

FEM and Constitutive Modelling in Geomechanics
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Lecture: 1
Introductory Lecture and Course Outline

So, hello and welcome you all to this NPTEL course on finite element and constitute modelling in geomechanics by way of introduction I am Professor Rajagopal from the Department of Civil Engineering IIT Madras Chennai. And this course is on the finite element analysis and also the constitute modelling of the soil and we will be combining both of them for some analysis of geotechnical problems.

I will be taking you through the entire theoretical concepts and then some practical aspects that you require for doing this analysis.

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Understanding geotechnical phenomena


- Laboratory model tests – small-scale, large-scale, centrifuge – scale effects, difficulty to replicate exact soil conditions, boundary conditions, etc.
- Full-scale field tests – expensive & time consuming, may not be possible all the time
- Numerical simulations – quick & easy; can examine the influence of different parameters on the response – good idea of expected deformations, forces in members, failure mechanism, etc. can be obtained quickly
- Easy to perform parametric studies to examine the influence of variation in soil properties
- useful for optimal design & helps in remedial measures

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And let us see; what are the different options that we have when we want to know how our structure is going to perform after we do the construction, say the one of the simplest ones is by constructing laboratory scale models and these could be small scale or large scale or centrifuge models. But the problem here is the scale effects. Say let us say your real structure is some 20 meters height but your laboratory model is only going to be some maybe 200 millimeters or maximum 500 millimeters or 1 meter height but then how do we extrapolate.

Because of the difference in the stress levels your soil in the lab may not perform similar to how it will perform in the field. So, we will have some problem in extrapolating and then we may not be able to replicate the exact soil conditions that we come across in the field. Then other difficulty with these laboratory tests is the boundary effects. Because of the necessity we take a small model and that boundary may be too close to the applied loading and because of that we may get a stiffer response than what it should be.

And the other option is we go in for full scale filters but this is going to be expensive and also time consuming and it may not be possible all the time. We may not have the luxury of having time there is a lot of extra money for constructing a real scale structure see how it is going to perform and then based on that take decisions on improvising the design for obtaining the required the safety margins.

So, this full scale filters may not be possible all the time and the other option is that going for numerical modelling or numerical simulations. These methods are very quick and easy and we can examine the influence of different parameters. So, special when it comes to soil we do not know exactly what are the soil properties it could be say the friction angle may range anywhere from 30 degrees to 40 degrees depending on how you compact the soil or or depending on the environmental conditions.

And the advantage with the numerical models is that we can simulate all these variations and then get a range of responses both the most conservative and then at the other end the most realistic or most idealistic response. And we can get a good idea of the the expected the deformations and then the forces in the members and then the failure mechanism all these things we can get a good estimate so, that we can improvise our designs.


And as I mentioned earlier it is easy to perform the parametric studies to examine the influence of the variation in the soil properties. And it is these numerical techniques are quite useful in optimizing our design and also to take some remedial measures. Like let us say you come across some observation in the field you can simulate that back in the numerical model and see what is the influence of different methods of ground treatment and then use the best method for remediating your distress.

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Limitations of conventional methods

- Engineering approximations
- Mostly hand calculations
- Limit stresses are satisfied ($\tau_{\max} = c + \sigma_n \tan\phi$)
- Deformations are not estimated
- Influence of neighbouring structures?
- Influence of complicated loading & boundaries?
- Differentiation between excavation & construction
- Terzaghi's consolidation theory vs. coupled consolidation theories (Biot's theory)
- Increase in pore pressures during consolidation (Mandel-Cryer effect) – not possible to simulate by Terzaghi or decoupled consolidation theories


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Then one of the simplest methods that we have are the conventional methods that is what we study in our undergraduate and postgraduate courses on design and all these things I can call them as conventional methods and although they are good for design. But they are only good for most ideal conditions and they have lot of limitations see the one of the first limitation is we make lot of engineering approximations in arriving at a solution.

And the reason why we want to approximate is we want to use only hand calculations. At the most we might write a small computer program and do some calculations but then these are not very rigorous as those methods that we will be discussing in this course. And we will be anywhere satisfying at the limit state of stresses that is the maximum shear stress,

$$\tau_{\max} = c + \sigma_n \tan\phi$$

But then in most cases we do not know what is σ_n ?

Because we may not know what is the failure plane or the upset plane and then what is the operating normal stress on that we do not know we need to make a lot of assumptions and then apply this equation for getting a solution and after doing all these analysis we can only get some fact of safety. We will not get any deformations we cannot obtain the deformations or we cannot also obtain the forces.

And then the most important thing is we neglect the influence of the neighbouring structures. Like for example when we estimate the bearing capacity of the footing we assume that this particular footing is far away from the rest of the footings and our entire rupture surface is not affected by the neighbouring buildings or neighbouring structures either for our bearing

capacity or for settlements and that is a major limitation because in reality we will never have any structure that is built on its own that is not affected by our neighbouring buildings.

Then we can only take care of some simple loading and simple boundaries in our analysis like for example our boundaries are far away that our loading and then their rupture surface are not affected by the boundaries. And when we do these simple calculations we will not be able to differentiate between the responses that we get under during excavation or construction. Like let us say you build a; you want to build a retaining wall or a slope.

And let us say we want to build a slope and we can build that slope by placing the soil and doing the compaction or doing the excavation of the existing soil. Both will look alike at the end of our construction but then the response may not be the same because one is a constructed slope and the other is an excavated slope. We will not be able to differentiate between these two when we have these hand calculations based on conventional methods.

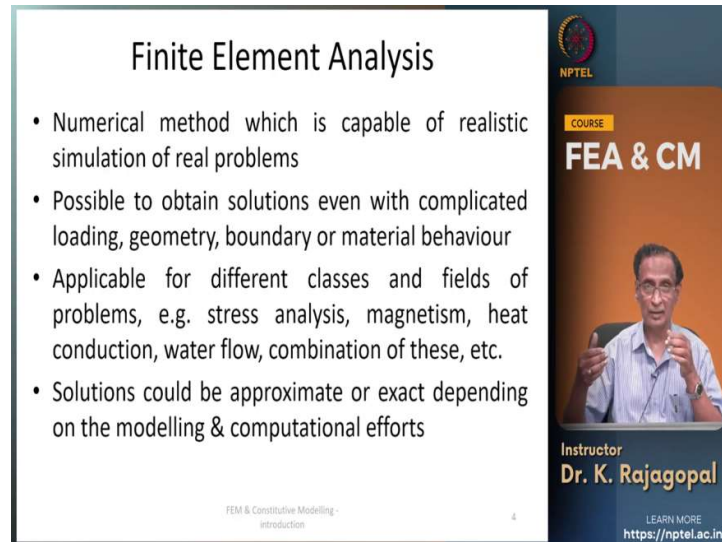
And the other limitation that we have is our Consolidation Theory these this Terzaghi's Consolidation Theory that is that assumes that the total stress remains constant the total stress being the pore pressure plus the effective stresses and the theory we call it as a decoupled theory wherein we predict the pore pressures separately. And then we predict or we estimate the degree of consolidation and then we estimate the settlement at different times based on the degree of consolidation.

