

# Finite Element Analysis and Constitutive Modelling in Geomechanics

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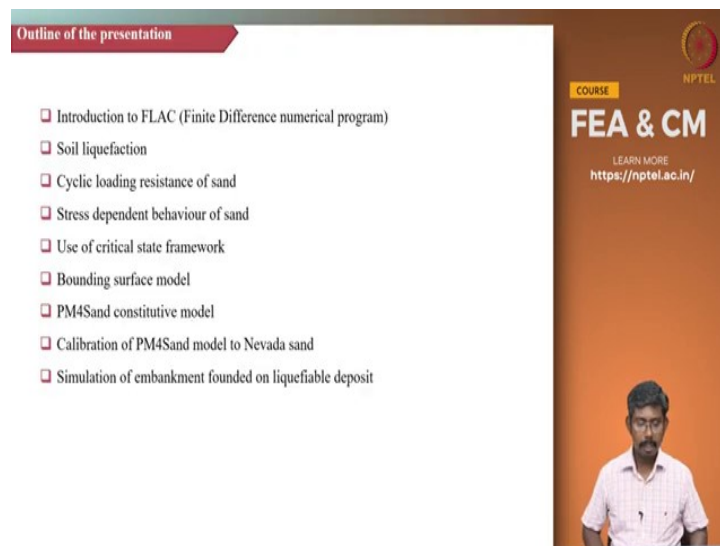
Indian Institute of Technology – Madras

## Lecture – 43

### Simulation of Soil Liquefaction Using FLAC

Hi, this is Dinesh working as a Project Officer in Geotechnical Engineering Division at IIT Madras. It is my pleasure to offer a guest lecture for the course of FE and Constitutive Modelling in Geomechanics. I would like to deliver my lecture on the topic of Simulation of Soil Liquefaction Using FLAC.

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The outline of presentation is like this so, a brief intro for FLAC followed by soil liquefaction. And then how to characterize the cyclic loading resistance of sand and the dependency of sand on stress dependent behaviour of sand. And the critical state framework to incorporate such stress dependent behaviour of sand. And more importantly the bounding surface model which is the topic of discussion today which is to be used for simulation of soil liquefaction in FLAC.

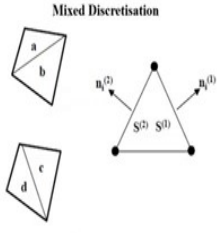
So, one of such bounding surface model is PM4Sand model. It would be discussed in this lecture and the calibration of PM4Sand model to Nevada sand. And finally, the simulation of a boundary value problem which is an embankment. So, resting on a liquefiable foundation soil.

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**FLAC – (Fast Lagrangian Analysis of Continua)**

Every derivative in the set of governing differential equations are replaced by algebraic expressions in terms of stress or displacement at discrete points in space (gridpoint)  
Stresses and displacements are undefined within the elements.


Mixed Discretisation



Itasca (2011)



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So, coming to FLAC it is a finite difference program. So, in this, the governing differential equations are replaced by algebraic expressions. So, the stresses and displacements the quantities of interest are obtained at discrete points in space. So, the stresses and displacements are undefined within the elements so which means normally in case of finite element.

The stresses and displacements will vary over the element because the shear functions are defined over there. So, whereas here the equations are solved at the node and there is no interpolation over the element, it is the interaction between the nodes, the geometric domain of the body or soil material is discretized in FLAC 2D using quadrilateral element. So, it can be seen that the quadrilateral elements are made of two triangular elements.

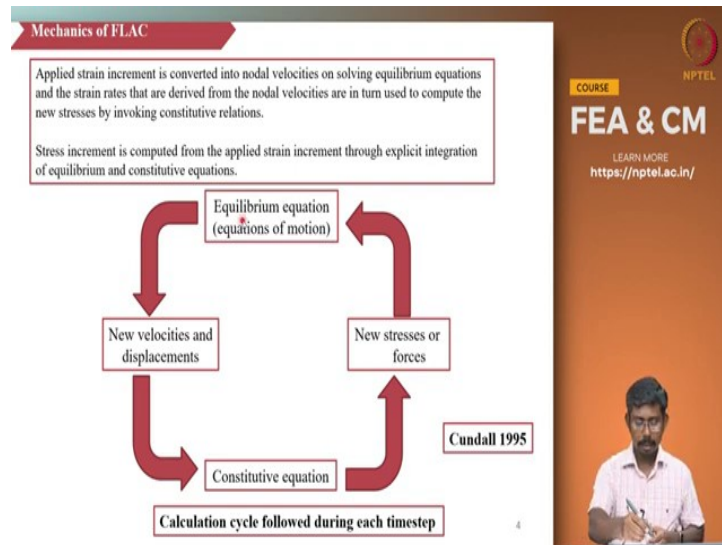
And each quadrilateral elements are not used each quadrilateral elements are composed of an overlay. So, two quadrilateral elements are overlaid. The quadrilateral elements are made of triangular elements. A pair of overlaid quadrilateral elements are used to discretize the body in FLAC and each quadrilateral element is made of two triangular elements. This particular form is called mixer discretization.

This is done to this is done in FLAC to the geometric domain of the body in FLAC is discretized using quadrilateral element. Each quadrilateral is made of two triangular elements. The quadrilateral elements are not used in a not just a single quadrilateral element it

is instead a overlaid set of quadrilateral elements are used to represent each element. So, this particular form is called mixer discretation.

So, this enables accurate prediction of plastic collapse load. It avoids volumetric locking and the hardness deformation can also be averted.

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So, basically in FLAC the strain increments are applied and the nodal velocities are derived by solving the equilibrium equations. So, these nodal velocities are further converted into strain rates. These strain rates are used by the equilibrium equations to get the new set of stresses. So, this is the typical, a cycle of calculation that is followed in FLAC. So, basically, the strain increment is applied to obtain the stress increment.

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**Explicit Vs Implicit scheme (Itasca 2011)**

| Explicit   | Implicit   |
|--|--|
| Timestep must be smaller than a critical value for stability.  | Timestep can be arbitrarily large, with unconditionally stable schemes.  |
| Small amount of computational effort per timestep.   | Large amount of computational effort per timestep.   |
| No significant numerical damping introduced for dynamic solution.  | Numerical damping dependent on timestep present with unconditionally stable schemes.   |
| No iterations necessary to follow nonlinear constitutive law.  | Iterative procedure necessary to follow nonlinear constitutive law.  |
| Provided that the timestep criterion is always satisfied, nonlinear laws are always followed in a valid physical way.  | Always necessary to demonstrate that the above-mentioned procedure is (a) stable, and (b) follows the physically correct path (for path-sensitive problems). |
| Matrices are never formed. Memory requirements are always at a minimum. No bandwidth limitations.                      | Stiffness matrices must be stored. Must find ways to overcome associated problems such as bandwidth. Memory requirements tend to be large.                   |
| Since matrices are never formed, large displacements and strains are accommodated without additional computing effort. | Additional computing effort needed to follow large displacements and strains.  |

FLAC adopts explicit scheme so, here so, the time step is very smaller when compared to the implicit case because the equations are solved at every time step. So, it is not that something the iteration that happens in finite elements in case of implicit schemes is not done over here. So, this reduces the computational effort and here the no iterations are necessary in case of explicit.

Since the elemental equations are not formed over here, the matrices are not used so which in turn reduces significant computation time. So, whereas in case of implicit the elemental equations are formed global equations and they are iterated further to achieve the desired tolerable solution. This is a tolerance for the solution, so, this reduces the computation time in case of explicit.

So, explicit can also handle the non-linear problems in a robust way. It can accommodate large strains as well.

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Mechanics of computation using FLAC

- **Finite difference** – converts the differential equations into equivalent algebraic form
- **Explicit integration** – incremental but not iterative
- **Newton central difference method**
- **Lagrangian** – grid deforms as material deforms
- **Mixed discretisation** - Avoid Mesh (volumetric) locking - to better predict collapse load
- Quadrilateral element divided into a overlaid pair of constant strain triangular elements.
- Shear stress in triangular elements resist hour-glass mode of deformation
- **Large strain** – Geometric non-linearity
- **Fully-coupled mode** incorporating both, mechanical behaviour of soil and groundwater responses
- **Smaller time step** – propagation of numerical information between the nodes should be faster than physical information (can be extremely small for liquefaction problem because of the large wave speeds in saturated soils)
- **Pore water pressure** – it is inherent to FLAC and computed according to the volumetric strains

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To put it up briefly, so, the finite difference converts the differential equations into equivalent algebraic form and the explicit scheme is adopted here. It is incremental because the problem of interest here it is non-linear problem, it is but it is iterative. The explicit integration is adopted over here, explicit scheme is used in FLAC. So, it is incremental but not iterative.

