PRESTRESSED CONCRETE STRUCTURES

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Module – 5: Analysis and Design for Shear and Torsion

Lecture-27: Design for Torsion (Part - 1)

Welcome back to prestressed concrete structures. This is the fifth lecture of Module 5 on analysis and design for shear and torsion. In this lecture, we shall study the design for torsion. First we shall study about the limit state of collapse for torsion. Next, we shall learn about the design of longitudinal reinforcement and transverse reinforcement.

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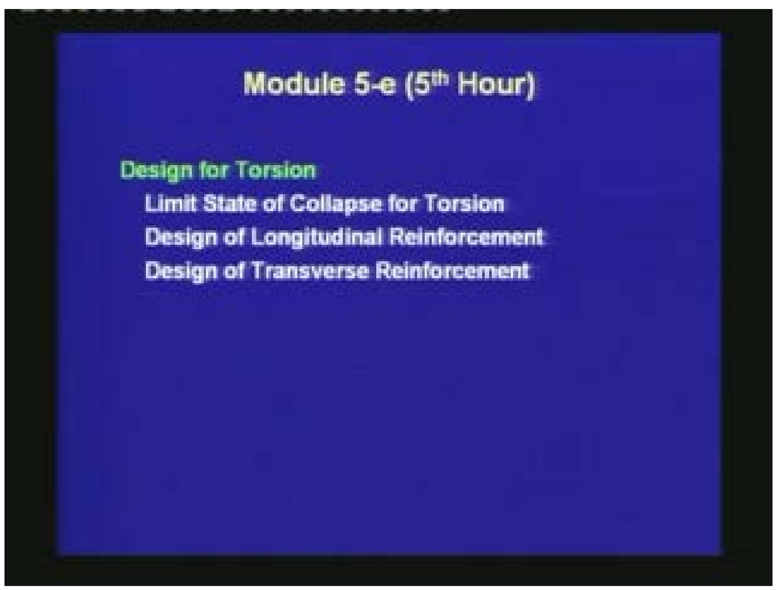

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The design for the limit state of collapse for torsion is based on the skew bending theory. For a beam subjected to simultaneous flexure and torsion, an equivalent ultimate bending moment at a section is calculated.

Last time, when we discussed about the analysis for torsion, we learnt that for the skew bending theory three modes of failure are defined at ultimate. These modes of failure are based on a concept of a resultant moment which comes from the flexural moment and the torsional moment. Based on this concept, an expression of equivalent ultimate bending moment is developed for the design for torsion.

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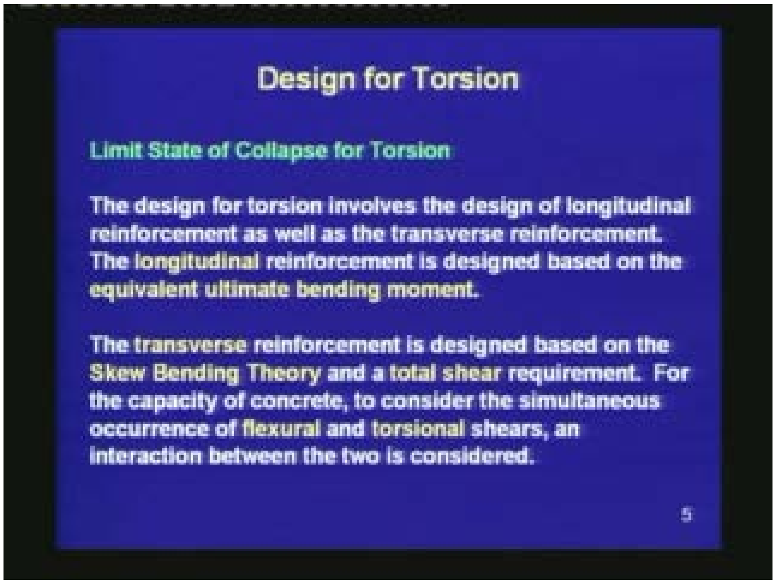

The design for torsion involves the design of longitudinal reinforcement as well as the transverse reinforcement. The resistance to torsion comes from a space truss action, where the concrete struts, the longitudinal steel and transverse reinforcement all take part in resisting the torsion. Hence, the design targets both the steel: longitudinal and transverse. For concrete, the design considers its capacity.

The longitudinal reinforcement is designed based on the equivalent ultimate bending moment that is defined based on the skew bending theory. The transverse reinforcement is designed based on the skew bending theory and a total shear requirement. For the capacity of concrete, to consider the simultaneous occurrence of flexural and torsional shears an interaction between the two is considered.

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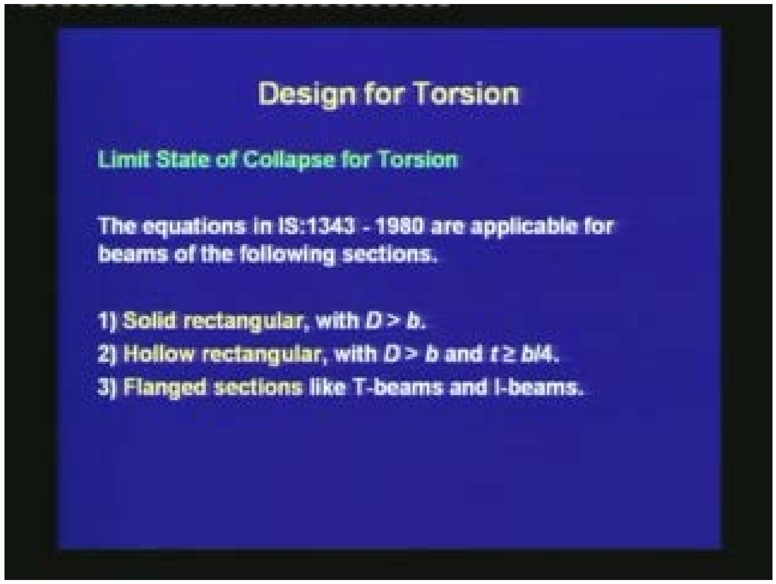

The equations in IS: 1343-1980 are applicable for beams of the following sections.

1) Solid rectangular, with D, which represents the depth, is greater than b, which represents the breadth. Incase if a member has a larger breath than the depth, then D will be equated to the larger breath and b will be equated to the smaller depth. Hence, D is always larger than b in the expressions that we shall see.

2) The expressions are also applicable to hollow rectangular sections, where the thickness is greater than one quarter of the breath; that means the hollow rectangular section should have adequate thickness for the resistance to the torsion.

3) Finally, the equations are also applicable to flanged sections like T-beams and I-beams.

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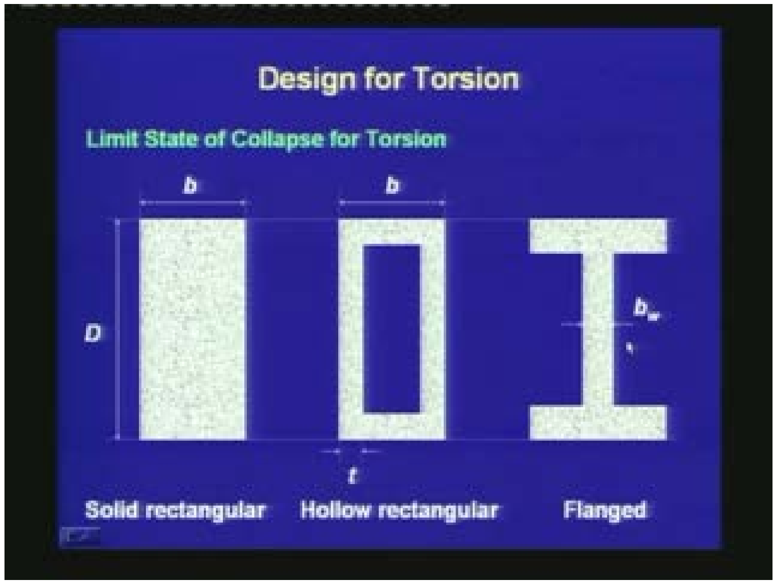

These are the three types of sections for which the equations are applicable. For a solid rectangular section D is larger than b. Similarly, for a hollow rectangular section, D is larger than b, and the thickness is greater than or equal to one quarter of b. For a flanged section, b has to be substituted by the breath of the web for most of the expressions.

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The variables are thus, b is the breath of the section which is equal to b_w for flanged section; D is the total depth of the section; t is the thickness of the section for a hollow section. The average prestress in a section at the level of CGC is limited to 0.3 $f_{ck.}$

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First, we are moving on to the design of longitudinal reinforcement. There are three expressions of the equivalent ultimate bending moment for the three modes of failure. We had seen these three modes of failure in our last lecture, and these are modeled by three expressions of the equivalent ultimate bending moment. The background of the codal provisions is given in a paper written by Professors Rangan and Hall. The title of the paper is “Design of Prestressed Concrete Beams subjected to Combined Bending, Shear and Torsion”. It was published in the ACI Journal, American Concrete Institute, in March 1975. The volume number is 72, issue number is 3 and the pages are from 89 to 93.

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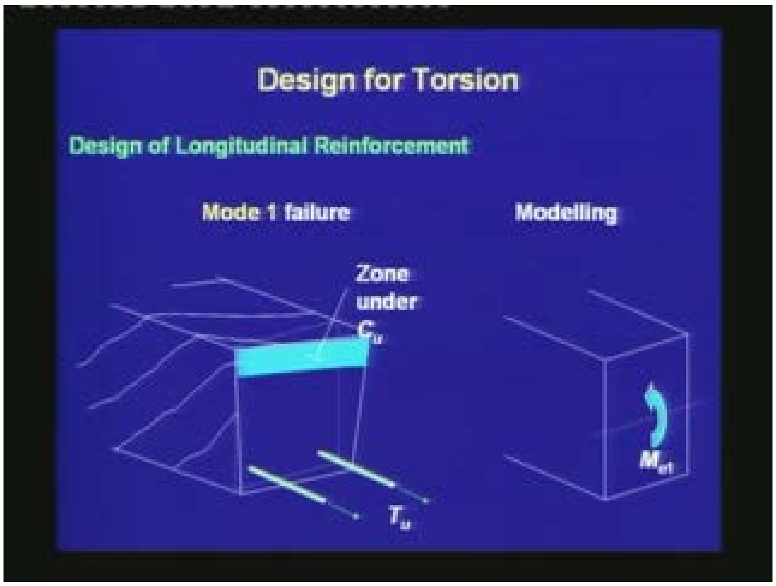

In the first mode of failure, there is an influence of the torsion on the bending. The plane of failure is skewed with the axis of the beam, but the zone of compression and the tension remain similar to conventional flexure. This is modeled by an equivalent moment which creates compression in the same face as that of the flexure, and it creates tension in the same phase as that of the flexure.