Later we have more rigorous coupled consolidation theories for example by Biot. And the one feature that will not be able to replicate by using the third Terzaghi's theory is our increase in the pore pressures the Tarzaghi's theory it is assumed that the pore pressure is maximum at the initial construction time. And then gradually the pore pressures will go on decreasing with the time.

But then if we look at more rigorous models like our Biot's Theory sometimes the pore pressures might increase with time initially and then start decreasing and that is called as Mandel Cryer effect and because of that our effective stresses may not always increase they might decrease. Because if our Mandel Cryer effect is too severe because of some constraint from either the boundary condition or because of too much of loading and that could become more critical.

And this type of things will not be able to take account of in our hand calculations are simple conventional methods. So, we need to go in for more advanced methods like our finite element analysis and some other things.

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The slide is titled "Finite Element Analysis" and contains the following bullet points:

- Numerical method which is capable of realistic simulation of real problems
- Possible to obtain solutions even with complicated loading, geometry, boundary or material behaviour
- Applicable for different classes and fields of problems, e.g. stress analysis, magnetism, heat conduction, water flow, combination of these, etc.
- Solutions could be approximate or exact depending on the modelling & computational efforts

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And so, let me just tell you what I mean by finite element analysis actually I am not going to explain everything now I will explain that later. So, it is one of the numerical methods that is capable of realistic simulation of the real problems. This is one of the numerical methods because there are other methods like the finite difference or discrete element methods boundary element methods and so on.

But finite element method is very well proven and it has the capacity to realistically model the real Geometry the Loading and then the response then of course the constitutive behaviour of the material. And it is possible to apply the finite element analysis for problems even with very complicated loading or the geometry or the the material behaviour and so on. And so, we can even have some problems where our boundary is moving with the time.

That is also possible okay and this finite element methods they are applicable for different classes of problems like we can use these finite element analysis methods for doing Simple stress analysis or we can apply in electrical engineering or we can apply in physics like magnetism, heat conduction at the water flow or we can combine all of them together or some of them together.


Like for example in our geotechnical problems we are going to combine a distress analysis with the water. So, we will have both the stresses and the soil and also the pore pressures and then we will have the deformations and then the pore pressures and so on. And it comes naturally and our solutions could be the approximate or exact depending on the efforts that you take in modelling and also the competition.

If you are willing to spend a lot of time on developing a more detailed model then we can have more accurate results at the same time we will be spending more time on the computations both time and effort. If you are willing to spend that we can get better results okay.

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Solution methods	Solution requirements				
	Equilibrium	Compatibility	Constitutive behaviour	Boundary conditions	
				force	displacements
Closed form-mathematical	Yes	Yes	Linear elastic	Yes	Yes
Limit equilibrium	Yes	No	Rigid-plastic	Yes	No
Stress field	Yes	No	Rigid-plastic	Yes	No
Lower Bound	Yes	No	Ideal plasticity	Yes	No
Upper Bound	No	Yes	Ideal plasticity	No	Yes
Winkler spring models	Yes	Marginally	elastic springs	Yes	Yes
FEM, DEM, Finite Difference methods	Yes	Yes	Any constitutive model	Yes	Yes

Potts & Zdravkovic (1999)



And this table shows the different methods of analysis and then what are the different conditions that we solve that we satisfy. See the closed form or mathematical methods they satisfy the equilibrium equations. So, basically the equilibrium is the applied force should be equal to the reaction force or the water coming in should be equal to the quantity of water that is going out that is another continuity equation that we have in geotechnical engineering.

And then the compatibility between the strains between different locations or the deformations at different points within a continuum it should be satisfied that this is also satisfied. Then the constituted behaviour because all these mathematical models they can only be applied for a linear elastic problems. Like if you open any elasticity textbook you will get a lot of solutions for the case of any say the loading on a surface or any type of loading and a surface and we can get the deformations and then the stresses.

And then we can satisfy both the force and displacement boundary conditions by these are closed form solutions but the main problem is our constituted equation and then the boundary should be simple and then we will be able to apply and then we have the limit equilibrium methods like the simple slip circle methods.

See these are they assume the rigid plastic type constitute model and we will not be able to satisfy most of our compatibility equations because of that we will not get any deformations. We cannot predict our deformations and then there are other methods like the stress field method but there is also very simple method where we can satisfy only sum and we will not be able to get the displacements.

And then the lower bound upper bound methods there also limited in their capabilities because we have only simple constitute model and we are not satisfying all the conditions like equilibrium compatibility and constitutive Matrix and so on. Then another popular method in geotechnical engineering especially in Foundation Engineering is the Winkler Spring approach.

Say this Winkler Springs the entire soil is represented by small springs that are equivalent that have some stiffness corresponding to the soil. We will be able to satisfy the equilibrium and compatibility will not be able to satisfy completely but to some extent we can satisfy the compatibility. And the constituted behaviour now we have only elastic springs or at the most we can put some non-linear springs.

And the displacements that we get may not exactly represent the reality because the Winkler Spring it has only one simple spring whereas the soil may have different layers say soft layer hard layer and another hard layer and so on. And we will not be able to replicate all these layers by means of one single spring. And so, the deformations that we get they are not exact then we have other numerical methods like the finite element method and then the discrete element method boundary element and finite difference methods.

And we can satisfy all types of equilibrium equations compatibility boundary conditions Force conditions and then when at the time varying boundary or the time varying forces. And we can also satisfy any type of constituted model. The constituted model for soil typically

will have one modulus for loading and unloading we may have some other modulus and then reloading we may have some other modulus or if you apply static loading you will have one modulus or if you apply cyclic loading you may have some other type of loading.

All these things we should be able to take care of by these advanced numerical methods and our finite element method that we are going to discuss it satisfies all the given conditions like equilibrium compatibility and then constitute the behaviour of the materials and then we will be able to satisfy both the force and displacement boundary conditions.

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The slide is titled "Constitutive behaviour" and contains a list of five bullet points. To the right of the text is a video thumbnail for the course "FEA & CM" featuring Dr. K. Rajagopal as the instructor. The NPTEL logo is at the top right, and the URL "https://nptel.ac.in/" is at the bottom right.

- Stress-strain response of materials
- How does the material respond to applied stress or strain?
- Simplest is linear-elastic i.e. stress and strain are proportional during loading or unloading
- Soil is a complicated material with inelastic and nonlinear behaviour
- Soil behaviour is dependent on water content, rate of loading, in situ stresses, initial void ratio, type of soil, age of soil deposit, etc. – highly complicated

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Then what is constituted behaviour of the material? Basically the stress strain response of the material. So, if you apply some stress what is the strain or if you apply some strain what is the stress that is that is what we mean by the constitutive behaviour and how does the material respond to applied stress or strain. And then also the manner in which you apply whether you are going to apply a gradual strain or an impact strain or impact stress.

