Analysis and Design of Bituminous Pavements

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Lecture - 04

Stresses and Strains in Bituminous Pavements - I

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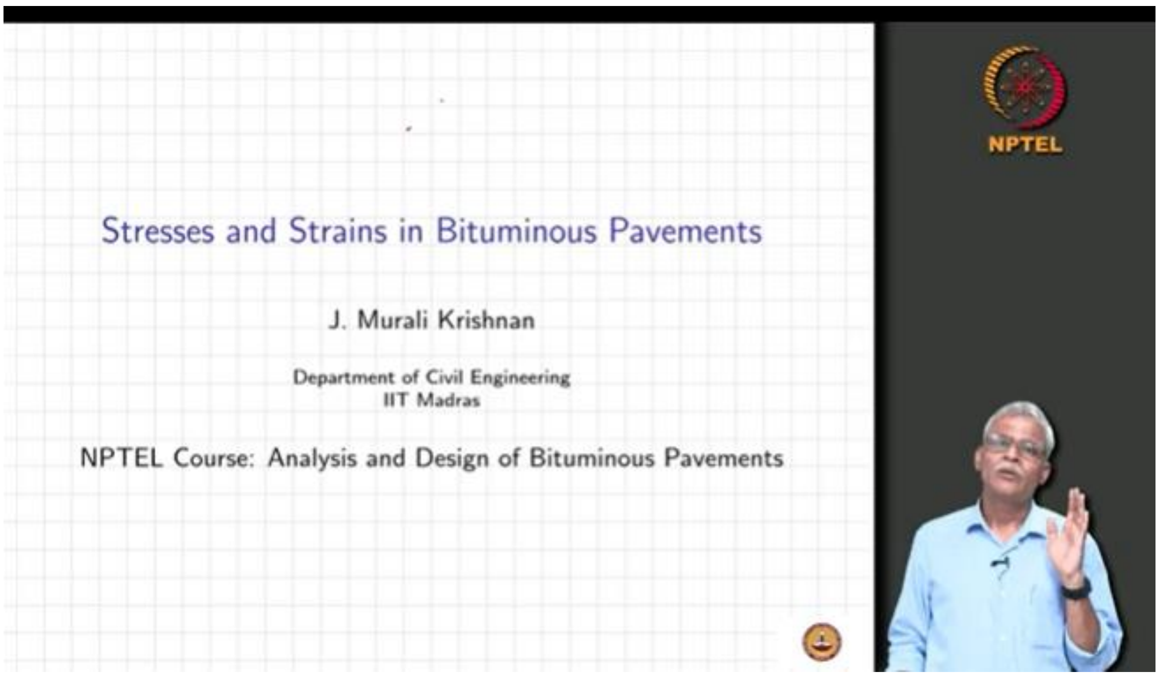

Hello everyone, so what we are now going to do in this particular portion of this course is to do a very important exercise which is to compute the stresses and strains in bituminous pavement. This particular portion of the work is very important, this particular portion of this course in which we are going to revisit some of the established principle in mechanics is very important because this is where you get an idea about the state of critical stress, critical strain and how it will influence the performance of the bituminous pavement. Now before I proceed further, I would request all of you whoever is registered for this course and listening to this is to revisit some of your at least the undergraduate level mechanics of materials course. And in fact there must be definitely lot of NPTEL course material available for this particular portion. So what I am going to do is I am going to assume that all of you have a decent background in mechanics. So when I say principle stresses or principle strain or octahedral shear stress, I am going to assume that you know it. But if some of you need some special help, I suggest that you write an email to me and I will try and see whether I can organize something for you or at least I can point you to some of the online material that is already available. So this is the basic background with which I am going to go.

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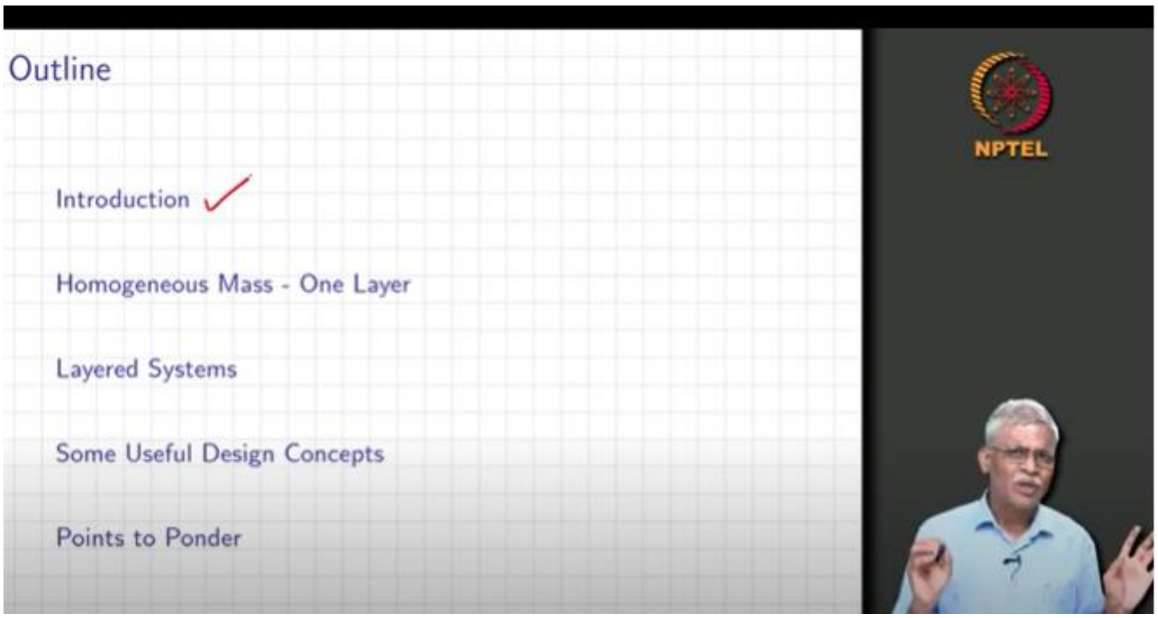

I am going to give some very brief introduction. Then we will talk about one layer. Then we talk about layered systems and some useful design concepts we will do. And in fact for some of the practicing pavement engineers, this will be very, very important. To just give a simple example, so let us say we have pavement 1, pavement 2, each of them having some given design life. You want to ask the question, should I use superior layer, less thickness or should I use regular layer, more thickness? How do we handle all these things? You may not have a straightforward answer during the course of this presentation, but at the end of the course you will have enough tools in your hand to understand this question. So if somebody comes and says you know what, let us use polymer modified binder and let us use a thickness of 4 cm or if somebody says let us use a VG20 binder and let us use

5 cm thickness? Are these two things equivalent because one is more pricey but less thickness and another is less pricey but more thickness. How do we answer some of these questions? So let us get on with the introduction.