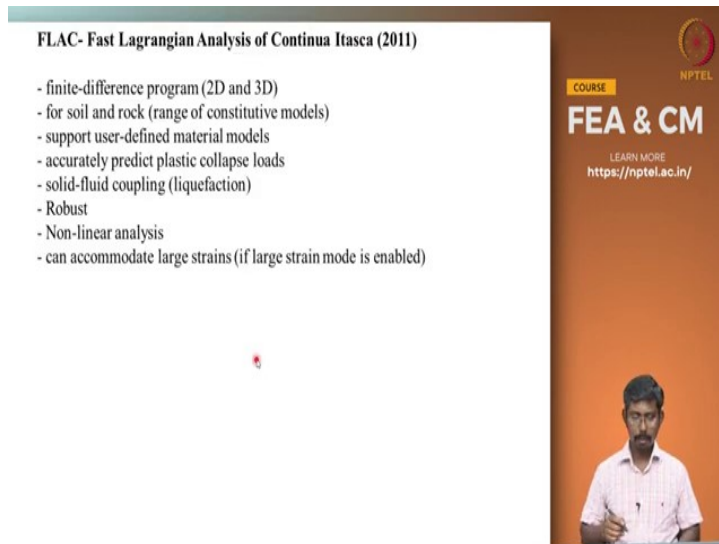
It is incremental because it is a non-linear form, non-lineous restrain relationship that is to be adapted for the soil material. So, it uses the Newton central difference scheme and it is a

Lagrangian. So, in case of Lagrangian the nodal displacements are updated and the grid D forms, along with the material that it represents.

And as explained earlier, so, the mixer discretization scheme is adopted over here, in which the quadrilateral elements are overlaid over each other and each quadrilateral elements are made of two triangular elements and this particular program can accommodate larger strains and fully coupled mode. This is most important aspect in case of liquefaction, wherein the volumetric changes due to shear loading and also the flow of water during such loading.

Both are equally important, so, this fully coupled mode can be performed in FLAC. So, it obviously uses a smaller time step and because of which the stability can be ensured and the pore water pressure is computed in FLAC based on the volumetric strains and the constitutive models used are effective stress models.

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**FLAC- Fast Lagrangian Analysis of Continua Itasca (2011)**

- finite-difference program (2D and 3D)
- for soil and rock (range of constitutive models)
- support user-defined material models
- accurately predict plastic collapse loads
- solid-fluid coupling (liquefaction)
- Robust
- Non-linear analysis
- can accommodate large strains (if large strain mode is enabled)

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So, both 2D and 3D programs are available. So, this is particularly very useful for soil and rock modelling and it supports the user defined material models. Anyone can develop a new constitutive model and it can be implemented in FLAC. So that is what it is user defined models. So, it can predict the plastic collapse load as a result of mixed discretization scheme that is used in FLAC and solid fluid coupling.

So, both mechanical and pore water, solid, fluid so, fully coupled simulations are possible because FLAC allows such options. And indeed is robust and it can simulate the non-linear.

The non-linear loss can be easily the non-linear material behaviour can be simulated using FLAC and it can accommodate large strains.

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**FLAC- Fast Lagrangian Analysis of Continua Itasca (2011)**  
Undrained/Drained/Fully coupled  
Undrained and drained are extreme cases idealized.  
In reality, it is fully coupled.  
Fully coupled Effective stress analysis  
Pore pressure generation and dissipation is modeled.  
Reduction in effective stress due to increase in pore pressure as a result of contractive volumetric behaviour of soil.  
Increase in effective stress due to dissipation of pore pressures.

**Solid-Fluid interaction**  
Mechanical volume changes leads to increase in excess pore pressure and thereby reducing the effective stress  
↑  
↓  
At the same time dissipation (consolidation) of excess pore pressure allows the soil Skeleton to recover the strength and stiffness of soil

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So, the drainage conditions are very important for modelling soil liquefaction, whether it could be undrained or drained or fully coupled So, undrained and drained are the idealized cases but fully coupled is the more relevant more realistic. Fully coupled is the more realistic drainage condition and fully coupled effective stress analysis is possible in FLAC. So, the pore pressure generation and dissipation can be captured.

So, when the soil experiences the shear loading the pore water pressure increases because the soil exhibits a contractive behaviour. And as the pore water dissipates as a result of the permeability of the soil and the drainage function the inherent drainage function, so, the effectiveness increases. So, both the reduction in effective stress due to the generation of pore water pressure.

And the increase in effective stress as a result of dissipation of pore water pressure, both are equally important to simulate the soil liquefaction. So, such solid fluid interaction is possible over here. So, the mechanical volume changes that leads to the increase in excess pore pressure. This leads to the reduction in effective stress. At the same time, the pore water pressure also dissipates as a function of the drainage possibility and that allows to recover the strength and stiffness of the soil.

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### FLAC- Fast Lagrangian Analysis of Continua Itasca (2011)

#### Undrained/Drained/Fully coupled

Undrained and drained are extreme cases idealized.

In reality, it is fully coupled.

#### Fully coupled Effective stress analysis

Pore pressure generation and dissipation is modeled.

Reduction in effective stress due to increase in pore pressure as a result of contractive volumetric behaviour of soil.

Increase in effective stress due to dissipation of pore pressures.

#### **FISH (in-built programming language)**

- Simple syntaxes
- Develop scripts to execute the analysis
- Allows to compute additional quantities
- User-defined constitutive model



Apart from this FLAC also has a inbuilt programming, language called FISH. So, it has simple syntaxes. So, these the scripts that are developed using fish can be used to execute the analysis or you can compute the additional quantities. So, for instance, excess pore pressure ratio which is of a key indicator for soil liquefaction. So, it is not a direct quantity that can be obtained in FLAC from the menu.

So, the functions can be used to obtain the  $r_u$ , the excess pore pressure ratio. And more importantly, the user defined constitutive models so that can be developed using the FISH program.

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**Liquefaction**  
Saturated loose sand deposit experiences liquefaction when it is subjected to seismic loading. The contractive nature of loose sand drives the pore water pressure to increase and subsequently the strength and stiffness of sand approaches zero.

**Failure of structures due to Earthquake induced soil liquefaction**

Four small images arranged in a 2x2 grid. Top-left: A multi-story building tilted significantly. Top-right: A large dam with a significant crack and water behind it. Bottom-left: A bridge structure that has collapsed into the water. Bottom-right: A road surface that has buckled and shifted.

Byrne et al. (2006)

A vertical banner for an NPTEL course titled 'FEA & CM'. It features the NPTEL logo at the top right, the course title in large white letters on an orange background, and a small photo of a man with glasses and a beard, likely the lecturer, at the bottom.

So, coming to the topic of discussion today, liquefaction so, the saturated granular soil experiences liquefaction when it is subjected to cyclic shear loading. This occurs during

earthquake. So, what happens is due to the quick nature of earthquake loading, so, the water present in the soil pores. So, experiences the pressure and this pore pressure could not be dissipated easily because the earthquake loading is very quick.

So, this leads to the accumulation of excess pore water pressure. So, altogether this excess pore pressure develops due to the contractive behaviour of soil that the way how it responds to the cyclic shear loading. And once this excess pore pressure reaches the initial effective stress of soil. So, the soil is said to have no strength and stiffness so which is the fluidized state of soil.

So, such liquefaction has caused a devastating damages in the past, for instance, the Nevada for instance Niigata earthquake in 1964, caused the damage to the buildings and the failure of dam is visible. Failure of bridge structure and damages to the harbour. So, these are all as a result of liquefaction. So, the consequence of liquefaction is catastrophic.

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**Liquefaction**  
Saturated loose sand deposit experiences liquefaction when it is subjected to seismic loading. The contractive nature of loose sand drives the pore water pressure to increase and subsequently the strength and stiffness of sand approaches zero.