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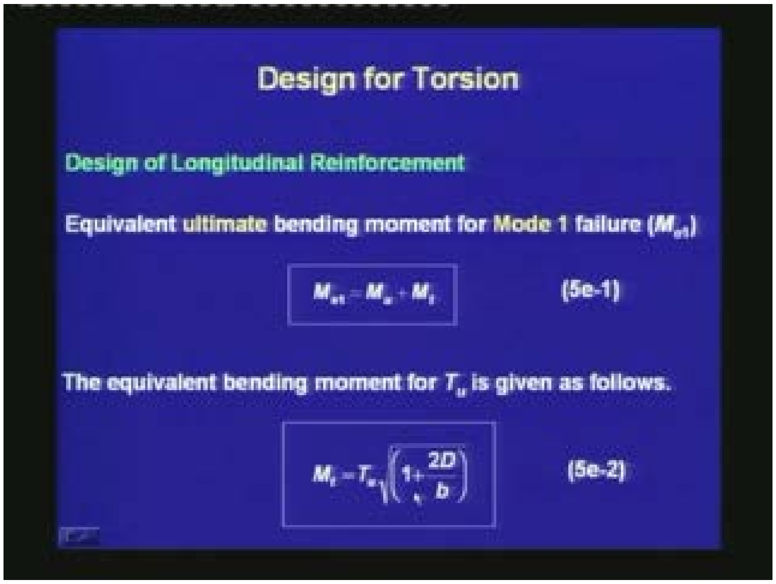

The equivalent ultimate bending moment for Mode 1 failure is denoted as M_{e1}. $M_{e1} = M_u + M_t$, where M_u is the ultimate flexural moment and M_t is the equivalent bending moment for the ultimate torsion. $M_t = T_u \sqrt{(1 + (2D/b))}$.

First, we calculate the ultimate torsional demand; from that we are calculating an equivalent bending moment. Then we are adding the equivalent bending moment to the flexural moment to get the total equivalent bending moment for Mode 1 failure.

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In this expression, M_u is the applied bending moment at ultimate; M_t is the additional equivalent bending moment for torsion; T_u is the applied torsion at ultimate. Since, the torsion generates tension in the reinforcement irrespective of the sign; the sign of M_t is same as that of M_u.

Remember that, whether the torsion acts clockwise about the beam axis or anti-clockwise about the axis, it always generates tension in the longitudinal steel. To account for this addition of stress due to torsion in the longitudinal steel, the sign of M_t is taken as the same as that of M_u. Thus, we have an increased demand in the longitudinal steel, due to flexure and the torsion.

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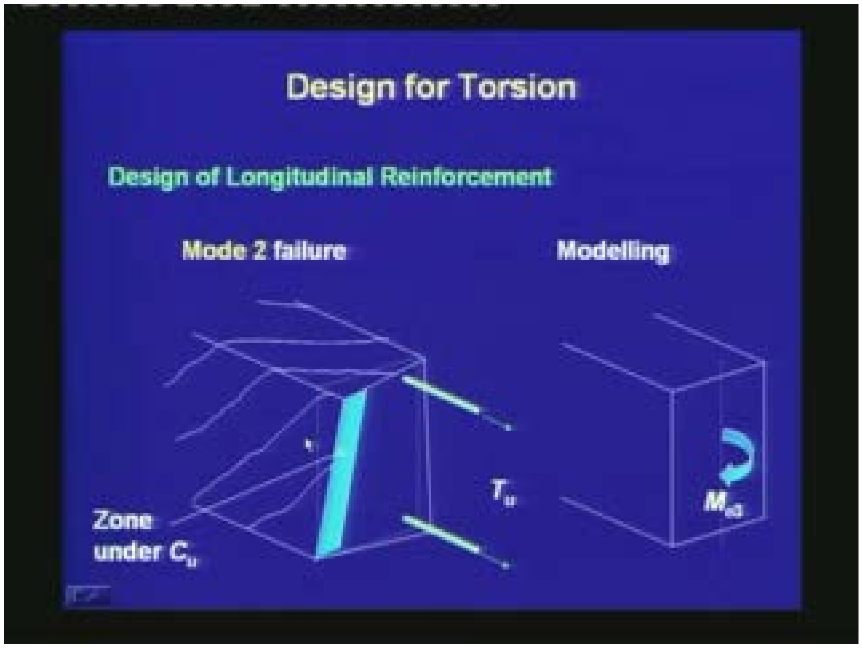

For the Mode 2 failure, the amount of torsion is significantly high and this type of failure is for beams with thin webs. Here, the compression of concrete is at one side face and the tension is in the other opposite side face. To model Mode 2 failure, the equivalent bending moment is considered to act about the vertical axis, which creates compression in one side face and tension in the other side face. For Mode 2 failure, the notation used in the codal expression is M_{e3}.

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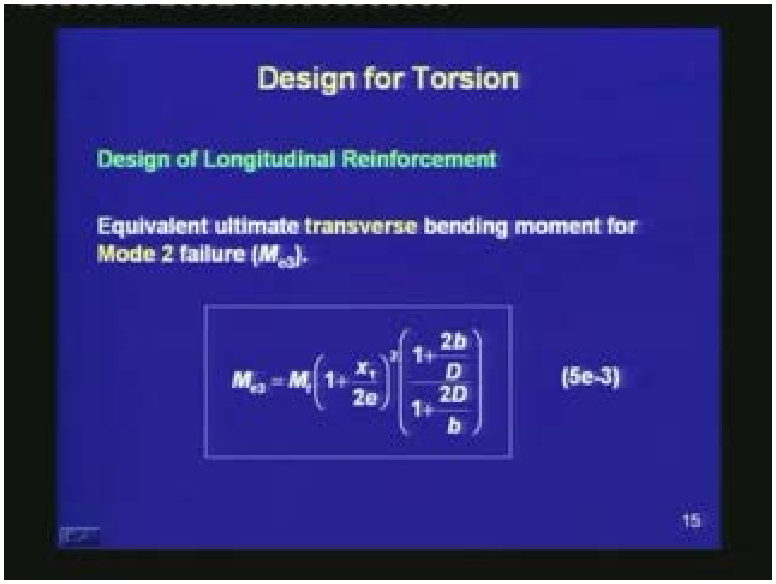

The equivalent ultimate transverse bending moment for Mode 2 failure is:

$M_{e3} = M_t\,(1 + (x_1/2e))^2\,((1 + 2b/D)/(1 + 2D/b))$.

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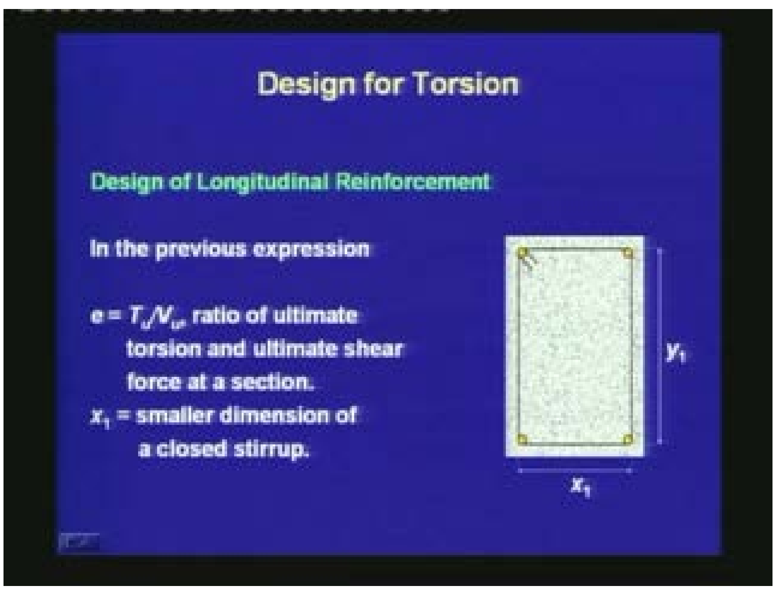

In this expression, $e = T_u/V_u$, ratio of ultimate torsion and ultimate shear force at a section, x_1 is the smaller dimension of a closed stirrup. For a rectangular section, let x_1 and y_1 be the smaller and larger dimensions of the closed stirrup, respectively.

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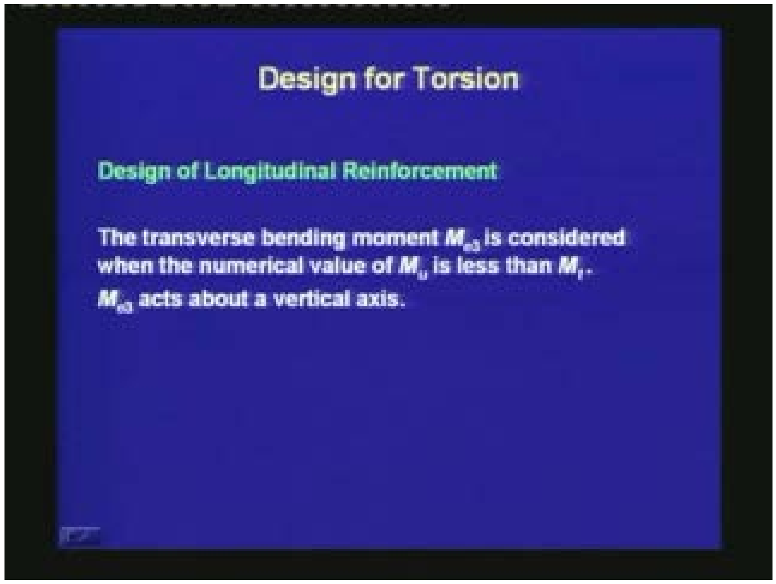

The transverse bending moment M_{e3} is considered when the numerical value of M_u is less than M_t. M_{e3} acts about a vertical axis.

The third mode of failure is generated when the torsion is substantial and the top steel is small. In that case, we may observe crushing in the bottom and substantial yielding in the top steel.

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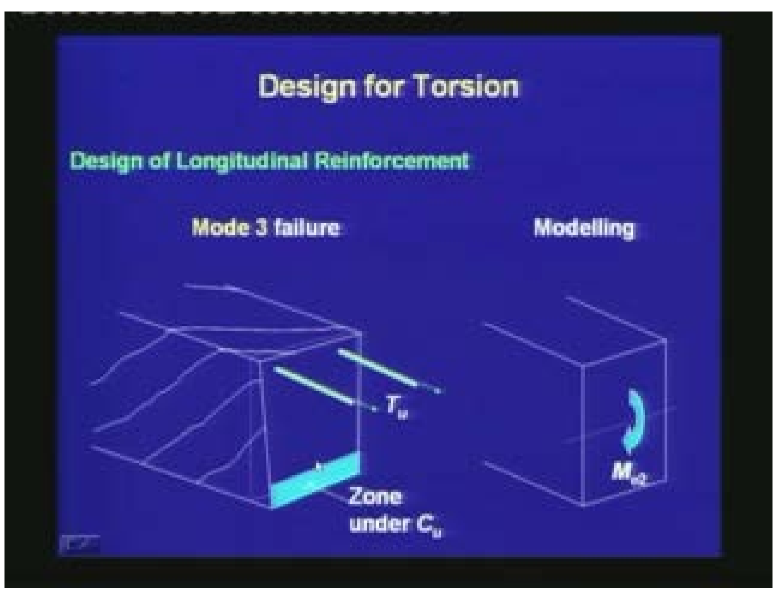

In this sketch, we can observe that the compression is occurring at the bottom and tension is at the top, which is in an opposite sense as that created by flexure. This is bending in an opposite sense to that of the flexural moment. This way, we check the requirement of the steel at the top.

