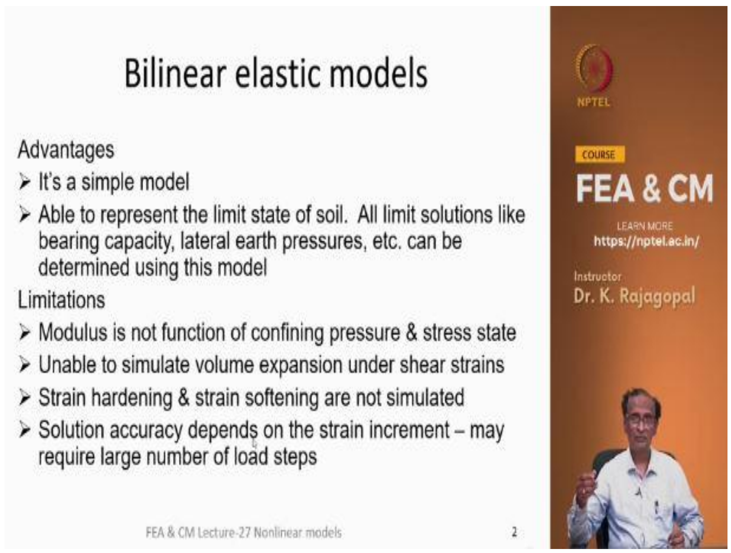
Finite Element Analysis and Constitutive Modelling in Geomechanics
Prof. K. Rajagopal
Department of Civil Engineering
Indian Institute of Technology-Madras

Lecture-31
Nonlinear Elastic and Hyperbolic Models

Welcome back to our lectures on the constitutive modelling. Till the previous class, we were looking at the linear elastic and then the bilinear elastic model.

(Refer Slide Time: 00:32)



And then bilinear elastic models, we have seen in terms of the tangent shear modulus and then the bulk modulus. And the advantage with those models is that we were able to put a limit on the loads that we can apply by introducing the Mohr–Coulomb yield criteria on the shear stresses. The advantages of these models, it is a very simple models either KG or EK model, they are very simple.

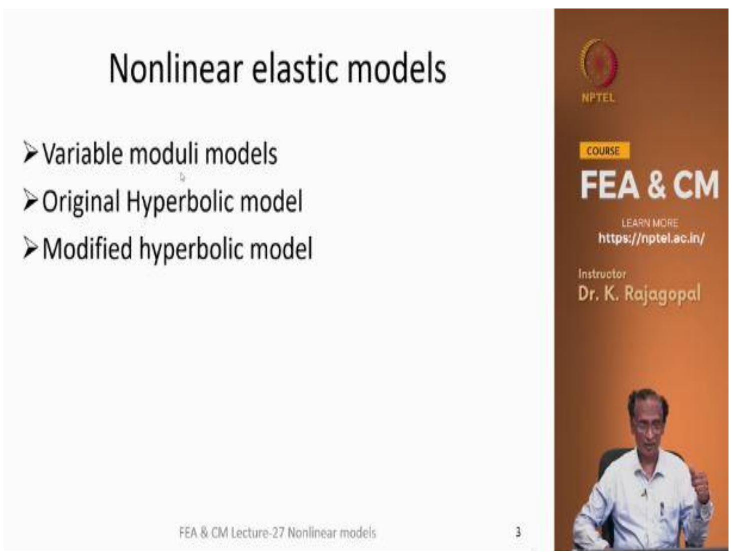
And they are able to represent the limit state of soil. And all the limit solutions like the bearing capacity of soils, the lateral earth pressures, now all these can be evaluated accurately using this simplistic model. But, several limitations are there. Are the modulus is not a function of the confining pressure. See this is one of the most important features of the soil. At a higher confining pressure the soil will exhibit a stronger response in terms of the strength and also stiffer response in terms of the modulus.

And that we are not able to simulate by using the bilinear elastic model. And these models are unable to simulate the volume expansion and another disadvantage that these models are unable to simulate volume expansion under the shear strains. That is another major feature of the soils and that we will see later. And the strain hardening and strain softening are not simulated by these bilinear elastic models.

And then the accuracy of the solution very much depends on the strain increment that we use, on the load increment that we use and we have seen with some numerical examples of the simulation of the triaxial compression test that with the coarse strain increments. We over predict the limiting stress by an order of even 50 percent, 60 percent and to get reasonably accurate results we have to use very small strain increments.

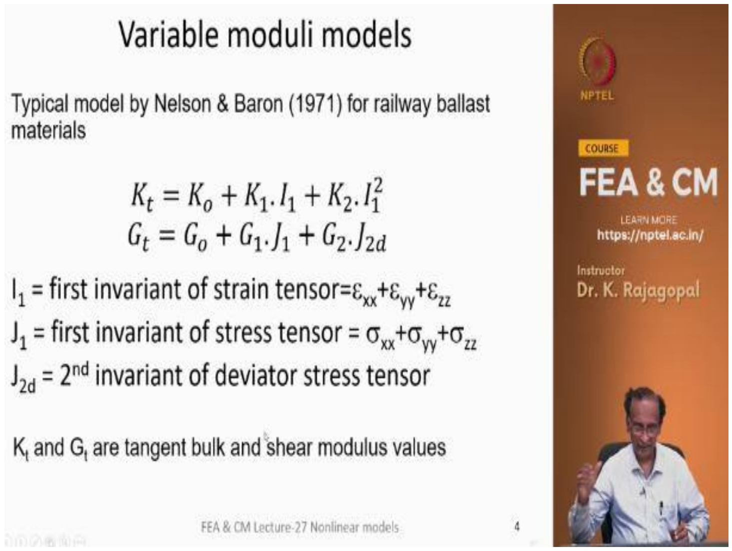
And that will increase our computational effort and also the time. And so we need to come out with better solution methods for improving our modeling capabilities.

(Refer Slide Time: 03:32)



And in that direction we have the non-linear elastic models. These are still elastic but non-linear. And one of the simplest non-linear elastic models is the variable moduli models and then we have the hyperbolic model and then modified hyperbolic model. And of course, there are several other intermediate types of models that I am not going to discuss because of time limitations but then I will give you some references that you can refer to for more details.

(Refer Slide Time: 04:16)



See one of the simplest variable moduli models was proposed by Nelson and Baron in 1971 for modeling of the railway ballast. And they formulated the constitute matrix in terms of tangent bulk modulus K t and then tangent shear modulus G t. And K t is expressed in terms of three parameters K naught, K 1 and K 2 like K t is K naught + K 1 I 1 + K 2 I 1 square and then G t is G naught + G 1 times J 1 + G 2 times J 2 d.