That also might affect our response it is our constitutive model should be able to represent or simulate this type of behaviour. One of the simplest constitutive models is our linear elastic that is the stress and strain are proportional to each other during both loading and also unloading and it is the simplest type of model and when especially when we are doing the any closet form solutions.

Like the elasticity solutions they are all based on simple linear elastic analysis. But then our soil is a highly complicated material it is not so, simple because under the same strain it

might behave differently at a low stress level low confinement or at high confinement or at a medium confinement. So, we have to also take into consideration these additional aspects. And then the soil is a highly non-linear material it is yeah there is no one-to-one correspondence between the stress and strain.

And also it is inelastic that is during the loading it might behave one way and during unloading it might behave in some other manner. and also the soil behaviour is dependent on the water content and then the rate of loading and the in situ stresses, initial void ratio, the type of the soil whether it is a loose sand dense sand or normally consolidated clay or over consolidated clay and so on.

And then the age of the soil deposit is, special in the case of clays the age might increase your bonds; the bond strength and because of that the strength may be more. And so, it is highly complicated but then through the finite element analysis we should be able to replicate all these of these behaviours.

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The slide is titled "Brief outline of the course" and contains the following bullet points:

- Both finite element analysis and constitutive modelling are combined into a single course
- Fundamental & practical aspects of finite element analysis
- Description of constitutive behaviour of soil and its simulation through numerical methods
- Understanding of the concepts through applications to typical geotechnical problems

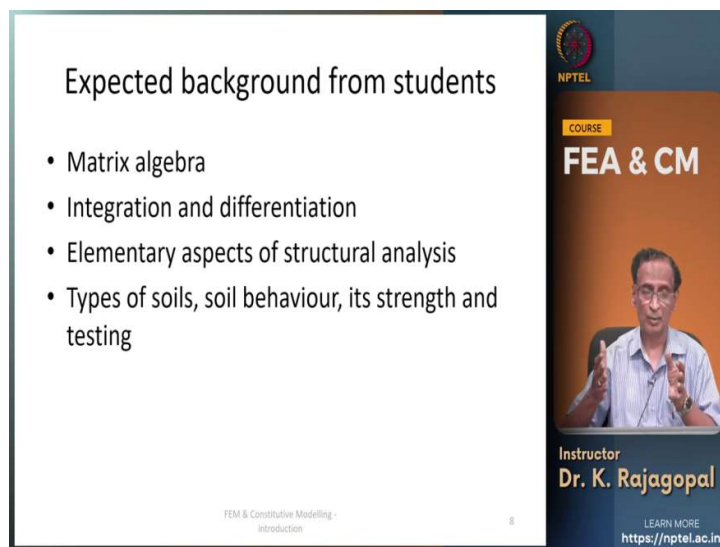
At the bottom left of the slide, it says "FEM & Constitutive Modelling - Introduction" and "7". On the right side, there is a video thumbnail for the course "FEA & CM" by "Instructor Dr. K. Rajagopal". The thumbnail also includes the NPTEL logo, the text "COURSE", and "LEARN MORE https://nptel.ac.in/" at the bottom.

And the just a brief outline of this course the main thing with this course is both finite element analysis and then the constitute modelling are combined into one single course. So, it is a bit ambitious course that I am going to cover both the finite element analysis and then constitute modelling and I will be discussing both the fundamental and then the Practical aspects of finite element analysis and then the constitutive modelling.

And I will be describing some of the constitutive models especially the behaviour of the soil and its simulation through the numerical models okay the because when we do any Laboratory test like a Triaxial compression test or anything we observe some things then what can we interpret from these observations and then how we can model them through this constituted models that that we will be seeing.

And we will be playing these concepts to some typical geotechnical problems like bearing capacity of a footing or lateral Earth pressures or the stability of a slope and so on so, that we can understand how we can apply our theory for practical situations okay.

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The slide is titled "Expected background from students" and lists the following prerequisites:

- Matrix algebra
- Integration and differentiation
- Elementary aspects of structural analysis
- Types of soils, soil behaviour, its strength and testing

The slide also features a vertical banner on the right side with the NPTEL logo at the top, the text "COURSE FEA & CM", a photo of the instructor Dr. K. Rajagopal, and the text "Instructor Dr. K. Rajagopal". At the bottom of the banner, it says "LEARN MORE https://nptel.ac.in/".

And what is the expected background of the students? It is not exactly necessary but it will be preferable if you have this type of background you should already know a little bit about Matrix algebra because all our calculations are going to involve matrices. We will be dealing with only matrices and I hope that you are familiar with some integration and the differentiation because these things we will require for our competitions and then some aspects of structural analysis.

At least one course on structural analysis you should have had undergone so, that we can you can appreciate the terms and other things that I am going to explain. And at least you should have had one course on the soil behaviour that describes about the different type of types of soils and then their behaviour and then their strength and then the different tests that we perform for determining these tests like the direct shear test or a simple shear test Vane Shear test track cell compression and so on okay.

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The slide is titled "Response quantities of interest" and lists the following items:

- Settlements
- Lateral deformations
- Pore pressures
- Time rate of settlements
- Forces in members
- Response under static/cyclic/impact/dynamic loading
- Dynamic amplification

At the bottom left of the slide, it says "FEM & Constitutive Modelling - introduction" and "10". On the right side, there is a video thumbnail for an NPTEL course titled "FEA & CM" by instructor "Dr. K. Rajagopal". The thumbnail includes the NPTEL logo, the course title, a photo of the instructor, and the text "Instructor Dr. K. Rajagopal" and "LEARN MORE https://nptel.ac.in/".

So, let us see what exactly we are going to come across. Say some typical geotechnical problems may might be related to foundations because in fact foundation engineering is one of the most fundamental of all the geotechnical engineering because we required these foundations. There could be shallow foundations or deep foundations. Then we might involve in deep excavations and if it is a deep excavations and a far away area where there are no structures nobody will care.

Nobody will care to do any proper analysis because the consequence of any collapse is not very severe. But let us say you are doing this deep execution in the middle of a city where you have already a lot of buildings nearby or your railway line or the road is there and any small mishap that happens because of your deep Excavations might lead to to to loss of economy because of the stoppage of traffic and or something.

So, we should be very careful when we are doing any deep excavations in city areas, construction of embankments and especially the embankments are constructed on a soft clay because these tend to to undergo a lot of settlements and then the soil might undergo lateral exponential leading to leading to slip circle failure. And we may be coming across retaining structures in the form of say retaining walls or sheet pile walls or anchored sheet pile walls and so on.