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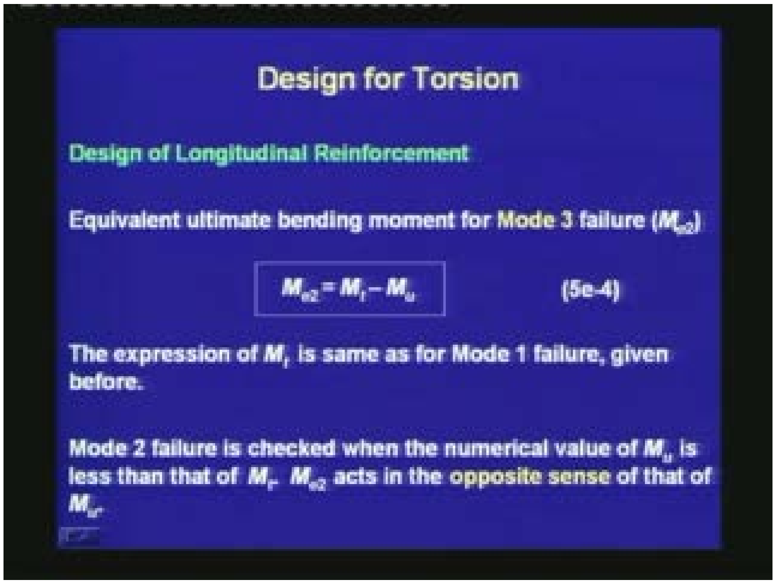

The equivalent ultimate bending moment for Mode 3 failure is denoted as M_{e2} in the code. $M_{e2} = M_t - M_u$. The expression of M_t is same as that for Mode 1 failure, given before. Mode 2 failure is checked when the numerical value of M_u is less than that of M_t.

Thus, when M_u generates compression at the top, M_{e2} will generate compression at the bottom. The top steel which is under compression due to flexure, may come under tension, when both flexure and torsion are occurring together.

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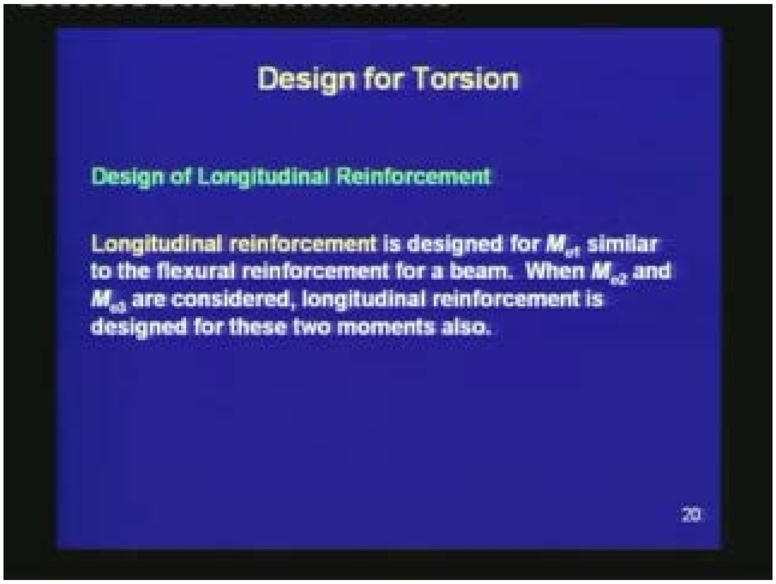

The longitudinal reinforcement is designed for M_{e1} similar to the flexural reinforcement for a beam. When M_{e2} and M_{e3} are considered, longitudinal reinforcement is designed for these two moments also.

Thus, the essence of design of longitudinal reinforcement for a beam under combined flexure and torsion is that we have defined an equivalent moment (M_{e1}) which is larger than the flexural moment (M_u) and acts in the same sense as that of M_u. We may also define two other equivalent moments (M_{e2} and M_{e3}) if the torsional moment is substantially high. M_{e3} is for the lateral bending. M_{e2} is for negative bending, which is in the opposite sense of the bending due to the flexural moment.

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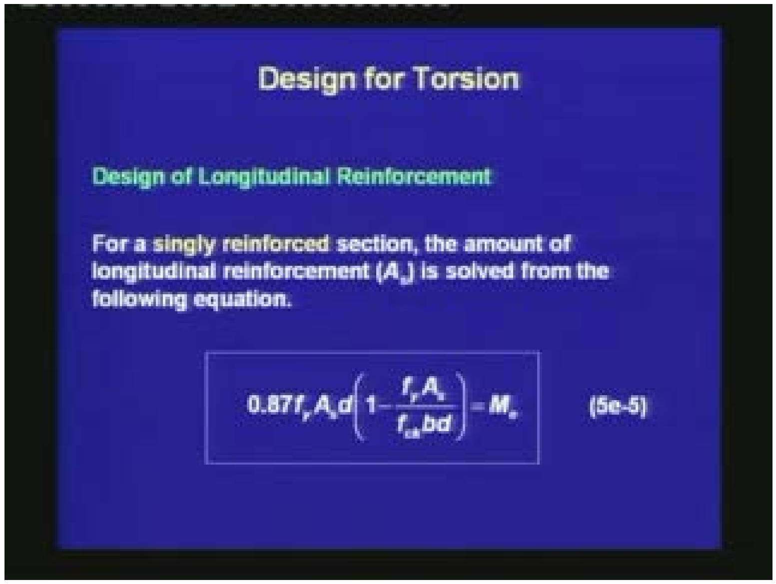

For a singly reinforced section, the amount of longitudinal reinforcement A_s is solved from the following equation.

$0.87\ f_y\ A_s\ d\ (1 – (f_y\ A_s / f_{ck}\ bd) = M_e$

This expression is available from reinforced concrete design, for an under-reinforced section. The moment is the force in the steel times the lever arm.

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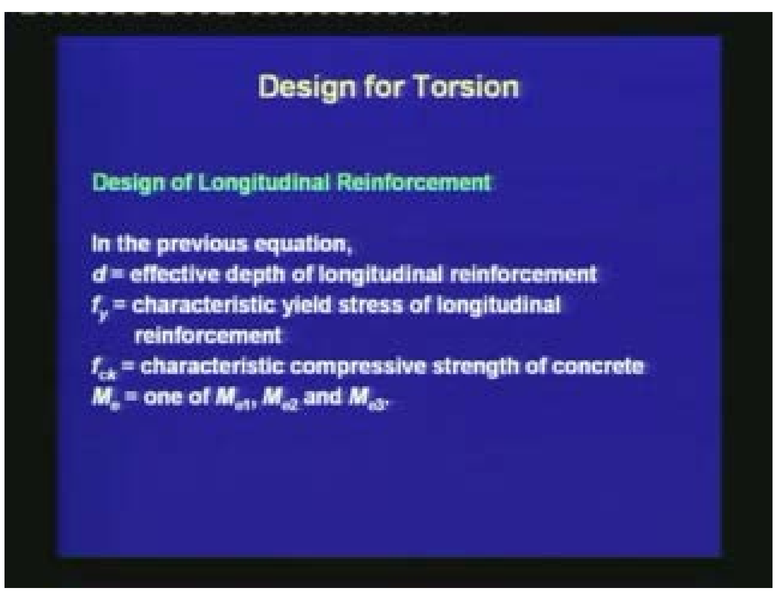

In this expression, d is the effective depth of longitudinal reinforcement, f_y is the characteristic yield stress of longitudinal reinforcement, f_{ck} is the characteristic compressive strength of concrete, and M_e is any of M_{e1}, M_{e2} or M_{e3}.

When we are substituting M_{e1}, we are designing the bottom steel; when we are substituting M_{e3}, we are designing the steel at the side; when we are substituting M_{e2}, we are designing the steel at the top. Hence, depending on the relative values of M_u, the flexural moment, and M_t, the equivalent moment due to torsion, we design for the longitudinal steel at the bottom, side or top.

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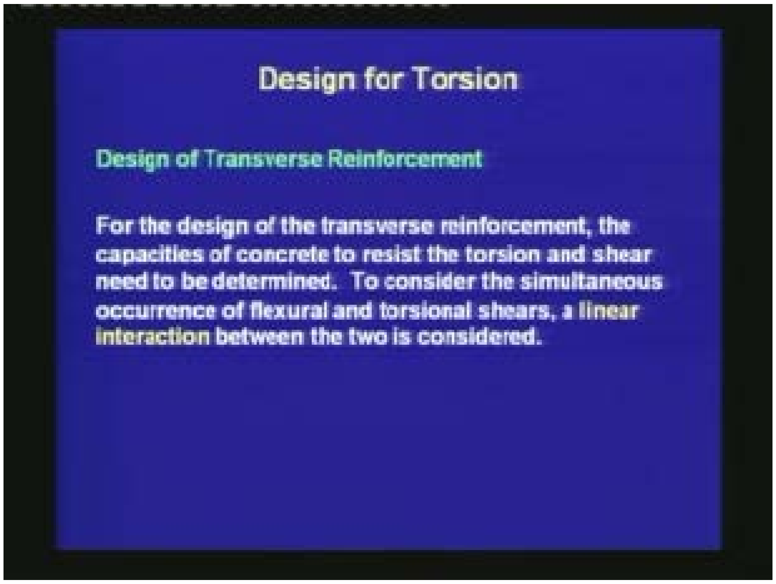

Next, is the design for transverse reinforcement. For the design of the transverse reinforcement, the capacities of concrete to resist the torsion and shear need to be determined. To consider the simultaneous occurrence of flexural and torsional shears, a linear interaction between the two is considered.

We have seen earlier in the design for shear that, before we can design the transverse reinforcement, we need to calculate the contribution of concrete to resist shear. Similarly, when torsion is present, we need to calculate the capacity of concrete to resist the torsional shear and flexural shear, simultaneously. To consider the occurrence of these two types of shear simultaneously, an interaction equation is assumed.