**Mechanism of soil liquefaction**

The diagram illustrates the process of soil liquefaction. On the left, a 3D block labeled 'Saturated granular layer' is shown under 'Cyclic shear load'. A cross-section of this layer shows 'Sand grain' and 'Pore Water' with 'Effective Stress' indicated by arrows between the grains. Below this, text states: 'Before liquefaction: There is contact between the soil particles'. An arrow points to the right, showing the state 'After liquefaction: Soil particles are floating due to increasing of pore water pressure'. The diagram is attributed to 'Araujo and Ledezma (2020)'. On the right side of the slide, there is a vertical banner for the course 'FEA & CM' with the NPTEL logo and the URL 'https://nptel.ac.in/'.

Coming to the mechanism of liquefaction, so, the cyclic shear loading causes the increase in pore water pressure. You can see the water present between the soil particles and here in this case there is a contacts between the soil grains. When the load is applied as a result of which as a result of it, pore pressure increases and it leads to the loss of contact between the particles.

So, it is known that the sand derives its strength and stiffness based on the contact between the soil particles. So once the contact is lost so, it is no longer support the structure.

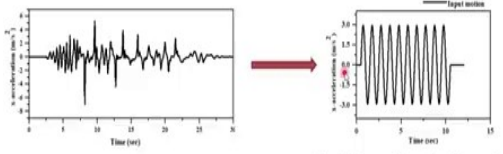


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**Basic aspects of element testing for liquefaction**

**Cyclic stress approach** - Evaluation of applied cyclic loading and resistance of sands  
Transformation of irregular loading history into equivalent uniform cycles of shear stress.

**Earthquake loading:** Equivalent number of uniform loading cycles (Seed and Idriss 1971)





**Irregular earthquake loading cycles**      **Equivalent uniform loading cycles**

(for illustration)

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To analyse the liquefaction the cyclic stress approach is the simplest approach. It is developed by Seed and Idriss. So, this enables the comparison of the cyclic shear loading, as well as the resistance of sand cyclic shear loading. We know that these are the earthquake loading has loading cycles which is non-uniform in nature it is quite irregular. So, its amplitude varies continuously but such kind of loading cannot be used directly in the lab to simulate the liquefaction.

So, it is necessary to convert the irregular loading cycles into equivalent uniform loading cycles. So, this earthquake loading and the sinusoidal loading plotted over here is not for scale. So, it is just meant for illustration purpose.

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**Basic aspects of element testing for liquefaction**

CSR (Cyclic Stress Ratio) is a ratio of uniform cyclic shear stress to initial effective stress

Cyclic triaxial :  $CSR = \frac{\sigma_{cs}}{2\sigma'_v}$

Cyclic simple shear :  $CSR = \frac{\tau_{cs}}{\sigma'_v}$

CRR (Cyclic Resistance Ratio) is the CSR for a specified number of uniform loading cycles causing liquefaction. (15 number of loading cycles for an earthquake magnitude of 7.5;  
reference stress = 65% peak shear stress amplitude)



Definition of CRR varies for different laboratory tests and for failure criteria (3% shear strain ;  $\tau_v > 0.98$ )

Excess pore pressure ratio : a ratio of excess pore pressure developed due to cyclic loading to initial effective stress

$$r_u = \frac{u_e}{\sigma'_v}$$

Liquefaction resistance :      Laboratory : cyclic triaxial  
   cyclic direct simple shear (CDSS)  
   cyclic torsional shear  
   Field test : SPT/CPT/DMT

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The cyclic shear loading is characterized in terms of CSR, cyclic stress ratio. It is the ratio of uniform cyclic shear stress to initial effective stress of soil. So, the CSR varies with respect to the kind of test that we perform. In case of cyclic triaxial so, the CSR is the ratio of the deviatoric stress to the confining stress. In case of cyclic direct simple shear, the CSR is defined as a ratio of cyclic shear stress to initial vertical effective stress.

CSR (Cyclic Stress Ratio) is a ratio of uniform cyclic shear stress to initial effective stress

$$\text{Cyclic triaxial : CSR} = \frac{q_{cyc}}{2\sigma'_{vc}}$$

$$\text{Cyclic simple shear : CSR} = \frac{\tau_{cyc}}{\sigma'_{vc}}$$

CRR (Cyclic Resistance Ratio) is the CSR for a specified number of uniform loading cycles causing liquefaction. (15 number of loading cycles for an earthquake magnitude of 7.5;

reference stress = 65% peak shear stress amplitude)

Definition of CRR varies for different laboratory tests and for failure criteria (3% shear strain ;  $r_u > 0.98$ )

Excess pore pressure ratio : a ratio of excess pore pressure developed due to cyclic loading to initial effective stress

$$r_u = \frac{u_e}{\sigma_{v0}'}$$

So, it depends upon the loading conditions in the test that we adopt. And this is about the loading applied to the soil element through the laboratory tests. But coming to the characterization of the liquefaction resistance of sand that is made in terms of cyclic resistance ratio CRR. So, CRR is nothing but the CSR that is, the cyclic stress ratio required to cause liquefaction attend applied uniform loading cycles.

The number of uniform loading cycles coming to the assessment of the liquefaction resistance of sand. That is the cyclic resistance, so that is made in terms of CRR cyclic resistance ratio. CRR is the cyclic stress ratio required to cause liquefaction at a specified number of uniform loading cycles. So, it has to be noted that it is uniform number of loading cycles because in laboratory we can only apply the uniform loading cycles.

So, normally the number of cycles is kept as 15. So, this 15 corresponds to the magnitude of earthquake of 7.5 and these both 15 and 7.5 are based on this particular aspect. Because the reference stress level chosen over here is 65 percentage of the peak shear stress amplitude. So, what is done is the for a given earthquake motion. The peak amplitude is considered and 65 percentage of that amplitude is chosen.

And the uniform loading cycles of that amplitude that is the 65 percentage of peak amplitude is applied to simulate the loading in laboratory. So, the definition of CRR varies with respect to the laboratory test and the failure criteria. So, how it is said to be to define the occurrence of liquefaction. The criteria's are based on both shear strain and as well as excess pore pressure ratio.

It is either 3 percent shear strain or excess pore pressure ratio of 0.98. So, the excess pore pressure ratio  $r_u$  is an important quantity a key indicator of liquefaction. So, it is defined as the ratio of excess pore water pressure to the initial vertical effective stress and apart from the laboratory tests. So, the cyclic liquefaction resistance can also be ascertained from the field as well.

So, it is standard penetration test, cone penetration test and dilatometric test can also be the data from these tests also enable us to determine the cyclic resistance of sand. This is beyond the scope of this particular lecture.

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**Basic aspects of element testing for liquefaction**

**Element tests on liquefaction**  
 Cyclic triaxial test  
 Cyclic direct simple shear test  
 Cyclic torsional shear test

**Contributors**  
 Seed and Idriss (1967); Ishihara et al. (1975)  
 Vaid and Finn (1979); Vaid and Chern (1985)  
 Mohamad and Dobry (1985);  
 Vaid and Sivathyalan (1997)

**Liquefaction related phenomenon**  
**Cyclic mobility** – soil with strain hardening behaviour  
**Flow liquefaction** - soil with strain softening behaviour  
**Lateral spreading** – sloping ground

**Cyclic mobility**

Graph (a) shows Shear stress (kPa) vs Shear strain (%) with a hysteresis loop. Graph (b) shows Shear stress (kPa) vs Effective vertical stress (kPa) with a cyclic loading path. Source: Bastidas (2016).

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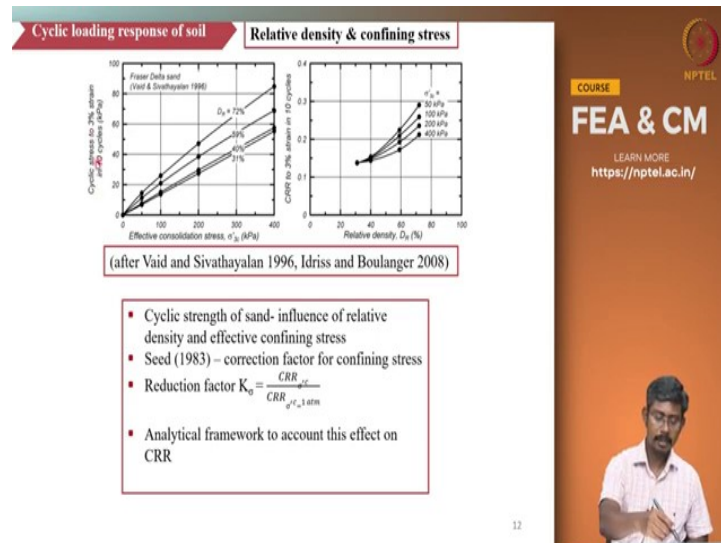
So, coming to the simulation of liquefaction in element is in laboratory. So, the most performed tests are cyclic triaxial, cyclic direct simple shear and cyclic torsional tests. So, various researchers have contributed for this particular aspect and the key phenomena's that are associated with the liquefaction are cyclic mobility flow liquefaction and lateral spreading.