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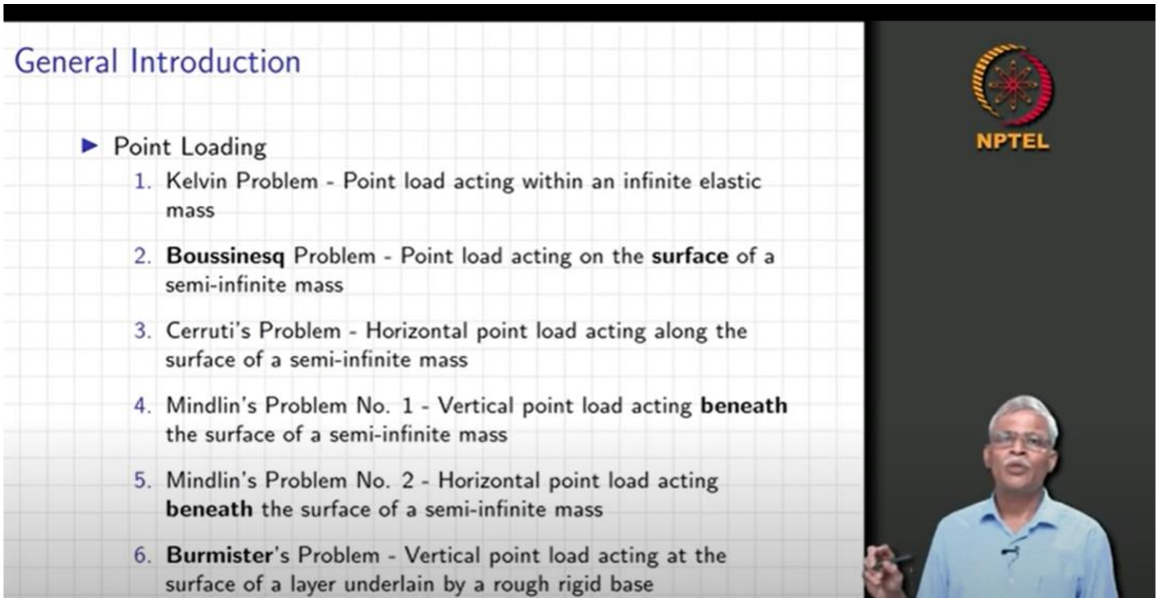

Now first and foremost thing you need to understand what is the problem that we really want to solve. The problem that we want to solve is I am just going to draw few lines. In fact, I will draw something like just two lines. Let us say we call this layer 1, layer 2 and layer 3. What we want to do is, we want to apply a wheel load and we want to ask the question what is the state of stress and state of strain at some significant points. This is what we want to do. Now what did people do in the earlier time? How did the original developments related to these things happened earlier?

So in the Kelvin problem the mass is infinite in the horizontal direction. So this is called as infinite elastic mass. There is some point load that is acting somewhere within this elastic mass. And given this what are the stresses and strains? This is what is really called as the Kelvin problem. These are problems that have been solved by some of the foremost mechanists 100 years or 150 years back.

The next problem which is very important to us which more or less is very useful to us for analyzing stresses and strains is what is really called as the Boussinesq problem. So what

Boussinesq did was he took this load which is there inside and placed it on the surface. Rest of the things are the same. Now this is semi-infinite because now I have a boundary here. So you really cannot say that this is infinite everywhere. I place it on the top of the earth. So he placed it and then again he was also interested in computing the stresses and strains. And in fact when Boussinesq published his results these results were very useful and used by the pavement engineers in the starting of 1910, 1920s and 1930s and all those things.

The next problem that was solved is what is really called as the Ceruti problem. What happens here is, if there is a load horizontal load that is applied on the surface of a semi-infinite mass. So this is what is really called as the Ceruti problem. Then you must be thinking where is this horizontal load coming. In fact, next time when you drive your car or two-wheeler or some of you happen to travel by a bus think about it. Let us say the driver brakes or you brake your car what happens? There is a load transfer from the rear to the front. That is what will really happen. And how does the load transfer take place? There is going to be a horizontal component and there is going to be a vertical component. Or think of it this way. You are negotiating a curve? So what will really happen? So if you go resolve the forces draw the free body diagram you are going to see one in the vertical, one along the direction and one in the transverse direction. So there are going to be three forces.

Think about this very carefully. So we need to talk in terms of horizontal traction along the direction of moment, traction across the direction perpendicular to it and vertical. And then you also will notice that I use this word traction. So traction is nothing but a vector. Basically you are talking in terms of load that is applied with a direction that is known to it. Normally in common language that we use in engineering we do not say it as traction. We keep saying it as I applied this stress. It is in a sense it is not correct because the stress is an internal state variable. So you do not apply the stress; you apply a traction. The body is in equilibrium and it results in the state of stress in the material.

Then comes the Mindlin's problem 1 and also Mindlin's problem 2. So what he did was vertical point load acting beneath the surface. So in fact one major problem that you will face when you are trying to solve the Boussinesq equation is the boundary conditions there.

What is the state of stress at the point below the point load exactly at that? So that becomes an indeterminate quantity. So what Mindlin did was talk about beneath the surface of it. So somewhere the load was applied and similarly he also solved the problem similar to what Ceruti solved. This is about horizontal point load acting beneath the surface of a semi-infinite mass.