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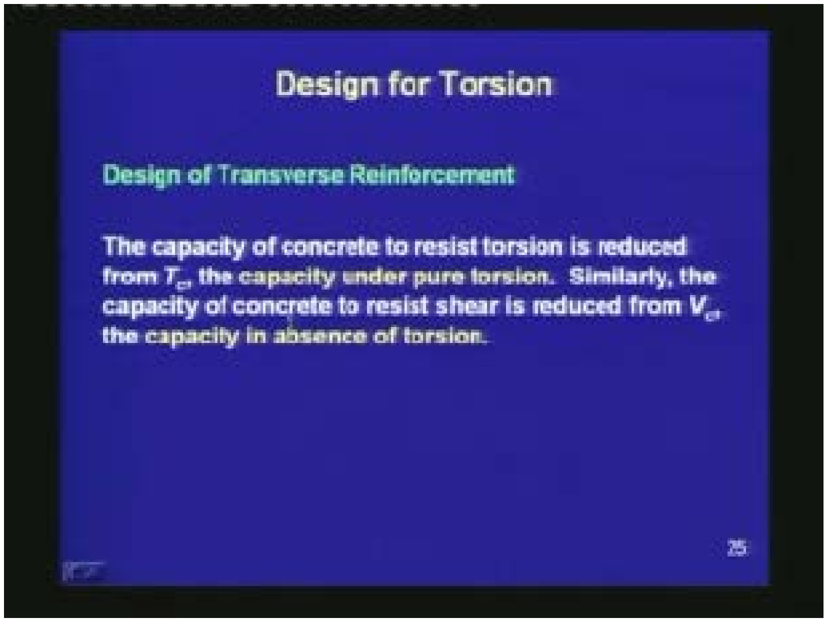

The capacity of concrete to resist torsion is reduced from T_c, the capacity under pure torsion. Similarly, the capacity of concrete to resist shear is reduced from V_c, which is the capacity in absence of torsion.

The meaning of interaction is that we have an expression of the capacity of concrete under pure torsion; we have an expression of concrete to resist shear in absence of torsion; these two capacities are represented by T_c and V_c, respectively. When both shear and torsion are occurring simultaneously, the capacity of concrete to resist torsion will get reduced from T_c, similarly the capacity of concrete to resist shear will reduce from V_c.

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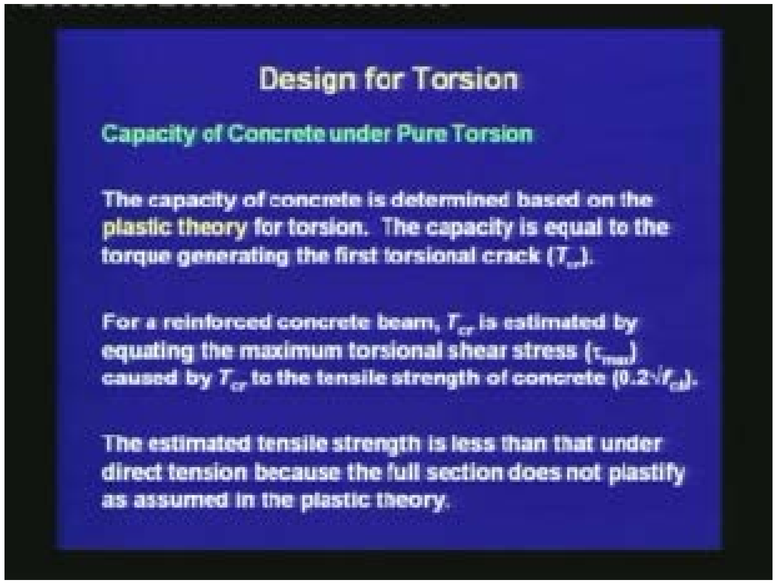

Next, we are developing the equation for the capacity of concrete under pure torsion, which will be represented as T_c. The capacity of concrete is determined based on the plastic theory for torsion. The capacity is equal to the torque generating the first torsional crack, which is represented as $T_{cr.}$

For a reinforced concrete beam, T_{cr} is estimated by equating the maximum torsional shear stress, which is τ_{max}, caused by T_{cr} to the tensile strength of concrete, which is estimated as $0.2\sqrt{f_{ck}}$.

The estimated tensile strength is less than that under direct tension, because the full section does not plastify as assumed in the plastic theory.

Thus, to get an expression for the torsional resistance of concrete, we refer to the plastic theory of the analysis of torsion. With that theory, we equate the expression of the maximum shear stress generated at the mid-depth of the longer face to a tensile strength of concrete, which is given as $0.2\sqrt{f_{ck}}$. Now, this estimate of the tensile strength of concrete is lower than the direct tensile strength. Because, in the plastic theory it is assumed that the full section is plastifying at failure. Whereas, when concrete cracks the full section does not plastify as assumed in the plastic theory. Hence, the tensile strength of concrete is reduced.

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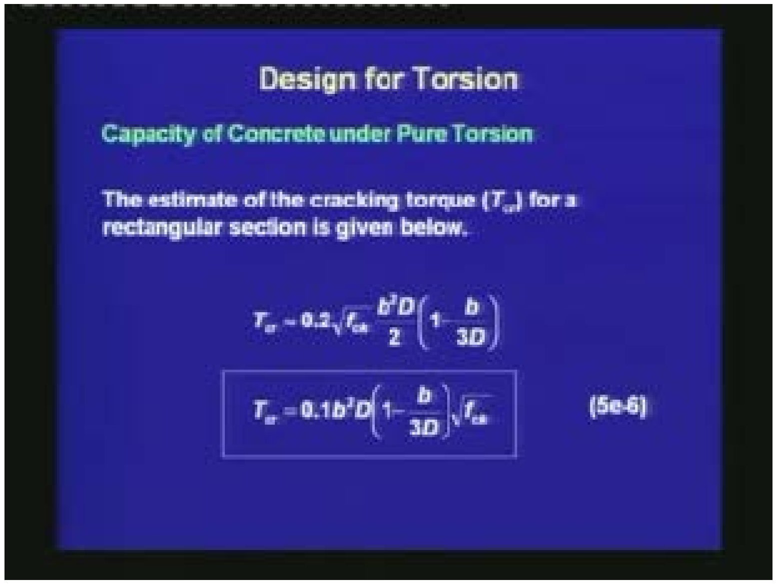

The estimate of the cracking torque T_{cr} for a rectangular section is given below.

$T_{cr} \approx 0.2\sqrt{f_{ck}}\ (b^2D/2)\ (1 - b/3D)$

When simplified,

$T_{cr} = 0.1b^2D\ (1 - b/3D)\sqrt{f_{ck}}$.

This is the expression of the torque that causes the first crack in the concrete member.

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Now, the expression that we had seen was for a rectangular section. For a flanged section, the section is treated as a compound section. A compound section is a summation of rectangular sections.

The cracking torque is estimated as a summation of the capacities of the individual rectangular sections. Since the interaction between the rectangular sections is neglected in the summation, the estimate of the cracking torque is a lower bound estimate.

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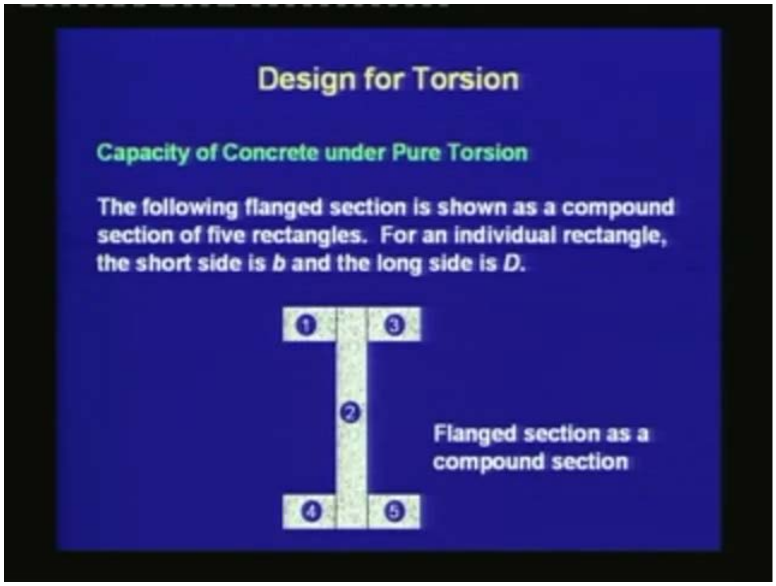

To explain, let us see this idea of compound section for a flanged section. The following flanged section is shown as a compound section of five rectangles. For an individual rectangle, the short side is b and the long side is D. For each individual rectangle, we are finding out the capacity which is given for a rectangular section. We sum up these capacities to get the T_{cr} for the flanged section. In doing this summation we are not considering any interaction between these rectangles, and hence the summation is actually a lower bound of the true value of T_{cr}. That is, whatever we calculate, will always be lower than the actual value of T_{cr} which can be observed from an experiment.

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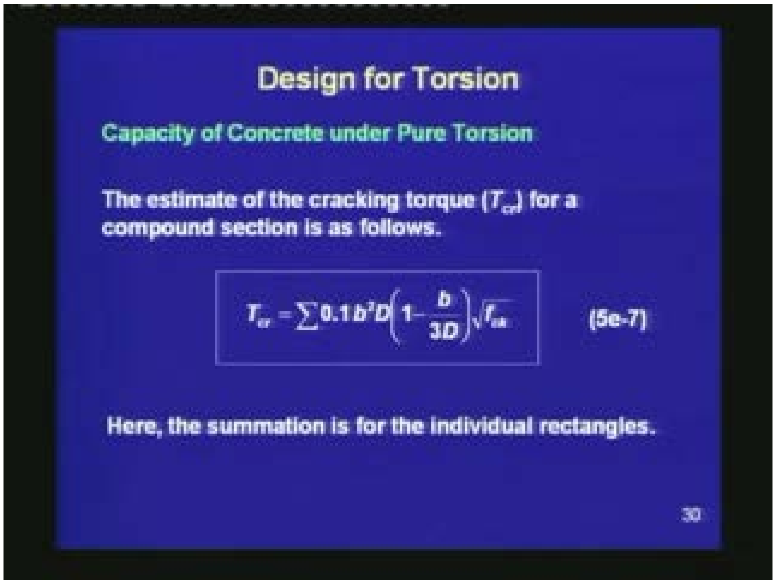

The estimate of the cracking torque T_{cr} for a compound section is as follows.

$T_{cr} = \Sigma 0.1 b^2 D\ (1 - b/3D)\sqrt{f_{ck}}$

Thus, once we calculate the capacities of the individual rectangles, we can sum them up to get the total capacity for the compound section.

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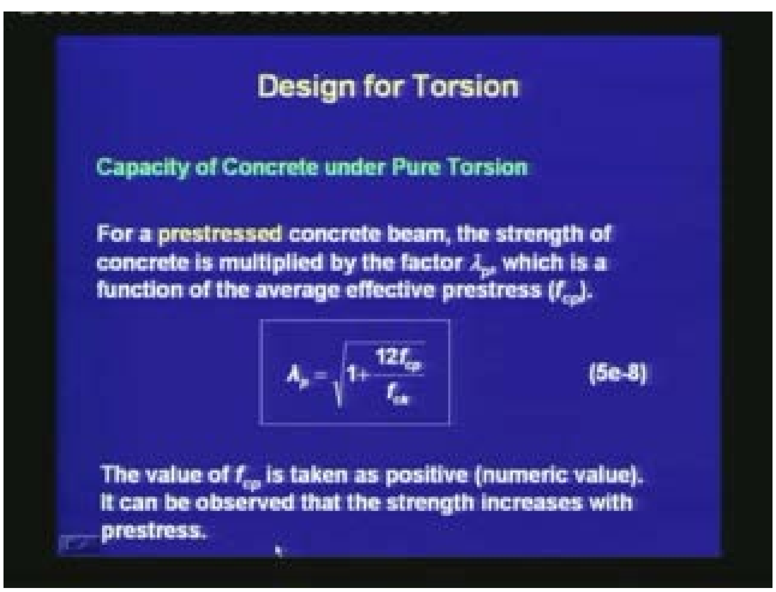

For a prestressed concrete beam, the strength of concrete is multiplied by the factor λ_p which is a function of the average effective prestress f_{cp}. We have observed earlier that the effect of prestressing is to reduce the principle tensile stress that is developed at the mid-depth of the longer side. Hence, the cracking torque is much higher and even after cracking, the strength of the concrete is retained as the aggregate inter-lock is retained due to reduced crack width. Thus, in presence of prestressing force the strength of concrete is increased by multiplying by a factor λ_p, which is a function of the amount of prestressing force at the level of CGC. λ_p is given by the expression

$$\lambda_p = \sqrt{(1 + 12\, f_{cp}/f_{ck})}$$

The value of f_{cp} is taken as positive that is, only the numeric value is considered. It can be observed that the torsional strength of concrete increases with prestress.