Then tunnels the tunnelling is one of the most complicated engineering activities and especially the tunnels done through city areas are really complicated because we need to be

doubly sure about the structures the safety of the structures that are there above the tunnel. And something unique to geotechnical engineering is the flow through the soil and then through Earth dams especially when you have an Earth dam if you have water impounded what will happen?

See the water is going to flow and if it is going to simply flow that is fine but as it is flowing it wants to take all the fine soil particles along with it. So, we need to be concerned about the gradient of the flow and then as compared to that the critical hydraulic gradient and so, that will not end up with any piping failure. Then what are the different responses that we are interested in determining through these analysis.

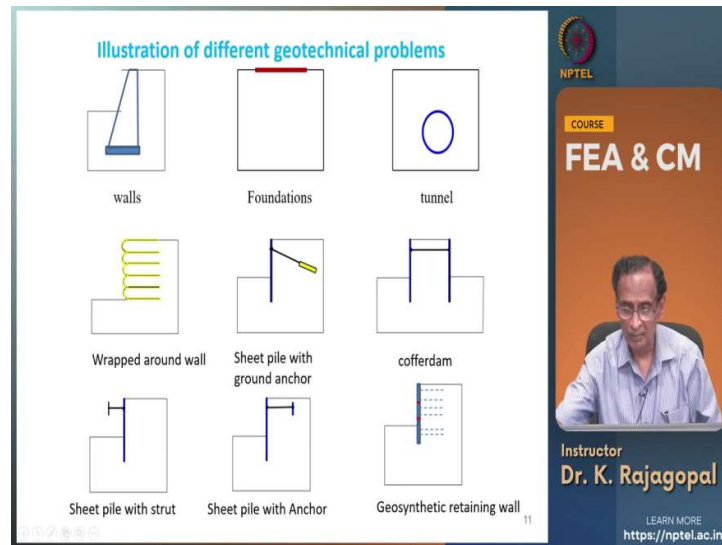
Obviously the first thing is we would like to know if you construct what will be the settlement both total and differential. Then what could be the lateral deformations especially if you are making any deep excavations what will be the lateral deformations. And if you are doing any construction on soil with water what are the pore pressures and then if you are dealing with any soft clay what is the consolidation what is the time rate of settlement?

Because this could govern our designs if you are not able to get rid of your settlements in a short time you might want to go in for some ground improvement. Then what are the forces in the members either a reinforcement layer or a sheet pile wall or your anchor what are the forces that are developed that need to be accounted for in the design. Then we should be able to get the response under static loading or cyclic loading.

Like our load is a repeated number of times cyclic repeated loading and then impact loading sudden up and suddenly applied load or a dynamic loading or a harmonic loading or a seismic loading and when we have the impact and dynamic loading we should be concerned about the dynamic amplification. So, if there is any resonance. So, if there is a frequency match between this structure and then the loading your response might be amplified that your forces may be higher and your displacements may be higher.

So, all these things we should be able to predict through our numerical modelling or the finite element analysis.

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So, I am just giving you some sketch of different numeric some problems that we might be dealing with. So, we may have a stability analysis of a retaining wall this could be either a gravity wall or a cantilever wall and we may have a foundation problem or we could have a tunnel or we may have a retaining wall built with geosynthetics by wrapping around. Or we may have a deep excavations supported by sheet pile wall with a ground anchor or we may have a cofferdam and the sheet pile wall with a strut especially.


If we are doing any narrow excavations the strut is something that we come across the strut is actually when our member is in compression, we call it as a strut and when it is in tension we call it as a as a type or okay. And you may have a sheet pile wall with an anchor wherein our sheet file wall is there and we have a tie rod then we have an anchor plate so, that we get some additional lateral load capacity to the to the retaining wall or we may have some geosynthetic retaining wall wherein we have some facing and that is connected to some layers of geosynthetic reinforcements so, that we can build a vertical soil slope.

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Course outline


Week number	Course content
1	Introduction, matrix algebra, prismatic elements (spring, bar & beam elements) and matrix structural analysis
2	Variational principles & Rayleigh-Ritz procedure
3	Analysis of continuum & different types of stress states
4	Generalized Coordinate & Lagrange methods for shape functions
5	Numerical integration techniques & derivation of isoparametric shape functions
6	Isoparametric element calculations – numerical calculations

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And so, the brief course outline is like this. This is a 12-week course and these are the topics that I am going to discuss in different weeks. The week number one that is this week this is the first lecture introduction lecture and then I will be going through a brief Matrix algebra and especially solving the simultaneous equations by the matrix inversion methods or the Gauss elimination method.

And we will look at the prismatic elements like the one dimensional elements like spring bar and beam elements and then how we can perform some simple matrix analysis matrix structural analysis. And basically the first week itself you will know what we are going to do in the rest of the course because I will be looking at all the steps that that we are going to have in the finite element analysis.

And then in the second week we will have some lectures on the mathematical aspects of the finite element analysis the variational principles and then the Rayleigh-Ritz procedures because there is a lot of mathematics behind this. And in fact the finite element technique was developed by mathematicians and initially when it was developed there were no computers. So, the way they had envisioned this finite element analysis it was totally different.

It was a more of a mathematical approach but later with the computers coming in and then their power increasing we came out with the different analysis methods. But the fundamental basis can be seen in the variational principles and then the Rayleigh-Ritz procedures that were developed long back. And then after this introduction we will go into the analysis of a continuum.

See continuum is our soil is a continuum and the continuum is different from a bar or beam element because bar or beam element we can imagine the shape and then the size whereas the soil it is there everywhere and it is not so, easy to imagine its shape. And what are the different types of stress states that that we can imagine in a continuum then what are the different equations all these things will be looking at the third week.

Then the fourth week we will be developing some methods for developing the shape functions for describing the variation continuous variation of stresses sorry the displacements and the strains within this continuum by classical methods like the generalized coordinate and the LaGrange methods. And then we will be moving on to do other methods like the numerical integration methods.

Because when we are doing any calculations through a computer we like to make best use of the computational power and we convert all our equations into some form that are easy to be solved by a computer. Because as human beings we have one method of analysis whereas a computer it cannot think like a human being it can only work like a mechanic it can just simply do repeated calculations.

And then we have to pose our problem in a form that is more suitable for competitions and that is achieved through this numerical integration methods. And the correspondingly the entire finite element analysis methods they have been changed from classical methods to isoparametric methods. And then in the after some introduction to developing the shape functions of the isoparametric elements we will be moving on to isoparametric calculations.

Because how can we do the different calculations and I will be demonstrating some hand calculations and then some simple computer programs I will be giving for doing these isoparametric calculations.


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Then we will also of course go through some equilibrium equations and then developing the governing equations for systems and solving. Then when it comes to geotechnical engineering we have to take care of in situ stresses that is the pre-existing geological stresses because sometimes our entire response might be dependent on the on the in-situ stress states. Like sometimes our initial at the pressures could be very very high especially if you are dealing with any over consolidated clay.