These phenomena occur at different soil conditions depending on the behaviour of soil to the shear loading, for instance cyclic mobility occurs in a soil which exhibits, strain hardening response to the shear loading. And flow liquefaction happens in case of the soil. If it exhibits, a softening behaviour under monotonic loading and lateral spreading is possible, in mostly in cases of sloping ground where static shear stress is predominant.

Here in this lecture, it is all about the cyclic mobility. So, here the typical the response of sand to cyclic simple shear loading is plotted here. Firstly, the sustained response can be seen for initial loading cycles. The soil behaviour is stiff and as the loading progresses can see, the strain accumulates with increasing number of loading cycles and this is evident from the stress path plot as well.

So, this is the initial state and as the loading is applied the effective stresses decreases progressively due to the generation of pore water pressure. And finally, the typical butterfly loop is apparent over here which is a typical behaviour. That is a typical cyclic mobility behaviour of sand.

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The behaviour of sand is stress dependent, so, the stress dependency is based on three factors, relative density confining stress and static shear stress. So, here it is seen that the variation of cyclic shear stress ratio CRR here it is 10 cycles is considered. So, as I mentioned earlier, it depends upon the stress level that we choose. So, here CRR is defined as number of uniform loading cycles required to cause 3 percent shear strain.

That is inferred as liquefaction in 10 number of loading cycles. On the other hand it is plotted against the consolidation stress. So, for relative density the influence of relative density can be seen here, with the increase in relative density for any given confining pressure can be seen that the CRR increases. So, this is the influence of relative density on cyclic resistance of sand.

When it comes to the influence of confining stress, it can be seen that for a given relative density, the cyclic resistance decreases with increase in confining stress. So, these are important aspects that are to be accounted to simulate the soil liquefaction. Normally, a correction factor is applied to incorporate the influence of the confining stress. So, the reduction factor  $K_{\sigma}$  is defined as the ratio of the cyclic resistance ratio at any confining stress to the cyclic resistance ratio.

At a confining stress of 100 kPa. Through this, the effect of confining stress can be represented on the cyclic resistance ratio.

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Cyclic loading response of soil
Coupling with Critical state

Critical state principles are invoked to address the influence of relative density and confining pressure on CRR

- State parameter  $\psi$  (Been and Jefferies 1985)
- Relative state parameter (Konrad 1986)
- Relative state parameter index  $I_{rs}$  (Boulanger 2003)
  - based on Bolton (1986) dilatancy relationship (empirical equation)

$$I_{rs} = D_r \left( Q - \ln \frac{100 p'_c}{P_a} \right)$$

$$I_{rs} = \frac{1}{Q - \ln \frac{100 p'_c}{P_a}}$$

From Data used  
 $\Delta \epsilon_v = 21, 40, 80 \& 170\%$   
 $\sigma'_c = 10, 15, 1.5, 6.4$   
(Irfan & Boulanger 1998)

(Irfan and Boulanger 2008)

Critical state line from  $I_{rs}$  relation  
(Bolton 1986) with  $Q=12$

(Boulanger 2003)

Critical state line from  $I_{rs}$  relation  
(Bolton 1986) with  $Q=12$

(Idriss and Boulanger 2008)

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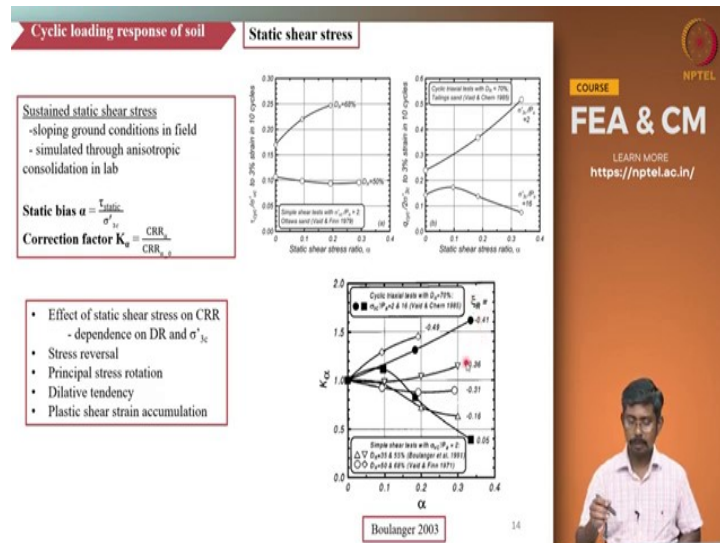
So, in order to represent the stress dependent behaviour of sand, it is necessary to invoke the principles of critical state. So, the main future of critical state is, it enables to use a single set of model parameters to represent the soil behaviour over a range of initial states that is at different relative densities and confining stress. So, the critical state begins with the state parameter.

It is the difference of current void ratio from the critical state void ratio. Here the relative state parameter index is focused over here which is based on the Bolton's dilatancy relationship. Here it can be seen that the critical state line plotted in void ratio and mean effective stress plot. So, this is a representation of density and this is a representation of confining stress.

So, if the critical state can be incorporated into the model, so, the soil behaviour at any given stress state can be simulated. Here the relative state parameter index is derived in terms of the relative density which is the difference of critical state relative density to the current relative density. So, the DRCS in turn is obtained in terms of the in the initial vertical effective stress and the other unknown parameter is  $Q$ .

It is a empirical constant it is based on Bolton's dilatancy index. So, this particular critical state framework is essentially an empirical one. So, with different  $Q$  values, the trends of  $x_i$  with respect to CRR is observed over here for two different testing conditions one is cyclic reaction. The other one is the simple shear.

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Apart from relative density and confining stress the other prominent factor is the static shear stress. So, this is the influence of this particular parameter is very much dominant in case of sloping soils. So, static bias is defined as that ratio of static shear stress to initial vertical effective stress or confining stress. This also has a prominent influence on the cyclic resistance, so that is apparent from the plot over here.

This plot is the static shear stress ratio  $\alpha$  versus the CRR cyclic resistance ratio. It can be seen that it varies with respect to relative density for a 50 percent relative density. There is not much of an effect of static shear stress on CRR but in case of 68 percentage of relative density. So, the CRR increases with  $\alpha$  and when it comes to the confining stress so, different contradictory responses.

Contradictory trends are evident with respect to confining stress as well. So, for higher confining stress the CRR is found to decrease with respect to  $\alpha$  on the other hand for a lower confining stresses the CRR increases with respect to  $\alpha$ . So, these trends are basically from the laboratory experiments performed by various researchers. So, these have to be embodied into the constitutive model if the sand behaviour is to be represented appropriately.

So that the liquefaction can be simulated realistically. So, for this purpose, the relative state parameter index. That is part which is previously discussed which is which for which paves the way for the critical state framework. For this particular case it can be seen that this IR is capable of capturing the trends exhibited by sand. For different static shear stress ratio and it is the correction factor over here  $K_\alpha$ .

It is the cyclic resistance ratio at any static shear stress to the cyclic resistance ratio when there is no static bias. So, this particular trends indicate that the  $x_r$ , the relative state parameter index can capture the effect of static shear stress on the cyclic resistance of sand.

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**Bounding surface plasticity**

Concept of bounding surface plasticity was developed by Dafalias (1975); Dafalias and Popov (1976)

**Yield surface and bounding surface**  
Material behaviour is elastic within yield surface  
Plastic straining occurs as stress state moves outside the yield surface, even within the bounding surface.  
Plastic modulus governing this plastic strain is a function of distance between the current stress state and its image point on the bounding surface.

Stress state cannot exceed bounding surface.  
Elastoplastic behaviour- If loaded when stress state is lying on bounding surface  
Elastic behaviour - If unloaded when stress state is lying on bounding surface  
Reloading causes plastic strain.