Now comes the most important name in pavement engineering Professor Burmister. So what he did was he said vertical point load P acting at the surface of a layer underlined by a rough rigid base. So now what happens all these were only one layer whereas now you have layer 1 and layer 2. So I hope this is clear. So you are talking about Kelvin problem, Boussinesq problem, Ceruti's problem, Mindlin's problem 1, Mindlin's problem 2 and comes the Burmister's problem.

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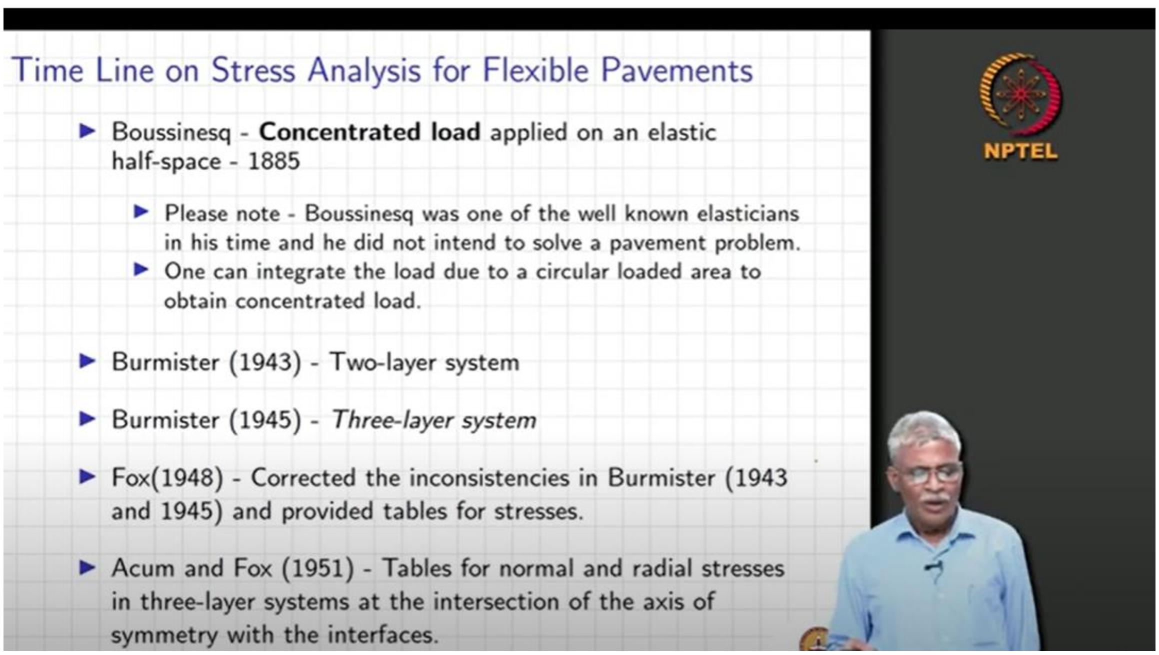

So what did he do basically? So let us now look at what are all the things that people did. What Boussinesq did was he applied a concentrated load on an elastic half space. So you are going to keep hearing this elastic half space all the time. So we will discuss what is elastic half space as we go along. So we need to understand very importantly one thing. He was a mathematician, elastician, person working in foundations in mechanics. He was not interested in solving a problem related to pavement engineering. And those days most of the time the pavement engineering concepts were quite empirical. What it worked in one location it might work here that is what it is. And in fact if you go and read some fascinating history about pavement engineering most of the time what you really wanted to do was to take a roller or something roll the existing road surface. Whatever is the road surface that you have gone repeatedly so that you get a very stiff rigid base surface. Now what can happen during the rain? In the rain the surface can become very slushy. And when you are driving your horse drawn vehicle you are going to have that small contact area of this steel wheel causing what is really called as the rut. And that is how you call it as rutting. So there was no big pavement engineering that happened. Most of the designs were quite empirical. So when Boussinesq wrote his paper, published it and immediately some of the very smart highway engineers realized that okay so now I have a solution to a problem. So I can actually compute the stresses and strain. So what you can do was to integrate the load due to a circular loaded area to obtain a concentrated load. This is something all of you have done in your strength of material engineering mechanics class wherein you talk about simply supported beam subjected to a point load or a uniformly distributed load. How do you go from one to the other? So all those things are known. I am not going to get into them.

So what Professor Burmister did in 1943 was to solve this two-layer system and then later he also extended it to the three-layer system. And in fact what exactly is this three-layer system? So if you recollect the original IRC cross section or any of the American cross section you saw that there were one or two layers mostly two layers of bituminous material. Then after that you saw two to three granular layers. So the granular layer could be base course, subbase course and maybe a compacted subgrade. Then after that you have the natural subgrade. So if you could club all the bituminous layers into one you have the first layer. If you could club all the granular layer into one you have the second layer and all of them resting on this subgrade. So this is layer 1, this is layer 2 and this is layer 3. So this is Burmister's 1943-1945.

Now when these solutions were released there was a quite a bit of excitement in the pavement engineering. You also need to understand the consequences of this year’s 1943-

1945; World War II is over. People were interested in rebuilding the country. Especially United States of America was kind of enthralled though they won the war and Germany was defeated. They were quite impressed with the autobahn system that was existing in Germany even those time. So they wanted to rebuild something like this and they were looking at how to go about it in a rational logical way. Because they had already the CBR method of pavement design which was very empirical. They had the Marshall method of mix design for bituminous mixtures which were very empirical. So there were many conferences, workshops were organized with the American Society of Civil Engineers. If you go back and search for this material, you will be able to find out if you are interested send me a mail I will be able to at least share some links of those proceedings.

So there was a big debate on how to go about doing it and then finally when this theory was published by Burmister everybody was thrilled. But Burmister is a professor in civil engineering and these are all detailed elaborate calculations and he had to do all these calculations without any recourse to a computer. So most of these calculations have to be translated into some kind of chart what we call it as Nomograph or set of table values. So that you could use a calculator no electronic calculator you know mechanical calculator and then interpolate and do it. So when Burmeister released it then in 1948 Fox from TRRL (Transportation Road Research Laboratory of United Kingdom) there were some minor issues related to the calculations and then he provided the complete table of stresses. And then in 1951 in the journal Geotechnique if you are familiar with this journal there were complete tables were provided by Acum and Fox and tables for normal and radial stresses in three-layer system at the intersection of the axis of symmetry with the interface. So what is the axis of symmetry? So this is the axis of symmetry. So I apply a load and at a point below the load what is the state of stress? So that it was released. So far so good.