Typical model by Nelson & Baron (1971) for railway ballast materials

$$K_t = K_o + K_1.I_1 + K_2.I_1^2$$
$$G_t = G_o + G_1.J_1 + G_2.J_{2d}$$

I_1 = first invariant of strain tensor=$\varepsilon_{xx}+\varepsilon_{yy}+\varepsilon_{zz}$

J_1 = first invariant of stress tensor = $\sigma_{xx}+\sigma_{yy}+\sigma_{zz}$

And in these I 1 is the first invariant of strain tensor epsilon xx + epsilon yy + epsilon zz and if the volumetric strain is compressive the volumetric strain is positive and that is our geotechnical sign convention. And so if you look at this equation as long as we have volumetric compression your bulk modulus is only going to increase, because it is K naught + K 1 I 1 + K 2 I 1 square and of course I 1 square will be very small compared to I 1.

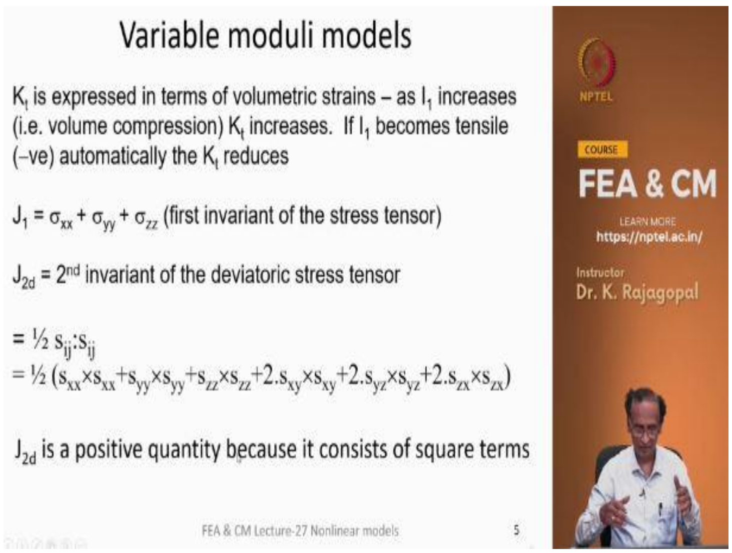
J_{2d} = 2nd invariant of deviator stress tensor

K_t and G_t are tangent bulk and shear modulus values

And then the shear modulus is expressed as G naught + G 1 times J 1 and J 1 is the first invariant of a stress tensor that we had seen earlier as sigma xx + sigma yy + sigma zz and J 2d is the second invariant of the deviator stress tensor. If you remember or if you recall we had split the total stress tensor into two parts. One is the spherical stress tensor in which all the stress tensor is a diagonal tensor with only the diagonal terms and all of them are equal and that represents the hydrostatic stress state.

And then we have the other component the deviator stress tensor and J 2d is the second invariant of that deviator stress tensor and deviator stress tensor represents your shear stresses. And our K t and G t they continuously change during the analysis.

(Refer Slide Time: 07:09)



And so as long as your volumetric strain is compressive the K t will go on increasing. And if I 1 is tensile our bulk modulus will reduce, because I 1 becomes negative**.** And then our J 1 is representing our volumetric compressive stresses and because of that the effect of the volumetric compressive stresses is to increase the shear modulus. Then J 2d is the second invariant of the deviator stress tensor.

$J_1 = \sigma_{xx} + \sigma_{yy} + \sigma_{zz}$ (first invariant of the stress tensor)

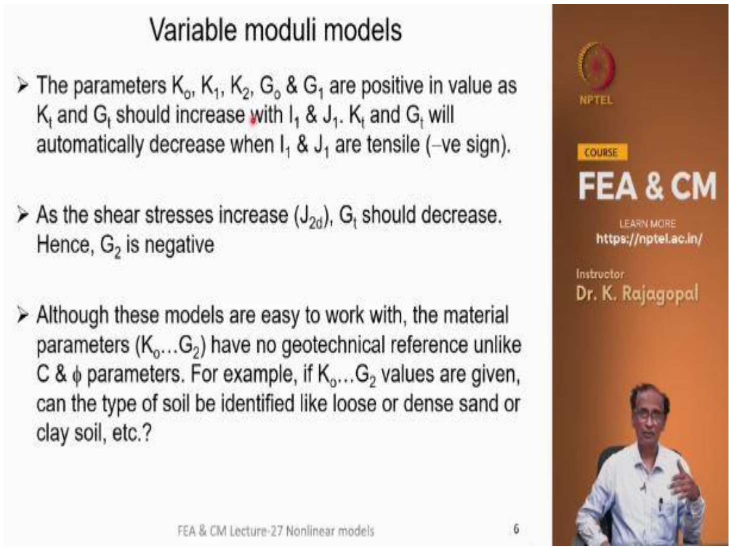
J_{2d} = 2nd invariant of the deviatoric stress tensor

$$= \frac{1}{2} s_{ij}:s_{ij}$$
$$= \frac{1}{2} (s_{xx}\times s_{xx}+s_{yy}\times s_{yy}+s_{zz}\times s_{zz}+2.s_{xy}\times s_{xy}+2.s_{yz}\times s_{yz}+2.s_{zx}\times s_{zx})$$

J_{2d} is a positive quantity because it consists of square terms

And if you recall that is written as one half of S ij multiplied by S ij term by term product. And the J 2d is a positive quantity, because it consists of all the square terms.

(Refer Slide Time: 08:12)

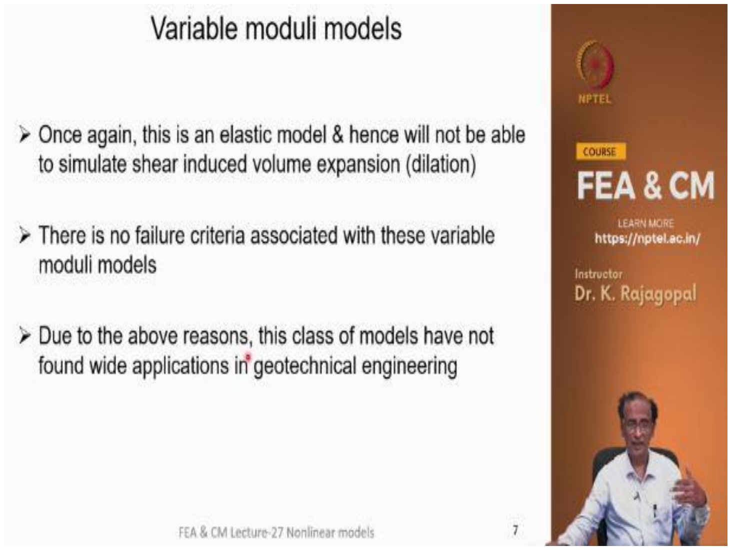


And the parameters K naught, K 1, K 2, G naught and G 1 or positive in value or K t and G t should increase with I 1 and J 1. And then as the shear stresses increase in magnitude our shear modulus should reduce. So, if we determine our parameters we will see that G 2 is negative and that is the only negative quantity of all these material parameters. And when we say these when we refer to these variable moduli models, the material parameters are K naught, K 1, K 2, G naught, G 1 and G 2.

