Course Name: An Introduction to Climate Dynamics, Variability and Monitoring

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ATMOSPHERIC CIRCULATION SYSTEMS - PRESSURE FORCES

Good morning class and welcome to our continuing lectures on climate dynamics, climate variability and climate monitoring. In the previous class, we discussed the first of the major forces that governs the velocity gradients that are seen in the wind circulation systems on the planet earth. That is the Coriolis force. Today, we will look at the second most important force that governs the wind circulation system and that is the pressure gradient force which is of course far more familiar to you. Now the pressure gradient force or the pressure force per unit mass can be expressed in terms of pressure gradients. So, pressure is force per unit area.

$$\frac{\vec{F}_p}{m} = -\frac{1}{\rho}\frac{\partial p}{\partial x}\hat{\imath} - \frac{1}{\rho}\frac{\partial p}{\partial y}\hat{\jmath} - \frac{1}{\rho}\frac{\partial p}{\partial z}\hat{k}$$

So, force per unit mass is basically the gradient of this pressure per unit density of the atmosphere at that point. So, the x component of the pressure force per unit mass is the gradient $-\frac{1}{\rho}\frac{\partial p}{\partial x}$. Similarly, negative of the pressure gradient in the meridional direction divided by the density is the pressure force in the y direction and negative of pressure in the z direction divided by density is the pressure force in the z direction. Now, this is reasonably clear. Pressure is basically Newton per meter square. So, $\frac{\partial p}{\partial x}$ becomes Newton per meter cube. Density is kg per meter cube. So, you get Newton per kg, which is force per unit mass. So, the units match up.

Now, when we are looking at the vertical component of air, we just looked at this expression here, which is the force per unit mass in the z direction. And from there, we were directly getting the hydrostatic balance relation of $-\frac{1}{\rho}\frac{\partial p}{\partial z} = g$, which is the gravitational force mg per unit mass. Correct? So, clearly, we see these two components are matching or consistent with how we are defining the thing. Now what about the force pressure gradients in the x and the y direction? How do we get them? Here, we can make some interesting simplification by drawing isopressure surfaces with height at any point above the ground. So let's understand how this works by first looking at the gravitational potential.

The gravitational potential per unit mass, so is basically g_z. The gradient of the gravitational potential per unit mass is $d\phi = -gdz$, which from this expression here is basically the gradient of the pressure in the z direction, the change in pressure in the z direction by density, $\frac{dp|_z}{\rho}$. So, the differential change in gravitational potential d phi is the differential change in pressure in the z direction by density. Then, at a given location where x and y values, the meridional position is constant, the zonal position is constant, change in gravitational potential with height z, assuming x and y are being held constant, is given by the magnitude is $g|\Delta z|$, which is the magnitude of $|\Delta\phi|$, which is the change in the pressure in the z direction by density, $\frac{|\Delta p_z|}{\rho}$.

$$g|\Delta z| = |\Delta\phi| = \frac{|\Delta p_z|}{\rho}$$

Now if we draw constant pressure surfaces, so suppose we have the surface of the earth at a certain height we find the pressure is 900 hectopascals.

Now we change the location a little bit in either the x or the y direction and again find the point above the ground where the pressure hits 900 hectopascals. So in this way, we can create a series of points along the X, that is the zonal and the meridional direction. So we have a two-dimensional surface. All of those points have the pressure of 900 hectopascals. That is the isopressure surface of 900 hectopascals above the ground.

Similarly, 800 hectopascals above the ground, 700 hectopascals above the ground. Now, these pressure lines are unlikely to be completely horizontal because pressure gradients are changing both in the x, y and the z direction. These isopressure lines will have waves. Somewhere, there will be dips. Somewhere, it will be moving upwards.