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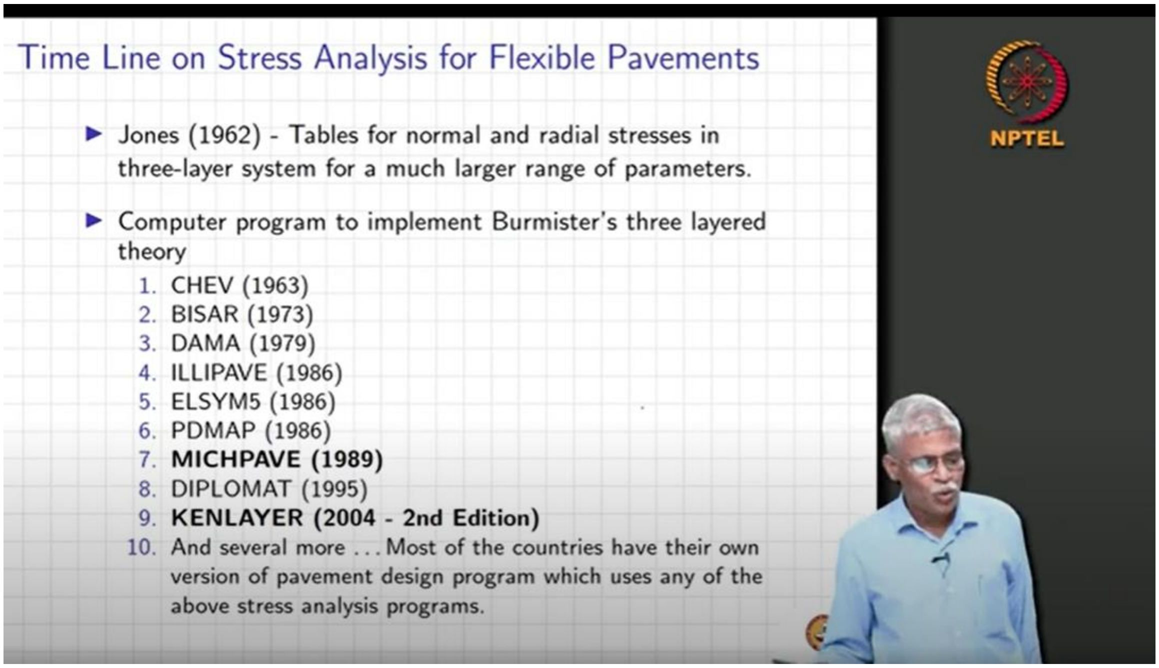

And in 1962 Jones in Highway Research Record publication published the table of normal and radial stresses in three-layer system for a much larger range of parameters. So now you could compute stresses and strains not only at the axis of symmetry but at different locations also. And once this was done this is also the time period in which you had computer programs coming in place. So there were many many computer programs that came in place and some of these programs that include SHEV, BIZARRE, DAMMA, ELIPAVE, ELSIM-5, PDMAP, MICH-PAVE, DEPLOMOT, KENLAYER and so forth so forth. And in India we use what is called IITPAVE. This is IITPAVE produced by IIT Kharagpur all those things are there. The interesting point is some of this are what you can say produced by Asphalt Institute. And there were some for instance companies such as Shell, Chevron which were making bitumen also released their own pavement design code. So you must be wondering like why would some an oil refinery or a bitumen producing company give code because since you are buying bitumen they will also give you a code of practice on how to use this bitumen to construct your pavement. So they also give software related to it. Most of the countries actually have their own version of pavement design program and any of this is there. And in fact you can go search for it. There are many freely downloadable software programs are used. In my opinion which is why I have highlighted this MICHPAVE and KENLAYER. These are very good programs and in fact you will be also using KENLAYER and you will realize how versatile it can be right.

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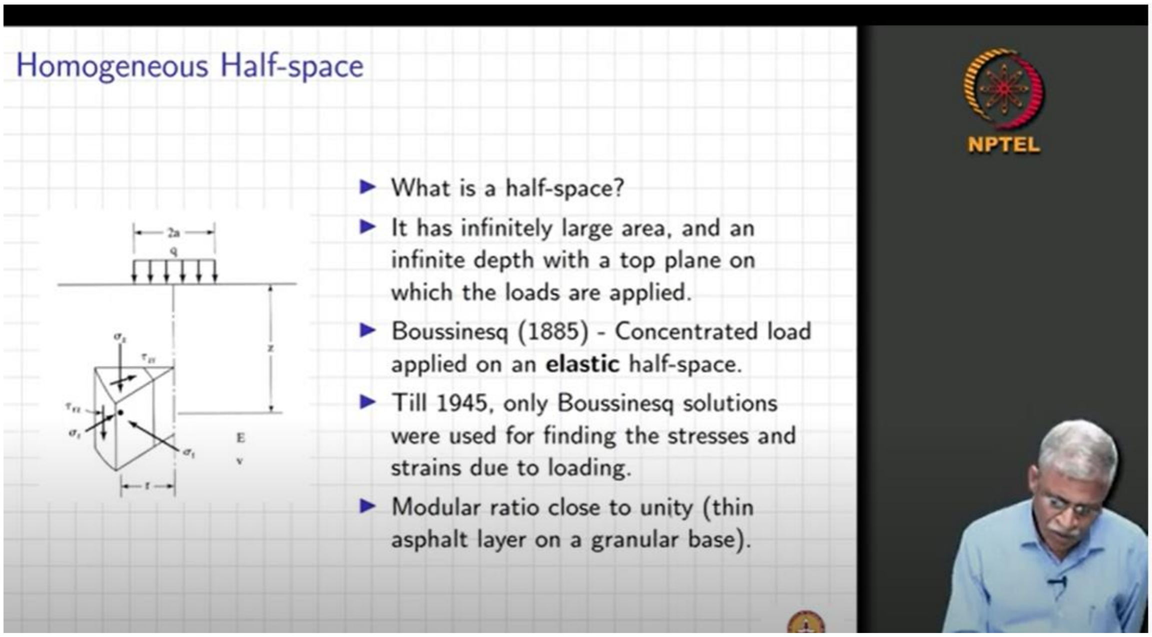