So, these are our material parameters. And one reason why these models have not become very popular compared to other models that we will see later. Is this K naught, K 1, K 2 and all these parameters they have no geotechnical context. These are obtained by regression analysis of the stress strain and volumetric strain curves. And so, if I give you the values of these parameters you will not be able to tell whether, the given soil is loose sand, dense sand or a clay soil or any other attributes of the soil we cannot guess by looking at these parameters.

And so that is one reason why although they were developed they have not become very popular in the context of geotechnical engineering and that is when other models started coming in.

(Refer Slide Time: 10:19)

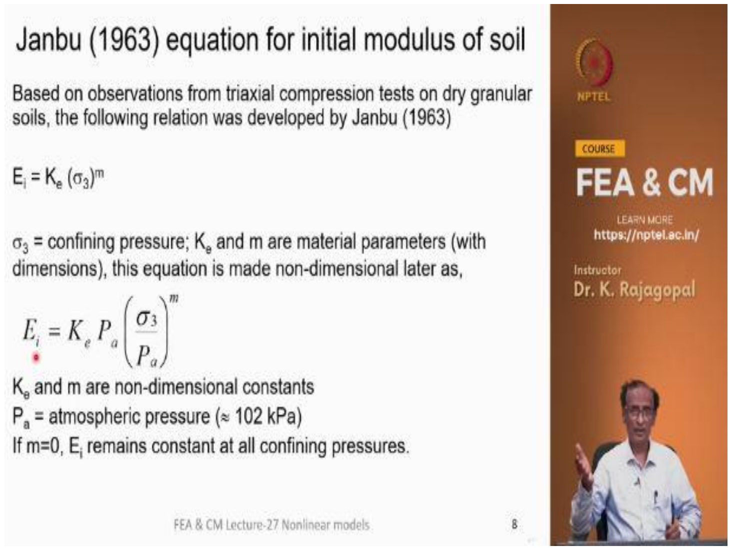


And these variable moduli models they are elastic models and hence they will not be able to simulate shear induced volume expansion that is dilation. And there is no failure criteria associated with these variable moduli models. There is no failure criteria like what we had seen in the case of bilinear elastic models. We had used the Mohr–Coulomb yield criterion for putting a limit on the shear stresses, but here there is no such limit.

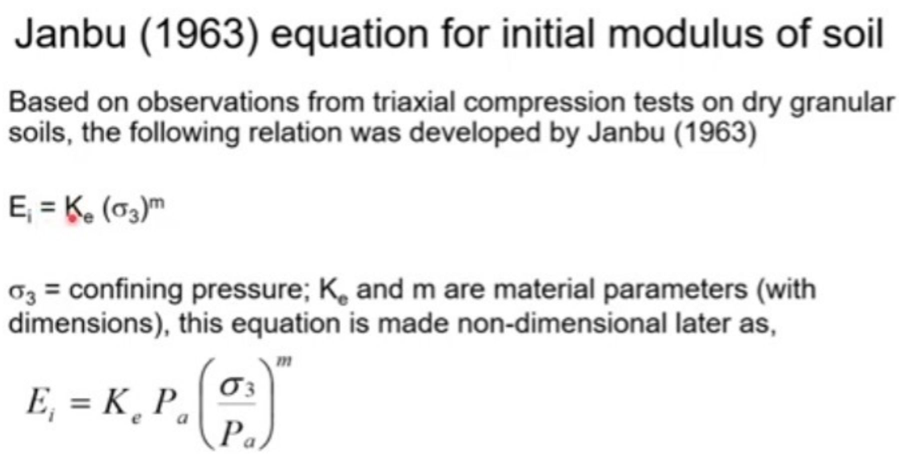
And our the K naught, K 1, K 2, G naught, G 1 and G 2 they automatically take care of the failure, because these are actually determined by regression analysis of the given stress strain curves. And because of these reasons mainly because there is no failure criteria that any geotechnical engineer can easily appreciate it is a modular model means every geotechnical engineer will know.

If you give the c and phi then we can know what type of soil we are dealing with if there is a predominant cohesion, then we know that the soil is a clay soil or if the c is negligible and the phi is predominant then we know that the soil is a sandy soil. And because of these reasons this variable modular models have not become very popular.

(Refer Slide Time: 11:57)



And then just about slightly before this variable, moduli models came into being in 1963 Janbu started working on expressing the relation between the initial modulus of the soil and then the confining pressure. So, he looked at a lot of test data from triaxial compression test. Then he gave an equation that the initial modulus can be expressed as some constant K e times sigma 3 to the power m where, sigma 3 is the confining pressure and then the K e is a constant K e and m are constants.



But then if you see this on the left hand side we have a modulus on the right hand side we have the stress and the units for both the modulus and the stress are the same but then we have these two parameters K e and m. So, these K e and m they have to have some units, because our left hand side and the right hand side should have the same units. So, this particular equation it is actually dependent on the dimensions that we use.

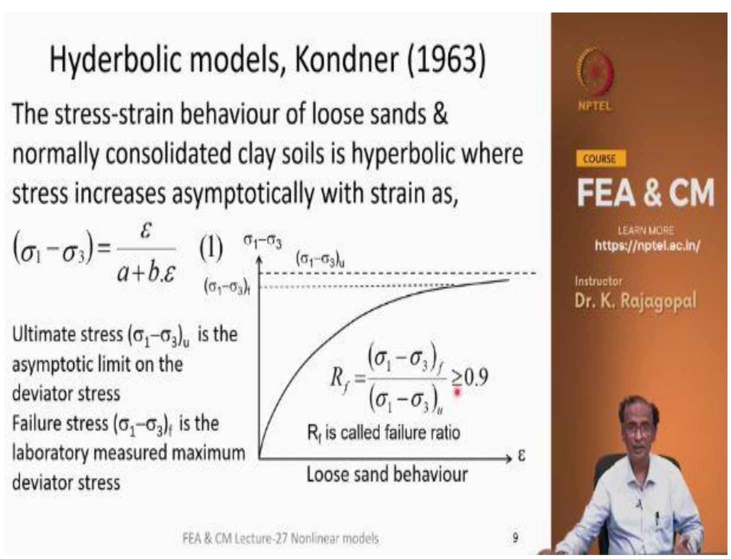
K_e and m are non-dimensional constants
P_a = atmospheric pressure (≈ 102 kPa)
If m=0, E_i remains constant at all confining pressures.

