Course Name: An Introduction to Climate Dynamics, Variability and Monitoring

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INTRODUCTION TO ATMOSPHERIC HEATING AND HEAT BALANCE

Good morning class and welcome to our lectures in climate dynamics, climate variability and climate monitoring. In the previous class, we looked and completed our discussion on how solar shortwave radiation and terrestrial longwave radiation gets absorbed and emitted as they pass through various atmospheric layers. We also derive the analytical expression that gives us the radiation intensity for a given frequency and in a given direction at any point in the atmosphere, whether it's for solar shortwave radiation or infrared longwave radiation, which is the terrestrial radiation going outwards into space or going downwards towards the surface. We also did a worked out example where we saw how a 10 kilometer layer of the stratosphere is able to effectively cut out all the UV radiation from the Sun using the expressions that we derived during the week. Once we have these, the question may arise What is the objective based on which we are deriving these expressions? What does it matter to the climate of the Earth as a whole? Before we begin our next section in this class, let me give you a brief picture of what is it that we are trying to do here. We had seen that if this is Earth, the surface of the earth, which is say at a certain temperature Ts, then between the surface and the top of the atmosphere, which say is at a certain temperature T infinity, there is a temperature gradient.

We have looked at this temperature gradient, say troposphere, then stratosphere, mesosphere, etc. And, while we have said what this temperature gradient is, the mean environmental lapse rate, we have not said how is it that this temperature gradient is established. We know that for the various layers of the atmosphere, that the various layers of the atmosphere, say a certain layer here, a certain layer here or a certain layer in the middle, they will be subject to both solar shortwave radiation and terrestrial longwave radiation. So as sun's rays passes through these atmospheric layers, a part of that energy will be absorbed by these individual atmospheric layers at a certain rate that we have derived in the previous week.

Similarly, as terrestrial radiation, let's call it blue, goes out and goes into space, certain part of that radiation and in fact, As we cumulatively, most of this radiation will get absorbed by the very same layers. But in return, they will also be themselves emitting radiation to the top and the bottom layers in its neighborhood. which in turn will be absorbed and again partly re-emitted back. And this process will go till a cumulative infrared radiation will be coming out from the top of the atmosphere. And we saw how this outward going flux, the F outward and the downward going flux F downward are evaluated for the infrared case and similarly we can do for the solar radiation case in which case it's mostly F downward, correct? Now, if you look at this, these layers individually and do an energy balance, what will you find? You will have, sorry about that, let's give a little color.

So if we take any one of these layers, this layer will be absorbing part of the solar shortwave radiation as they pass. It will also be absorbing infrared radiation coming from the top and the bottom and itself will be emitting infrared radiation outwards and downwards. So, if you recall from basic thermodynamics, the temperature of any gas is primarily dependent on the energy that is coming into the system and going out of the system. the temperature of this layer, say Tai, determines how much radiation it's emitting, correct? And this, if the temperature is going to remain constant through time, will be balanced by how much radiation this layer is absorbing. So, at a steady state, energy emitted must be equal to energy absorbed, correct.

And energy emitted is roughly proportional to the emissivity sigma into T ai to the power 4. And the energy absorbed is the fraction of solar shortwave absorbed and infrared longwave radiation absorbed. So, this balance determines the temperature of any atmospheric layer at a certain location z above the atmosphere. So, if we know how much solar radiation is getting absorbed and how much infrared radiation is getting absorbed, And if we know the emissivity of this body of gas, then we can know what temperature it is going to be using the steady state energy balance equation. This is called radiative energy balance equation.

Okay. The derivation that we will be doing is a more sophisticated version of this, but based on this idea, we can identify uniquely what should be the temperature of the atmosphere as we go upwards, based on how much radiation it is absorbing in the short wave and the long wave range, which in turn is based on the concentration of the absorbing gases like CO2, like ozone, like water vaporization. Most interestingly, emission of any of these absorbing gases like CO2, like methane as we are doing now is going to increase the concentration of these gases in these atmospheric layers, will be increasing the solar shortwave radiation that is being absorbed if say suppose ozone is increasing and increasing the infrared longwave radiation that is also being absorbed because of the presence of CO2, methane and water vapor. An increase on the right hand side necessarily means the left hand side has to increase to keep the steady state working. What this means is the temperature of that atmospheric layer will have to increase. and this is in fact what is causing the warming of the atmosphere.