So, it can be shown that the pressure force in the x direction, that is the zonal direction per unit mass, is the negative of the gradient of gravitational potential along the zonal direction, along an isopressure surface.

$$\frac{F_p^x}{m} = -\frac{\partial\phi}{\partial x}\bigg|_{p=const}$$

So, what do we mean by that? Suppose we take a point here, what is the x component of pressure force at this point? x component of pressure force, suppose this is the x direction, is the gradient or the tangent of the gravitational potential or the geopotential gradient along the isopressure line surface in this point. So, you have a isopressure surface heating this specific point. We take the gradient in the x direction along this isopressure surface of the gravitational potential. So, that value negative of that is the pressure force in the x direction.

$$\frac{F_p^y}{m} = -\frac{\partial \phi}{\partial y}\bigg|_{p=const}$$

Similarly, the pressure force in the meridional direction is the meridional gradient of the gravitational potential along the isopressure surface at that point. So, this can be shown. And so, if you know the isopressure surface, its slope, then the geopotential gradient along this isopressure surface in the zonal and the meridional direction, negative of that gives you the x and the y components of pressure force, while the z component is given by this expression here. So, you can just put g here, all right. When the derivatives of geopotential need to be taken along the constant pressure surface in the x and y direction.

Thus, the pressure gradient force is given by the slope of the geopotential in the x and y direction along a constant pressure surface. This means that steep slopes of constant pressure surfaces, so here the slope is very steep for example, as well moves in the x or the y direction will result in large pressure gradient in the zonal and the meridional direction. Remember phi is basically gz. So, $\partial \phi$ is gdz which is $\frac{|\Delta p_z|}{\Delta \rho}$. So, in a sense this is the gradient of the vertical component of pressure along the x direction, along the geopotential, along the isopressure surface.

So, here because the isopressure surface is very steep vertically, the dpz is changing very quickly. So, here your F_p^x will be very large whereas here it is horizontal. So, dpx is not changing at all. So, F_p^x will be very small. So, the slope of this isopressure surface gives you the magnitude of the pressure force in the x or the y direction.

So, in this way this expression can be evaluated. Now that we know Coriolis force and pressure force there is a very interesting concept that comes up especially away from the surface in the high altitudes and this is called the geostrophic balance and geostrophic velocity. What does this mean? So we will discuss. For large spatial scales length scales of 100 kilometers or more, away from the surface, so that the frictional effects are small, and far from the equator where the correlates force are weak. So, large spatial scales, scales of 100 kilometers or more, away from the surface, that is not very close to the ground where frictional forces are important, and away from the equator, so where correlates forces are strong enough, the only horizontal forces of importance that drive a parcel of air are the horizontal pressure gradient forces and the Coriolis forces.

So, if you have a large enough spatial scale, far enough away from the equator and away from the surface, the other forces drop out and you only have the horizontal pressure gradient which is the most important force and the Coriolis force. These are the only two forces of importance. When these two forces balance each other, the resultant flow is said to be in geostrophic balance. So let us understand what this means. So, what we are saying is, if you go back to the momentum equation, this is your say zonal momentum equation.

For large scales away from the surface and where Coriolis force is strong, the only force of importance is F_{Cor}^x and F_p^x, the pressure force and the Coriolis force. If these two forces are balanced, equal and opposite to each other, then of course because frictional force is negligible, $\frac{Du}{Dt}$ itself is zero. That is the velocity has reached a steady state. It is no longer changing with time or with space within that region. That is called geostrophic balance.

Similarly, in the meridional direction also if the y component of Coriolis force balances the pressure force, again the velocity $\frac{Dv}{Dt}$ becomes equal to 0. So, there is no effective change in velocity. The velocity has reached asterisk. This is the geostrophic balance in the meridional direction. So, when the Coriolis force and the pressure force balance each other, the resultant flow is called the geostrophic balance, and the corresponding steady state velocity is called the geostrophic velocity.

