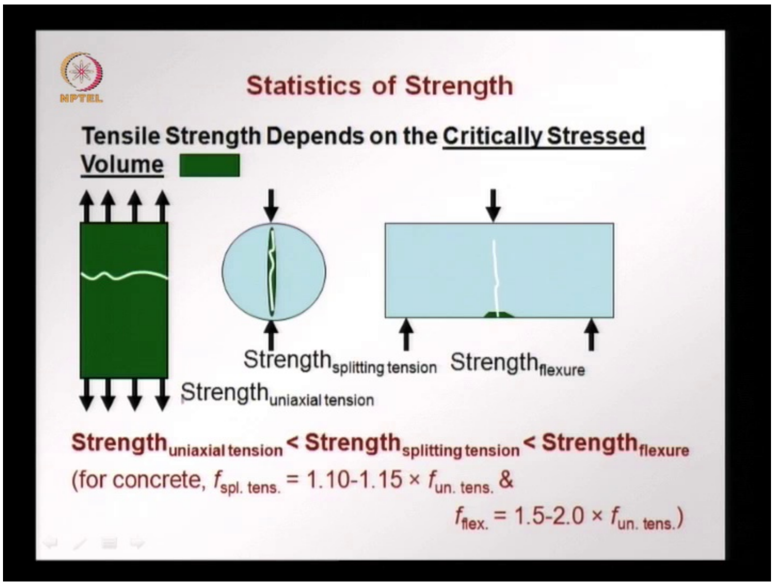

This also explains to us the effect of the Critically Stressed Volume; when we have a specimen failing in a certain way and many times we have tension occurring not in this manner, where we have that panel that we have discussed until.

Now, where we have a specimen subjected to uni-axial tension and here any flaw anywhere can control failure; however, if we have a splitting tension test only the flaws in this region, because this is where the high stresses will control failure. Further, if we have a beam only the flaws here will control failure, because this is where the high stress is; here we have a uniform stress state.

So, any flaw anywhere here can lead to failure; here a flaw here will not control the failure only those flaws, which are in this region again a flaw here in a beam will not cause failure in this configuration only here. And here this critically stressed volume is very small compared to this and this is an intermediate case. So, what we find is this case will give the highest strength, because the critically stress volume is the least followed by this. And in the case of concrete this is evident from test that the splitting tensile stress is about 10 to 15 percent higher than the uni-axial strength; and the strength from a beam can be 50 to 100 percent higher than what we get in uni-axial tension. And this is all duet to the effect of the size of the critically stressed volume as a function of the loading configuration on the strength.

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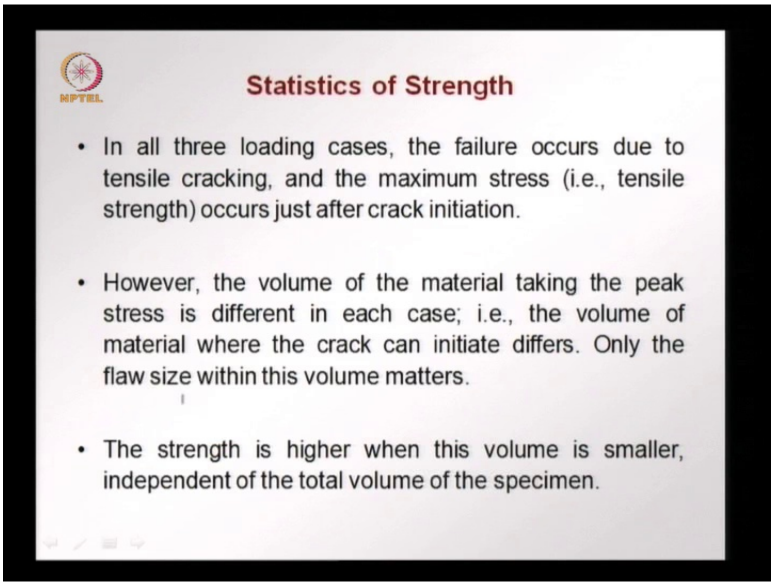

So, how do we model this we saw that in all the three cases failure occurs due to tensile cracking and the maximum stress occurs just after cracking initiation; the crack does not propagate lot, but it fail suddenly. The volume of the material taking the peak stress is different in each case that is the volume of the material, where the crack can initiate differs; only the flaw size within this material matters. So, we have to find out what is this volume and use this volume as a measure of the variability of the flaws. Strength is higher when this volume is smaller and it is independent of the total volume of the material; especially when the stresses are non uniform.