Potts and Zdravkovic (1999)

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Coming to the constitutive modelling for simulating soil liquefaction. Here, the bounding surface plasticity model is focused in this lecture. The concept of bounding surface plasticity was developed by Dafalias in 1975. This is a bit different from the classical elastoplasticity models here. The bounding surface is present and yield surface is present. So, if the stress state lies within this smaller yield surface, the behaviour is elastic.

And once the stress state exceeds this yield surface. So, the plastic training occurs and the stress state is bounded within the surface, so, it is for a reason it is bounding surface. So, the stress state cannot exceed the bounding surface. So, when the stress state lies on the bounding surface and if the material is loaded. So, it exhibits elastoplastic behaviour so which is typically a conventional yield surface.

But the main feature of this model is it can simulate plasticity when the stress state exceeds this inner real surface during loading itself but in classical elastoplasticity models. So, if the loading happens within the yield surface, so, the behaviour is said to be elastic. So, in order to capture the plasticity behaviour smaller yield surface is defined and the stress states are bounded by a much larger surface.

That is the bounding surface and the cyclic loading can be simulated through this. The loading and unloading, unloading is elastic in this case. So, during loading itself I mean the during loading within the bounding surface can represent the plastic strain as well. So, this plastic strain is governed by the plastic modulus. So that is a function of distance between the current stress state and its image point on the bounding surface.

So, for instance, so, the  $\sigma_{naught}$  is the point on the yield surface and this particular point is its projection on the bounding surface. So, the directly the distance between these two points governs the plastic modulus and as this distance decreases, the plastic modulus decreases. So, this enables the continuous variation of plastic modulus and it enables this particular framework very much suitable to simulate the cyclic loading.

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**2D Soil constitutive model**

**PM4Sand model**

**Loading surfaces in p-q space**

**Loading surfaces in stress ratio space**

(Boulanger and Ziotopoulou 2015)

$$M^b = M \exp(-n^b \xi_R)$$

$$M^d = M \exp(n^d \xi_R)$$

$$M = 2 \times \sin(\phi_{cp})$$

**Merits of the model**

1. Stress ratio controlled bounding surface plasticity model for plane strain condition.
2. Capable of simulating the seismic loading-induced soil liquefaction and cyclic mobility.
3. This model includes a damage index called 'fabric variable' which evolves in response to plastic shear strain during dilation.
4. With this fabric variable, PM4Sand can simulate Gradual accumulation of shear strains rather than lock-up in to a repeating stress-strain loop.

This model is available in FLAC-2D as a user defined constitutive model.  
The simulations were conducted using the finite difference numerical program FLAC-2D.

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So, PM4Sand model is from the family of bounding surface model. Here in addition to the bounding surface, this model is a critical state compatible model. So, to define the material behaviour. The critical state principles are invoked so that the material parameters can be used to represent the soil behaviour over a range of stresses. And other than critical state the dilatancy surface is also present.

So, this surface enables the transition in soil behaviour. If the stress state remains within the dilatancy surface so, the behaviour is contractive and when it crosses this dilatancy surface, though the behaviour becomes dilative. So, this dilatancy line is identical to the phase transformation line proposed by Ishihara. So which is basically the transition in volumetric behaviour from contractive to dilative.

So, here, in line with the discussed bounding surface concept. So, a smaller yield surface is present over here and the behaviour is elastic within this yield surface. And this yield surface can translate in these trust space, so that is called kinematic hardening, so, the seal surface can either it can translate as well as it can expand or contract in size. So, here only the translation is considered, so, the kinematic hardening occurs over here.

And the plastic modulus is computed as a distance based on the plastic modulus is computed in terms of the distance between the stress point on the yield surface and it is projection on the bounding surface the image point. So, for this purpose the alpha b, alpha d. So, these are the measure of distances from the current stress point. So, this in turn allows to compute the alpha b allows to compute the plastic modules.

So, it is essentially a stress ratio controlled constitutive model. So, the stress ratio is it is the ratio of shear stresses to mean effective stress. So, the mean effective stress decreases progressively due to the applied loading cycles because the pore pressure increases. So, the stress ratio is said to increase and it increases forth and back because it is a cyclic loading. So, the possibility of capturing the plastic strains within the bounding surface allows to capture the response of sand to the cyclic loading.

So, this is the concept behind the PM4Sand model. And it can be seen here this bounding surface, dilatancy surface and critical state surface. They are all defined as a function of the relative state parameter so which was discussed earlier. So, this  $\xi_r$  can account for the effect of confining stress and effect of static shear stress on the relative density. This  $\xi_r$  can account for the effect of static shear bias and the overburden stress on the cyclic resistance ratio.

$$M^p = M \exp(-n^p \xi_r)$$

$$M^d = M \exp(n^d \xi_r)$$

$$M = 2 \times \sin(\phi_{cv})$$

And this critical state is defined with the critical state friction angle. The other important aspect present in the PM4Sand model is the fabric variable it is a damage index. So, this fabric variable evolves when the stress state exceeds a dilatancy surface because the dilatancy the dilative behaviour of soil leads to the changes in orientation of the soil particle. So, once the stress state comes back from dilation to the contraction.

So, the contraction gets enhanced so, when to capture this enhanced contraction, the fabric variable is used in this model. And it is made as a function of plastic shear strain in this PM for sand model is available as a user defined constitute model in FLAC 2D and it is limited to plane strain conditions as of now.


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| Properties of Nevada 120 sand (Arulmoli et al. 1992) |         |         | Properties of embankment |                           |
|--|---------|---------|--------------------------|---------------------------|
| Relative density (%)                                 | 40      | 90      | Properties               | Embankment (Adalier 1996) |
| $G_{max}$ (MPa)                                      | 65      | 100     | Shear modulus $G$ (Mpa)  | 20                        |
| Dry density ( $kN/m^3$ )                             | 15.08   | 16.91   | Dry density ( $Kg/m^3$ ) | 1630                      |
| Permeability (m/sec)                                 | 6.6E-05 | 2.3E-05 | Cohesion (kPa)           | 22                        |
| $\nu_{max}$  |         | 0.887   | $\phi_{cv}$ (degrees)    | 31                        |
| $\nu_{min}$  |         | 0.511   | Poisson's ratio $\mu$    | 0.3                       |

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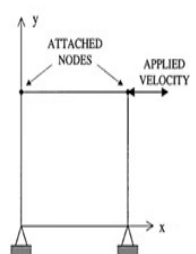


Here, Nevada sand is chosen as a material to simulate in FLAC using the PM4Sand model. And with respect to different relative density these are the values of shear modulus, maximum shear modulus, dry density and permeability. The  $\nu$  ratio, maximum minimum values for Nevada sand it is inferred from the literature. And the properties of embankment that is constructed over the liquefiable sand is said to have the shown properties maximum shear more or less of 20 mega Pascal and dry density of 1630 cohesion of 22 kPa and  $\phi$  of 31.

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**Calibration of PM4Sand model to Nevada sand**

Calibration of PM4Sand model is carried out by performing single element simulations Of Cyclic Direct Simple Shear Loading



Puebla (1997)

| Relative density (%) | 40   | 90    |
|----------------------|------|-------|
| $G_s$                | 726  | 950   |
| $h_{v,0}$            | 0.05 | 0.003 |
| $h_s$                | 0.5  | 0.5   |
| $n_b$                | 0.5  | 0.5   |
| $n_d$                | 0.1  | 0.1   |
| Q                    |      | 9.5   |
| R                    |      | 0.7   |

**Estimation of parameters**

$G_s$  - constant representing elastic shear modulus

$h_{v,0}$  - contraction rate parameter (controls volumetric strain accumulation when stress ratio is below dilatancy surface) (**trial-and-error**)

$h_s$  - controls the shear modulus degradation and hysteretic damping (**resonant column test data**)


$n_b$  and  $n_d$  - controls peak and phase transformation friction angles (**monotonic loading**)

Q and R - Bolton's dilatancy constants to establish critical state

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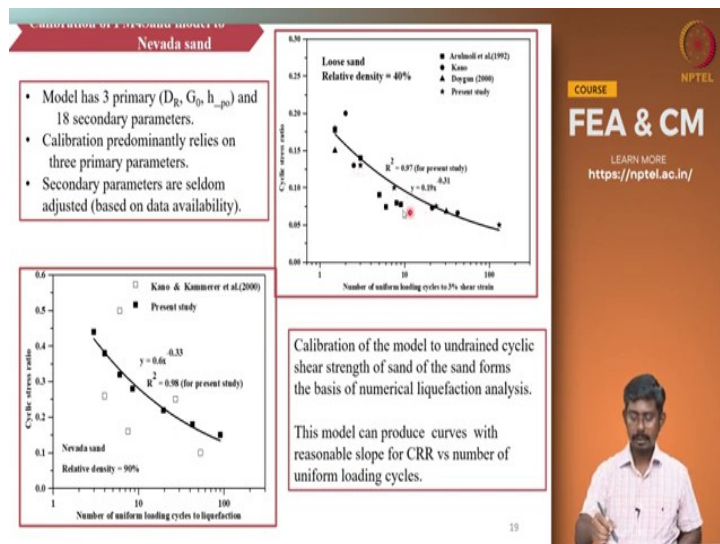
To calibrate the PM4Sand model to Nevada sand a series of cyclic simple shear simulations a single element simulations of cyclic simple shear loading was conducted. So, this shows the boundary conditions adopted. So, the base of the element is fixed and the top nodes of the

element are attached. So that it experiences the same amount of deformation. So, this is a typical boundary conditions that are describing the cyclic simple shear loading.

