Project Planning & Control

Prof. Koshy Varghese

Department of Civil Engineering

Indian Institute of Technology, Madras

Lecture - 52

PERT Background and Assumptions, Step wise Procedure

(Refer Slide Time: 00:16)

Now, let us go on to PERT. So, like we said, the people would developed PERT faced challenges in computing powers. So, they decided they should develop some kind of technique which can model uncertainty and at the same time, be a closed from kind of approach. And this was initiated, as you can see here, for the Polaris missile project by the US Department of Defense and this, as you can see is a defense project, if you are going to build a Polaris missile in the 1950's, there were a lot of uncertainties and you know the project could not be delayed by a large number of years. So, they decided they wanted to kind of take a CPM like technique and try to bring uncertainty modeling in it. So this was done, like I said, with a US Department of Defense along with the consultants Booze Allen Hamilton, and they were very successful in having developed and deployed the technique and they reported to have saved 2 years on the project duration by using a PERT approach.

(Refer Slide Time: 01:26)

APPLICATION OF A TECHNIQUE FOR RESEARCH AND DEVELOPMENT PROGRAM EVALUATION

D. G. Malcolm,* J. H. Roseboom, C. E. Clark*

Booz, Allen and Hamilton, Chicago, Illinois

and

W. Fazar,

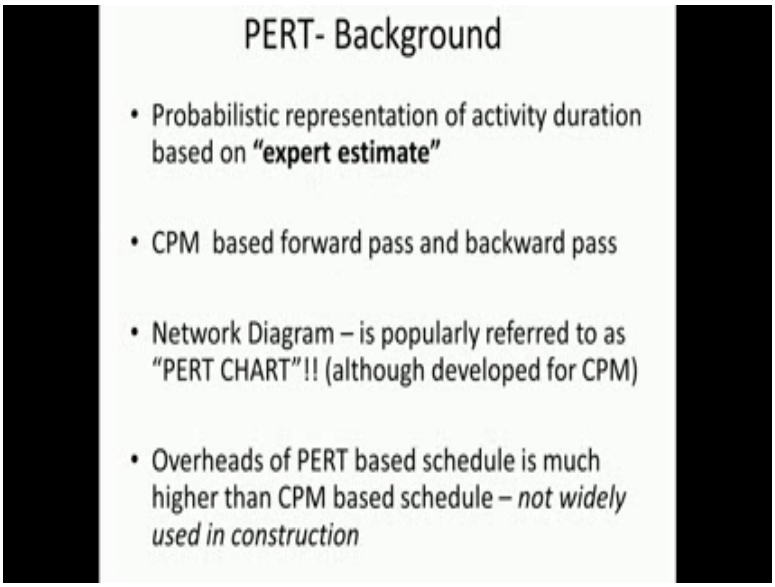
Special Projects Office, U. S. Navy, Washington, D. C.

(Received April 27, 1959)

This paper describes the development and application of a technique for measuring and controlling development progress for the Polaris Fleet Ballistic Missile program. Special Projects Office, Bureau of Ordnance, U. S. Navy. Project PERT (Program Evaluation Research Task) was set up to develop, test, and implement a methodology for providing management with integrated and quantitative evaluation of: (a) progress to date and the outlook for accomplishing the objectives of the FBM program, (b) validity of established plans and schedules for accomplishing the program objectives, and (c) effects of changes proposed in established plans. In the PERT model, the R and D program is characterized as a network of interrelated events to be achieved in proper ordered sequence. Base data for the analyses consists of elapsed time estimates for activities which connect dependent events in the network. The time estimates are obtained from responsible technical persons and are subsequently expressed in probability terms. This model is described. Use of the model on a specific component, design of a management control system properly related to existing management systems, reduction to the NORC computer, difficulties in implementation and preliminary results to date are discussed. Limitations of the model, and possible refinements and use of the computer model for testing schedules and for management experimentation in resource and performance tradeoffs are described.

The original paper by the developers of PERT is available online, I will be giving a link to this, in the hand out page, and for those of you is interested in getting into more details, I would recommend that you would read this because you will understand the thought process and the case study they used when PERT was originally developed.