The geostrophic balance will also hold for oceanic circulation. So, we will discuss this, below the wind agitated mixed layer far away from the coast of the equator. So, even in the oceans away from the coastlines where frictional forces are strong and away from the surface where there is a drag force from the wind. So, down below the initial wind driven layer of the ocean, there also geostrophic balance is satisfied. So, this is an expression that works both for oceanic circulation and wind circulations.

$$fv = -\frac{1}{\rho}\frac{\partial p}{\partial x} = 0 \; or,$$

$$v_G = \frac{1}{\rho f}\frac{\partial p}{\partial x} = \frac{1}{f}\frac{\partial \phi}{\partial x}\bigg|_{p=const}$$

So, if the x component of Coriolis force and the x component of pressure-forcer balance, then it means fv which is the x component of Coriolis force and $-\frac{1}{\rho}\frac{\partial p}{\partial x} = 0$ or this v is now the geostrophic velocity, the meridional component of geostrophic velocity is $\frac{1}{\rho f}\frac{\partial p}{\partial x}$. Now, $\frac{\partial p}{\partial x}$ is $\frac{\partial \phi}{\partial x}$. as we can see here, alright. So, we can also write this as $\frac{1}{f}\frac{\partial \phi}{\partial x}$ because $\partial \phi$ is basically $\frac{dp|_z}{\rho}$, right. So, the change in geopotential is basically the change in the vertical pressure gradient divided by velocity rho.

So, this $\frac{1}{\rho}\frac{\partial p}{\partial x}$ becomes equals to $\frac{\partial \rho}{\partial x}$ along the constant pressure surface. So, that is the most important part. So, here we are replacing this $\frac{1}{\rho}\frac{\partial p}{\partial x}$ which is the F_p^x with $-\frac{\partial \rho}{\partial x}\Big|_{p=const}$. So, this is the change in the geopotential gradient in the zonal direction along a isopressure surface. This is the geostrophic velocity meridional component.

$$-fu = -\frac{1}{\rho}\frac{\partial p}{\partial y} = 0 \; or,$$

$$u_G = -\frac{1}{\rho f}\frac{\partial p}{\partial y} = \frac{1}{f}\frac{\partial \phi}{\partial y}\bigg|_{p=const}$$

The y component is $-fu = -\frac{1}{\rho}\frac{\partial p}{\partial y}$. This is the y component of poreless force. This is the y component of pressure force. So, ug, the geostrophic velocity in the zonal direction is $-\frac{1}{\rho f}\frac{\partial p}{\partial y}$, which is $\frac{1}{f}\frac{\partial \phi}{\partial y}\Big|_{p=const}$. So, note here that the meridional geostrophic velocity is generated by the zonal pressure or the zonal geopotential gradient, whereas the zonal meridional velocity, this one, is generated by the meridional pressure or the meridional geopotential gradient.

If you combine these two, what you are getting is the geopotential velocity vector $\vec{V}_G$ is $u_G\hat{\imath} + v_G\hat{\jmath}$. And this can be written here as $\frac{1}{\rho f}\hat{k}$, the vertical vector cross, it is the vector cross product of gradient of pressure. Or the gradient of pressure by rho is basically the gradient of the velocity. Geopotential gradient phi.

$$\vec{V}_G = u_G\hat{\imath} + v_G\hat{\jmath} = \frac{1}{\rho f}\hat{k} \times \vec{V}_p = \frac{1}{f}\hat{k} \times \vec{\nabla}\phi$$

So, $\frac{1}{f}\hat{k} \times \vec{\nabla}\phi$. Because this is the gradient of pressure which has gradient xp, gradient yp, gradient zp. But gradient zp k cross k is 0. So, you just get the gradient xp and gradient yp and you are getting these two components. So, this is $\frac{1}{f}\hat{k} \times \vec{\nabla}p$ or $\frac{1}{f}\hat{k} \times \vec{\nabla}\phi$, cross product. So, what does this mean? The geopotential velocity, this velocity vector is hence perpendicular to the plane containing the pressure gradient. The pressure gradient $\vec{\nabla}p$ or $\vec{\nabla}\phi$ is one vector.

Because it is a cross product with the k, the vertical vector, the geopotential velocity will be perpendicular both to the vertical vector and the gradient p vector. It will be in a plane perpendicular to these two values. Hence, the geopotential velocity vector will be perpendicular to the pressure gradient. At any given location, okay. Or it also means it will be tangential to the pressure gradient lines.