The loading is applied in the lateral direction. In the top surface of the specimen, so, these simulations are performed with a different set of model constants and these are the final set of model constants that are arrived based on the calibration performed. So, the PM4Sand model has three primary parameters. That is  $G_{naught}$  which is a representative of elastic shear modulus,  $h_p$  naught this governs the contraction rate of sand.

This  $h$  naught is a parameter to represent the plastic modulus and this can be obtained from the date of resonance column test which is performed for different strain levels  $n_d$  are the model constants  $n_b$  represents the bounding surface  $n_d$  for dilatancy surface. So, we just defined earlier as can be seen  $n_b$  and  $n_d$  over here. These are the input parameters to make the surface in the constitute model and  $Q$  and  $R$  are the Bolton's dilatancy parameters. So, these are empirical constants for a material.

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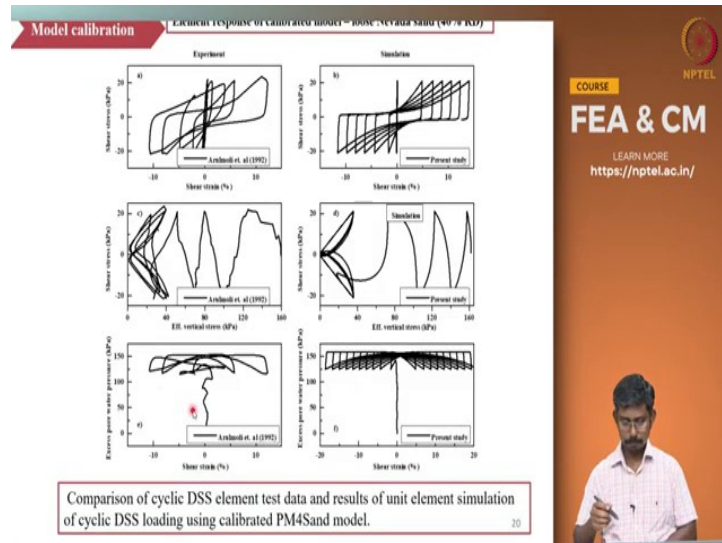
And so, by choosing a set of model constants listed in the table shown previously. These are the plots that are obtained for different relative densities. Say for this one is this plot corresponds to 40 percent relative density can be seen that the experimental data are sourced from the literature from various researchers. And the current study plot is here. So, the present study so by establishing a plot between the relationship between the number of uniform loading cycles required to reach 3 percent shear strain against the applied.

The calibration is performed by having a series of simulations unit elements simulations of cyclic simple shear loading. Here the number of uniform loading cycles required to reach 3 percent shear strain is plotted against the cyclic stress ratio. That is the loading applied to represent the shear loading. So, this expression  $y = 0.9$  into  $x$  rise to  $-0.31$ . So, this 0.31 is a factor.

So, this indicates the calibration of PM4Sand model to Nevada's sand, was performed by conducting a series of unit element simulations and the corresponding plot is here. It can be seen the data points from various sources from literature for 40 percent relative densities plotted over here. It is the number of uniform loading cycles to reach 3 percent shear strain that is liquefaction versus the applied cyclic stress ratio.

And this is the plot obtained and for 90 percent relative density so, this is the plot that are obtained.

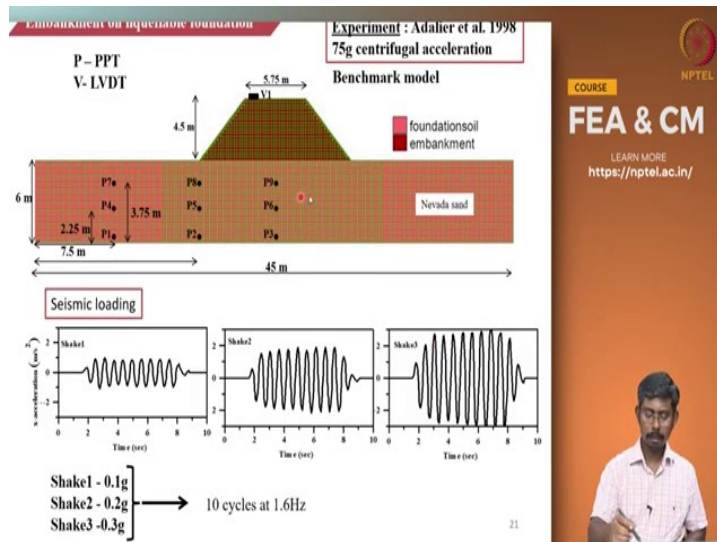
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So, the element response is the response of the calibrated element single element is compared with the respective experimental data. Experimental data is sourced from Adalier et al., 1992. The experimental data is sourced from the laboratory data of Arulmoli et al., 1992. It can be seen the stress state behaviour comparison first. So, here the model captures the progressive accumulation of shear strains with increase in number of loading cycles when it comes to just as path response.

A typical butterfly loop is captured and the increase in pore water pressure. With respect to shear strain is also captured.

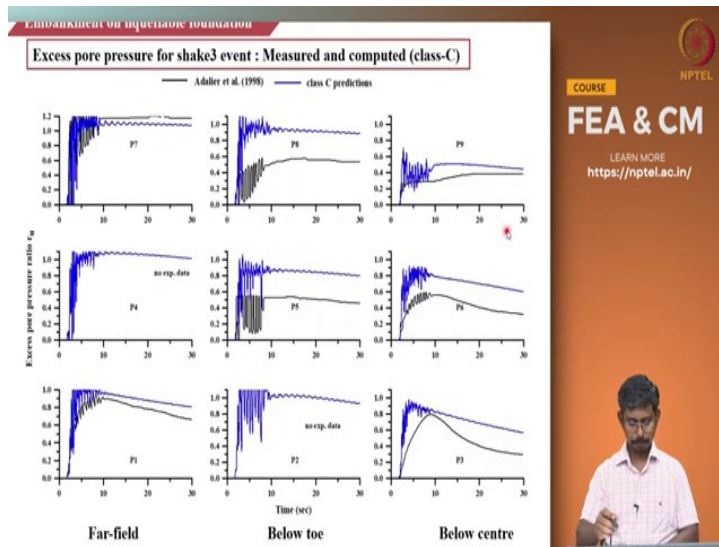
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So, coming to the boundary value case which is simulated in FLAC that is the loose foundation deposit that is, the liquefiable deposit, supporting an embankment clay embankment. So, this model is subjected to three different shaking events, their harmonic in nature, so, their amplitudes of 0.1G, 0.2 G and 0.3 G. So, each has 10 cycles and the frequency of 0.16 hertz.

Here the nodes of the left boundary is attached to the right boundaries to simulate the one directional shear beam loading. It is a common boundary condition adopted to simulate the cyclic loading of soil deposit.