The K e and m they are not absolute constants but they depend on the dimensions that we use. So, later this equation was modified into a non dimensional form by writing it like this E i is K e times P a times sigma 3 by Pa to the power m is actually it is basically the same equation but slightly rewritten by introducing the atmospheric pressure P a. And now, the sigma 3 by P a is a non dimensional constant.

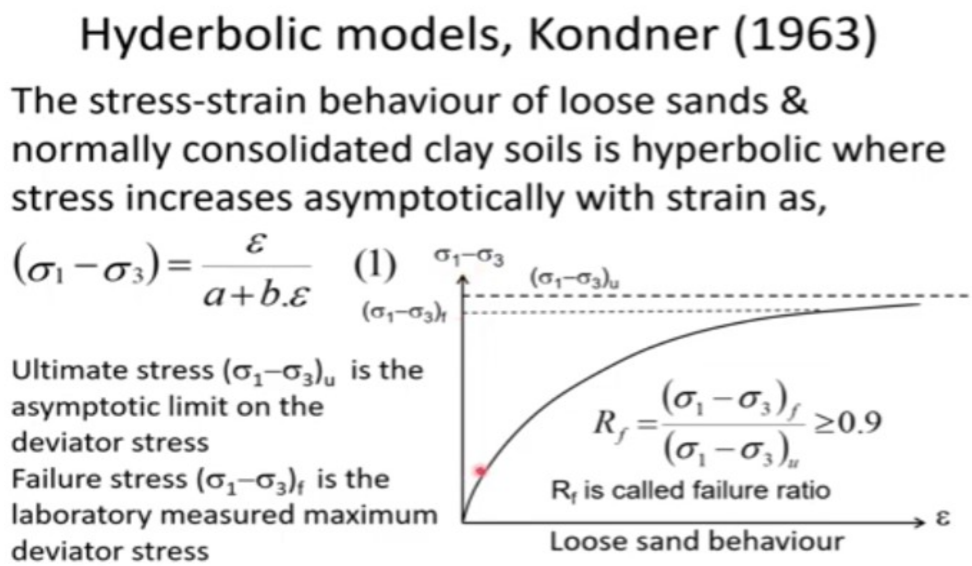
So, our m is non dimensional and then our P a has the same units as the modulus. So, our K e is also a non dimensional. So, our K e and m they are non dimensional and they can be used in any system of units. So, once you determine the K and m for a data given in one system of units. Let us say FPS units. Can use the same K and m even in the SI units and the only thing is this Pa and the atmospheric pressure that we have should be expressed in that particular units.

So, for example in the SI units P a is approximately 102 kilopascals, whereas it is about 14 psi in the FPS units. And if m is 0 in this equation your E i becomes constant. So, that is the advantage by choosing different parameters. We can make the modulus constant during the analysis.

(Refer Slide Time: 15:24)



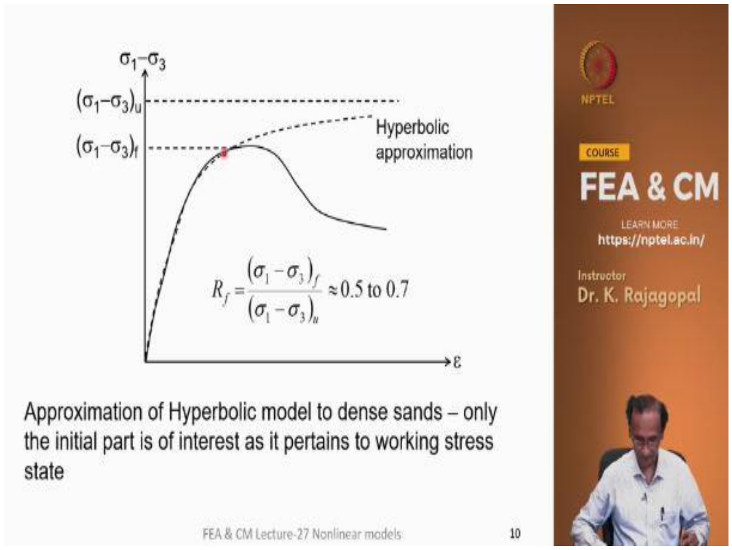
And Kondner, he proposed the hyperbolic relation between this strain and then the deviate stress. So, he looked at all the data that we get from triaxial compression test. And then like typically our loose sand has a stress strained behaviour something like this. On the x axis we have the axial strain; on the y axis we have the deviator stress. And symptomatically the deviated stress will go on increasing with strain that is for an ideal loose sand and that a very large strain it could reach some asymptotic limit.



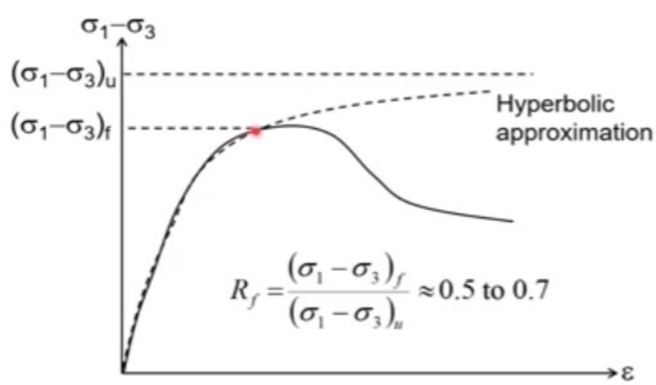
And that limit where it is achieved may be beyond our experimental range of strains. Say for example, if you are laboratory operators can perform only strains up to about 15 to 20 percent and sometimes for some soils within the 20 percent strain you may not be able to achieve your peak stress or the limiting stress and the sigma 1 - sigma 3 ultimate is actually it is an asymptotic limit whereas, the sigma 1 - sigma 3 f is the failure limit that we define based on our own limitations.

And Kondner, he gave this relation the deviate stress is epsilon by a + b epsilon and this is a typical hyperbolic equation. And this a and b they are constants but then they have some meaning and that we will see. And then they defined one ratio called as failure ratio R f as sigma 1 - sigma 3 f by sigma 1 - sigma 3 u. And this is actually the sigma 1 - sigma 3 f is what we determine from the laboratory test and this ultimate stress is a theoretical limit that could be achieved at very large strain. And for ideal loose sands this ratio will be more than about 0.9 and for a dense sand it could be very low about 0.5 to 0.6.

