Course Name: An Introduction to Climate Dynamics, Variability and Monitoring

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Lecture 12

UNDERSTANDING ATMOSPHERIC STABILITY ADIABATIC RELATIONS AND LAPSE RATE

Good morning class and welcome to our continuing lectures on climate dynamics, climate variability and climate monitoring. In the previous class we started our discussion on the concept of atmospheric stability and instability and we defined the idea that an atmosphere is stable if large convection currents cannot arise within such an atmospheric condition, whereas an atmosphere is unstable if the conditions are such that you can have large vertical convection currents that can transport moisture from the surface to the upper atmospheres where they can form cloud fronts, thunderstorms, etc. So in this context, then a stable atmosphere is one where if a parcel of air is forced to rise for any reason, it will tend to sink back down when the lifting force is removed. So let me explain this idea a little bit more for recall. Suppose you have a air parcel near the surface, ok. Now, suppose for whatever reason the air parcel moves a little bit upwards, alright.

It will, it will have a little bit of different pressure forces being applied to it, correct, when it moves from one region in the altitude to another region in the altitude, ok. Now, whatever force that was caused this air parcel to move upwards was removed for example, Now the question is are the pressure gradient such that because the air parcel has moved upwards a spontaneous pressure differential comes into existence because of which the air parcel can move further upwards. If not, if the air parcel is in a region where the pressure forces are such the air, the air parcel will tend to move back down that is a downward restoring force is applied on it so that if it moves up it again comes back down then the atmosphere is stable and convection currents cannot rise because convection currents will only rise If a upward moving air parcel continues to feel an accelerating force that moves it up into the upper levels of the troposphere and a air parcel from the top of the troposphere, if it starts to move down, it will feel an accelerating downward force that will drive the air parcel downwards. Only then a vertical convection cell can truly form, ok.

So if such forces are not present and instead if an air parcel moves up the forces are such that it tries to move back down again and if an air parcel at the top atmosphere tries to move down the forces are such that it tries to move back up again. Then we have a stable atmosphere. So in short A stable atmosphere is one where if an air parcel is forced to rise for any reason, it tends to sink back down when the lifting force is removed. In a stable atmosphere, buoyancy driven convection currents are weak and it is difficult for energy and moisture to move vertically from the surface. And the tendency therefore of cloud formation is also low.

Thus, a stable atmosphere is linked with fair weather conditions. Contrast this to the case of an unstable atmosphere where if a parcel of air is displaced upward, it automatically creates a condition where a force is generated that tends to move it further upwards. So, if a parcel of air is displaced upwards, it continues to rise even after the lifting force is removed as the buoyancy force exceeds the gravitational force. So, the buoyancy force is increasing as you move up in altitude so that moves upwards continuously. In such a case, strong convection currents can spontaneously form in such a condition and you will get an unstable atmosphere with a rapid rise and fall of air currents creating large convection cells.

Now, these large convection cells will move moisture in the upper atmosphere where because of lower temperature it will condense into clouds and hence increase the chances of precipitation. Another important aspect of why precipitation events are also related to strong weather phenomena like storms, thundershowers, which has a lot of wind energy with it is because when the air parcel is cooled and the water vapor in it condenses, the condensation releases latent heat of condensation into the air around it. So, the air gets energized because of this heat release and this excess energy is converted into the kinetic energy of winds, alright. So, that is why large scale convection currents not only create precipitation events but violent storm events as well. So in general, the vertical stability and instability of the atmosphere is an important measure which indicates the presence of either fair weather or precipitation.

And how, and we will see what are the conditions that encourage the creation of unstable atmospheric conditions. And how the amount of evaporation that is happening plays a very key role in converting a stable atmospheric region into an unstable atmospheric region. And this has direct link to current climate change issues in terms of precipitation because hotter air, the rates of evaporation and the amount of moisture in air is also increasing. And this creates, encourages the formation of unstable atmospheric events more frequently, which is why you are seeing more violent weather events associated with storms, flash floods, etc. So, we will see all of these aspects now.