(Refer Slide Time: 01:48)

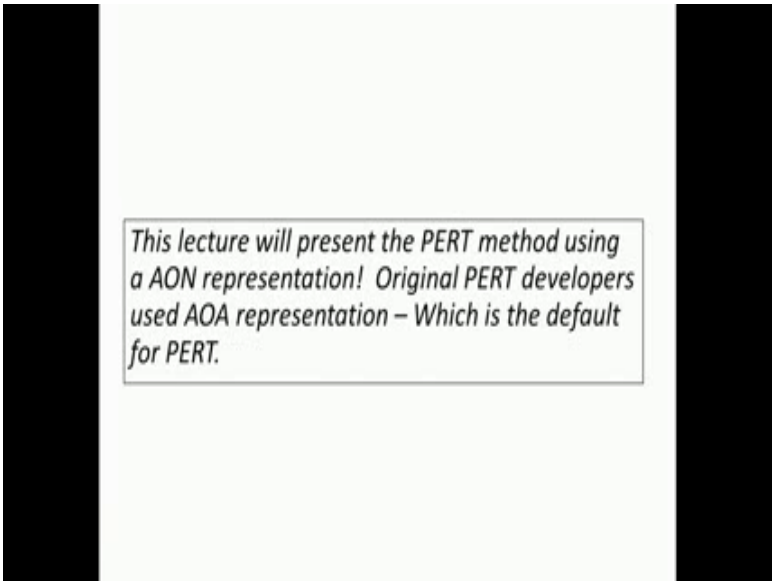


Now, little bit a background on PERT. So, like we have been discussing, PERT will try to represent probabilistic by get by an expert estimate. So, we do not have data to remember that the project, we are doing it is mostly like on the first time, we do not have any historic data in this case. So, the PERT developers had to go with an expert estimate. So, there was no option and you when we go to the technique, will see that on projects,

which are really novel, this is fairly a good way to start off. Now, after you get this estimate, and you represent the uncertainty with the appropriate distribution, PERT uses basically the CPM type of forward pass, backward pass analysis. So, that way it is very familiar to.. very familiar process, you know as per CPM. And you will see that the network diagram is also almost the same with, it is actually exactly the same, and although the CPM developed the network diagram first, the CPM diagram is the network diagram is very popularly refer to as the PERT chart. So, it is you know if you look up Google and say PERT chart you will get the network diagram.

And even in practice most people use the term PERT chart refer to a network diagram, although it was the CPM developers whose has originally brought out the network as a way of planning. Now the last point you have on the slide is that yes, doing PERT is certainly much more challenging and CPM, because we are trying to deal with uncertainly based issues. And as we discussed in construction, you know most the activities we do are not really that novel, you know we should be able to get reasonable time estimates on it, we might have historic data on it, you know we should be able to estimate the durations with the reasonable amount of certainty, and given that and that the overhead of PERT is quite high, you will find it PERT is not widely used in construction today. Like we discuss some the other areas which have a large range of uncertainties might be using PERT, but construction does not use it widely.

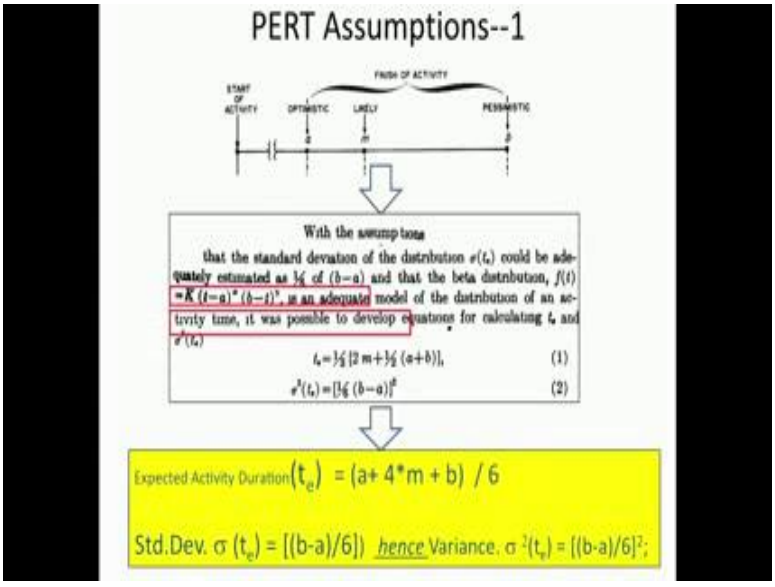
(Refer Slide Time: 04:04)



Now, when we go forward to this lecture, I will presenting PERT using the AON representation, because that is what we have been familiar with, and you used in most of

the other lectures. The original PERT developers used AOA; the AOA is the activity on arrow technique which is still the default for PERT. We do not see too many references with AON and PERT; there are some references, but most people would still use AOA, but I am going ahead with a AON representation, because that is what we are use throughout this lecture, and I do not think there is too much of a concept loss, if we continuity use a AON. And I think it will be a easier for you all to follow.