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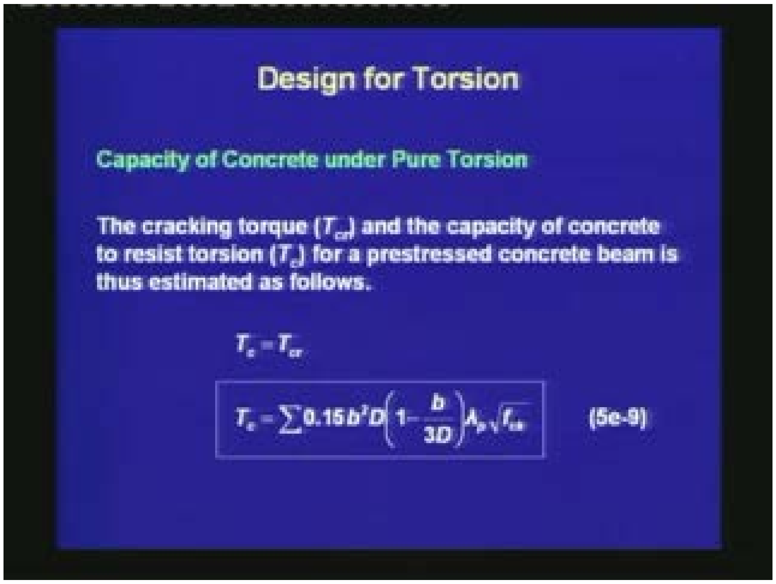

The cracking torque and the capacity of concrete to resist torsion, which is given as T_c for a prestressed concrete beam is thus estimated as follows:

$$T_c = T_{cr} = \Sigma 0.15 b^2 D\, (1 - b/3D) \lambda_p \sqrt{f_{ck}}$$

This is the expression given in the code for the torsional strength of concrete under pure torsion and in presence of prestressing force.

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In this expression, b is the breadth, and D is the depth of the individual rectangle in a compound section.

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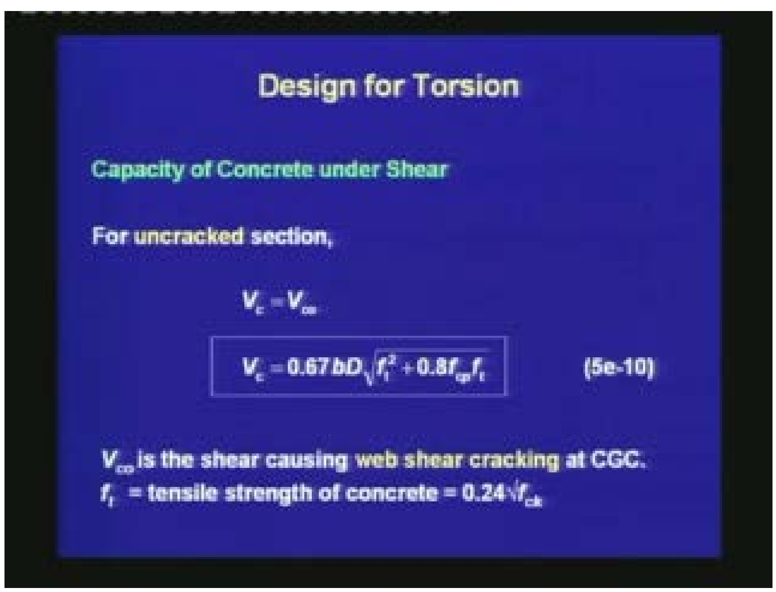

Next, we are finding out the capacity of concrete under shear. The following expressions we had seen in the analysis for shear, and the same expressions we are revising once again. There are two expressions of the capacity of concrete for shear: first for an uncracked section, which is the amount of shear generating web shear crack; second for a cracked section, which is the amount of shear that converts a flexural crack to a flexure–shear crack. For an uncracked section,

$V_c = V_{c0} = 0.67bD \sqrt{(f_t^2 + 0.8f_{cp}f_t)}$

V_{c0} is the shear causing web shear cracking at CGC, f_t is the tensile strength of concrete equal to $0.24\sqrt{f_{ck}}$. Note that the tensile strength is larger than that used for finding out the torsional strength of concrete.

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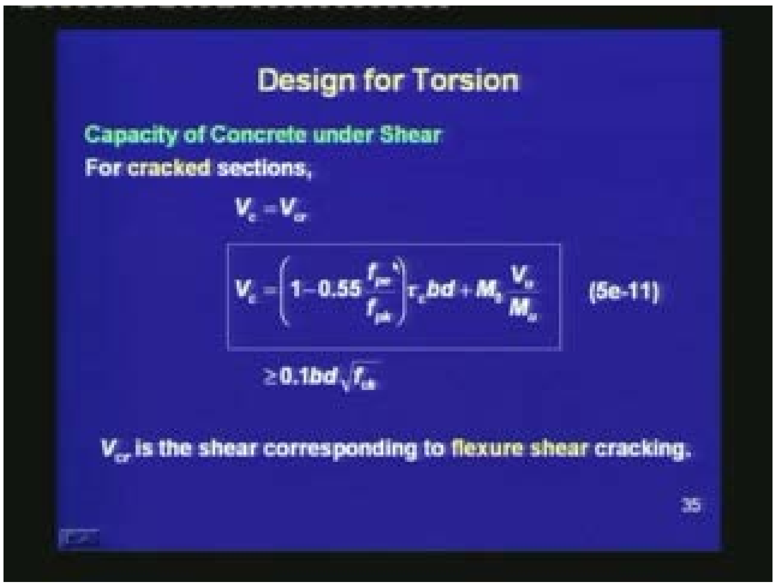

For cracked section,

$V_c = V_{cr} = (1 – 0.55 f_{pe}/f_{pk})\tau_c bd + M_0 (V/M) \geq 0.1bd\sqrt{f_{ck}}$.

There are two terms: the first term denotes the shear which converts a flexural crack to a flexure–shear crack, and the second term is the shear which generates a flexural crack.

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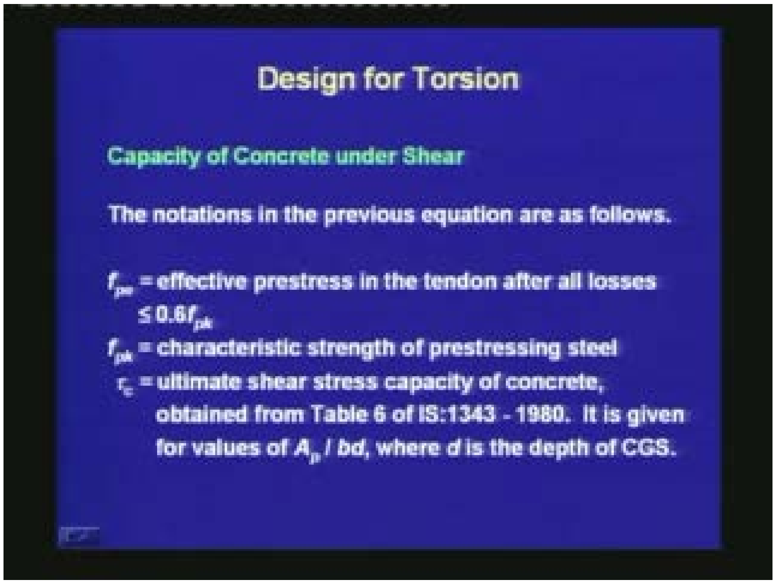

The notation in the previous equation are as follows: f_{pe} is an effective prestress in the tendon after all losses and it should be less than or equal to 0.6 f_{pk}, where f_{pk} is the characteristic strength of prestressing steel; τ_c is the ultimate shear stress capacity of concrete, obtained from Table 6 of IS: 1343 – 1980. It is given for values of A_p/bd, where d is the depth of CGS.

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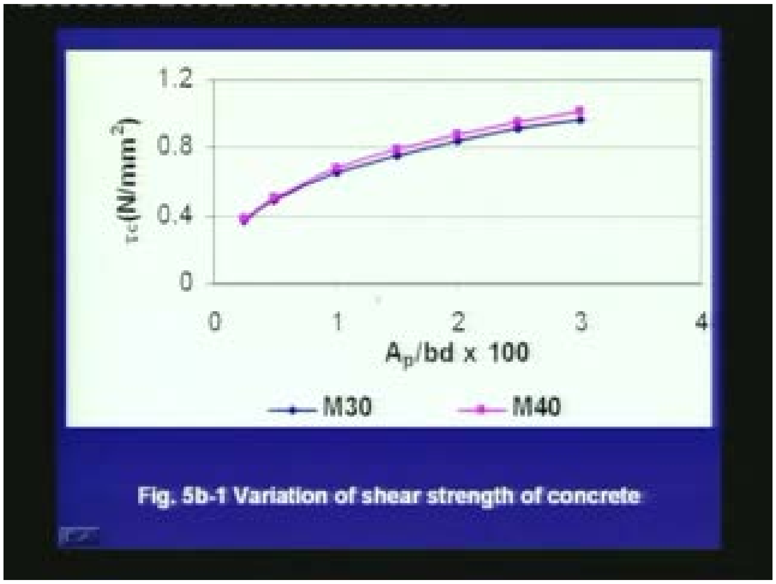

This is the plot of the variation of τ_c with the amount of prestressing steel. What we observe is that as the prestressing steel is increasing, the capacity of concrete to resist shear is also increasing. In presence of prestressing force, the cracking occurs at higher load and the resistance after cracking is retained because the aggregate inter-lock is retained, and the zone of concrete under compression is larger. Also, the dowel action is better in presence of prestressing force.

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The other variables in the equation are: d is the distance from the extreme compression fibre to the centroid of the tendons at the section considered; M_0 is the moment initiating a flexural crack which is estimated as $0.8f_{pt}$ I/y; f_{pt} is the compressive stress in concrete at the level of CGS due to prestress only; I is the gross moment of inertia; y is the depth of the CGS from CGC.