So, first let us look at the simpler case where we have a dry air condition only. So, we have a parcel of air which is mostly dry. So, the water vapor is a small insignificant fraction in this parcel of dry air. And we will see what are the conditions of stability and instability for a parcel of dry air with small amounts of water vapor which is far below the saturation limit. Now, air is of course an ideal gas and hence for ideal gas we have the ideal gas relation of Pb equals to mRT where R is the gas constant for air which is 0.287 kilojoules per kg Kelvin. Now, instead of density it is better to define the inverse of density which is called the specific volume alpha as volume by mass. So, it is meter cube per kg. So, density is mass per unit volume, specific volume is volume per unit mass which is inverse of density. So, this unit is we called alpha.

So, if you put, if you divide both sides of the ideal gas law by m, what you get is p alpha equals to RT, where alpha is V by m. Now, what we do is we take a differential of this ideal gas equation. So, we are looking at what happens if there are small changes in pressure, specific volume of or temperature within a parcel of dry air. And because to understand what are the what happens when this small things occur, we take a differential of this equation. So, we get P d alpha plus alpha dp equals to R dt.

$$Pd\alpha + \alpha dP = RdT$$

Remember R is gas constant, so it does not change. This is standard chain block pressure into differential change in specific volume plus specific volume into differential change in pressure equals to gas constant into differential change in temperature. However, we have already said that R is Cp minus Cv. So, the gas constant R is the difference between the specific heat at constant pressure minus specific heat at constant volume. This is a very standard relationship for ideal gas constant.

Where Cp is the specific heat of air at constant pressure, its value is 1.005 kilojoules per kg kelvin and Cv is the specific heat of air at constant volume which is 0.718 kilojoules per kg kelvin. So if you don't know what specific heat is, very briefly, specific heat is the amount of heat transfer required to increase the temperature of a specific mass of a substance by 1 degree. So if you take say 1 kg of air and keep it at constant pressure and heat it, it will require 1.005 kilojoules to increase the temperature of this 1 kg of air by 1 Kelvin. That is what specific heat at constant pressure means. While specific heat at constant volume means if you take that same 1 kg of air and heat it at constant volume, so volume is not changing, pressure may be changing. Then, it will cost 0.718 kilojoules to increase its temperature by 1 Kelvin.

So, this is Cp, this is Cv and the difference is the gas constant R. So, what we do here is we replace this R by Cp minus Cv. So, then T d alpha equals to Cp dt minus Cv dt minus alpha dt.

$$Pd\alpha = c_p dT - c_v dT - \alpha dP \qquad (35)$$

So, this expression we have derived by just the manipulation of the differential form of the ideal gas law. Now, we look into the first law of thermodynamics.

Now, the first law of thermodynamics basically says that the heat transfer into a substance minus the work output from the substance is equals to the change in internal energy of the substance. So, Q in minus W out equals to change in internal energy U of the substance. Heat transfer is heat transfer. Work transfer for a ideal gas is defined by the change in volume at a given pressure. So, the work transfer is basically integral of P d alpha.

So, this is work per unit mass. So, this is integral P into the change in specific volume per unit mass. So, integral P d alpha basically if you say heat up a gas its volume will specific volume will increase at a given pressure. So, that integral P into change in differential change in specific volume integral of that over an entire process is the work done by that parcel of air during this process. So, this is the expression for work here which we are written we are writing.

And the change in internal energy for an ideal gas where we can assume that the specific heat is relatively constant. So, for small changes in temperature, we can assume that the specific heat Cv is constant. The change in internal energy can be shown as change in internal energy delta u equals to Cv into change in temperature of that gas. This is also kilojoules per kg. So, this change in internal energy equals to specific heat into change in temperature delta T.

