

Course Name: An Introduction to Climate Dynamics, Variability and Monitoring

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**POTENTIAL TEMPERATURE, ADIABATIC LAPSE RATE OF MOIST AIR,
THREE POSSIBLE STABILITY RELATIONSHIPS**

Good morning class and welcome to our continuing lectures in climate dynamics, climate variability and climate monitoring. In the previous class, we were discussing the rate at which an air parcel cools as it moves up adiabatically in a vertical convection curve. And this rate depends on whether the air parcel is dry that is contains very little water vapor or wet that is its near saturation or not. First we discussed the adiabatic lapse rate for dry air which is given by γ_d . This is the rate at which the temperature of that air parcel is falling with altitude under an adiabatic condition where that air parcel is not exchanging heat with the surrounding atmosphere. And we saw that the adiabatic lapse rate for dry air called the dry adiabatic lapse rate was 9.8 kelvins per kilometer which is greater than the mean atmospheric lapse rate of 6.5 kelvins per kilometer. So, in general the dry even in a given location in general the dry adiabatic lapse rate will be greater than the mean atmospheric lapse rate in that location. What we also saw is that if this is true, that is if the γ_d is greater than γ , the dry adiabatic lapse rate greater than the prevalent atmospheric lapse rate at that location, then we have a stable atmosphere because the air parcel is cooling quicker than the surrounding atmosphere with height and hence will be colder and denser and hence the gravitational force will eventually cause the air parcel to sink back down.

So we have a restoring force of gravity that is acting against the buoyancy force that was initially causing the upward movement and hence we will have a stable atmospheric condition where the air parcels would not rise up or down too significantly and we will have weak convection cells. In contrast, if in a certain location the dry adiabatic lapse rate is less than the atmospheric lapse rate at that location, then we will have a vertically unstable condition for dry air as well because the rising parcel of air will be cooling at a rate which is slower than the mean atmospheric cooling rate with altitude and hence a rising parcel of air will be hotter and less dense than the surrounding air and hence the buoyancy force will allow that parcel to move upwards even further causing the formation of large convection cells which will result in an unstable atmospheric

condition. So, in general, if γ_d greater than γ , then the atmosphere is vertically stable for dry conditions. If γ_d is less than γ , then the atmosphere is vertically unstable for dry conditions. Another measure of atmospheric stability that is often used is similar, it is called the potential temperature θ .

So, this is what the nomenclature is potential temperature and the symbol that is being used is θ . So, let us look at the definition of this potential temperature. Suppose we have a parcel of air at a certain altitude and at that altitude it has a certain temperature T and pressure P . Now, potential temperature is defined as the temperature this parcel of air will attain if it were to adiabatically be brought from its original pressure P to the reference pressure P_0 which is equal to the 1 bar or 1000 hectopascals which is close to the sea level pressure. So, suppose we start with a given parcel of air at a certain temperature and pressure. And we do an adiabatic process by which the pressure of that air parcel moves from its original pressure P to the final pressure P_0 which is 1000 hectopascals or 1 bar.

$$\theta = T \left(\frac{P_0}{P} \right)^{R/c_p}, \quad P_0 = 1000 \text{ hPa}$$

Then the final temperature of this parcel of air after the end of this adiabatic process is the potential temperature value θ . Now, if you recall from the previous class, we evaluated this expression that in an adiabatic process on ideal gas, the initial temperature and initial pressure and the final temperature and final pressure related by this relation. In our case, the initial temperature is T , the initial pressure is P , the final pressure is P_0 and the final temperature is θ . So, we can directly apply this expression to evaluate the value of θ which becomes equal to T into P_0 by P into R by C_p , where θ is the final potential temperature of this parcel of air. T is the initial temperature of this parcel, P_0 is the standard reference pressure of 1 bar, P is the initial pressure of this air and R by C_p is basically the gas constant by C_p of air.

So, this expression gives you the potential temperature of any air parcel whose initial temperature and pressure we know. Using this potential temperature concept, we can create an additional and equivalent criteria for atmospheric stability. We can show that if the potential temperature increases with altitude, that is $\frac{d\theta}{dz}$ is greater than zero in any location, then the atmosphere is stable under unsaturated or dry conditions. So, let me explain what this means. We have in an atmosphere a certain temperature to pressure values from sea level to going up upward.