(Refer Slide Time: 04:46)

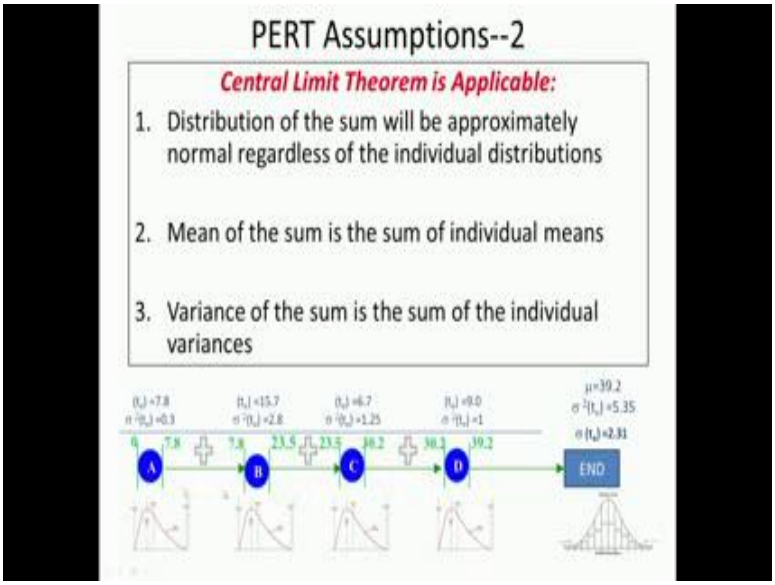


Now, let us take up the assumptions of PERT. So, one of the things is PERT is, there are several assumptions. So, we have to understand the assumptions also cause a lot of limitations in the applicability of PERT, and that is one another reason it is not that widely use, because the values we get are not too meaningful, then you know why go through the whole process. But starting of the assumptions, the first assumption is basically on the estimate at values, how we actually convert this into duration. So, as you can see this is a quote from the original paper, this is the assumptions that the standard deviation could be adequately estimated as one sixth of b - a. So, that is one assumption which is there in PERT that, and we are using a beta distribution.

So, they have given the formula of the beta distribution that is being used, they saying.. they assuming that adequate model for the distribution, and based on this distribution, and this standard deviation we get the basic 2 formulas in PERT which is very widely used, which says that which is here that is the expected time is equal to the optimistic time 'a' + 4 times the most the likely time 'm' plus 'b' the whole the sum of all this is divided by 6 will be the expected time. Now similarly for this distribution the standard

deviation is given the pessimistic time minus the optimistic time b minus a divided by 6, and we know the square of the standard deviation is a variation.. is a variance, and this is the variance equation which is this is just the square of the standard deviation. So, what we have tend to see in PERT is standard formula for the PERT activity and the PERT, standard deviation calculating the activity in the standard deviations itself are the first assumptions which the authors of PERT have said that they are making.

(Refer Slide Time: 06:43)