This is integral P d alpha and this is heat transfer q. So, the heat transfer into this air parcel minus the work output from this air parcel due to its expansion is equals to the change in specific internal energy which is equals to C v into change in temperature of air parcel. what we do here is this is the first law for a entire process. So, suppose we take a differential process, so a very differential change in the state of a parcel of air, then we can change this expression into a differential expression. So, the differential expression is differential heat transfer delta q equals to minus P d alpha plus C v into dT differential change in temperature.

$$q - w = \Delta u \quad or,$$
$$q - \int Pd\alpha = c_v \Delta T \qquad (36)$$

If you move the P d alpha on this side, the differential heat transfer into the air parcel equals to the differential change in internal energy du plus P into d alpha pressure into change differential change in specific volume. which is equal to Cvdt plus Pd alpha, okay.

$$\delta q = du + Pd\alpha = c_v dT + Pd\alpha \quad (37)$$

So, this becomes a differential form of the first law for an ideal gas and this is for our case for the air parcel in question. Now, if you look at this equation and compare it with this equation that we have seen, we are seeing that Cvdt is present, okay, Cvdt, okay. So, we can replace P d alpha plus Cv dt will be equal to Cp dt minus alpha d t. P d alpha plus C v d t equals to C p d t minus alpha d t. Here we have C v d t plus P d alpha. So, we replace this expression by the expression here in equation 35 to get final expression as del Q equals to C p d t minus alpha d p. So, C v d t plus P d alpha has been replaced by C p d t minus alpha d p. Now, what is alpha? From the ideal gas law expression here, this expression, alpha is RT by P, correct? So, the specific volume alpha of air is equals to R into T by P.

So, we replace alpha by RT by P. So, we get delta Q equals to CpDT minus RT into dP by P.

$$\delta q = c_p dT - \alpha dP = c_p dT - RT\frac{dP}{P} \quad (38)$$

So, we are just doing manipulations using the various forms of the ideal gas law and combining it with the first law expression. Now, we can go for a special case. Suppose this air parcel is experiencing a process where the heat transfer into the system or out of the system is zero.

So, delta Q is zero. Such a process is called an adiabatic process. So, an adiabatic process is one where the heat transfer into or out of the system is 0, ok. So, if this is the case then we can replace delta Q with 0 and we get for an adiabatic process Cp dt equal to RT into dP by P, ok. or dt by T equals to R by Cp into dt by T. So, we have taken the T here and R by Cp on this side.

$$c_P dT = RT\frac{dP}{P}$$
$$or, \frac{dT}{T} = \frac{R}{c_P}\frac{dP}{P} \quad (39)$$

So, this is a differential equation that we can easily integrate because this is d of log T and this is d of log P. So, doing this we get as the final expression temperature into pressure to the power minus R by Cp is a constant. This is the expression for an ideal gas which is experiencing an adiabatic process where there is no heat transfer between this ideal gas and its surroundings. In this case, the temperature and pressure of this ideal gas will have this relation according to equation 40.

$$\boldsymbol{TP^{-\frac{R}{c_P}} = constant \quad (40)}$$

So, if you have an adiabatic process which changes the state of that air parcel from some initial state which is at T1 and P1 to some final state given by temperature T2 and T2, we will have T1, T1 to the power minus R by Cp equals to T2, P2 to the power minus R by Cp.

$$\boldsymbol{T_1 P_1^{-R/c_P} = T_2 P_2^{-R/c_P} \quad (41)}$$

So, this expression we have derived and we will use this to understand what happens to an air parcel if it is moving up and down across a vertical air column. So, here then we will discuss what is called the adiabatic lapse rate for unsaturated air. So, air is dry or unsaturated and it remains unsaturated during the process. So, here we want to explain something. If the water vapor content is very low, we can approximate the air to be dry.

As long as the water vapor is not close to the saturation limit, we do not have to consider saturation or condensation processes happening or changes in the gas constant due to the presence of moisture. So, this is mostly for dry air where the water vapor content is quite low. So what happens to a dry parcel of air as it is moving up and down a vertical column of air? That is what we want to understand. So suppose a parcel of air is rising through the atmosphere in a convection current. Usually the air parcel is rising quite quickly and the heat transfer between the parcel and the surrounding is something that we can neglect.