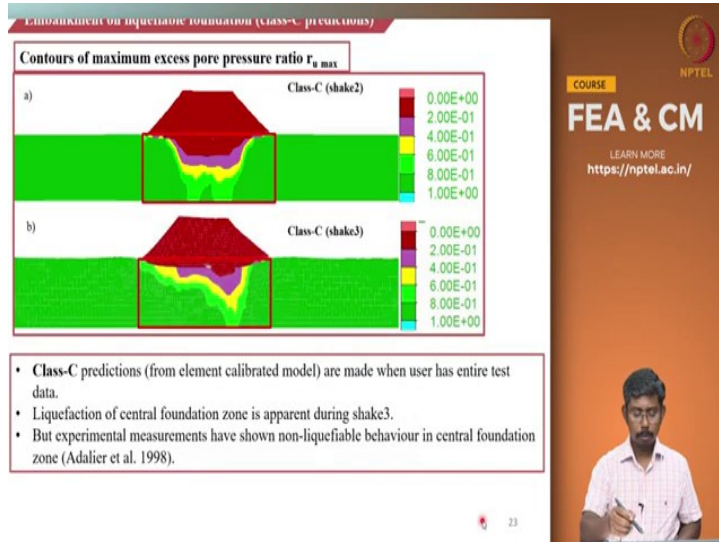
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So, the predicted the computed excess pore water pressure is plotted against the experimental data. Here it can be seen the locations three vertical array of pore pressure transducers are considered here. So, these are the locations where the experimental data is available and the pore pressure for these locations. This is P9, P6, P3 corresponds to the locations below the centre of embankment.

P8, P5, P2 corresponds to the soil deposit lying below the toe of embankment and this P7, P4 and P1 represents the far field case. So, it can be seen that for far field case a reasonable agreement exists between the computed excess pore pressure ratio and the measured excess pore pressure ratio. And when it comes to the foundation location below the toe of embankment, it can be seen the computed excess pore pressure is largely over predicted. The same trend is reflected at location below the centre of embankment as well.

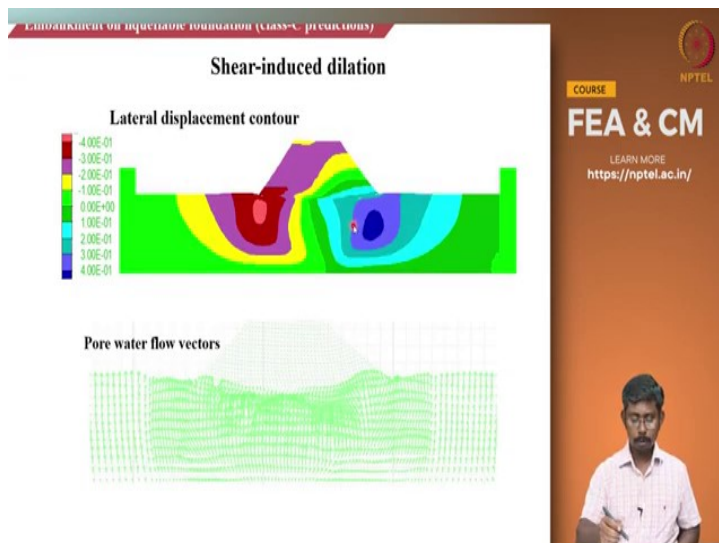
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So, apart from the individual pore pressure plots to gain additional insights the contours are plotted here. It can be seen that this is the excess pore pressure ratio contours maximum value of excess, pore pressure ratio. It can be seen that the far field region completely liquefied and the soil below the embankment also reaches liquefaction. And only a certain region below the embankment is still away from liquefaction.

This corresponds to the shaking 2 event and for shake 3 event. It can be seen that it is like the further the soil below the embankment reaches liquefaction. So, here this is reported as class C prediction. So, this prediction is nothing but it is undertaken when the experimental data is available to the modular already. So that is categorized as class C simulation. It is categorized as class C simulation.

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So, the discrepancy in computed excess pore water pressure at the central foundation region is attributed to shear induced dilation. So, here the lateral displacement contour is shown here. So, here it can be seen that the soil below the toe of embankment experiences, large lateral deformations in both the sides. So, this leads to the dilation of soil over here, as soil moves laterally in this direction, in the horizontal direction, under both the toes.

So, the soil below this region experiences dilation. So, this dilation is responsible for preserving the strength and stiffness of the soil deposit. That is lying below the embankment, so that is what the most literature has attributed to but the predicted excess pore pressures are high but the pore pressures are over predicted by the numerical model developed and in another instance.

The pore water flow vectors are shown here it can be seen that the vectors points upwards. So, this is the dissipation of excess pore water pressure and here in this region, it can be seen that the vectors are pointing towards inward direction, as opposed to the outward direction. So, this is said to be the dilation but the dilation occurs but not to a necessary level. So that the whatever the experimental data could not be predicted by the developed numerical model.

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Model parameters used for class-C and class-C1 predictions

| Model     | Class C prediction |     |       | Class C1 prediction |     |         |         |
|-----------|--------------------|-----|-------|---------------------|-----|---------|---------|
|           | n_b                | n_d | h_po  | n_b                 | n_d | h_po    |         |
|           |                    |     |       |                     |     | Shake 2 | Shake 3 |
| Benchmark | 0.5                | 0.1 | 0.005 | 0.2                 | 0.4 | 0.005   | 0.005   |

**Classes of predictions :** Class-A or Class-B or Class-C & Class-C1

- Class-A – Prior to the experiment
- Class-B – After the experiment (experiment results not known)
- Class-C – Experiment results are known
- Class-C1 – Model parameters adjusted to improve quality of predictions (Ziotopoulou 2017)

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For this class C1 simulations were performed. So, what are these classes of predictions are class A, class B, class C and class C1. Class A is the blind prediction, so, the computation is done prior to the experiment. And class B is done after the experiment but the results are not

known to the modular who conducts the simulation. And class C case represents that the experimental results are known by the person who is undertaking the numerical simulations.

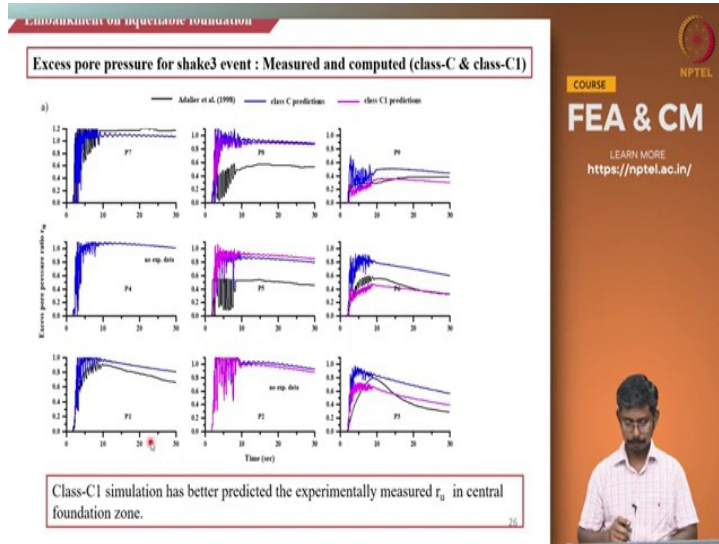
And class C 1 is a particular case where the soil model parameters are adjusted to improve the quality of the predictions. So, in this case, class C simulations are performed by adjusting the model constants  $n_b$  which governs the bounding surface in turn, the  $n_b$  governs the bounding surface,  $n_d$  governs the direct energy surface and  $h_p$  is a contraction trade parameter.

So, these values were changed for class C1 prediction because this  $n_b$  and  $n_d$  are adjusted so that the dilation is said to occur earlier than what it actually, predicted before in order to improve the simulation. So, for this the class C parameters were switched to class C1 parameters and the  $h_p$  for both striking event were kept same. In fact, all these parameters were same for both shake 1, shake 3 event.

Following the shake 1 event since the shaking event one is of smaller amplitude. It did not lead to liquefaction, so that is why the results of shaking 1 are not included here for class C1 predictions again. The element simulations were conducted and it is plotted over here. So, it can be seen that this is the bold line corresponds to class C case. The dotted line corresponds to class C1.

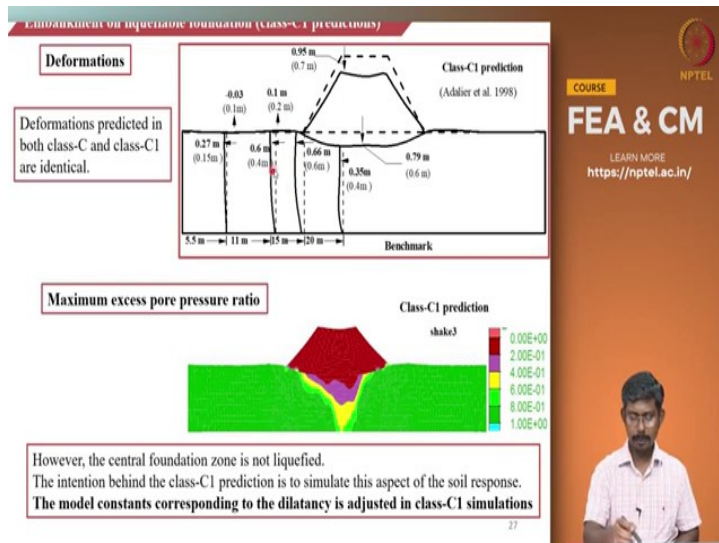
So, from this plot it can be seen that the cyclic resistance that are represented by class C1 cases higher than the class C case. So which is the response that is indeed for class C1 simulation.