So, it is T_0, P_0 at the sea level, then $T_1, T_2, T_3, P_1, P_2, P_3$, the temperature is varying, the pressure is varying. At each altitude, there will have a certain temperature and pressure value T and P . From that, we can calculate the potential temperature θ at that altitude.

So, we can create an expression of theta with respect to Z. So, let us just write this expression here.

So, at any altitude, we can evaluate the potential temperature of air at that altitude Z as the same expression basically, but we can write this in this format. theta at a given altitude z is temperature in that altitude, at that altitude and pressure at that altitude. So, let us just call this 44a because it is basically the same expression, we are just making the z dependence explicit. The temperature of air is varying with altitude Z in a certain location, the pressure of air is varying with the certain altitude Z at that same location, so along the vertical direction. So, we can write an expression of theta with respect to z in that location along the vertical direction as well.

And we can then differentiate this expression with respect to z. Then what we are saying is if in a certain location $\frac{d\theta}{dz}$ is greater than 0, that is the potential temperature is increasing with altitude, then the atmosphere is stable under unsaturated conditions. So, if the potential temperature increases with altitude, that is theta is increasing with z, then the atmosphere is stable at that location under these conditions for dry or unsaturated air. If in contrast, $\frac{d\theta}{dz}$ is less than 0, that is the potential temperature theta is decreasing with altitude, then the unsaturated, under unsaturated conditions the atmosphere is unstable. the atmosphere will be unstable under unsaturated conditions when the potential temperature falls with altitude.

So, $\frac{d\theta}{dz} > 0$, stable. $\frac{d\theta}{dz} < 0$, unstable for dry conditions. And these are basically another way of expressing this gamma dependency. So, $\gamma_d > \gamma$, you have vertically stable conditions. And we can show this condition is the same as $\frac{d\theta}{dz} > 0$.

Similarly, $\gamma_d < \gamma$, which is a vertically unstable condition is the same as $\frac{d\theta}{dz} < 0$. So, it is just another way of explaining the stability or instability condition for dry or unsaturated air. However, one needs to note that in most locations, the adiabatic lapse rate for dry air remains greater than the atmospheric lapse rate. So, this expression is usually satisfied. Hence, dry conditions rarely give rise to convection currents and indicate fair weather conditions.

However, the presence of large amounts of water vapor will change the situation significantly and this is what we will go next time. So, let us look at the adiabatic lapse rate when we are looking at a moist air or saturated air. Now let's understand the basic physics first and then we will look at the expressions. In the next class, we will give a derivation of this adiabatic lapse rate as well if time permits. So when moist air is rising and cooling, initially nothing happens because the air is still under unsaturated conditions.

But eventually the air reaches its saturation point at that temperature and pressure, given the amount of moisture it contains, correct. So, eventually the temperature falls enough that the actual vapour pressure equals the saturation vapour pressure for that air parcel. As the air continues to rise further, it cools beyond this point and then water vapor starts to condense out of this parcel of air. This condensed water vapor creates fog or cloud and can be thought to be removed from that rising parcel of air. However, As the water vapor condenses to liquid water, it releases the latent heat of vaporization or condensation into air.

So, there is a certain amount of heat that is released during the condensation process and it is released into that parcel of air. Okay, so water as it condenses releases heat to air, that parcel of air rising up and hence because there is this internal heat addition due to the continuous condensation of water vapor into liquid water, you have an extra source of heat within this parcel of air that was not there for the dry air case. Because of this, the rate at which the temperature of that parcel of air falls as it rises up in altitude also decreases. So, the temperature decrease is less for a saturated parcel of air because of the heat released due to condensation of water vapor compared to the temperature fall for an unsaturated parcel of air under the same conditions. So what this means is that the saturated parcel of air would cool at a slower rate than an equivalent adiabatically moving unsaturated parcel of air.

This is called the saturation adiabatic lapse rate. or adiabatic lapse rate for saturated parcel of air, γ_s . s means saturated. This is $-\frac{dT}{dz}$ adiabatic under saturation conditions for that parcel of air. And we have found out from the basic logic of physics that γ_s will be less than γ_d .

That is the rate of decrease of temperature with altitude will be lower for a saturated parcel compared to an unsaturated parcel. we can also derive an explicit expression of this γ_s . So, the derivation of γ_s is given by after the entire derivation is done, the final expression looks something like this. And in an additional lecture, we will give the derivation of this γ_s as well, though it will be part of the bonus content, we will not, you will not have it explicitly in the syllabus, but it will be, it is good to for you to know what this γ_s is going to be. So, this γ_s is equals to γ_d by $1 + \frac{L_v}{C_p} \frac{DQ}{DT}$.