So, suppose we have a air parcel of brief rectangular volume like this and this rectangular volume contains certain mass of air and this mass of air is rising upwards through in altitude in a vertical column of air. Now this movement is usually quite rapid and there isn't a significant heat transfer between this air parcel and the surrounding air. Hence we can neglect the heat transfer between this air parcel and the surrounding air in which this air parcel is in. So this is a simplification but let's go with the simplification and see what is called, what happens. So this is called the adiabatic motion of air where we are neglecting heat transfer So, air is whatever process that air parcel is undergoing, it is only happening in adiabatic process.

Then we can define something called the adiabatic lapse rate, which is given by del t del z subscript adiabatic. This basically tells you the rate of change of temperature of this air parcel with altitude as it rises adiabatically from altitude to a higher altitude or vice versa. So, basically the rate of change of temperature of this rising air parcel or conversely a falling air parcel with altitude under an adiabatic condition, that is this parcel is not exchanging any heat with the surrounding air which is assumed to be stationary, ok. So, this definition is called the adiabatic lapse rate. It is a rate at which the parcel of air's temperature will fall as it moves up in the atmosphere in an adiabatic motion.

Now, let us recall our hydrostatic balance equation which was g equals to minus 1 by rho dp by dz. Now, 1 by rho is specific volume alpha again. So, g equals to minus alpha dp dz or alpha into dp. alpha into dp equals to minus g dz.

$$g = -\frac{1}{\rho}\frac{dP}{dz} = -\alpha\frac{dP}{dz}, or\ \alpha dP = -gdz \qquad (42)$$

So, the hydrostatic balance equation can be expressed in this format.

The specific volume into differential change in pressure equals to negative of the acceleration due to gravity into differential change in altitude. Okay. However, remember what we have evaluated in equation 38. Del q is equals to Cp dt minus alpha dt.

Now, for an adiabatic condition delta Q is 0. So, Cp dt will be equal to alpha dp. So, for an adiabatic process Cp into dt will be equal to alpha into dp. Now, here we have the expression alpha dp in the hydrostatic balance equation. So, we can replace alpha dp by Cp dt if the air parcel is moving adiabatically. So, the hydrostatic balance equation can be written as Cp dt equals to minus G dz for an adiabatic process, correct? Hence, the adiabatic lapse rate condition can be written as dt by dz, which is our adiabatic condition, equals to minus g by Cp.

Just one clarification, this will be a minus sign. So, lapse rate is the decrease of temperature with altitude. So, you always have a minus delta dz term in the lapse rate equation. So, this is definition of minus delta delta. So, this is minus dT dz equals to minus g by Cp for an adiabatic process.

So, which is our adiabatic lapse rate condition. So, the adiabatic lapse rate for a parcel of dry air is given by gamma D. So, D is for dry air is equals to minus del T del Z adiabatic. So, this is a partial differential because we are keeping heat transfer as 0. So, it is not a complete differential, it is a partial differential.

We do not have to worry about this. It is basically this expression, equals to G by Cp.

Now, g is known 9.8, Cp is 1.005. 9.81 by 1.005 is approximately 9.8 Kelvins per kilo meter.

$$\boldsymbol{\Gamma_d = -\left(\frac{\partial T}{\partial z}\right)_{ad} = \frac{g}{c_p} = 9.8\,\frac{K}{km}} \qquad (43)$$