The atmosphere is warming up because we have to maintain the energy balance that is the amount of radiation absorbed must equal the amount of radiation energy emitted and the amount of radiant energy being absorbed is increasing because the absorber concentration rho i is increasing due to human caused emissions. So, in the first section we have been able to deduce how much solar shortwave radiation will be absorbed given a certain amount of rho i and given the absorption mass absorption coefficient of that molecule which is primarily determined by its molecular structure. Now, we will do this steady state energy balance as we go along and evaluate the steady state temperature given a certain amount of concentration of these absorbing gases. With that introduction, let us now go back into our notes to understand how we will be doing this in this class and the next few classes. Atmosphere absorbs and emits radiation and heats up or cools down as a result.

Which in turn determines the temperature profile of the atmosphere we talked about. That is what I was explaining. So, first we need to know how much heating and how much cooling is happening due to the emission and absorption of the radiation. In the first part, we saw how much radiation is being emitted or being absorbed. Next, we will look how much energy is being added or taken out of the system due to the emission and absorption of this radiation.

Once we know that, then we will put in the final piece of how that energy is translating into the temperature profile of that atmosphere. So, there are three effects happening, heating due to absorption of solar shortwave radiation, and two other effects heating and cooling due to the absorption and emission of thermal long wave radiation. Of course to do that we will be using simplified methods because primarily here we are looking at analytical results when in a more advanced class of course we will be doing this numerically where we don't have to do these approximations. So, we will use what is called the plane parallel approximation, where we assume that the atmosphere consists of parallel horizontal slabs of gas layers as we have been showing, flat round slabs of parallel horizontal gas layers. And the movement of the radiation fluxes occur in the vertical direction.

And these movement of radiation fluxes in the vertical direction through these horizontal layers is the only process going on. So there is a lot of simplifications here which I need to draw your attention towards. The ground is flat and plain. The atmospheric layers are like parallel layers at different temperatures. The curvature of the earth is being neglected.

The vertical motion of air, something that we have discussed earlier when we were discussing vertical stability and vertical instability that is being neglected. horizontal motion of air, atmospheric wind circulation systems are being neglected. We will discuss those later in the class and any energy balance should also look at the vertical and the horizontal transport of energy due to the vertical movement of the gases. But that is something that we are neglecting in this very simple first order model. So, with that in mind, consider a horizontal slab of the atmosphere of area A and thickness del z situated at an altitude z from the surface.

The net upward radiation power entering the slab from the bottom is A into Fz. A is the area of this slab of horizontally layered air. Delta z is the thickness of this layer now Fz is basically this flux let me show you the flux somewhere here this radiation flux, the net IR flux due to upward moving radiation flux and downward moving radiation flux. Upward moving radiative flux, downward moving radiative flux. The sum of this and this at any location z from the ground.

That is what Fz is. That is the net flux that is being emitted or absorbed because Fz can be positive or negative, is a net flux at the horizontal location z and A is the area horizontal to the ground. So, the surface at any location z is A and the flux watt per meter square is the flux through that surface. So, the total radiation power in watts entering the slab from the bottom is A into Fz, ok. and that is the bottom at z. Now, this is at thick, so the layer is at a thickness delta z, okay.

So, the net radiation leaving from top of the slab is A into F z at z plus delta z. So, whatever is the net upward flux at z, Fz z plus delta z is the net upward flux at z plus delta z. So, this layer A into Fz is going in as the net energy per unit time a into Fz plus delta z is going outwards at from the top of that slab as energy going out per unit time. So, the net energy net change in the radiative power within this volume. What is this volume of the slab? The cross-sectional area A into the thickness delta z.