And in such cases if you make any excavation you have to expect a significant release of stresses that might cause problems the neighbouring buildings or our own support structures. Then we will be dealing with the joints and then the joint elements or the weak joints within the materials okay and that is default planes and so on. Then when it comes to soils we can call it as a semi-infinite material.

That is however deep you dig you will come across soil or however far you go you travel on the land you will come across the soil. Then how do we account for that in the finite element analysis because we have the semi-infinite extent that is when we do the mathematical analysis we can do the integration from minus infinity to plus infinity and take care of the semi-infinite nature but then how do we do that in finite element analysis.

Especially our computers do not like the word infinity it they will just simply crash they cannot accommodate this Infinity. So, we need to look at some special elements that are called as infinite elements or mapped infinite type elements that are specially developed for taking care of this infinite soil mass. And then we will be moving on to our non-linear and

constitutive models. So, mean time we will discuss the different non-linear finite element techniques and the different type of constitutive models.

And in the week 10 we will be looking at the non-linear constitutive models like the variable modular models, hyperbolic model, Mohr-Coulomb model. Then what are the different stress correction methods because when we are dealing with any constitutive model like for example our Mohr-Coulomb model the $\tau_{max} = c + \sigma_n \tan \phi$ or if you draw any Mohr Circle it should be just tangent over yield surface.

What if our stress state exceeds real surface that we should not allow because our soil strength is only so, much? So, at the maximum the Mohr circle that we draw for the current stress state should be tangent and if it is exceeding or if it is violating the yield condition then we have to apply some correction. And there are different methods for doing this correction that we will be discussing.

And then elastoplastic constitutive models then how to incorporate the dilation of the soil because the dilation is the volume increase under shear strain. And if you have a linear elastic material or a non-linear elastic material this shear and the normal strains and the shear stress and normal stresses are decoupled they are not related to each other. If we apply a pure shear strain you will develop only pure shear stress but then the soil we know even if you apply some shear strain.

Like for example in our direct shear box test we could get some volume expansion especially when you have a dense sand or a highly angular certain soil particles. Then we need to have some hardening soil models which differentiate between the loading and unloading parts and so on. Then the last week I will be introducing the soil consolidation and then how to calculate the response under impact and the dynamic loading.

How do we modify the finite element equations for this type of loading because most of the course will be dealing with only static loading but then in last week I will introduce some aspects of soil consolidation water flow and then the dynamic loading okay.

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Stabilisation under progress



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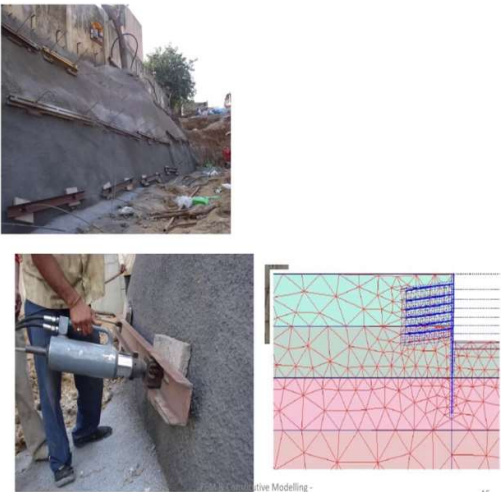


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So, now let us see what are some problems that we can come across and then how we can do some numerical simulations. See here what you are looking at say it is a deep excavations and there are some buildings just outside of this construction area. And this particular deep excavation is supported by soil nailing and especially some of these rows they are pre-stressed rows.


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So, here you see and the surface itself is protected by spray concrete and then some rows of these nails they are pre-stressed and. So, how do we model this? See in the finite element analysis is very simple you see here you have your slope either a vertical slope or an inclined slope. Then we have these nails that are installed at different depths and then we gradually excavate the soil.

And then look at as we are excavating the soil we install the nail exactly we can follow the same sequence as we follow in the construction okay. And after you install the first row of nails then you go for further excavation and then install the second of nails and so on.

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The slide displays a photograph of a deep excavation site on the left, showing a sloped wall supported by a grid of steel reinforcement and concrete. To the right is a finite element analysis (FEA) plot showing a cross-section of the excavation with a color-coded stress distribution. A vertical color bar on the right of the plot indicates stress levels from 0 to 100 kPa. Below the plot is a table of 'Total displacements (mm)' with a maximum value of 201.3 in Element 81 at Node 1758. The slide also features the NPTEL logo, the course title 'FEA & CM', and a video of the instructor Dr. K. Rajagopal. The bottom of the slide includes the PLAXIS logo, project details (Project: Zone-C-final section-4-galions 398, Date: 06/2012, Location: Indian Institute of Technology Madras), and the slide number 16.

Or we may have some other excavation like this here is another it says sloped excavations and then the slope itself is stabilized by grouted nails then the spray concrete. Then what should be the length of these nails to achieve a required factor of safety or what is the effect of any rains and all these things we should be able to take care by finite element analysis. Here you see some soil nails and here you see this free phreatic surface that we can simulate the different phreatic surfaces based on the rainfall that we have.

And we can see what is the most likely possible rupture surface and then we can also get our fact of safety.

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Identification of Failure mechanism through $c-\phi$ reduction in Plaxis program

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And here is another close-up of the same thing because once you have this volume of soil that is going to be subjected to active deformations we can take care. Because in at the ground level our structures should not be built within the volume of soil that is subjected to any ground movements and that is one information that we require for our construction purposes.

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Typical failure surface from FE analysis

possible slip surface

Slip surface is not bi-linear wedge or circular surface

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And here is another picture of the same thing close up picture.

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Aerial view showing the runway embankment at Kannur airport

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And here is you are looking at a at a rail at a Runway embankment it is a it is about 70 meter site and it is consisting of number of slopes number of slopes and number of bumps. Because data can one continuous slope and then because of the tight land situation we have to have very steep slope and so you need to use some layers of geosynthetic reinforcement so, that we can construct a steep slope.