So, this will be the adiabatic lapse rate for dry air. What does this mean? It means that if an adiabatic, if a parcel of air is moving upwards through an adiabatic process, that is there is no heat transfer between it and the surroundings, then it will cool at the rate of 9.8 Kelvin per kilometer increase in its altitude. So, if the parcel moves from 0, from the sea level and its temperature is say 298 kelvins and it goes 1 kilometer up to an adiabatic process, no heat transfer with the surrounding, then it will cool by 9.8 kelvins. So, approximately its temperature will be 288 kelvins after 1 kilometer rise and so on and so forth. So, every kilometer rise an adiabatic parcel of air will cool by 9.8 kelvins. However, if you recall the mean lapse rate in the tropospheric system, we had a temperature gradient in the troposphere, right? How the temperature of the air is actually changing with altitude, okay? So, if you measure the air temperature from sea level upwards, what is the mean rate of decrease of this temperature? That was 6.5 kelvins per kilometer. The mean lapse rate in the troposphere, the rate of decrease of temperature with altitude is 6.5 kelvins per kilometer, whereas the rate of decrease of a dry parcel of air as moves upwards is 9.8 kelvins per kilometer. This means that the rate of decrease of temperature of an adiabatically rising parcel of air is larger than the mean rate of decrease of temperature of air with altitude. So, let us understand what this fundamentally means. This means, suppose you are at sea level and the temperature is 30 degree centigrade, around 303 Kelvin. In general, if you go up 1 kilometer, the air temperature will decrease by 6.5 Kelvin, which is 6.5 degree centigrade only. So, from 30 degrees, it will go down to say around 23.5 degree centigrade, 1 kilometer above the sea level at that certain point. However, if a dry parcel of air moves upwards from the sea level to 1 kilometer by a convection current, under the simplified condition that is not exchanging any heat with the surrounding stationary air, then it will cool by approximately 10 degree centigrade. So, its temperature will be 20 degrees, whereas the surrounding temperature is 24 degrees almost. So, that rising parcel of air will be cooling faster than the surrounding stationary air.

So, it will become colder and colder compared to the surrounding air at the same altitude. What does it mean? A colder air is of course less dense at the same pressure. Remember the pressure is changing similarly. So, the rising parcel of air has the same pressure in general compared to the surrounding air.

So, the pressure is not changing. So if you go back to your ideal gas law, PV equals to MRT or P alpha equals to RT, the pressure is constant, so alpha is proportional to temperature, the specific volume is proportional to temperature, ok. So if the temperature is lower, its specific volume is lower. or inversely specific specific volume is the inverse of density so if the temperature is lower the density is higher so because we are in a constant pressure system the air parcel pressure and the surrounding air is at the same pressure at any given altitude okay so its density will be higher than the density of the surrounding the colder air will become denser and denser compared to the surrounding air and a denser air will have a higher gravitational force compared to the buoyancy force. So whatever was the original buoyancy force that was allowing that air parcel to move upwards that would soon be overwhelmed by the increased gravitational force because of the higher density of this relatively colder parcel of air and hence eventually the air parcel will try to move back down because it is too dense, it cannot move up anymore. This is basically the stability condition that as you move up there is a restoring force that tries to move it back down and we see here for the dry case at least that as the air parcel moves upward it cools faster and hence becomes more dense in the surrounding so the restoring gravitational force tries to move it back down.

So in general What you are getting for a dry parcel of air is that because of its faster cooling tendency, it will mostly the case that gamma dry, the lapse rate for dry air will be greater than the mean atmospheric lapse rate or the prevailing atmospheric lapse rate. And this is the condition for vertical stability. So, in general, if gamma equals to minus del T del Z is the actual lapse rate of the air temperature and if gamma dry, the adiabatic dry air lapse rate is greater than gamma, then the atmosphere is vertically stable under unsaturated condition. This is only dry air condition.

However, of course, this is the mean lapse rate only. There may be cases at a certain atmospheric location, the mean, the local lapse rate of the surrounding air is actually greater. That is possible. It is rare, but it is possible. So, in case your atmospheric lapse rate gamma is greater than the dry lapse rate, then your buoyancy force, the air surrounding air is cooling faster and it is more dense than the rising parcel of air.

So, your buoyancy force will only increase. So, in that case you will have a unstable or an unsaturated condition. So, if gamma d is greater than gamma, then you have a stable atmosphere. If gamma d is less than gamma, then you have an unstable atmospheric condition and convection currents can sustain themselves and this is for dry air phase. So, in the next class, we will look at the moist air condition. So, thank you for listening and see you in the next class. Thank you.