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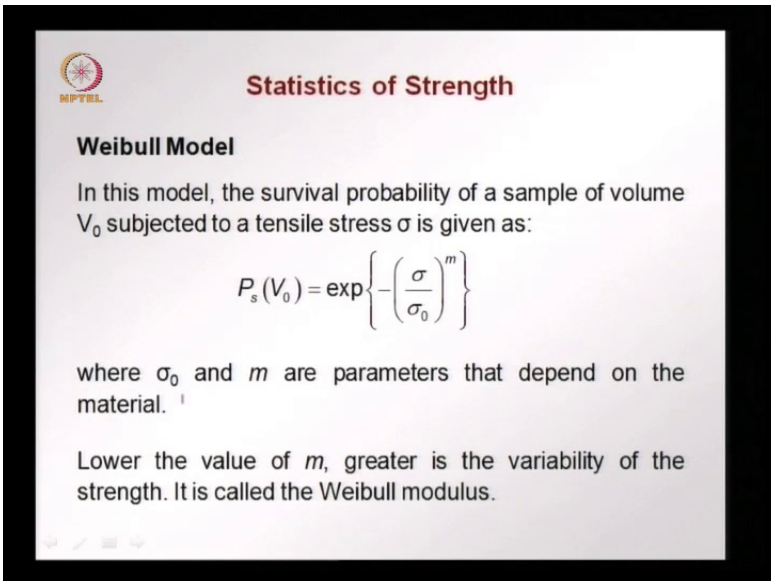

One of the ways of tackling this problem is through what is called the Weibull Model; here we look at survival probability of a specimen of a certain volume V 0 subjected to a tensile stress sigma, and this is the survival probability. The probability that a sample subjected to sigma, stress of size V 0 that it will not fail is given by this. Sigma 0 is the mean strength of the material m is what is called the Weibull modulus lower. The Weibull modulus higher is the variability and lower is the probability of survival at that particular stress.

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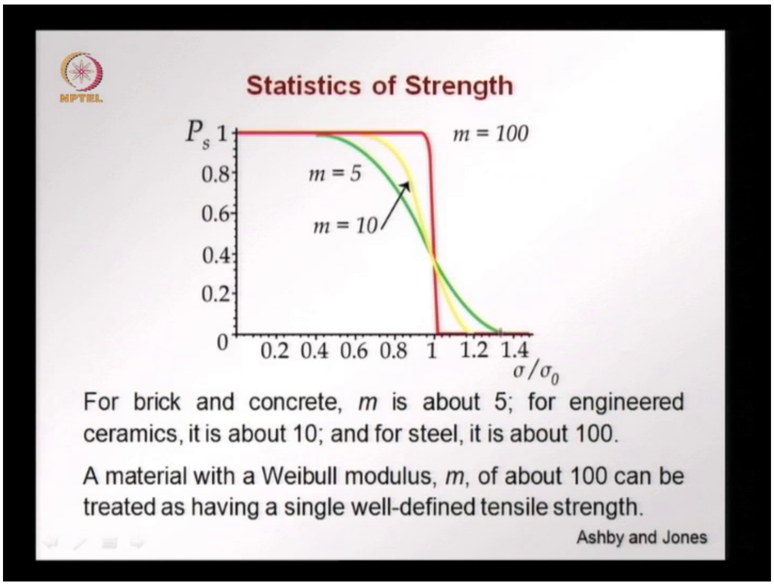

This distribution is shown here, P s is the survival probability on the x axis we have a ratio between the applied stress and the mean strength of the material. And what we find is that when m is very large there is a very definite drop; there is a very good probability that the failure strength is going to be sigma 0. However, when m is smaller there is also a probability that the strength is smaller or larger the variability is there; m for brick and concrete is about 5 for ceramics it is about ten and for steel it is about hundred; and when we have a material with a Weibull modulus of 100 we can say that it has a well defined tensile strength.

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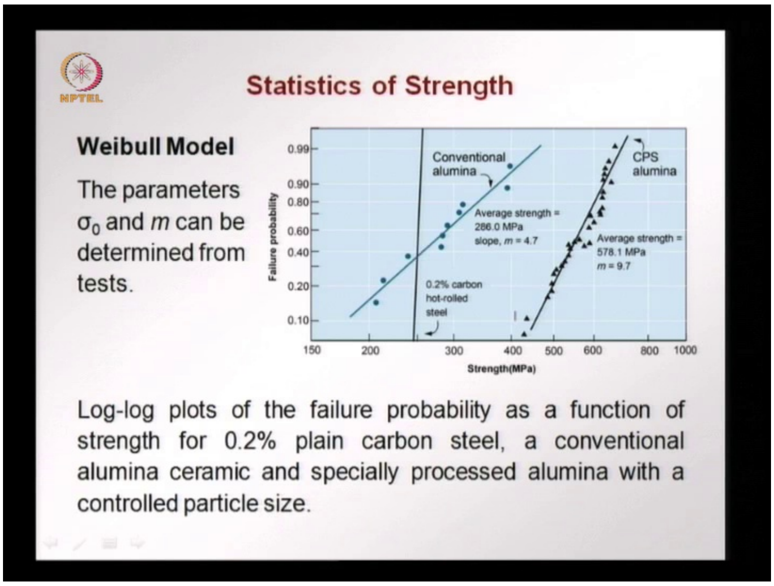