So let us now start talking about what is this one-layer theory okay. So few notations have to be understood by all of you. So this is 2a; where a is the contact radius. So 2a is the diameter, q is load per unit area. So we are talking about one layer and we want to talk about what is called as homogeneous half space. What is a half space? It has infinitely large area and an infinite depth with a top plane on which the loads are applied. So there is a line that is given here. So you cannot say it is infinite. It is finite here but surrounding it everywhere it is infinite. So that is what we really call it as a half space. It is not a full space. So the whole thing if it is surrounding it then you call it as a full space. So this is half space. Now there are few terminologies that I am going to use. These terminologies are not really consistent and correct within the framework of mechanics. See for instance I am since I am following the textbook by Huang I am using the same terminology used by Huang.

Since it is a circular area you can actually talk about one sector here. You can just take one sector and we are going to look at few stresses here σ_z. Ideally you should be writing it as

σ_{zz} but he writes it as σ_z, σ_t and σ_r. So let me write it here σ_t, σ_r, σ_z and then the shear stresses are τ_{zr} and τ_{rz}. And so this is at any given point which is at a distance r from the center line. So this is your symmetry and this is at z. So any point is notified by two coordinates r from the center line and z. So what kind of coordinate system is used? You are basically using a cylindrical polar coordinate system.

In fact, before we get into all these things you need to understand till 1945 that means till Burmister gave his solutions only these solutions were used. And most of the time you will be asking so what is this one there is no one layer no because they would have given some asphalt layer people started using in 1930s. So the idea is so let us say this is your pavement full pavement the asphalt layer will be very thin and let us say this is the bottom of the subgrade. So you can take this thickness h1 and let us say total thickness is h. So this proportion of h1 to h is so small that you can consider it to be one layer.

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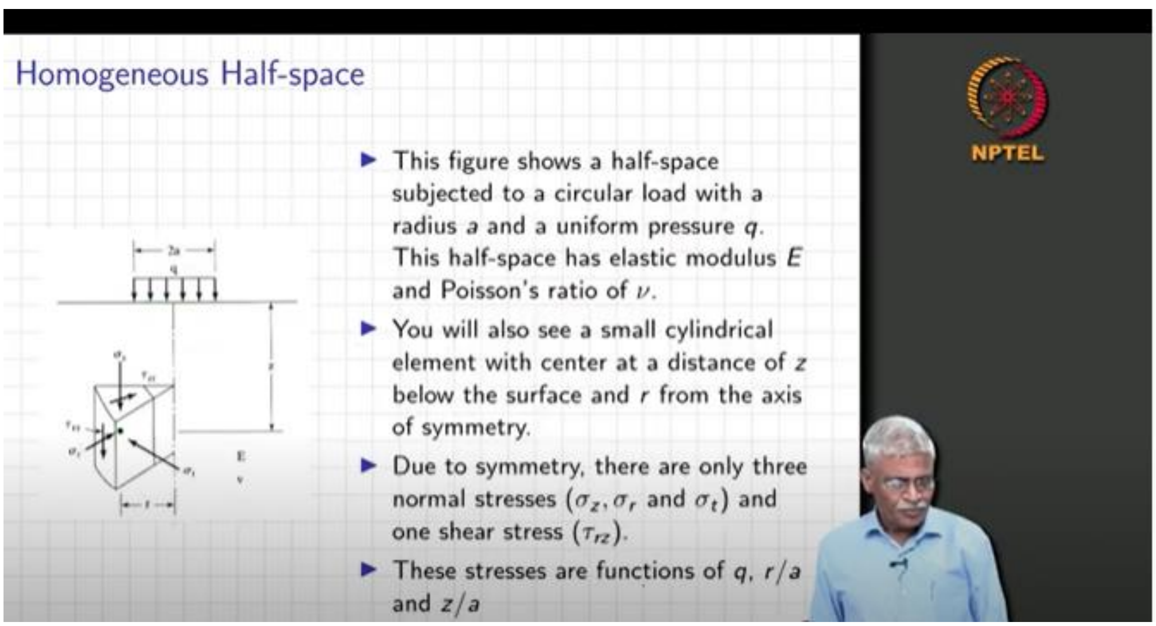

So let us read it out very carefully this particular figure that you see shows a half space subjected to a circular load with a radius a radius and a uniform pressure q this has an elastic modulus E and Poisson's ratio ν. And now you will also see there is a cylindrical element with a center which is at a distance z and r from the axis of symmetry. Now I am not actually getting into the complete and systematic procedure related to stress analysis here. What I am going to do is I am going to give these solutions or these equations as it is and not deriving them. Sometimes what we do is when we teach this in bituminous pavement engineering we find it easier if we show them the derivation show the students the derivations. And in a real life class what I normally do is I use this Saturdays to show it to the student, that this is how the derivation is actually done. But that kind of takes it outside the scope of this course so I would not do it but if any of you are interested again I can at least share you some handwritten notes and lectures but you need to have a little bit good background in mechanics to really understand that.

Now why did I say all these things if you understand something about symmetry because this is a circular area so it is enough if I just draw one radial line and compute the stresses and strains there as it is the same everywhere. So because of this there are going to be only three normal stresses and one shear stress. See ideally you will be talking about a stress tensor; a tensor has 3× 3 = 9 terms. Now due to balance of angular momentum we say that this is symmetric $\sigma_{ij} = \sigma_{ji}$ so what will really happen this 9 terms will reduce to 6 terms. When the stress tensor is symmetric what does it mean is $\sigma = \sigma^T$. So you can actually look at this half diagonal element will be identical so you will get only 6 components. Now if you appeal to symmetry this 6 terms will become 4 terms. Now how it will become 4 terms that is a different story for a different time. Now the next thing that you need to understand is these stresses that you see are the stresses that are used all the time; σ_z , σ_r and σ_t. They are going to be functions of q and we non-dimensionalize the coordinates. So we are going to write it as r/a and z/a that is what we are going to do.