That is the net volume of this layer of air and the net change in the energy within this volume is A into Fz energy coming in minus A into Fz plus delta z. And this we can write as almost equal to minus a delta z dfz dz. dfz dz is the gradient of the net upward flux with altitude. It is negative here because this is At z plus delta z, so this is the final, this is the initial, so this is minus a fz plus delta z minus fz, that will be delta fz, correct. So, delta fz by delta z at the limit this becomes a delta z dfz dz.

$$A[F_z(z) - F_z(z + \Delta z)] \approx -A\Delta z \frac{dF_z}{dz} = \Delta \dot{E}$$

So, this is the gradient of the flux, this is the volume of this air layer, okay. This is the net loss in energy per unit time of this horizontal slab of air. This loss of radiative power implies that radiating heating, radiative heating per unit volume is given by minus dfz dz. This is volume into the volume. change in flux and this value is equal to minus dE dt basically.

$$\frac{\Delta \dot{E}}{\forall} = -\frac{dF_z}{dz}\ \frac{W}{m^3}\ .$$

So, the net change in the radiative flux is equals to the change in the energy per unit time within this volume. So, radiative heating per unit volume is the change in energy divided by the volume. So, this is equal to minus dFz dz. So, let me just write it here so that it becomes clear, equals to delta E dot, the change in the rate of power leaving the system, delta E dot. So, the net radiative heating So, this is minus delta E dot.

$$\dot{q}_{rad}(z) = -\frac{1}{\rho(z)}\frac{dF_z}{dz}\ \frac{W}{kg}$$

So, the net radiative heating is therefore, minus dFz dz. Then the heating rate per unit mass is given by this is delta E dot. dFz dz by rho z. So, let us understand this. This is the net radiative heating per unit volume.

So, this is basically delta E dot by the total volume V which is equals to A delta z. coming in minus going out is the net energy into the system that is why delta E dot in. This delta E dot in is the radiative heating, total radiative heating in this volume. So, radiative heating per unit volume is delta E dot by A dot del z which is minus dfz dz.

So, this is watt per meter cube. If you divide watt per meter cube by density which is kgs per meter cube you will get watt per kg. So, the heating rate per unit mass is given by q dot radiative heating per unit mass equals to minus 1 by rho z dFz dz. This unit is watt per kg and this unit here is watt per meter So, the heating rate per unit volume is minus dfz dz, the negative of the gradient of the net upward flux with altitude. And the heating rate per unit mass is 1 by rho times that value.

That is 1 by rho z dFz dz. Where Fz is the net upward radiative flux minus net downward hemispherical radiative flux. So, I hope that much is clear. So, now let us try to find this expression for at least the short wave heating piece. So, for short wave heating, The optical depth for the shortwave radiation at a frequency nu and altitude Z is given by this expression here.

We have already derived this. Z to infinity, partial density of the absorbing species, the mass-based absorption coefficient of the absorbing species and dZ. The downward irradiance of solar radiation at frequency nu, irradiance is basically radiation intensity, is given by, again this comes from the original expression, Fz This is downward radiative. Yes. Irradiance is radiative flux. So, Fz downwards at z equals to Fz downwards at infinity, top of the atmosphere, into e to the power minus down uz.

Okay. Here, the expression is very simple because there is no emission of shortwave radiation by any of the atmospheric layers. So, it is just attenuation that is happening. So, Just as we have done this for the radiation intensity, here we are doing it for the irradiance or radiation flux. The radiation flux at an altitude z is the radiation flux at the top of the atmosphere into the exponential to the power minus the optical depth. Here we are assuming that the sun is directly overhead and hence we are neglecting the angle at which the solar rays are incident, we are assuming all the solar rays are coming normal to the ground.

So, this is a very simplification, simplified approximation. We will see if this approximation is not valid what happens at a later time. The atmosphere does not radiate any energy at the short wave wavelength. So, the upward going short wave flux is zero. Hence, the net upward spectral irradiance is Fz nu at z is minus Fz nu downwards which is expressed by this term here.

Okay. Now we know that the spectral heating rate per unit volume which is rho into this, basically this term here. can be also written as a density into that skew dot radius mass based heating rate. The spectral heating rate per unit volume is minus dFz at further frequency by dz, basically this expression here, but based on the frequency, which now Fzw we can replace by ddz of this term here and ddz of this term If we do a differential of this term, this tau nu term can be differentiated d tau nu dz which is rho i k nu absorption i, correct. This term comes out and then the original term f z nu downwards at infinity e to the power minus tau nu z. So, basically we are differentiating this term.