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So now, the computed excess pore pressure ratio for both class C prediction and class C1 prediction are compared with the experimental result. So, it can be seen that now, the excess pore pressure shows a better match. The computed excess pore pressure, so, it shows a better match to the experimental results so, it is about class C1 prediction.

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And it can be seen that in case of class C1 prediction, so, the here it has the response had improved slightly. But still the soil has liquefied in this region but not like the case that was seen for class C1, not like the case that was shown for class C can be seen here over here. And the deformation plots are compared over here. It is corresponds to the C1 prediction and Adalier et al., 1998 corresponds to the experimental data.

So, it can be seen that the numerical model had over predicted the displacements vertical settlement. And it has under predicted the  $e$  over here and this lateral displacements are also over predicted.

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**Variable permeability in soil permeability**

Soil permeability increases with the increase in excess pore water pressure.  
**It is not based on experimental data.**

Governing equations (Shahir et al.2014)  
 $k_v = k_i \times (1 + (\alpha_k - 1)r_u)^{\beta_1}$   
 $k_h = \alpha_k \times k_i$

$K_v$  - permeability of soil when  $r_{u,max} < 0.95$   
 $K_h$  - permeability of soil when  $r_{u,max} > 0.95$   
 $\alpha_k = 10$ ; rate of increase in permeability during pore pressure build-up.  
 $\beta_1 = 2.0$ ; variable permeability factor

Husmand et al. 1987;  
 Arulanandan and Muralieatharan 1989;  
 Taiebat et al. 2007  
 Shahir and Pak (2010)  
 Shahir et al. (2014)

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Depth (m)

Maximum excess pore pressure ratio ( $r_{u,max}$ )

$\alpha_k = 1$   $\alpha_k = 3$   
 $\alpha_k = 5$   $\alpha_k = 8$   
 $\alpha_k = 10$  class C1

The plot illustrates the effect of **constant** soil permeability and the **variable permeability** on excess pore pressure generation.  
 Reduction of peak excess pore pressure is with

Other than class C1 considerations other than considering class C1 simulations temporal variation of soil permeability is also considered as an option. So, for this, the relationship proposed by Shahir et al., 2014 was chosen. So, in this case the soil permeability is made to increase as with respect to the excess pore pressure ratio  $r_u$ . So, as  $r_u$  increases which means that the soil approaches liquefaction.

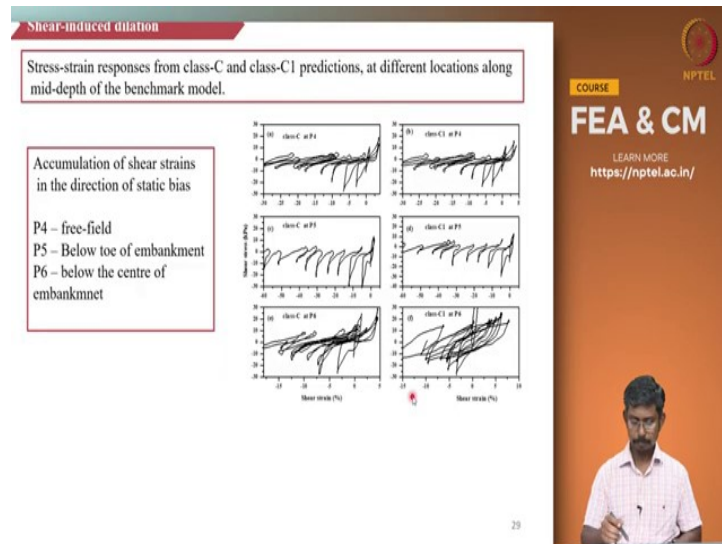
So, as we know that the soil particle loses contact, so, it can be the increase in permeability can be justified and in fact that has been suggested by several researchers as well. So that has been incorporated in this numerical model and the respective plots are shown here. So, it can be the alpha K corresponds to the factor by which the permeabilities increase. So, maximum value of alpha K chosen here is 10.

So, it can be seen that for alpha K 1 it can be seen that the excess pore pressure are lower and as you increase the permeability say for this case it is 3 and for 5 and for all the increments. So, the excess pore pressure ratio increases when compared to alpha K = 1. On the other hand, it can be seen that the plot for class C1 is also included here which is based on the adjusted dilatancy parameters.

So, this shows the lower pore water pressure ratio. So, from this exercise, it is found that the increase in permeability will lead to the increase in excess pore pressure ratio computed. It is because the soil in the free field, region, liquefies first and once the permeability of the soil is increased. So, obviously, pore water can come out of from the central region towards the far field.

So, this inhibits the dilation. So that in turn leads to increase in the excess pore pressure that are computed below the centre of the embankment.

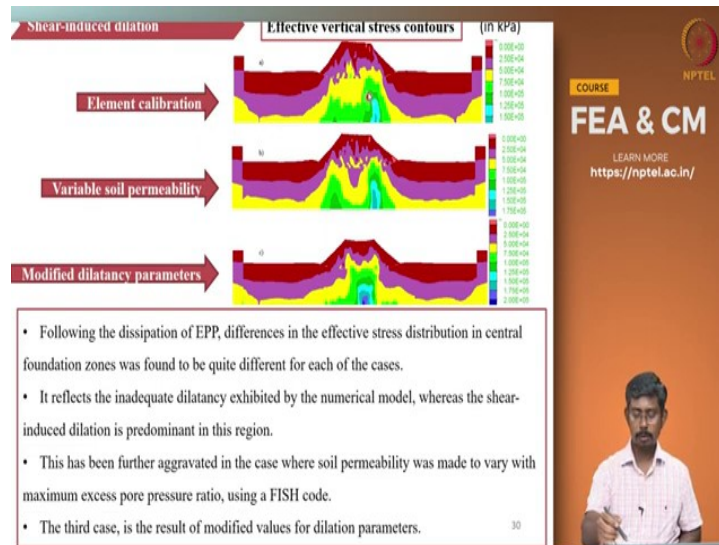
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This is also reflected in terms of reflected in the shear strain plots so, where it can be seen that P4 corresponds to the free field P5 corresponds to below the toe of embankment and P6 corresponds to centre of the embankment. It can be seen that for these cases these are the ones that correspond to class C and the right hand side it is class C 1. So, there is not much change, much difference in case of the free field scenario.

And for the case of 2 also, there is not much difference. But when it comes to below the centre of the embankment. So, it can be seen that some dilation is apparent in class C1 prediction when compared to class C prediction. So, this implies the difference between the class C and class C1 predictions that were performed in the study.

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This is also demonstrated with the help of vertical effective stress, so, this element calibration, so, this corresponds to class C case. So, these contours are the ones that were obtained following the dissipation of excess pore water pressure that were generated as a result of application of shaking event 2. So, here it can be seen that the effective stresses did not recover completely here in class C case and when the permeability of the soil is varied.

Further discrepancy was observed it is in fact, worse than this case class C case and when it comes to the modified dilatancy parameters, this is the class C1 case. It can be seen that the effective stresses are required are recovered to the levels that are actually expected. So, this summarizes the class C case and the variable permeability case and class C1 case.

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**Summary**

- ❑ Basic aspects of liquefaction were discussed
- ❑ Concept of finite difference computation using FLAC
- ❑ Bounding surface model – PM4Sand to simulate liquefaction
- ❑ Boundary value problem of embankment-foundation soil system

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So, summary of this lecture is the basic aspects of liquefaction. The cyclic resistance of sand and the element calibration of constitutive model to Nevada sand was discussed and the overview of FLAC program was provided. The use of bounding surface model that is the PM4Sand model to simulate liquefaction, was demonstrated with the case of a boundary value problem that is an embankment resting over a foundation soil. Thank you.