Like we see here these are test showing data from steel alumina and a processed alumina. We find that the slope is higher for a conventional alumina ceramic for a steel it is almost vertical; that means, m is very high there is a very high probability that failure will occur only here. For a conventional alumina there is a probability that they could be failure anywhere here; alumina the modulus is higher for C P S alumina, a processed alumina the modulus higher for conventional alumina the modulus is around 5. So, from test like this we can calibrate the Weibull Model to get the value of m.

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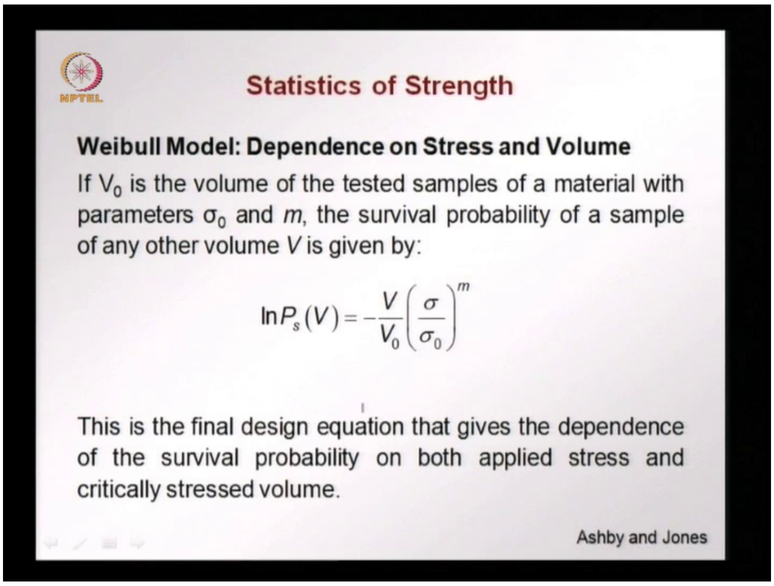

Further we have to bring in the effect of the critically stressed volume and we modify now this equation to say that if V 0 is the volume of the tested sample that we tested say in uni-axial tension. And the parameters of the Weibull Model for this material are sigma 0 being the mean strength and m being the Weibull Model. The survival probability of a sample of any other volume other than V 0 or a volume with a critically stressed volume other than V 0 say V is given by this. Log natural of the survival probability of this volume V is minus V by V 0 times sigma by sigma 0 elevated to the power m.

So, this is now the final design equation that gives both the effects of applied stress and critically stress volume on the survival probability; and this can now be used to compare different materials and find out what would be an acceptable stress value for a specimen with having a certain volume or a critically stress volume, and knowing the stress that is applied on it.

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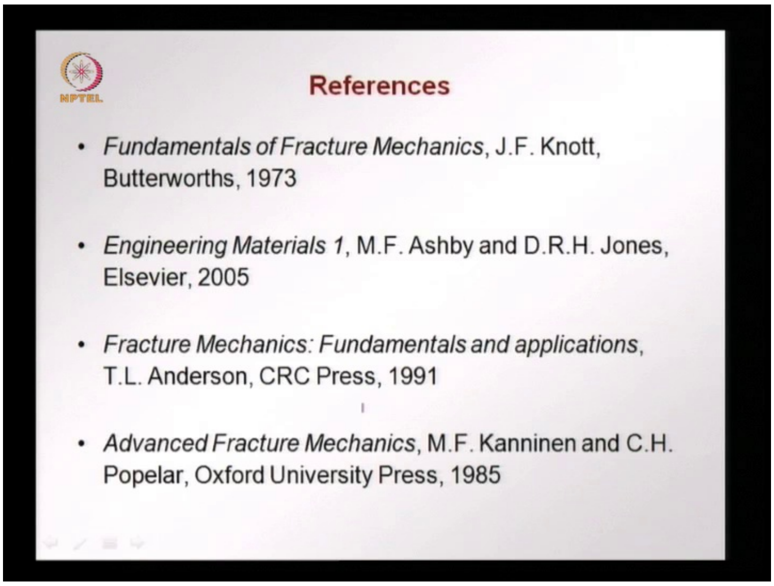

So, to conclude we have looked in this lecture at different fracture mechanisms how we deal with fracture, how we model fracture and at the end we also looked at probabilistic fracture, where we saw how the defects and flaws can come into play. There are lot of references that you will see on the screen that you can look at there are many good books on fracture mechanics and lot of information on the web also.

Thank you.