(Refer Slide Time: 18:02)

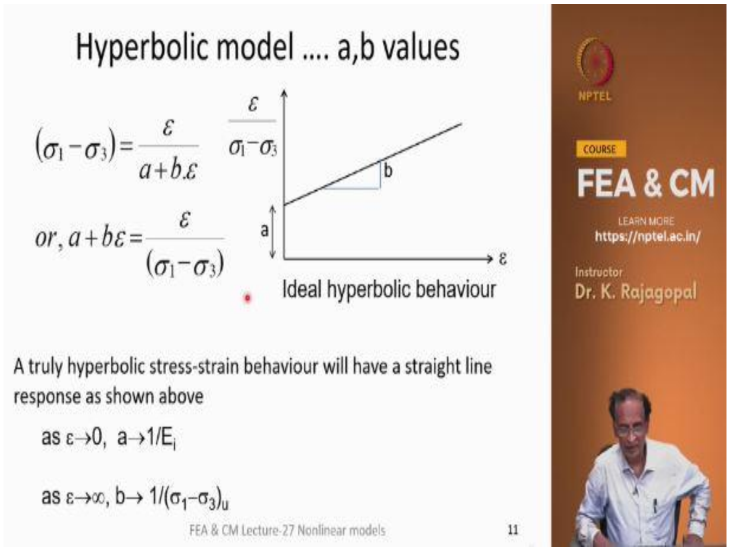


And for a dense sand if we try to fit a hyperbolic equation it is something like this, we can only fit this hyperbolic equation only up to in the hardening part. And beyond that your stress is going on increasing but then your laboratory determined deviator stress will continue to fall in this train softening part. In this your failure ratio R f could be in the range of about 0.5 to 0.7 and we only represent the initial part of the stress strain behaviour of the dense sands through this hyperbolic model.

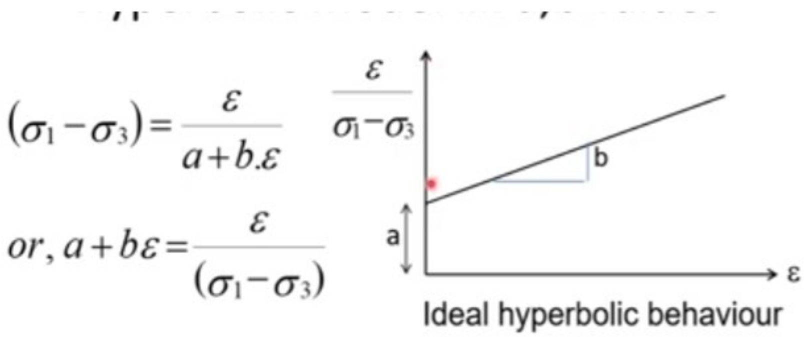


Approximation of Hyperbolic model to dense sands – only the initial part is of interest as it pertains to working stress state

(Refer Slide Time: 18:55)



And what are these a and b? We can easily guess by reformulating this equation as a + b epsilon is as a epsilon by sigma 1 - sigma 3. And if you plot a graph between epsilon and the x axis and then epsilon by sigma 1 - sigma 3 and the y axis and if it is a truly hyperbolic behaviour all the data points should fall in a straight line like this or in the worst case they will all be falling very close to this line and then we can fit a best fit line on the intercept on the y axis is your a and the slope is b. That is what we can easily see from this equation.



And says epsilon tends towards 0 or this thought your a tends to inverse of the initial modulus. So, actually as epsilon tends to 0, b epsilon cancels out on the left hand side and on the right hand side we have epsilon by sigma 1 - sigma 3 that is the inverse of the initial modulus. So our a that is the intercept on the y axis is the inverse of your initial modulus and then what is the slope of this line b?

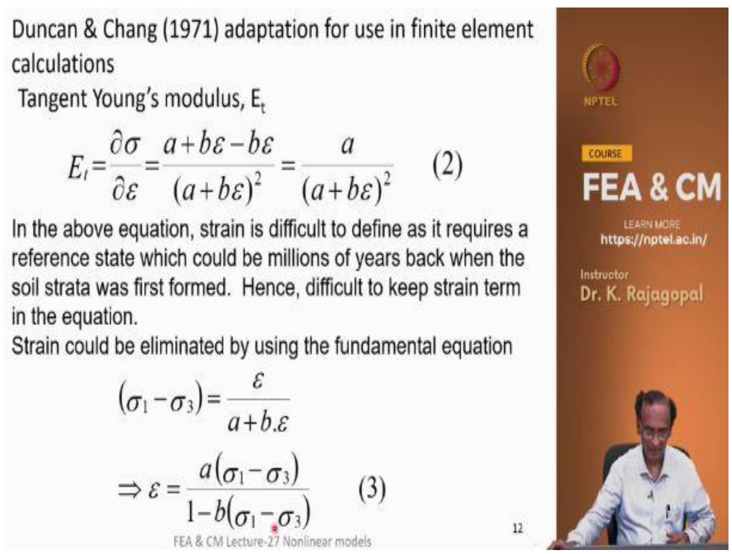
$$\text{as } \varepsilon \to 0,\ a \to 1/E_i$$

$$\text{as } \varepsilon \to \infty,\ b \to 1/(\sigma_1 - \sigma_3)_u$$

So, as epsilon tends to infinity the effect of a can be neglected and then on the right hand side you have epsilon by sigma 1 - sigma 3 and that could be ultimate because we are dealing with ultimate limit state or at very, very large shear strains epsilon gets canceled out and your b tends towards 1 by sigma 1 - sigma 3 ultimate. So, the b is the reciprocal of the ultimate stress or the asymptotic limit on the deviator stress and a is the reciprocal of the initial modulus. So, this is the meaning of these two terms a and b in the hyperbolic model.

And Kondner, he just gave this equation and said that you can represent the stress strain behaviour of the loose sands and then the normally consolidated clays using this equation.

(Refer Slide Time: 21:45)



And beyond that he did not go, but in the year 1971 Duncan and Chang they have taken this model and adapted it for finite element analysis. Fact the finite element analysis became quite common by late 60s. Then at that time people were looking at different methods of modeling the non-linear behaviour and then the ultimate strength of the soil and so on. And Duncan and Chang they have slightly modified this hyperbolic model suitable for finite element implementations.

And they defined the tangent Young's modulus as dou sigma by dou epsilon this is just basically the tangent slope of the stress strain curve and if you differentiate your hyperbolic equation epsilon by a + b epsilon with respect to epsilon we get a + b epsilon - b epsilon and divided by the square of the denominator and this comes out as a by a + b epsilon. So, in this equation your tangent modulus is a by a + b epsilon and the whole square is a very simple equation.