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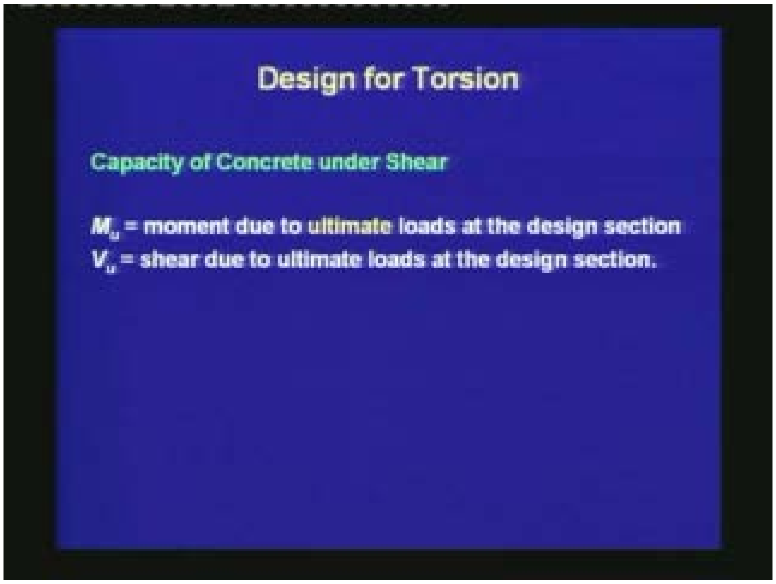

M_u is the moment due to ultimate loads at the design section, and V_u is the shear due to ultimate loads at the design section.

Thus, given the expressions of V_{c0} and V_{cr}, we can calculate V_c, which is the capacity of concrete to resist shear in absence of torsion. Now, we are reducing the capacities of concrete to resist torsion and shear, when both of them act together. This is done by the help of an interaction equation.

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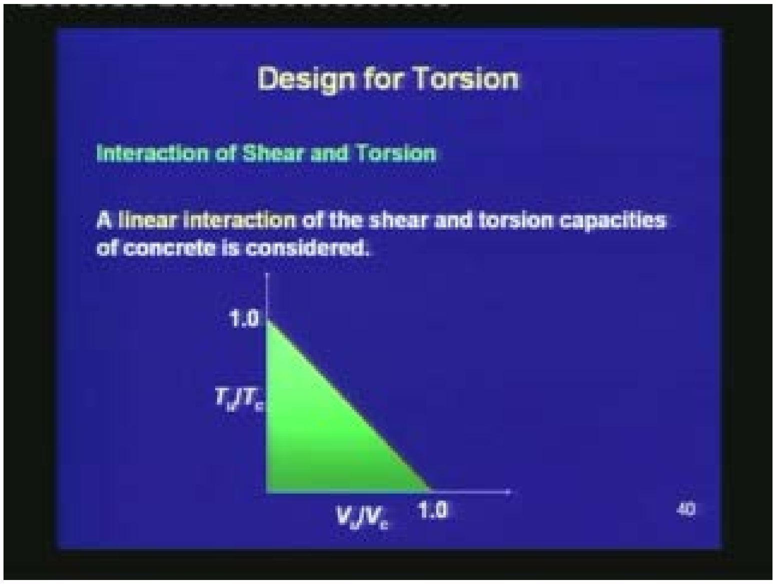

A linear interaction of the shear and torsion capacities of concrete is considered. In the graph, the ultimate shear V_u is normalized with the capacity of concrete to resist shear in the absence of torsion; this variable is in the x axis. In the y axis, the ultimate torsion T_u is normalized with respect to the capacity of concrete to resist torsion under pure torsion; this variable is plotted in the y axis.

A straight line is joined from the point where $T_u = T_c$ to a point where $V_u = V_c$. Thus, the two extreme points correspond to the cases of pure torsion, and shear in absence of torsion, respectively. When both of them occur, the capacity is reduced base along this straight line and this is a linear interaction. Thus, if the demand is somewhere within this straight line, in the green area, then the section will be safe; if the demand lies outside the interaction line, then it will be unsafe.

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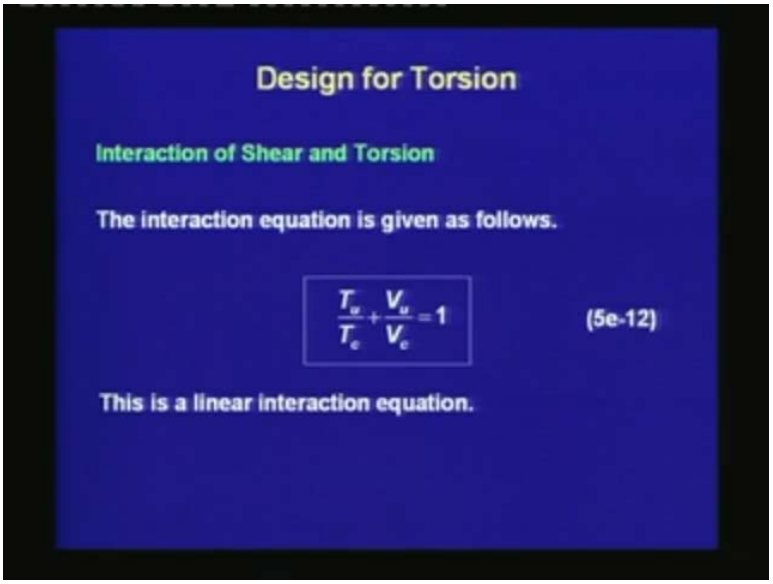

The interaction equation is given as follows:

$T_u/T_c + V_u/V_c = 1$

This is a linear interaction equation. It is the plot of the straight line that joins the two points in the axes, as we had seen in the graph. From this linear interaction, we are calculation the capacity of concrete to resist torsion and shear, when both torsion and shear are acting.

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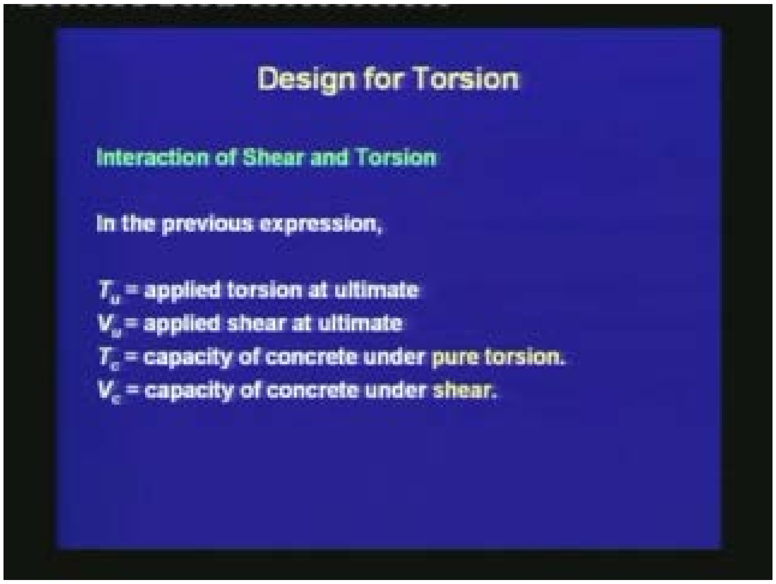

In this expression of the interaction equation: T_u is the applied torsion at ultimate; V_u is the applied shear at ultimate; T_c is the capacity of concrete under pure torsion; V_c is the capacity of concrete under shear.

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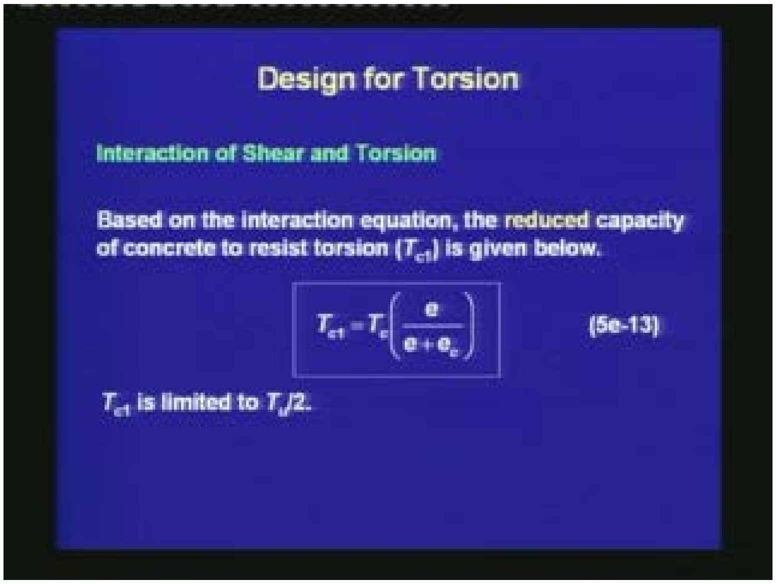

Based on the interaction equation, the reduced capacity of concrete to resist torsion, which is denoted as T_{c1}, is given below:

$T_{c1} = T_c (e/e+e_c)$

The code recommends to limit T_{c1} to half of the ultimate torsional moment. Thus, the concrete should not carry more than half of the torsional moment that acts in a beam.

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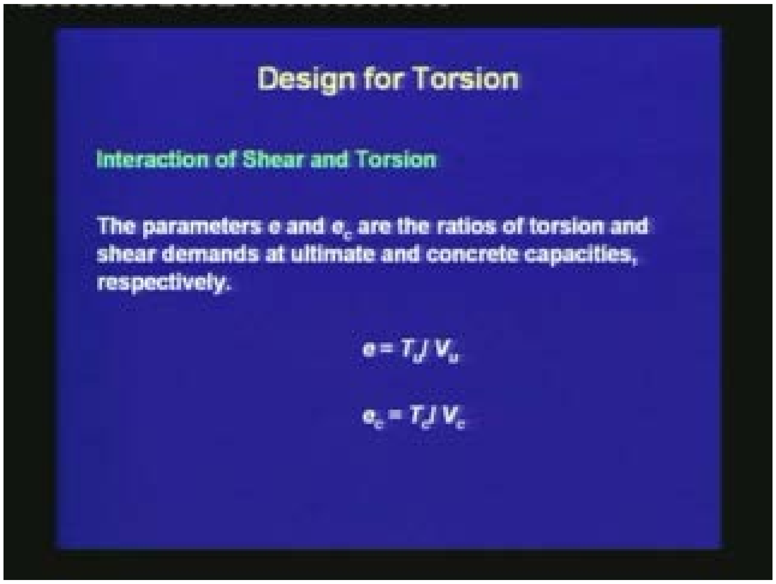

The parameters e and e_c are the ratios of torsion and shear demands at ultimate, and concrete capacities, respectively.

$e = T_u/V_u$, which is the ratio of the torsion and shear demands at ultimate.

$e_c = T_c/V_c$, which is the ratio of the concrete capacities to resist torsion and shear.

These are the two variables, which are used to write the expressions of the capacities of concrete to resist torsion and shear in a compact way.