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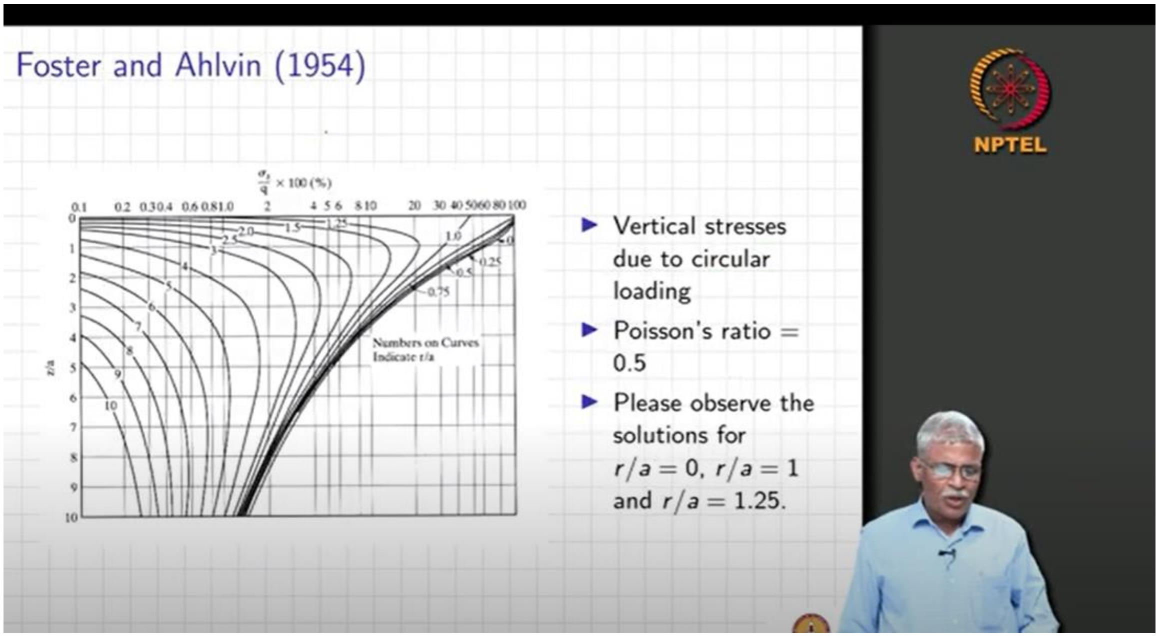

So I am just going to show you some important stats charts and we are going to look at it very very carefully. So this is a chart that is based on Foster and Alvin 1954 and now there is a Poisson's ratio that is given here. So the question that you want to ask is what should be the Poisson's ratio value. The Poisson's ratio for bituminous mixtures is roughly around 0.35. Now if you use 0.35 in the solutions or if you keep this Poisson's ratio as a variable the equations become very tedious. Then what these people did was there were many terms in which you had something like 1 - 2ν. So then you realize that if ν was taken as 0.5 there were lot of terms that will get cancelled out. So you will have a clean closed form solution but that is one thing. Second thing is you should be able to answer this when ν = 0.5 what does that mean? What kind of materials will have a Poisson's ratio equal to 0.5? You should be able to find it out. I am not going to give the answer. I might even ask this question in one of the weekly assignments. You can find it out because you already know answers to some of these questions. So we will keep this as 0.5. So then when we draw this picture; now we are in a position to clearly draw this kind of chart. And this kind of chart when they are given to a pavement designer, highway engineer or a student taking this course, life becomes very simple and easy because you can immediately compute the stresses and strain.

So let us first focus the attention on what is there on the x axis. On the x axis you see that σ_z/q. Now you know what is σ_z. Now why do we write it as σ_z/q? When we write it as σ_z/q we normalize it, it becomes dimensionless. So σ_z/q ×100. So basically you are writing in terms of percentages or you can think of it this way. How much of the load that is applied results into the stresses that you are really looking at σ_z. What you see here is the z axis vertically that is z/a, it goes here. So this is z/a and now you see lot of family of course. These figures indicate r/a. So now when r/a = 1 what does that mean? You are basically looking at the portion at the edge of the contact area where r = a. When r/a = 0 you are looking at the center portion. So let us first look at 0. So this is the center portion. Center exactly at the center of the point of load application. What you see here is the stress variation. So that means at the surface the load is 100%, the stresses are 100%. As you go down, go down, go down at different layers you can see how the proportion of the stress is reducing. Now let us take a look at 1. This is at the edge. So that means you have a pavement. So this is one layer. You apply a load. So this is your tire edge. So this is the tire edge you are looking at; r/a is 1. This seems to be having this kind of a variation. And the interesting part that you will notice here is this portion; from r/a = 0 to r/a = 1, the variations are something like this.

But as you go out of it, let us say 1.25 somewhere here or 2 or 2.5 what you really see here is the stresses actually are less in the surface and then slowly they start moving. So you are really looking at some kind of a bulb kind of an effect that you see here. So that is what you get here 1.25 or maybe 2 or something. So what it means at z/a, so near the surface the stresses are less. As you go down, you are basically seeing that the stresses slightly start increasing. So this is as far as σ_z is concerned.