Here we will look at a simple case where **the sun is directly overhead**. We will also neglect scattering. The optical depth for a SW radiation of frequency v at an altitude z is given by,

$$\tau_v(z) = \int_z^{\infty} \rho_i k^i_{v_{abs}} dz'$$

The downward irradiance of solar radiation at frequency v is

$$F^{\downarrow}_{z,v}(z) = F^{\downarrow\infty}_{z,v} e^{-\tau_v(z)} \quad (113)$$

The atmosphere does not radiate energy at the shortwave wavelengths. Hence $F^{\uparrow}_{z,v}(z) = 0$. Hence the net upward spectral irradiance is

$$\boldsymbol{F_{z,v}(z) = -F^{\downarrow\infty}_{z,v} e^{-\tau_v(z)}} \quad (114)$$

The **spectral heating rate per unit volume** at height z due to shortwave radiation is given by,

$$\boldsymbol{\rho(z)\dot{q}^{SW}_{rad,v}(z) = -\frac{dF_{z,v}}{dz} = F^{\downarrow\infty}_{z,v} e^{-\tau_v(z)} \rho_i(z) k^i_{v_{abs}}} \quad \mathbf{(115)}$$

Usually, mass absorption coeff. is independent of z and for many species the partial density can expressed as

So, this term comes out, then we get d tau nu z dz, d tau nu z dz is basically rho i at z K nu absorption i. So, we get this term here. And the negative cancels out because this is negative and this is negative as well. So, this becomes the spectral heating rate. The spectral heating rate is the net downward solar flux for that frequency at the top of the atmosphere, exponential to the power minus the optical depth at z, the density of the absorbing medium at z and the mass absorption coefficient value of the absorbing medium. usually this term does not depend on z at all, rho i z can of course be evaluated.

$$\rho_i(z) = \rho_i(0)e^{-\frac{z}{H_i}} \quad (116)$$

Where H_i is a constant related to the scale height. Then

$$\tau_v(z) = \int_z^\infty k_{v_{abs}}^i \rho_i(0) e^{-\frac{z'}{H_i}} dz'$$
$$= H_i k_{v_{abs}}^i \rho_i(0) e^{-\frac{z}{H_i}}$$
$$= \tau_v(0) e^{-\frac{z}{H_i}} \quad (117)$$

Where

$$\tau_v(0) = H_i k_{v_{abs}}^i \rho_i(0) \quad (118)$$

Hence (113) becomes

$$F_{z,v}(z) = -F_{z,v}^{\downarrow\infty} exp\left[-\tau_v(0)e^{-\frac{z}{H_i}}\right] \quad (119)$$

And the **volumetric heating rate for SW radiation of frequency v at altitude z w**hen sun is directly overhead is

$$\boldsymbol{\rho(z)\dot{q}_{rad,v}^{SW}(z) = F_{z,v}^{\downarrow\infty} k_{v_{abs}}^i \rho_i(0) exp\left[-\frac{z}{H_i} - \tau_v(0)e^{-\frac{z}{H_i}}\right] \frac{W}{m^3}} \quad \mathbf{(120)}$$

we have evaluated that in the example before but we can also make small It can either be given, we can evaluate rho i z based on the known mole fraction values or we can also have certain analytical models. So, one simple analytical model is rho i z is the rho i value at the sea level, the partial density of the absorbed medium at sea level e to an exponential decaying term of the form e to the power minus z by h i. where h i is a constant which is related to the scale height. It is not the scale height, but it is related to the scale height. So, we can put this expression in the tau nu z term. See tau nu z is this expression here, we can put that expression here rho i naught e to the power minus z prime h i dz prime and then integrate this term. Remember k is a constant for our purposes rho i naught is a constant, so we just integrate this from z to infinity and we can get this expression here and we can write the constant initial constant as tau nu 0 as thus this h i which is a scale height for the absorbing species, mass absorption coefficient and the initial density value rho 0. So we can put this and we get the final expression. We will continue this in the next lecture to see the final derivation that we will be getting. Thank you for listening and see you again in the next class.