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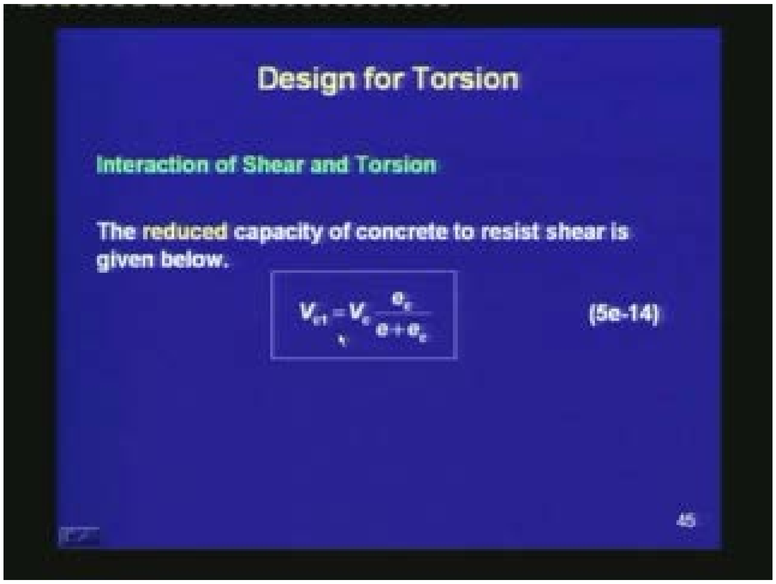

The reduced capacity of concrete to resist shear is given below.

$V_{c1} = V_c (e_c/e+e_c)$.

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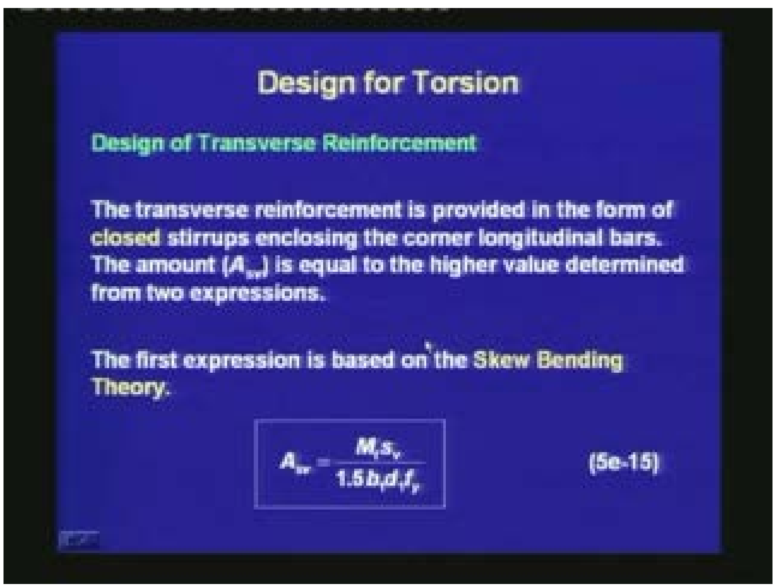

The transverse reinforcement is provided in the form of closed stirrups enclosing the corner longitudinal bars; this is the important requirement for the design for torsion. The torsion generates a circulatory shear, and hence the stirrups need to be closed. Open stirrup is not adequate for torsional resistance. The amount of transverse reinforcement A_{sv} is equal to the higher value determined from two expressions. The first expression is based on the skew bending theory. A_{sv} is given in terms of the equivalent moment generated due to torsion, which is denoted as M_t.

$A_{sv} = M_t s_v/1.5b_1d_1f_y$

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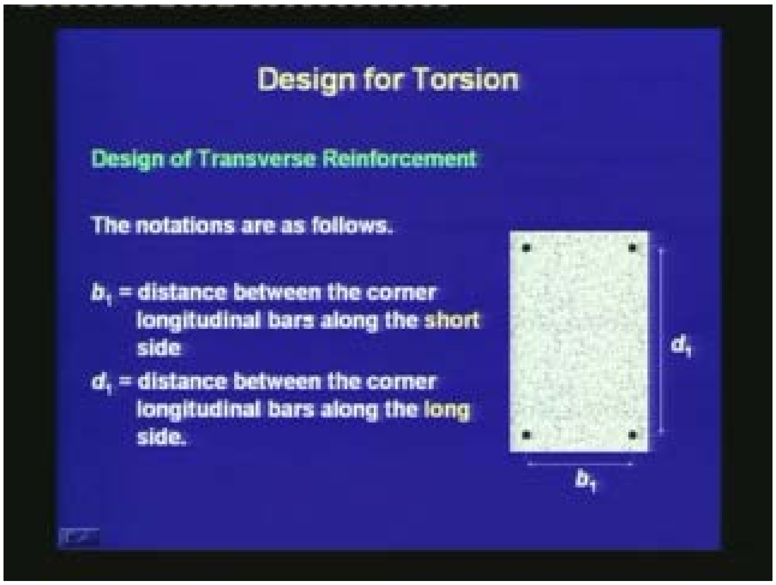

In this expression, b_1 is the distance between the corner longitudinal bars along the short side; d_1 is the distance between the corner longitudinal bars along the long side. Thus, for the rectangular section b_1 is the shorter distance between the longitudinal bars and d_1 is the larger distance between the longitudinal bars.

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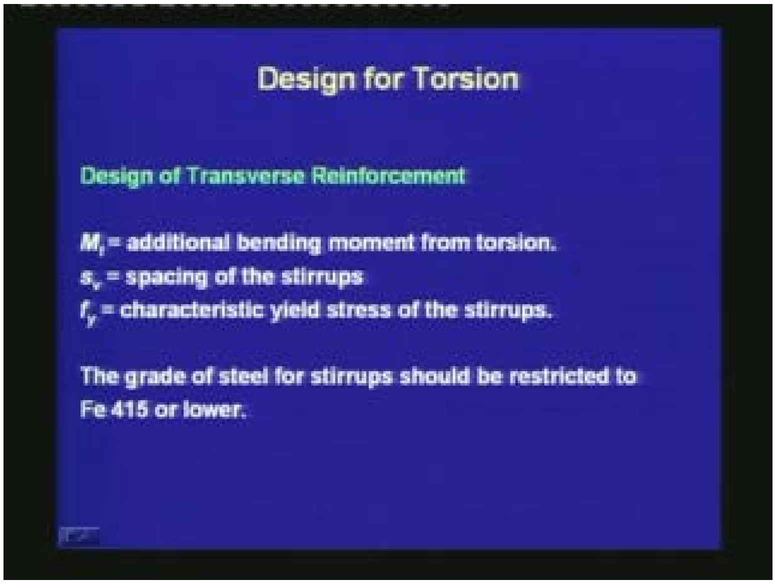

M_t is the additional bending moment from torsion; s_v is the spacing of the stirrups; f_y is the characteristic yield stress of the stirrups. The grade of steel for stirrups should be restricted to Fe 415 or lower to have adequate ductility in the stirrups. Thus, we have found one expression of the transverse reinforcement, which is in terms of the equivalent moment due to torsion, and this is derived from the skew bending theory.

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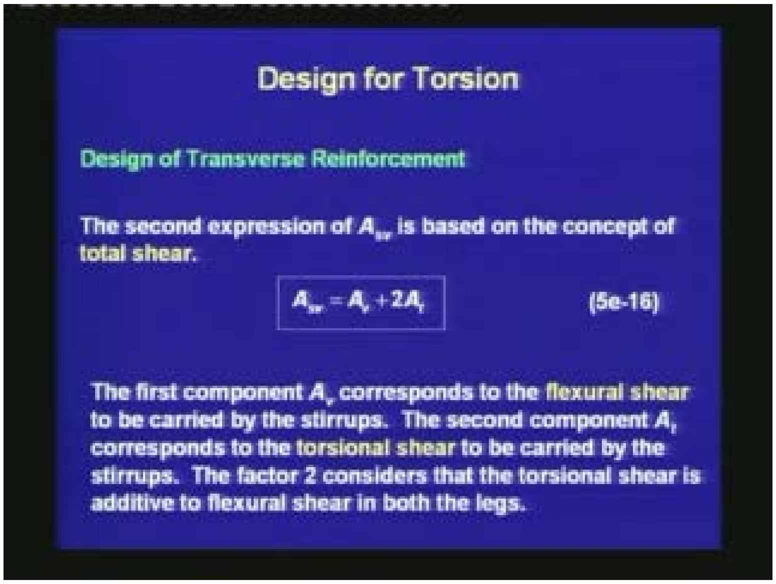

The second expression of A_{sv} is based on a total shear requirement.

$A_{sv} = A_v + 2A_t$

The first component A_v corresponds to the flexural shear to be carried by the stirrups, which we can calculate from conventional shear design. The second component A_t corresponds to the torsional shear to be carried by the stirrups. The factor 2 in front of A_t considers that the torsional shear is additive to flexural shear in both the legs. Let us try to understand this equation for different types of sections.

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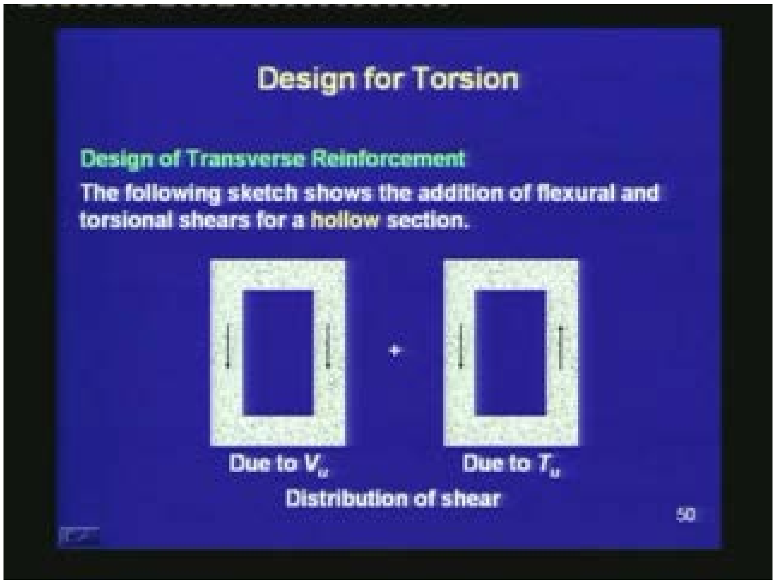

The following sketch shows the addition of flexural and torsional shears for a hollow section. Due to the flexural shear V_u there is vertically downward shear occurring in both the legs of the hollow section, whereas due to torsion there is a vertically downward shear in the left web and vertically opposite shear in the right web. Thus, when both shear and torsion are occurring then the left web will see a higher shear as compare to the value generated from V_u. Similarly, the right web will observe a lower shear compared to the value generated due to V_u.