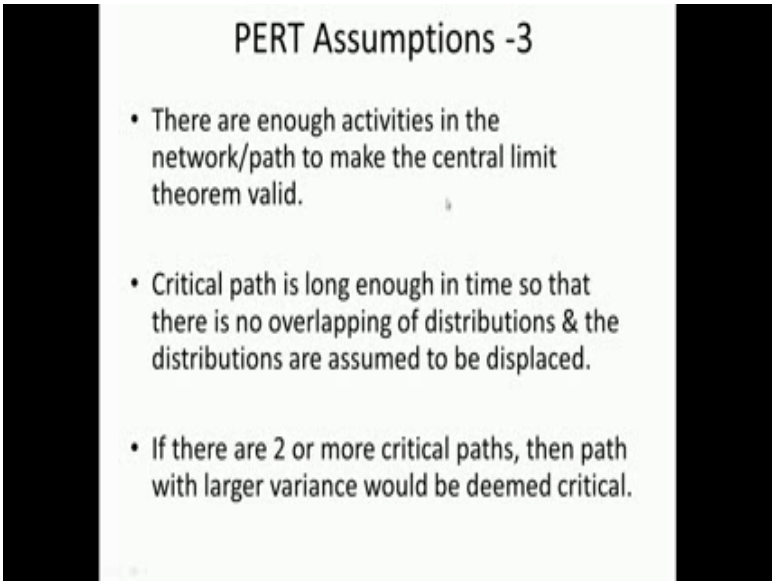
As we move on, the second assumption and PERT is that the central limit theorem is applicable. Now, this is very basics statistical theorem and what central limit theorem says that if I have a set of activities or the way if you would apply to PERT is, if I had a series of activities that we have earlier seen that each of these activities will be modeled, the duration will be modeled with the distribution right, you can see the distribution you can say we modeled the distribution. And we know this is not a normal distribution, this is the beta type distribution or a PERT distribution, that is what we are going a model it when we use PERT approach. And what the first point the central limit theorem says is the distribution of the sum will be approximately normal regardless of the individual distributions.

So, as I go down this link in the network and I calculate my early start, early finish, and I sum it up and I go down here, I might be taking specific values from not normal beta distributions or other distributions, but in the end when I sum up my.. the distribution at the end is a normal. My project duration will have a normal distribution although my activity durations here can be a beta.

So, this is basically what the first point says. The second point says, the mean of the sum is the sum of the individual means; that means, when I calculate the mean of all of these values, so, when I coming here and I add up the early, here is the early start, here is my duration which I calculated, I get 7.8, here is the early start of b; the duration is 15.7, so I get my early finish, and so on, and here I get the end of d is when I am ending 39.2. So, my mean of this normal distribution will be 39.2. So, the mean of the sums is the sum of individual means. So, here is my mean value that is going to be 39.2. And the third point says, the variance of the sum is the sum of the individual variances.

So, as we saw earlier, we had the standard deviation and we squared the standard deviation to find the variance. So, here I have various variance value which I have put here and if I sum these variances, so, the variance will give the sum of the variances will give me the variance to this distribution, and the root of this variance will give me the standard deviation. So, by the end of this exercise by normal distribution I have got a mean of 39.2 and the standard deviation of 2.31 and once I characterize this with these 2 values, you know that we can do a lot of computations based on the standard normal curve. So, you might wonder how I got these values, they are basically values which I have substituted from another problem and I have not given you the base values to be able to get this. So, do not try to calculate this, I just want you to, to illustrate with these values .. to show how the central limit theorem applied.

(Refer Slide Time: 09:53)

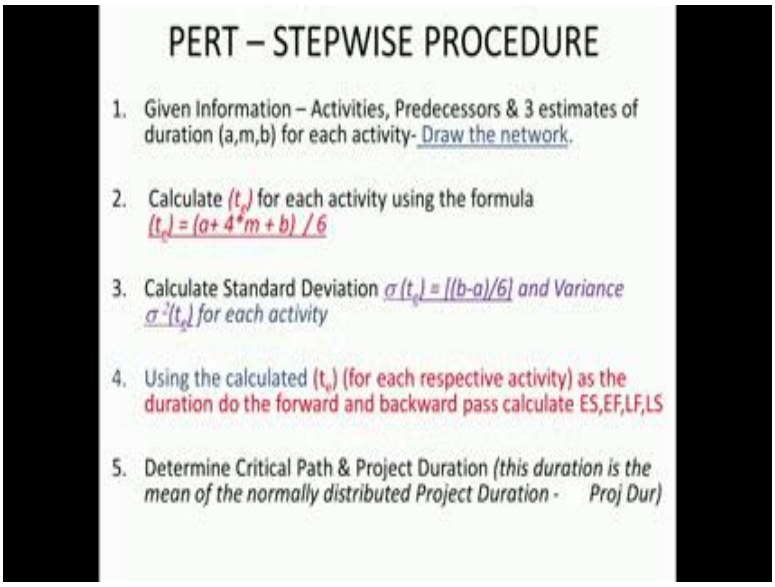


So, now we go on to the third set of assumptions for PERT, the first point here is there are enough activities to make the network path, to make the central limit theorem valid.