Let us understand this. γ_d is the dry adiabatic lapse rate. It is divided by $1 +$ an additional term and this term is usually positive. Because it is positive, γ_s is less than γ_d . And what does this term contain? The latent heat of vaporization of water vapor, so this is the heat that is released when you condense water vapor into liquid water. It is the latent heat of vaporization L_v .

C_p here is the C_p of air and $dQ_s dt$. Q_s is the saturation point specific humidity. Remember when we were discussing the specific humidity, Q_s is the mass of water vapor divided by the mass of total mass of air. So, mass of water vapor by the mass of moisture that is the specific humidity Q_s . The rate of change of specific humidity Q_s with temperature is $dQ_s dt$.

Remember this term depends on the partial pressure P_s value which in turn depends on temperature. So, this will have a gradient. Now, this gradient will be positive. The reason is as you increase in temperature, a certain mass of air can hold more water vapor. So, Q_s , the saturation point specific humidity, the maximum mass of water vapor in a mass of moist air which leads to saturation conditions will be increasing as the temperature of this moist air parcel increases.

So, the rate of change of Q_s with increasing temperature will be greater than 0. So, $dQ_s dt$ will be a positive gradient. C_p is the C_p of air, L_v is the latent heat of vaporization. So, this additional term ensures that γ_s is less than γ_d . we can also define an equivalent potential temperature θ_E .

How do we define this? Here the definition is a little bit more complicated than the initial conditions, but let us look at it a little bit carefully. It's the temperature that a moist parcel of air at a given state T and P would have if all its moisture were condensed out at adiabatic conditions.

So, if firstly... We are condensing away all the water vapor that was present in the saturated parcel of air under adiabatic conditions. So, we have removed all the water vapor and all the heat has been deposited in within the mass of this parcel of air and this parcel of air is not rejecting or accepting any heat from the surrounding. So, that is the adiabatic conditions. After this has been done, then this dried parcel of air is adiabatically brought to the reference pressure P_0 . So, the potential temperature for a dry parcel of air we just did an adiabatic process in which that dry parcel of air moved from its initial temperature and pressure to a final reference pressure P_0 and the corresponding temperature was the equivalent temperature.

Here for a saturated parcel of air we are doing a different process, we are condensing all the moisture out of the saturated parcel of air through an adiabatic process and then this dried parcel of air is brought to its final reference pressure. Then the temperature that this after this two-stage process is done, that temperature is the equivalent potential temperature. Now, since condensation releases heat to the parcel, so the equivalent potential temperature is greater than the potential temperature for the dry parcel of air. So, θ_E will be greater than θ and we can write this expression in forms of this exponential function. θ_E is equals to θ into exponential of L_v by C_p into Q_s by T .

So, latent heat of vaporization by C_p , the specific heat of air into Q_s by T , Q_s is the again the specific humidity and T is the temperature of that parcel of air, initial temperature of that parcel of air. So, this is the expression and in general this expression is greater than θ . So based on this analysis of what is happening for a moist parcel of air, we are seeing two trends. The saturation lapse rate, adiabatic lapse rate is greater than the dry adiabatic, is less than the dry adiabatic lapse rate. And the equivalent potential temperature is greater than the potential temperature for dry air.

So what is the condition for a vertically stable atmosphere? under all conditions. So, we are looking now at an atmosphere which can be moist, which can be dry. We are asking what would ensure that the atmosphere is stable regardless of whether it is dry or moist. That will only happen if the environmental lapse rate γ is less than the saturation adiabatic lapse rate γ_s , which obviously is less than the dry adiabatic lapse rate. So, the environmental lapse rate is lower than both the saturation lapse rate and the dry adiabatic lapse rate.

Originally what we said was $\gamma < \gamma_d$ then it is a vertically stable atmosphere. Here we are saying if γ is also less than γ_s , then this atmosphere is vertically stable under both dry and saturation conditions. So, this condition $\gamma < \gamma_s < \gamma_d$ has to be satisfied for this to happen. And we will get the equivalent expression that $d\theta_e/dz$ and $d\theta/dz$ both are greater than 0. So, both the equivalent potential temperature and the potential temperature is increasing with altitude.