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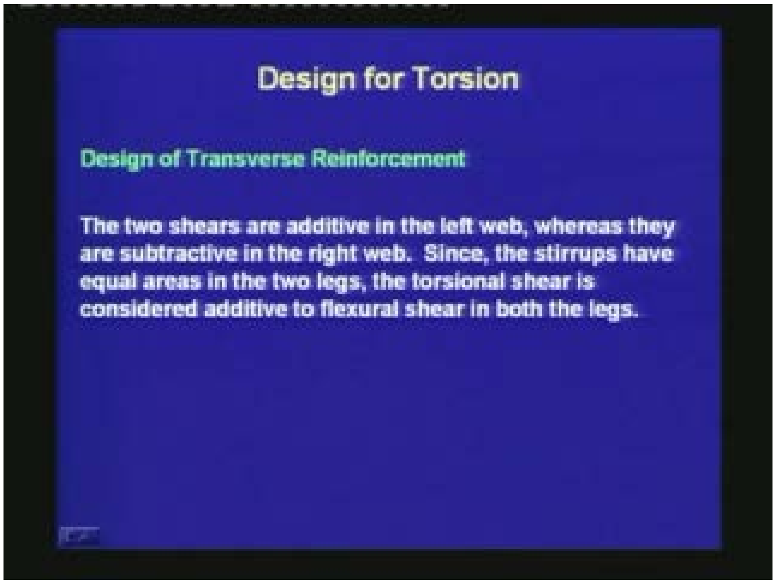

The two shears are additive in the left web whereas they are subtractive in the right web. Since, the stirrups have equal areas in the two legs, the torsional shear is considered additive to flexural shear in both the legs.

In the expression $A_{sv} = A_v + 2A_t$, the assumption is that the torsional shear is additive to the flexural shear in both the webs of a hollow section. This is necessary because we are providing same area of the torsional reinforcement in the two webs of the hollow section.

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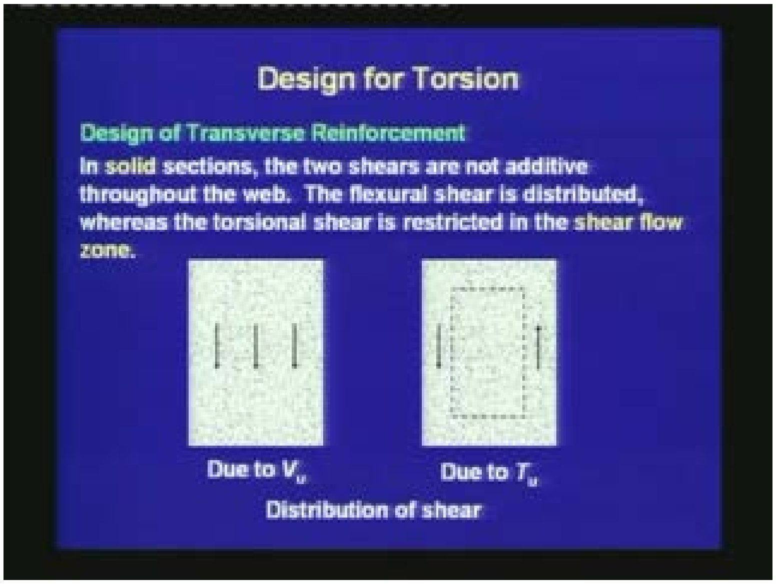

In solid sections, the two shears are not additive throughout the web. The flexural shear is distributed and acts downwards in the web, whereas the torsional shear is restricted in the peripheral shear flow zone; in one side it acts downwards, in the other side it acts upwards. In the central region, the shear is insignificant due to torsion. Thus, when shear and torsion act together they are not additive throughout the width of the web.

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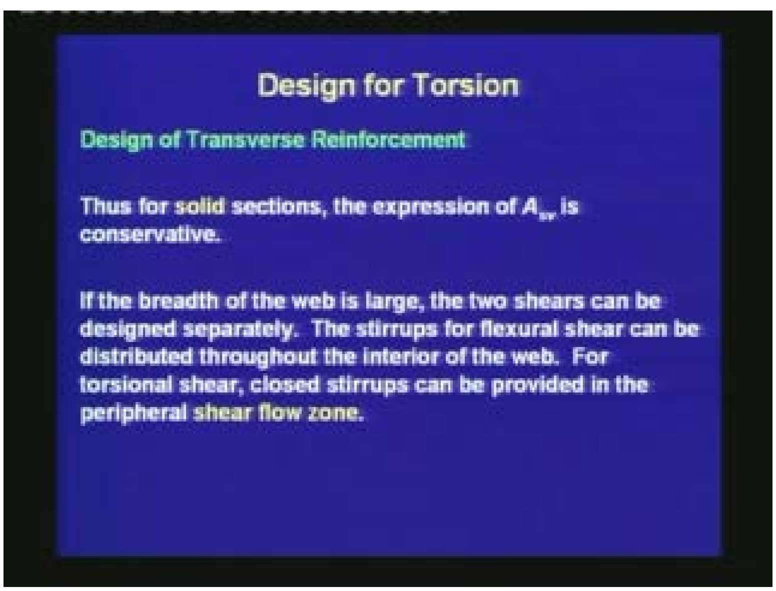

Thus for solid sections, the expression of A_{sv} is conservative. If the breadth of the web is large, the two shears can be designed separately. The stirrups for flexural shear can be distributed throughout the interior of the web. For torsional shear, closed stirrups can be provided in the peripheral shear flow zone. This is adopted to avoid clustering of the transverse reinforcement.

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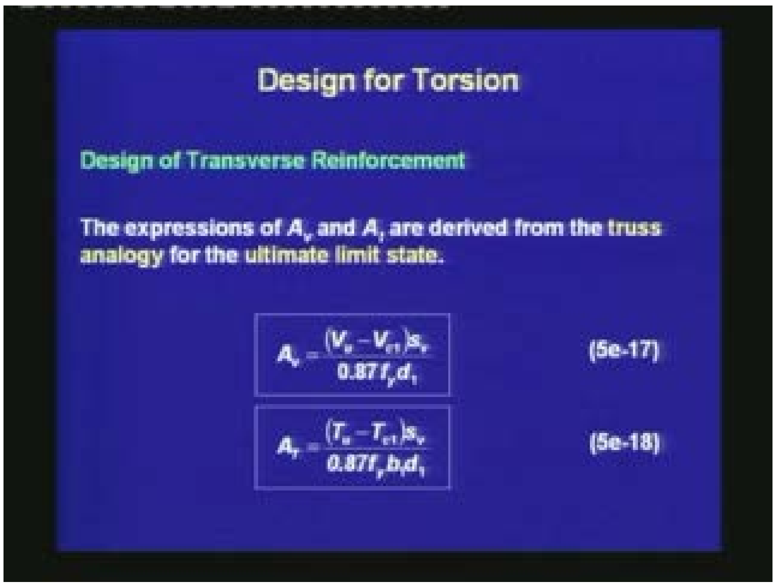

The expressions of A_v and A_t are derived from the truss analogy for the ultimate limit state. A_v is calculated from the shear that will be generated in the stirrups which is (V_u – V_{c1}), where V_u is the shear demand and V_{c1} is the concrete capacity.

$A_v = (V_u - V_{c1})s_v/0.87f_yd_1$, where d_1 is the larger depth between the longitudinal bars.

In the expression for A_t, T_u is the torsion demand, T_{c1} is the capacity of concrete to resist torsion, (T_u – T_{c1}) is the torsion to be resisted by the transverse steel.

$A_t = (T_u - T_{c1})s_v/0.87\ f_y\ b_1\ d_1$

This expression is derived from the space truss analogy for torsion. Thus, once we know A_v and A_t we are combining them to get the value of A_{sv}, which is provided in the form of closed stirrups. Otherwise for a solid section, we can design for A_v and A_t separately.

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The transverse steel should satisfy a minimum requirement. The minimum amount of transverse reinforcement is same as that of shear and it is given by the following equation:

$A_{sv}/b_{sv} = 0.4/0.87f_y$

This equation considers a minimum stress in the transverse reinforcement equal to 0.4 N/mm^2. The minimum amount of transverse reinforcement needs to be provided to avoid any diagonal tension failure due to the combined effect of torsion and shear.

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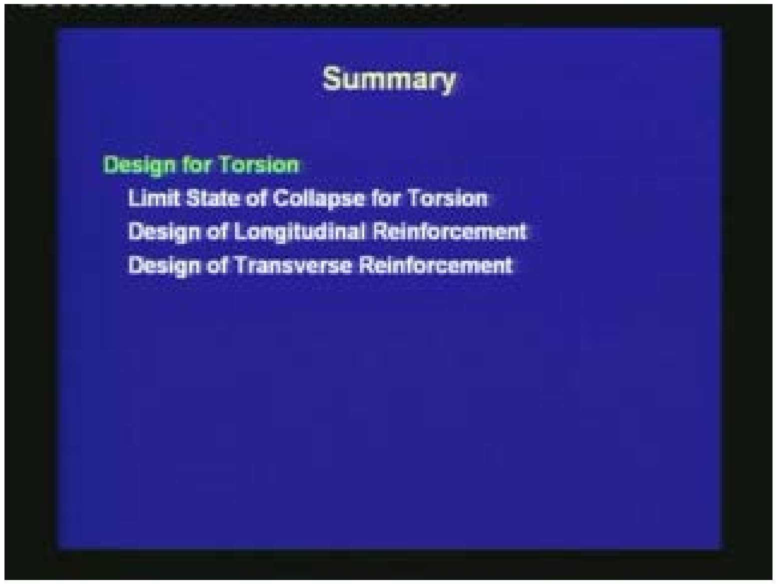

In today's lecture, we first studied the background of the design equations for the limit state of collapse for torsion. We identified three modes of failure for a beam under combined flexure and torsion. Corresponding to each mode of failure, an equivalent ultimate bending moment was defined based on the skew bending theory.

In Mode 1 failure, the equivalent ultimate moment acts in the same direction as that of the flexural moment, but the magnitude of the equivalent moment is higher than that of the flexural moment. We design the longitudinal reinforcement for the equivalent moment, which is given as M_{e1}.

If the torsional effect is large, which is measured as M_t greater than M_u, we should also design for the transverse bending and negative bending. The transverse bending is modeled by the expression M_{e3}, and for that there is compression in one side face and tension in the other side face. The designed steel is for one side face and we provide a symmetric amount of steel in the other side face. For the negative bending moment, the equivalent moment M_{e2} acts in an opposite sense to that flexural moment. Thus, if the flexural moment causes compression at the top, then the equivalent moment will cause compression at the bottom, and we design for the steel at the top.

Next, we moved on to the design of the transverse reinforcement. The area of transverse reinforcement is calculated based on two concepts, whichever gives the higher value is selected. The first expression is based on the skew bending theory, and is related with the equivalent torsional moment M_t. The second expression is calculated from the requirement of total shear. Before we design the transverse reinforcement, we should know the capacities of concrete to resist torsion and shear. The capacities get reduced in presence of torsion and shear acting together. The expressions are based on a linear interaction equation for shear and torsion.

Once we know the capacities of concrete, we can deduct them from the torsion and shear demands to get the forces to be carried by the steel. Based on truss analogies, the expressions of A_v and A_t are derived. Then we get the total amount of transverse reinforcement that is required for torsion design.

In the next class, we will be looking into the design steps.

Thank you