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Limit equilibrium analysis of the stability of reinforced soil slopes by sliding & slip circle failure mechanisms

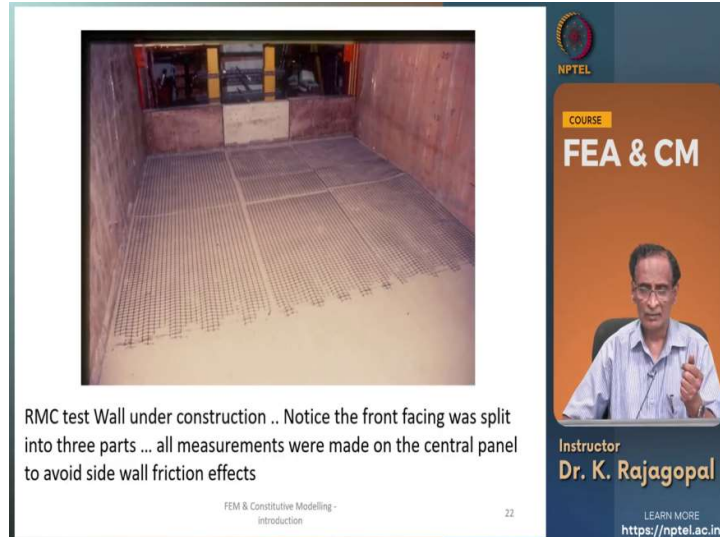
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And so we can do the analysis either by limit equilibrium method like our slip Circle analysis either by assuming the slip circles or by bilinear sliding mechanism like this and we can calculate our factors of safety and so on. Or we can go in for the full-fledged finite element analysis that will tell us the forces developed in the reinforcement layers and then what are the deformations and then we will be able to include the actual geometry.

We do not need to simplify any geometry any specific say the weak spots of the soil or anything then we can easily incorporate the phreatic surface and the external loading and so on. And then we can do the finite element analysis and get our factors of safety.

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Let us say we can apply this finite element analysis for analysis of retaining walls. Here you see one test retaining wall this was built at Raw Elementary College of Canada and this is about three meters high wall and this was an experimental wall that was in fact I was part of this research work long time back. And we were interested in knowing what will be the failure mechanism.

And here you see the front panels actually these you can see this the sum length of reinforcement and then on the front facing there are some panels that are provided during the construction.

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Soil after removal of facing panels in RMC trial wall.. Internal rupture surface is close to Rankine active surface

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And after the full construction was over these panels were removed to look at the internal rupture surface and whether the soil will stand on its own and so on.

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displacement potentiometer

surcharge

E

load ring

3m

D

0.5m

C

B

A

pin

load cell

3m

3m

Layer 4

earth pressure cell

strain gage

Layer 3

geosynthetic reinforcement

Layer 2

extensometer

Layer 1

earth pressure cell

Experimental Set up to be simulated through FEA

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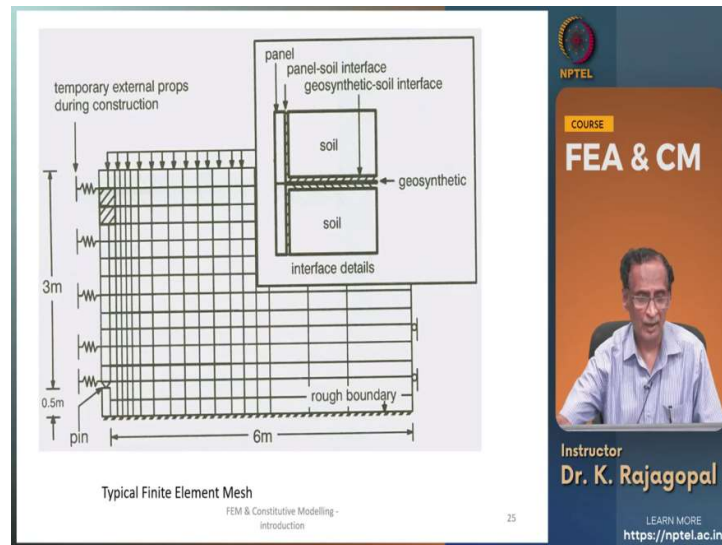
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And this is this complete schematic of the cross-sectional view of this of this testing. This soil is 3 meters height and the length of the soil is six meters and these are these dashed lines are the reinforcement layers there are totally four reinforcement layers each at 750 millimeter spacing and number of measurements were made for measuring the strain within the reinforcement layers are the deformations and then the earth pressures. And then the loads that are developed within this within the reinforcement layers and so on.

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And this is the finite element mesh actually this is the close-up of this the problem that we have here is the facing panels are made of one material these are hard materials and then the soil is there and then we have the reinforcement geosynthetic reinforcement. And each one has its own stiffness. The soil is the weakest among all these three materials and at the junction between these different materials what do we do?

We have to allow for separation or sliding otherwise we will be will be unduly increasing the strength of the soil because if you allow the soil to bond with the facing panel. It will behave as if it is as strong as the facing panel. The facing panel itself is it is very strong material. Then similarly the geosynthetic it has a much higher tensile strength whereas the soil has only the compressive strength sorry it does not it has only some shear strength it does not have any tensile strength.

So, here you see an interface provided between the panel and then the soil and then the soil and then the geosynthetic. And then the geosynthetic is connected to the facing panels through some mechanism and we should be able to simulate all these through our finite element models. And then in the construction itself there are four panels and each of them they are supported during the construction.

And then after the construction is over these supports were removed then external pressure was applied for failing them. And this is a finite element model for simulating the response of this retaining wall.

(Refer Slide Time: 53:03)

Element	Type	Nodes
1	quadrilateral	9-1-3-11-6-2-7-10
2	quadrilateral	11-3-5-13-7-4-8-12
3	interface	16-14-9-11-15-10
4	interface	20-18-11-13-19-12
5	quadrilateral	25-14-16-28-21-15-22-27
6	interface	17-29-28-16-23-22
7	uniaxial bar	17-29-23
8	interface	18-30-29-17-24-23
9	quadrilateral	30-18-20-32-24-19-25-31
10	nodal link	11-17

Close-up details of the mesh at junction of grid and panels

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And this is a close-up of the facing panel then the soil the reinforcement and then the interface between the different materials like the wall panel and then the soil the soil and then the reinforcement and so on.

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Full-height panel support to reinforced soil walls - rigid body rotation of facing

Incremental panel support to reinforced soil walls - bulging type deformation at front end

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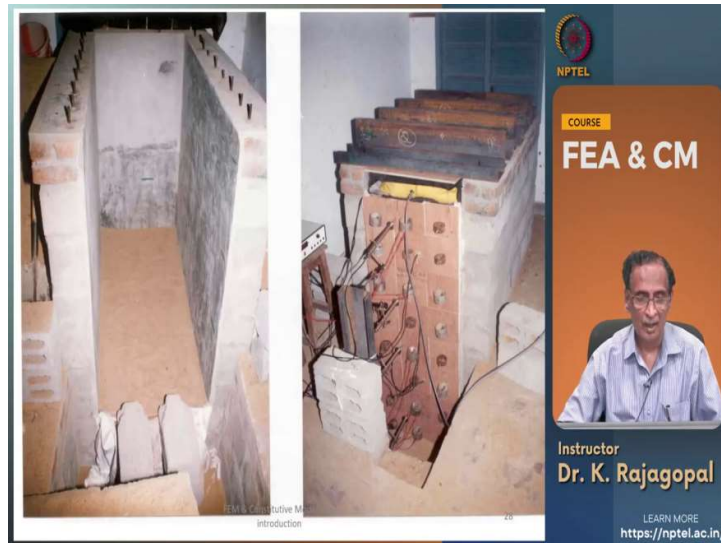
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And here is a I am showing two different types of retaining walls one is called as a full height panel wall where the entire height of the wall is made up of only one single concrete panel and in a manner of deformation it is going to deform like a rigid body. It is going to rotate around the bottom it is like this and this is the upper picture is the actual picture taken from the site whereas this is the bottom one is from our finite element analysis.