Duncan & Chang (1971) adaptation for use in finite element calculations

Tangent Young's modulus, E_t

$$E_t = \frac{\partial \sigma}{\partial \varepsilon} = \frac{a + b\varepsilon - b\varepsilon}{(a + b\varepsilon)^2} = \frac{a}{(a + b\varepsilon)^2} \qquad (2)$$

And the strain is increasing your shear modulus is going to reduce, because that is what is meant by this denominator; the epsilon being in the denominator and also the square you will get a rapid decrease in the modulus as the strain increases. But then philosophically speaking what is strain? Say for defining strain we require some initial state and unless you know the initial length you cannot define your strain like the strain for a one dimensional cases.

The length minus original length divided by the original length is your strain. Similarly for soils, if you want to define strain you require some initial state and that is impossible to get now, because the initial state will correspond to some long time in the geological past. And that could be millions of years depending on the soil straight up and so it is better to remove your strain from this equation.

Strain could be eliminated by using the fundamental equation

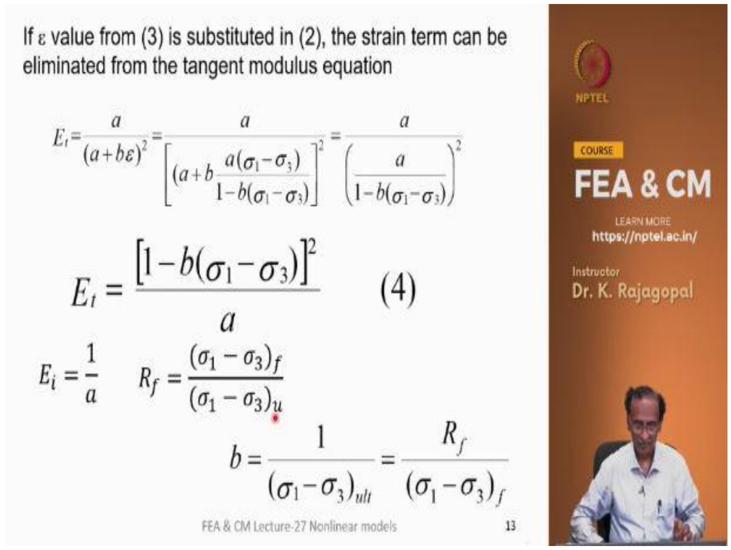
$$(\sigma_1 - \sigma_3) = \frac{\varepsilon}{a + b.\varepsilon}$$

$$\Rightarrow \varepsilon = \frac{a(\sigma_1 - \sigma_3)}{1 - b(\sigma_1 - \sigma_3)} \qquad (3)$$

1

So, it is and then we can actually use our governing equation, the fundamental equation to replace epsilon by other terms. So, by rewriting this equation here epsilon can be written as a times sigma 1 - sigma 3 divided by 1 - b times sigma 1 - sigma 3. And we can take this epsilon and substitute here.

(Refer Slide Time: 25:05)



And then we will get a very interesting equation tangent Young's modulus is a by a + b epsilon whole square. And epsilon is this from our fundamental equation. So, your a be times 1 - b sigma 1 - sigma 3 + a b sigma 1 - sigma 3. So, if you simplify you will get a by this whole square. And so your tangent Young's modulus is actually this denominator will go to the numerator because it is 1 by 1 by denominator.

If ε value from (3) is substituted in (2), the strain term can be eliminated from the tangent modulus equation

$$E_t = \frac{a}{(a+b\varepsilon)^2} = \frac{a}{\left[\left(a + b\frac{a(\sigma_1-\sigma_3)}{1-b(\sigma_1-\sigma_3)}\right]^2} = \frac{a}{\left(\frac{a}{1-b(\sigma_1-\sigma_3)}\right)^2}$$

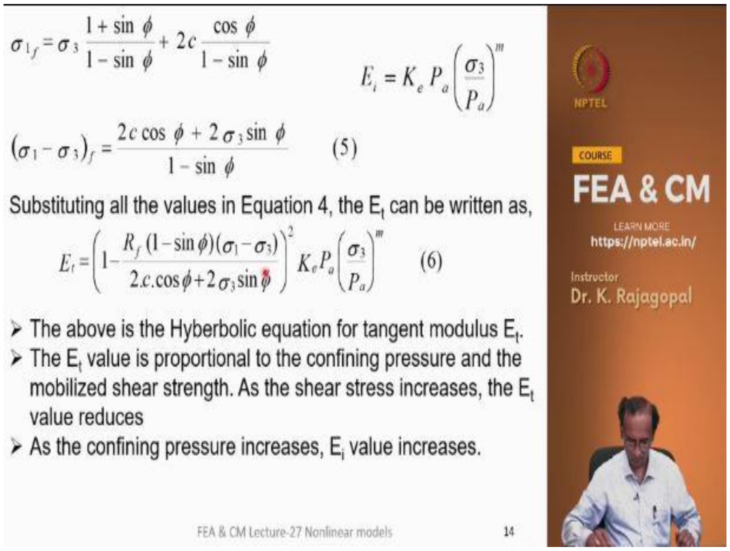
$$E_t = \frac{[1-b(\sigma_1-\sigma_3)]^2}{a} \qquad (4)$$

$$E_i = \frac{1}{a} \qquad R_f = \frac{(\sigma_1-\sigma_3)_f}{(\sigma_1-\sigma_3)_u}$$

$$b = \frac{1}{(\sigma_1-\sigma_3)_{ult}} = \frac{R_f}{(\sigma_1-\sigma_3)_f}$$

So, 1 - b sigma 1 - sigma 3 whole square divided by a because your a square is here and a is there, and a means the inverse of the initial modulus. So, it in fact our E t can be written as E i times 1 - b sigma 1 - sigma 3 whole square and our b is related to failure ratio R f. So, R f is sigma 1 - sigma 3 f by sigma 1 - sigma 3 u and our b is sigma 1 by sigma 1 - sigma 3 u. So, that is R f by sigma 1 - sigma 3 failure.