So, remember that for the central limit theorem to be valid, if I have 1 or 2 activities were just one activity; obviously, the duration still remains to be beta distribution. So, we need a large sum of or a large set of activities down the network for the distribution, for the project duration to be normal. So, that is the first assumption, that there of enough activities in the network to make the central limit theorem valid. The second assumption is, the critical path is long enough in time so that no overlapping of distributions and are there in the distributions are assumed to be displaced.

So, if you have overlapping distributions, computation of PERT probabilities do not become valid. So, this is quite a limiting assumptions especially when you have multiple critical paths which are close to each other. And, in case there are 2 or more critical paths then the larger variance would be deemed critical. So, PERT does not take in,to fact by default, the fact in multiple critical paths can exist and there can be overlapping durations, overlapping distributions here which will then cause you know one critical path can be critical for some time and another critical path another time, and this is certainly a limiting assumption in PERT.

(Refer Slide Time: 11:26)



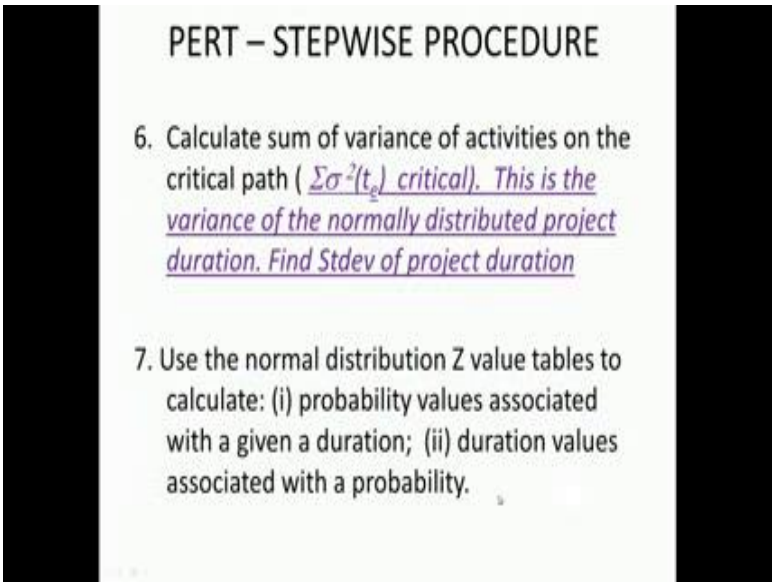
Now, will go to the step-wise procedure for PERT. So, typically when you are given information to do PERT value, we will not start with trying to make an expert estimate, because that is beyond the scope of this course. We will typically give you the activities and predecessors like in CPM and 3 estimates of duration. So, we will give you the pessimistic, the most likely, and the optimistic as 3 estimates of duration. So, once with this information, the first thing you can do the draw the network which is very similar to

CPM. The second step, is to be able to calculate the expected value for each activity and we see in this formula before, this is pessimistic plus 4 times most likely plus optimistic divided by 6, and we will calculate the expected value of duration for each activity.

And then, the standard deviation of the activity duration using this formula here, and the variance is just the square of the standard deviation. Now, when we go to step 4, basically we will use this expected value for each activity, and we do the forward pass and backward pass and calculate early start, early finish, late start, late finish ,usually late finish and late start as we did for CPM. So, this is very similar to the CPM step except that we are using the expected value.

And in step 5, we just us again very similar to CPM, we determine the critical path in the project duration, and now we should realize that although we.. we have.. we have a project duration, but this duration is not a single value, but it is the mean of the normally distributed project duration. So, when we did CPM, we did not have any more information other than the single value. Here we have not just a single value, but that single value represents the mean of the normally distributed project duration. So, this is the very important aspect.

(Refer Slide Time: 13:35)



We now go to step 6, here we have to calculate the sum of variances of activities on the critical path. So, this I will illustrate in the example. So, we take once we can calculate the variances, we take the sum the variance only along the critical path, and this sum is actually the variance of the normally distributed project duration. So, in the earlier step we found the mean, and here we found the variance and from the variance, we find the standard deviation, now we have actually characterized the project duration as a normal distribution with a mean value and the variance.

So, once we have characterized the normal distribution of the mean value and variance, we are, we'll be able to use a normal the z value tables, to calculate either the probability values associated with a given distribution or the duration values associated with a probability, this also we will illustrate in the example as to how these are done. But basically, these 5 steps form the core of... these seven steps from the core of your PERT procedure.