Then on the right hand side you see some other type of reinforced soil retaining wall where the entire height is not consisting of one single panel but multiple panels like this. And here

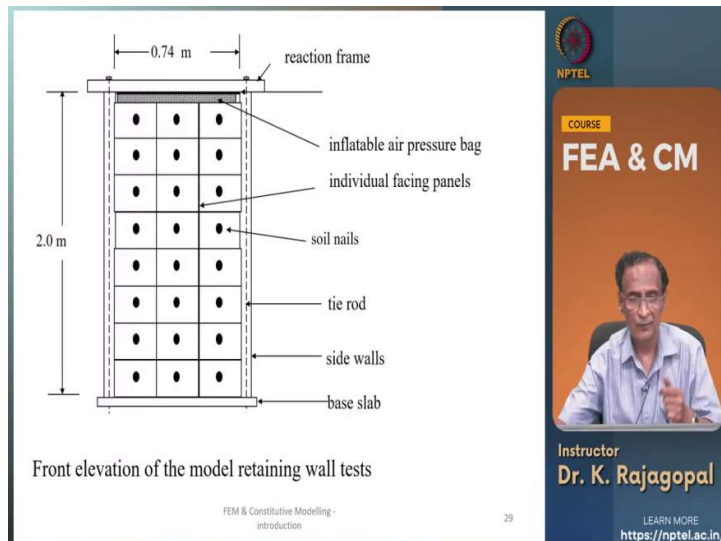
you see the deformed shape is totally different here it is rotated around the bottom whereas here there is some rotation but then more of a bulging type failure. And this bulging type failure might give rise to some difference in the in the stress distribution between reinforcement and then the soil compared to this. And our finite element model should be able to replicate this type of behaviour.

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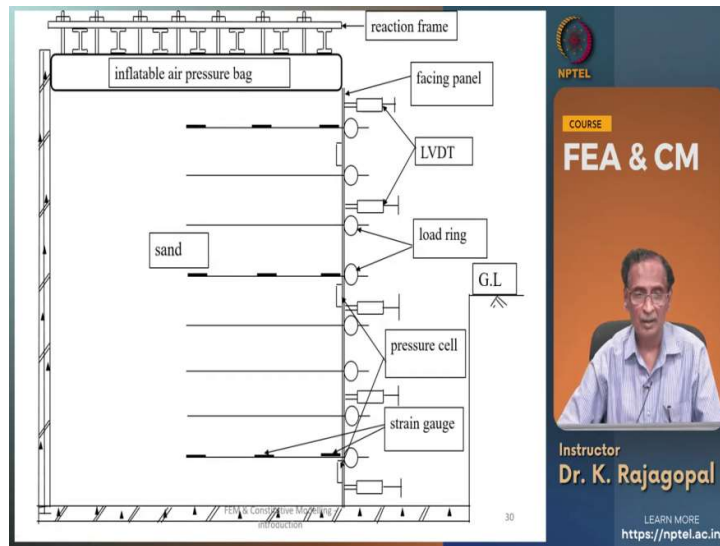
And here you see one more small scale model test actually. This particular one is built at IIT Madras. The height of this wall is about 1.6 meters and this width is about 750 millimeters and this is the space for placing the soil and this entire height was filled with the dry sand and supported by soil nails. And you see all these loading rings that are placed only for measuring the load developed in the in these soil nails.

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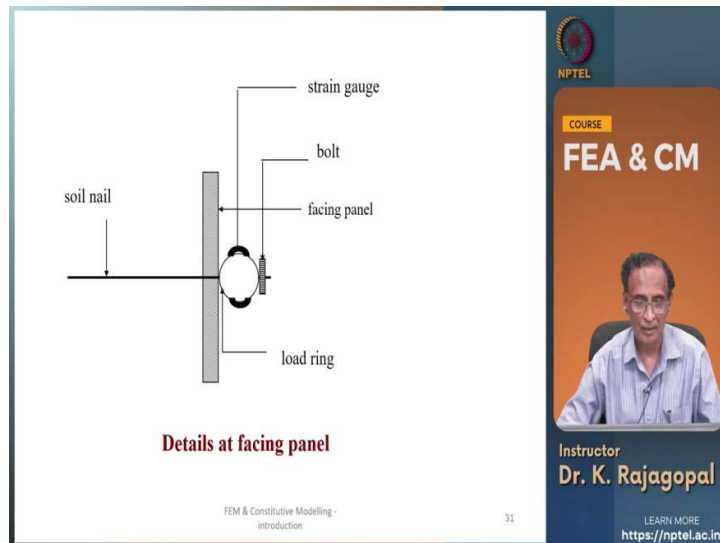
And the front part is like this.

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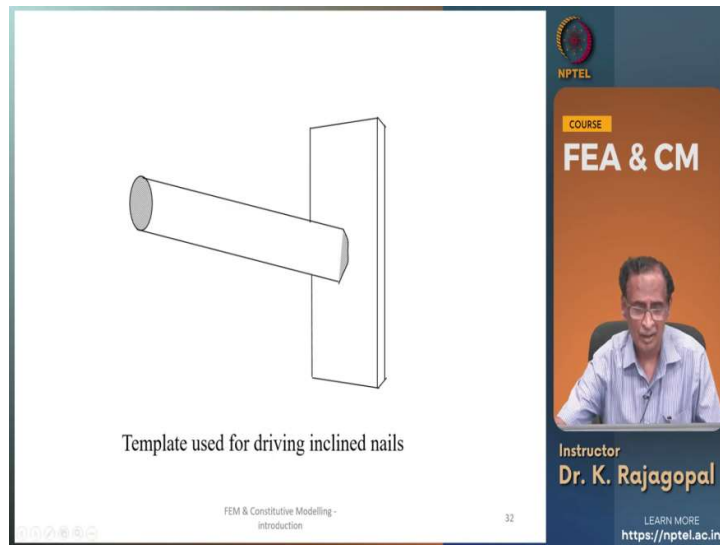
And the soil was subjected to failure by applying uniform pressure through an inflatable air pressure bag.