(Refer Slide Time: 26:49)



This is the limiting deviator stress and our sigma 1 f can be related to sigma 3 c and phi through our Mohr–Coulomb relation and if you subtract sigma 3 from the right hand side you can simplify this equation as 2c cosine 5 + 2 sigma 3 sin phi by 1 - sine phi. And our initial modulus E i is a K e times P a times sigma 3 by P a to the power m and by substituting our equation for sigma 1 - sigma 3 f.

$$\sigma_{1_f} = \sigma_3 \frac{1+\sin\phi}{1-\sin\phi} + 2c\frac{\cos\phi}{1-\sin\phi} \qquad E_i = K_e P_a \left(\frac{\sigma_3}{P_a}\right)^m$$

$$(\sigma_1 - \sigma_3)_f = \frac{2c\cos\phi + 2\sigma_3\sin\phi}{1-\sin\phi} \qquad (5)$$

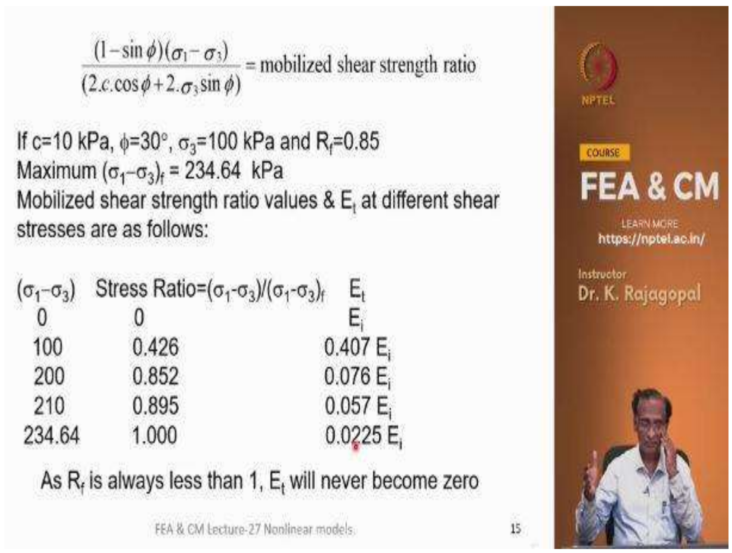
Substituting all the values in Equation 4, the E_t can be written as,

$$E_t = \left(1 - \frac{R_f(1-\sin\phi)(\sigma_1-\sigma_3)}{2c.\cos\phi + 2\sigma_3\sin\phi}\right)^2 K_e P_a \left(\frac{\sigma_3}{P_a}\right)^m \qquad (6)$$

And then our R f we get the tangent Young's modulus as 1 - R f 1 - sine phi sigma 1 - sigma 3 by 2c cosine phi + 2 sigma 3 sine phi this whole square multiplied by K e P a times sigma 3 by P a to the power m. So, in one single equation Duncan and Chang they were able to incorporate the influence of the confining pressure sigma 3 and then the influence of the shear stress in relation to the shear strength.

So, as the shear stress is increasing your modulus is going to decrease. Say this is what constant sigma 3, say if you are shear stress sigma 1 - sigma 3 is going on increasing then this ratio will go on increasing and then this is 1 minus this, so our this bracket will go on decreasing as the shear stress is increasing. And in one single equation Duncan and Chang they have given a beautiful equation that can represent the failure of the soil because we have basically introduced the Mohr–Coulomb relation for the strength of the soil.

(Refer Slide Time: 29:07)



So, this parameter 1 - sine phi times sigma 1 - sigma 3 by 2c cosine phi + 2 sigma 3 sine phi is actually it is called as the mobilize the shear strength. Basically, this sigma 1 - sigma 3 f is this and the sigma 1 - sigma 3 is the shear stress. So, this ratio is called as the mobilized shear strength ratio. And initially in the triaxial compression test when sigma 1 and sigma 3 are equal, this is 0 and then towards ultimate state when we are at sigma 1 f this ratio will be 1.

$$\frac{(1-\sin\phi)(\sigma_1-\sigma_3)}{(2.c.\cos\phi+2.\sigma_3\sin\phi)} = \text{mobilized shear strength ratio}$$

Let us look at a numerical example just to illustrate what is the effect of the shear stress on the initial modulus. Let us take a soil with a c of 10, phi of 30 degrees, confining pressure of 100 kPa and then failure ratio is 0.85 and corresponding to these values sigma 3 of 100 c and phi of 10 and 30 degrees, the maximum deviator stress as per the Mohr–Coulomb relation is 234.64. And let us say at sigma 1 - sigma 3 of 0 that is at the start of our triaxial compression test.

If c=10 kPa, ϕ=30°, σ_3=100 kPa and R_f=0.85
Maximum $(\sigma_1-\sigma_3)_f$ = 234.64 kPa
Mobilized shear strength ratio values & E_t at different shear stresses are as follows:

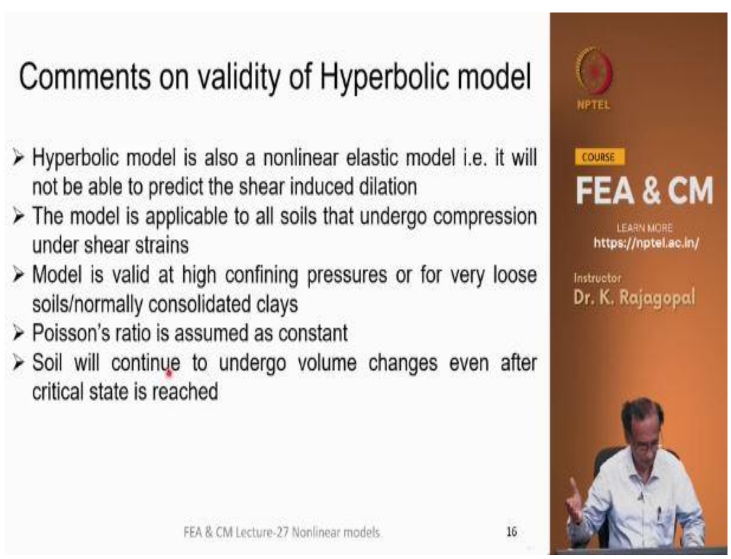
$(\sigma_1-\sigma_3)$	Stress Ratio=$(\sigma_1-\sigma_3)/(\sigma_1-\sigma_3)_f$	E_t
0	0	E_i
100	0.426	0.407 E_i
200	0.852	0.076 E_i
210	0.895	0.057 E_i
234.64	1.000	0.0225 E_i

As R_f is always less than 1, E_t will never become zero

Our stress ratio is 0 and your initial modulus is E i because we have this sigma 1 - sigma 3 is 0 this bracket will become 1. And as this sigma 1 - sigma 3 is increasing; this is the stress ratio 0.426. That means at deviator stress of 100 you have mobilized 42.6 percent of the of the shear strength of the soil and a 200, 210, 234.64 we have mobilized at the full strength of the soil.

And if you look at the tangent Young's modulus at 100 it falls down to 0.4 times E i. And the limit state the E tangent is only 2 percent of the initial modulus. So, in a way we are able to represent failure. So, after the limit state the Young's modulus reduces so much that your further increase of strains may not result in further increase of shear stresses. So, this is one way of modeling the failure through the hyperbolic model. And as R f is always less than 1, we will never get a tangent Young's modulus of zero.