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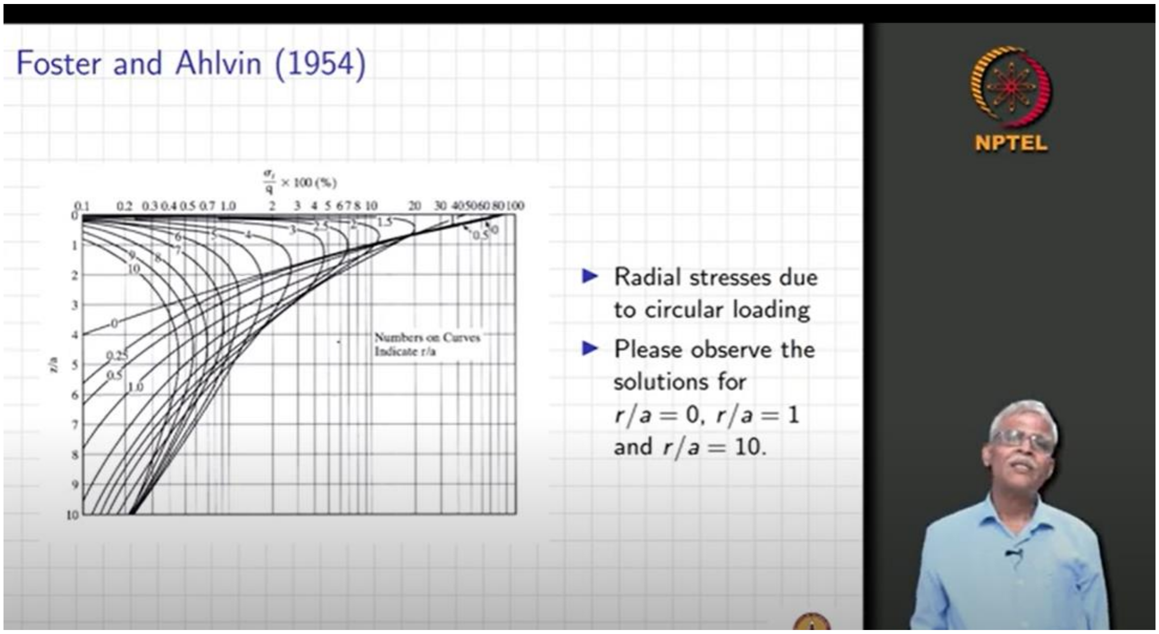

Now let us take a look at σ_r/q. Now what is σ_r? You can say call it as the radial stress. If you really want to use such kind of a terminology and again you are going to see. What is 0? 0 is at the center. What is 1? 1 is at the tire edge. And let us take the case of 2, something like this. So this is what you are going to get here. So you need to also take a look at, let us say r/a = 10. The stresses are considerably lower. Not only that, you do not actually see any stress in the surface in the first portion.

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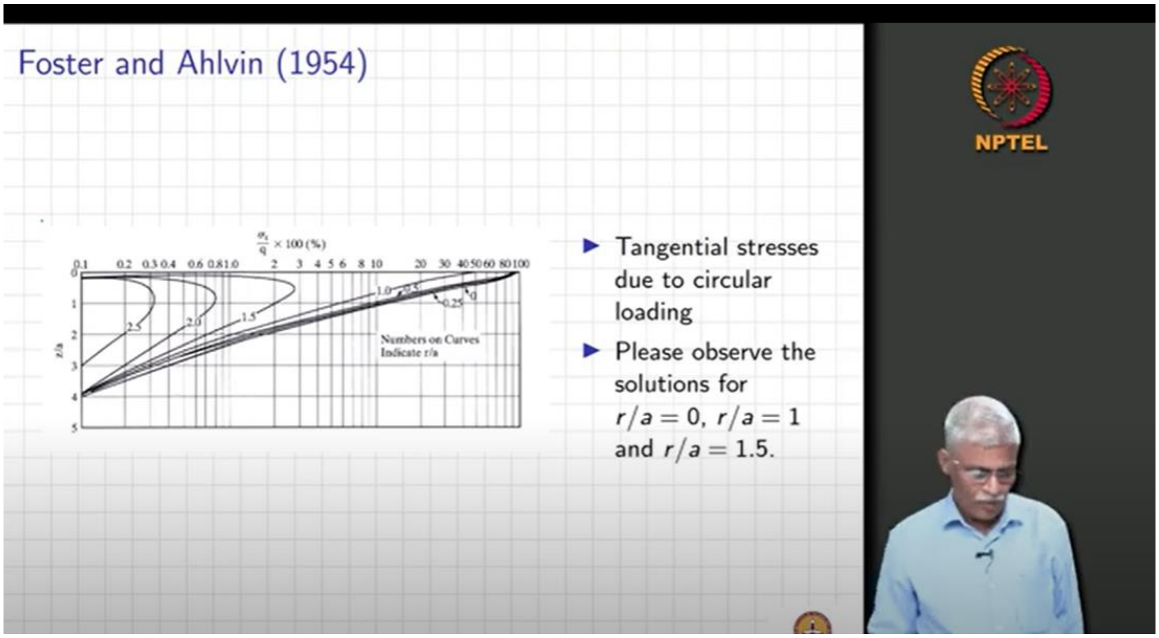

Now let us take a look at σ_t/q. What is σ_t/q? σ_t is your tangential stress. So again you see here, we will trace this. So this is 0. Then this is going to be 1 and let us do 2. So this is 2. So tangential stresses due to circular loading, so take a look at it. Now these are some of the stresses. Now how it is easy to solve this problem? It will be easy when you do it. So you will be given q. What is the load? That is, you will also know what is a. Where do you really want? You also will know what is really r. So knowing q, knowing a and knowing r, you will be able to find out the proportions and then you will be able to compute what is σ_t, that is the whole idea.

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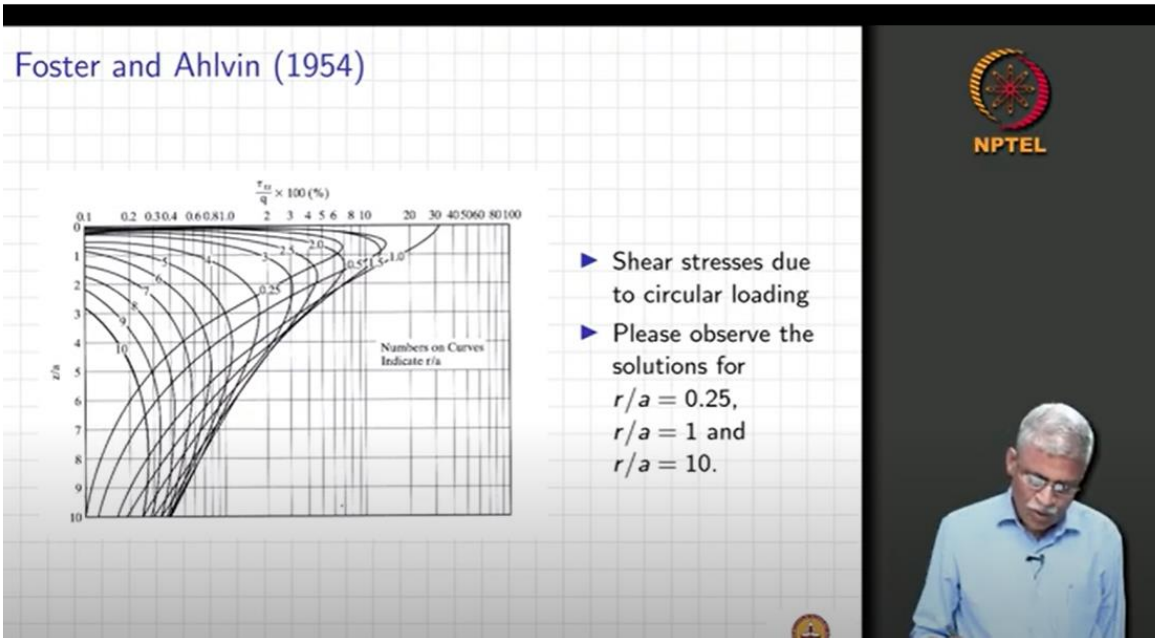