So, $d\theta/dz$ is greater than 0, $d\theta_e/dz$ is greater than 0. Then if this condition, so this is an either or, either you can write it in this term or you can write it in terms of potential temperature. If the air parts are rising upwards, satisfies this criteria, then it cools at a faster rate both for the unsaturated and the saturated case and hence tends to sink back down. This is shown in this figure here, the figure D. The green line is the environmental lapse rate, ELR.

the dotted red line is the unsaturated lapse rate, unsaturated adiabatic lapse rate, the dotted the solid blue line is the saturated adiabatic lapse rate. And you can see that this is a very stable condition because at all the cases the temperature of the air of the moving air moving upwards is less than the environmental temperature. So, you have a very very stable condition. Both for moist and for dry air, the rising air parcel will be cooler than the surrounding and hence will tend to sink back down. The next condition is very interesting, it is a vertically unstable atmosphere under all conditions.

So, regardless of whether the air parcel is dry or wet, a rising air parcel will be hotter than the surrounding atmosphere and the buoyancy forces will tend to move it further upwards. So, the condition is reversed here, $\gamma_s < \gamma_d < \gamma$

gamma. So, the environmental lapse rate is greater than both the dry adiabatic lapse rate and the saturation lapse rate. Under this condition, both for dry and moist air, the air parcel will be hotter than the prevailing atmospheric case at any altitude and will tend to rise even upwards. And here for the potential temperature case $d\theta_e/dz$ is less than 0, $d\theta/dz$ is less than 0.

So, both equivalent potential temperature and potential temperature are increasing with altitude. Figure A is a condition of a very unstable atmosphere where the temperature is less than the environmental, greater than the environment temperature throughout the zone and the slope is also such that it will remain that way throughout the entire region of the atmosphere. Finally, we have a conditionally vertically unstable atmosphere or a conditionally stable atmosphere, both names are given. Here, the environmental lapse rate is in between the saturation lapse rate and the dry adiabatic lapse rate. So, the environmental lapse rate is lower than the dry adiabatic lapse rate.

So, the atmosphere remains stable under dry conditions. However, the environmental lapse rate is greater than the saturation adiabatic lapse rate and hence if there is a lot of water vapor in the parcel of air, then it reaches a condition where the moist air parcel is cooling at a slower rate than the surrounding air and so remains hotter than the surrounding air and hence buoyancy force can move it further upwards. So, if the air is moist and near saturation point, then we will have an unstable atmosphere with convection cells forming. If the air is dry, then it is a stable atmosphere with no convection cells forming. And here we have a change of sign between the potential temperature and the equivalent potential temperature. $d\theta/dz$ is greater than 0, but $d\theta_e/dz$ is less than 0.

So such a situation is conditionally stable in that stability is preserved when the air is unsaturated but atmosphere becomes unstable as the humidity in the air increases. And what is very interesting is conditionally stable regions are frequently encountered in the tropics where increasing humidity creates instability and vigorous convection and cloud formation. So as the air becomes more humid with evaporation, you will see a shift from a stable to unstable region when convection cells will form, cloud formations and storm fronts will begin to appear. Okay.

Now here, in this figure, a very interesting point is shown. The dotted red line is the unsaturated adiabatic lapse rate. The blue line is the saturated adiabatic lapse rate. Because initially, air is unsaturated. As it goes upwards, it is saturated. cools and eventually the saturation point is reached at the given temperature and then it becomes a saturated parcel of air and the slope decreases.

So, this is the blue slope is the saturation adiabatic lapse rate and the transition between the red and the blue happens in a specific altitude and this is called the lifting

condensation level. The altitude at which at a given location a parcel of air transitions from being unsaturated to being saturated. So, here there are two cases of partially unstable atmospheric conditions. This is somewhat unstable because the atmospheric temperature is lower than the rising parcel of air.

So, convection cells are forming. However, for the dry slope is actually higher than the green slope. So, the red slope is higher than the green slope. So, if this convection transition point the lifting condensation level was higher up then this could have reached a stable point where the temperature is actually lower than the surrounding temperature. So, this is a somewhat unstable. Similarly, this is somewhat stable here at all points the temperature of the rising parcel is lower than the environmental temperature.

So, it is stable. However, if you see γ_s has a lower lower slope than γ . So, this point if you extend there will reach a crossover point beyond which you will begin to see instability. So, it is a somewhat stable condition. So, conditional instability gives rise to very interesting features of dynamics of this shift from a stable point to a unstable point with altitude or with the time of the day.

So, I will stop here today. We will continue this discussion in the next class. Thank you for listening and see you in the next class.