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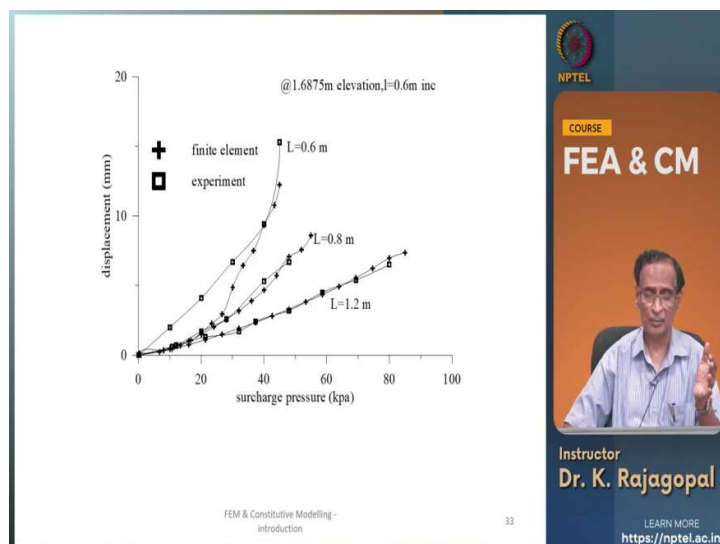


These nails are placed horizontally or at some inclination.

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By using some template
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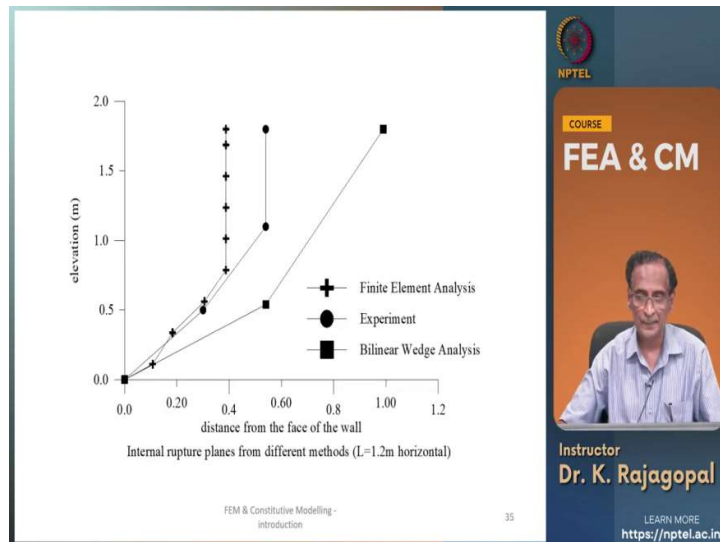


Then a lot of laboratories were performed by changing the length of the reinforcement or the diameter of the rods are the inclination and these are different responses that are obtained and the comparison between the finite element analysis and then the laboratory test. In all the cases there is a good response a good comparison and obviously with a shorter length reinforcement the wall fails at a smaller applied pressure and that is what is replicated by finite element models.

And then the influence of the inclination the actual design of these soil nail walls is done by empirical methods by using some approximate term rupture surface. And then when we

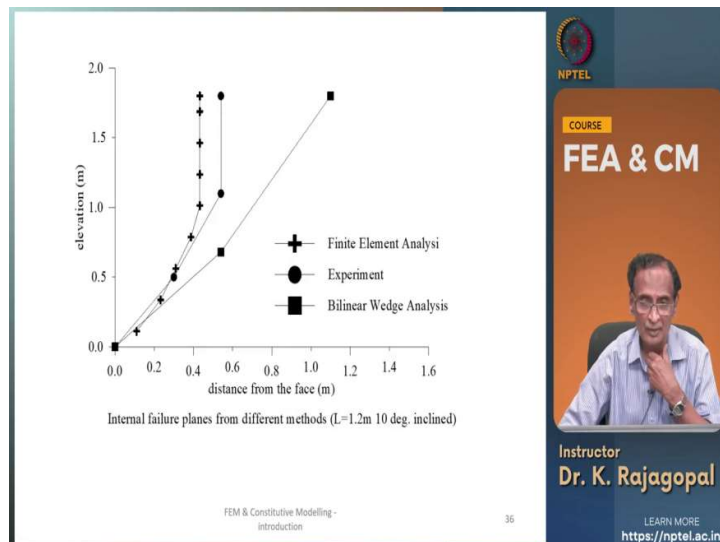
compare the finite element analysis with the approximate methods there is a lot of difference that brings out the advantage of using numerical methods.

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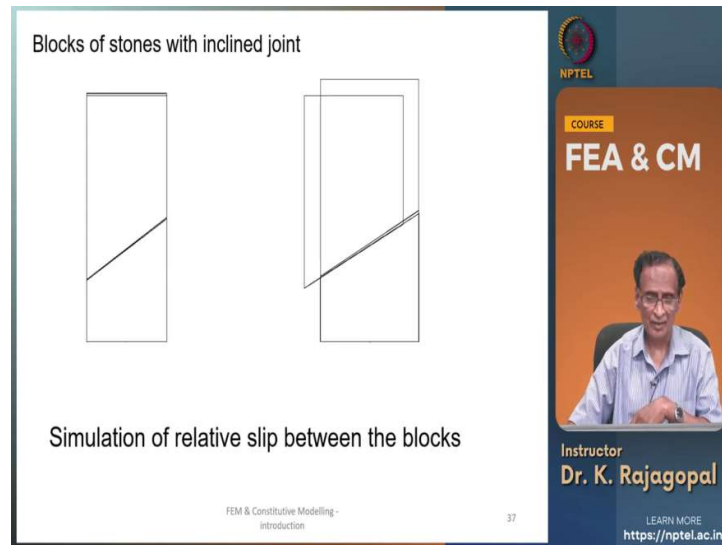
And these are the locations of the of the peak strains within the soil nails and this particular one is from bilinear wedge analysis it is totally different. And this particular one is of the measured one from Laboratories and this is from our finite element analysis.

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And the different for different and inclinations and different length of reinforcements and here we see that our finite element models they are able to replicate the real behaviour and they are able to show the influence of the length of the soil nails and then their inclination and their load capacity okay.


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Then the other thing that we need to model is we should have the capacity to model the sliding between different parts of the of the soil or between the soil and reinforcement or between the soil and the panel. And here I am showing you some example of a rock joint. Let us say you have a rock collected from the site with a joint here and if you test it in compression test this is what is going to happen.


Initially both the upper and bottom ones they are going to deform together but then after the failure the upper block is going to slide along this surface and whatever analysis that we are going to do should be able to replicate this sliding okay. And so, that also we are going to discuss as part of this course. So, from next lecture onwards will be going more into the theoretical details and we will be doing a step by step so, that you will not have any difficulty.

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FINITE ELEMENT ANALYSIS AND CONSTITUTIVE MODELLING IN GEOMECHANICS

INTRODUCTORY LECTURE & COURSE OUTLINE



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And if you have any questions you please do write to me at this email address profkr@gmail.com and I will be able to respond back to your queries okay. So, thank you very much this is the end of the first lecture and from next lecture onwards we will be dealing with the theoretical aspects and other practical aspects. So, thank you very much.