So now finally we look at the shear stresses. So what is the shear stress that you are really looking at? This is the shear stress that you are looking at. This is the shear stress that you are looking at. So let us take a look at, now what happened at 0? There seems to be nothing that is shown. Correct, no? There will not be anything. So let us take a look at 1. So this is for 1. So let us take a look at, let us say 0. So this is 0.5 and this is 1. So you will see some interesting trends here. 0.25, 0.5, 1, 1.5, 1 will be of some variable, one side whereas 0.25, 0.5 and 1.5 will be slightly different and the interesting point is again 10. And you will see that from the surface, let us say up to 3 times the load contact radius, you are not going to see any stresses here and even if there are stresses, they are going to be minimal. Whereas on this direction, in the r/a direction, you are going to see something sufficient. So if you draw now a tire, you are going to see that when r/a is 1, this is where you are going to have at the tire edge, you are going to have shear stresses.

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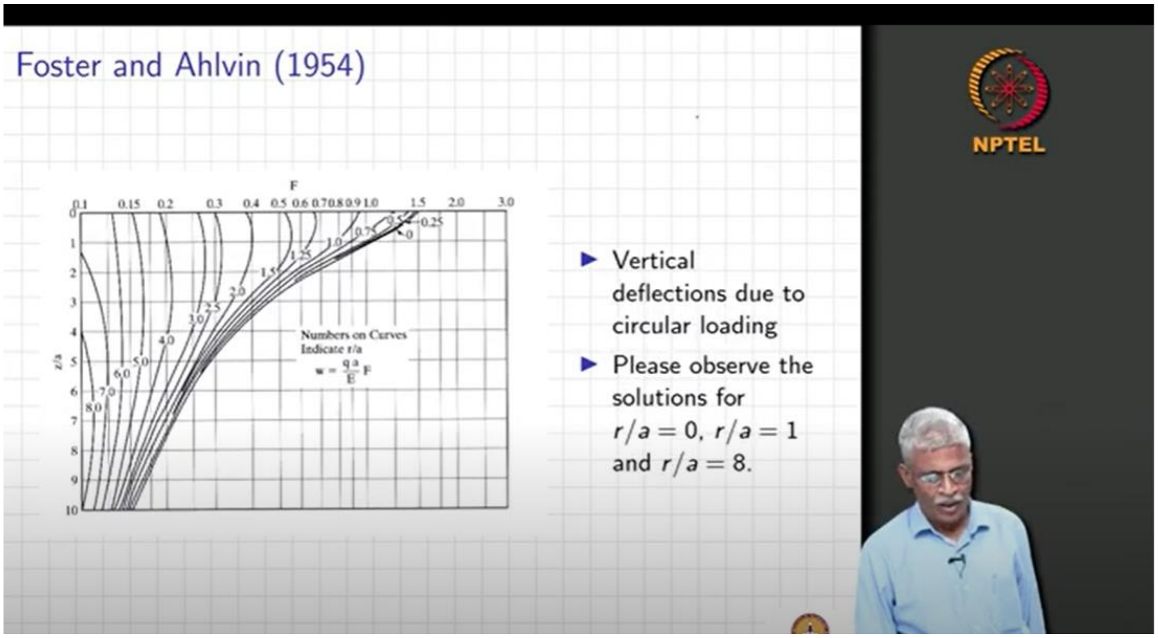

So now comes the deflections. Now what do you really want to know? You want to say that this is my pavement and I want to find out the deflections. So how do we really find out the deflections and you recollect your u, v, w. So w is reserved for the deflection in the vertical direction. So this is given by this formula. Let me write it as:

$$w = \frac{qa}{E} F$$

Now these charts and in fact I will also be sharing these charts with you and in exam most of the time, at least when I took my pavement design course, we will be carrying these charts only. So we will be basically using the scale and reading it and computing it. You will also be doing it because you just need to get an idea of how to go about doing this. So what you see here, you see that in the x axis the deflection factor is there. In the y axis what you see here is z/a and these are all the curves that indicate r/a. So let us say you are applying a load of 600 kPa and if a = 10 cm and you are interested in finding out let us say deflection at a point let us say 20 cm from the surface which is at let us say 20 cm from the center, what you will be doing here? You need to compute the deflection because q is known to you which is 600 kPa, a is known to you. I am going to give you E, so let us call this as 2000 MPa. You need to find out F. How will you find out F? You need to use this chart. So for using this chart you know how to find out z/a. z is given, a is given you can find out and similarly r/a that is also known to you. So what you do, you use this appropriate portion here, compute the deflection factor and then let us say the deflection factor is 0.5. So substitute it in equation to compute what is really the deflection factor.

The interesting part here is you need to know is when r/a is 8 or r/a is 0, this is where the maximum deflection is there. Then you are going to see r/a = 1 where is the deflection and as the r/a keeps increasing, because that means you move away from the center, the deflection keeps reducing which makes lot of sense here. So let me stop it here because we are going to continue in the next session on the layered system.