(Refer Slide Time: 32:24)



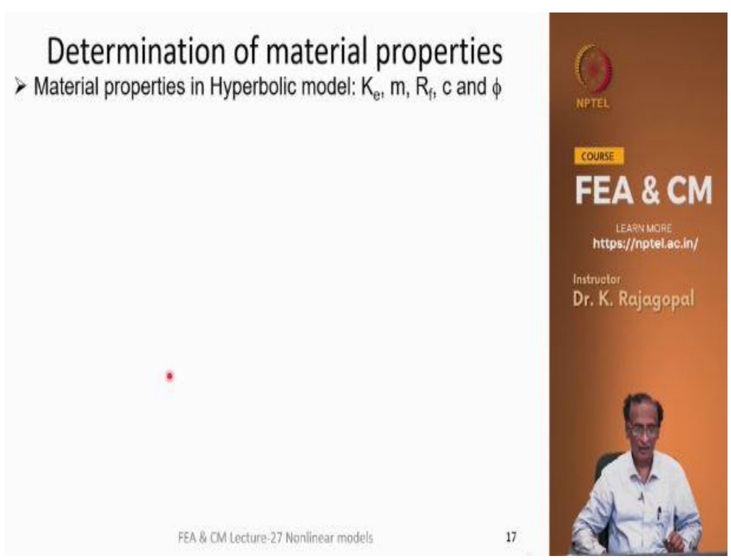
And this hyperbolic model is also a non-linear elastic model and it will not be able to represent the shear induced dilation, but this is able to represent the effect of the confining pressure and then the effect of the applied shear stresses and the modulus. And this model is applicable to all the soils that undergo volumetric compression, because basically it is an elastic model.

So, we cannot really represent the shear strain induced the dilation volume expansion. So, this model we can safely apply to all the soils that undergo compression that is our loose sands are normally consolidated clays or even dense sand with a very high phi but at very, very high confining pressure the soil will only undergo compression, because of the suppressed dilatency and even that very high confining pressures we can apply the hyperbolic model.

And this model is applicable at high confining pressure for very loose soils are normally consolidated clays. And the Poisson's ratio is assumed to remain constant because, there is no specific equation. Of course there are some models with an equation for Poisson's ratio but then I have not included those topics in this course, because we are going to see a modified hyperbolic model wherein we represent the variation of the Poisson's ratio and because of this reason that the Poisson's ratio remains constant.

The soil will undergo volumetric compression even after limit state. If you increase the strain the soil will only undergo volumetric compression.

(Refer Slide Time: 34:36)



And the determination of material properties I think that we will see later. And now, let us look at our hyperbolic equation and let me demonstrate through one excel program, this is the hyperbolic original model. **(Video Starts: 34:58)** So, this program is a very simple excel program that implements our hyperbolic model for determining the stress strain behaviour of the soils.

And the basic input that you need to give are the cohesive strength c, friction angle phi and then the confining pressure sigma 3 and then the K e and then atmospheric pressure and then the exponent in the equation m and then our initial modulus K e times P a sigma 3 by P a to the power m. This is the Janbu's equation and our incremental strain that is the increment that which we apply the axial strain and then the R f.

We will see how to determine these parameters in the next lecture**.** And then the Poisson's ratio is given as 0.35 and then the volumetric strain increment is very simple 1 - 2 nu times epsilon axial strain. Axial strain is our incremental strain. So, initially when the strain is 0, your bracketed term is 1 that is sigma 1 - sigma 3 is 0 that is the initial state or that is when they apply the confining pressure.

Your tangent Young's modulus is only the initial modulus and then our stress is 100**.** And then as you go on increasing the strain your stress will increase the axial stress. Axial stress is delta epsilon times E tangent**.** And you see here your transit modulus initial modulus is 25198 and then it is a gradually reducing, as your strain is increasing and your stress is increasing.

In this particular case you are maximum sigma 1 is 334.64 and so here we see at an axial strain of 0.02 our axial stress is 314 and then if you look at the tangent modulus it is gradually decreasing. Initially it was 25198 then it has decreased to 8989, 5184, 3463, 2506 and so on. It is decreasing as your stresses are increasing. And then, if you see this basically our maximum sigma 1 is 334.64 as per hour Mohr–Coulomb relation.

But then, our stress is going on increasing, because our R f is a 0.75 and because of that the bracket never becomes 0. Unless you give it as 1 let us say I give it as 1 and after failure. So, as you are approaching the shear strength, your tangent modulus has reduced so much. And your further increase of shear stresses will decrease and after sometime after 334 we will see that is actually it requires a very large number of iterations because beyond certain stress it is not increasing.

Like for example, if I look at the stress line equation see here it is initial it is increasing very fast but beyond that the shear stress is increasing whereas, gradual and it has gone up to about 325 that is still below our yield limit and let me just show you another let us set it back to 0.75**.** And our maximum shear stress theoretical limit is 334 and if you look at this stress strain graph.

You see this initially our sigma 1 is 100 that is the starting state when we applied the confining pressure 100 it has increased to 25 and then 275 and so on. It is going on increasing and at an axial strain of 20 percent your sigma 1 has become 395, which is more than our shear strength of 334 it is beyond our limit like 385 and let me see where we reach the 334 and I think at about maybe 3 percent strain let me go back to the calculation sheet.

So, at about 3 percent strain we have reached a failure stress of 334, but it is still the stress is increasing, because our R f value is 0.75. So, our tangent modulus it will never become 0. It is always some value, see it is the tangent modulus even at very large strain it is not 0 and because of that you will get some increase in the shear stress and let me just increase it to 0.9 just for illustration. Say here we will see the slope will reduce**.**

And let me just give you an extreme example. Let us take an R f of 0. So, what will happen with an R f of 0? We should get a straight line and let us see whether we get that I think the program is not able to plot it because your incremental stress will go on increasing and then the total stress is too high, it is your R f let me just take it some value of 0.5 and then you see that you get significant slope.

And because of that you are shear stress is increasing much beyond your shear strength. Then if you look at the volumetric strain, the volumetric strain continues to increase, the soil continues to undergo volumetric compression with increasing axial strain because your Poisson’s ratio is constant and the slope is constant. All through even during the critical state and that is what we see here, this is all in the critical state.

Let me just let me put it to some reasonable value of about 0.85. So, any stress beyond 334 is in the critical state or the limit state. And we see that our axial strain continues to increase, the axial strain is increasing and then even the volumetric strain is increasing in the compression direction that is what we see here. So, this original hyperbolic model the way we have it is not able to represent the constant volume state after the failure.

Although it is very good in terms of limiting the yield stresses to some value based on their Mohr–Coulomb limit. **(Video End: 44:56)** So, this is the major limitation of the original hyperbolic model and we will see in the next lecture how to incorporate the effect of the tangent Poisson's ratio in our analysis. So, that we will do in the next class.