So, if you draw the lines of the pressure gradient how the how the pressure gradient vector is oriented it will be tangential to those, okay. Isopressure gradient lines it will be tangential to the isopressure gradient lines. Note also that the Coriolis force parameter f is positive in the northern hemisphere because sin phi is positive and negative in the southern hemisphere where sin phi is negative. Hence, the direction of the geostromic velocity switches by 180 degrees as one moves from northern hemisphere to southern hemisphere. Because f is negative, in the magnitude terms, it is $-\frac{1}{|f|}\hat{k} \times \vec{\nabla}\phi$ in the southern hemisphere.

So, if the pressure gradient and the vertical gradient is like this, the Coriolis force is towards this direction in the northern hemisphere and away from this direction in the southern hemisphere. So, you can do what is called the right-hand thumb rule, k cross the Coriolis vector going this way in the northern hemisphere, negative of this in the southern hemisphere. The change in the sign of the Coriolis parameter explains why winds are oppositely directed in the northern hemisphere and southern hemisphere and why cyclonic winds rotate counterclockwise in the northern hemisphere and clockwise in the southern hemisphere. Let us understand this idea. In a cyclone, what we have is a centralized low pressure surrounded by a high pressure.

So, the isopressure lines are circles and the pressure gradient is radial. So, the pressure is high here, low here. So, pressure is increasing in the radially outward direction. So, gradient of P is in the radial direction. So, suppose it is along the ground, gradient of P is in this direction, this is the vertical.

So, you have vertical cross the gradient of P. So, the velocities will be along the isopressure lines and tangential to the radial direction. So, the wind will be also rotating in a circular fashion. So, basically what is happening? The pressure gradient vector is perpendicular to the isopressure lines. So, clearly in most cases then the geostrophic velocity will be along the isopressure lines. Of course, it will also be perpendicular to the k vector.

So, if the pressure gradients are perfectly horizontal, then it will be along the isopressure line. Otherwise, it will be slightly different. But what we see here is then because the gradient is radially outwards and the k vector is coming out of the page, by the right hand thumb rule you are getting clockwise rotation of the geostrophic wind, in the northern hemisphere. In the southern hemisphere, this is anti-clockwise, anti-clockwise rotation. In the southern hemisphere, it is a negative of that. So, it will be left-hand thumb rule. So, it will be clockwise rotation of the winds in a cyclonic circulation. In an anticyclone, you have a high pressure at the center and low pressure at the outside. So, you have clockwise rotation of circulation system in an anticyclonic system in the northern hemisphere and anticlockwise rotation in the southern hemisphere for an anticyclone. So, cyclonic winds rotate counterclockwise in northern hemisphere and clockwise in southern hemispheres. Anticyclonic winds rotate clockwise in northern hemisphere and counterclockwise in the southern hemisphere.

So, this is the reason why we have cyclonic circulations in the first place. The geostrophic balance ensures the winds move in a circle along this low pressure, high pressure gradient. We can also express this in terms of temperature. we are discussing this in terms of pressure forces, but pressure gradients are related to temperature gradients using the ideal gas law where$p = \rho RT$. So, gradient of pressure is related to the gradient of temperature.

$$\frac{\partial u_G}{\partial \ln{(p_z)}} = \frac{R}{f}\frac{\partial T}{\partial y}\bigg|_{p=const}$$

$$\frac{\partial v_G}{\partial \ln{(p_z)}} = -\frac{R}{f}\frac{\partial T}{\partial x}\bigg|_{p=const}$$

So, we can derive the gradient of geostromic velocity with vertical pressure in terms of temperature and these are the final expressions. We are not deriving this here. The gradient of the zonal geostrophic velocity with respect to the log of the vertical component of pressure is equals to the gas constant for air into the Coriolis force constant F, force parameter F into the gradient of temperature in the meridional direction along a constant pressure surface. So, the gradient of temperature in the meridional direction along a constant pressure surface divided by the Coriolis parameter into the ideal gas constant for air is the gradient of geostropic velocity with the vertical component of pressure, log of the vertical component of pressure.

This can be derived, I am not deriving it here. Similarly, the gradient of the meridional component of geostrophic velocity with respect to log of the vertical component of pressure is equals to $-\frac{R}{f}$, the gradient of temperature in the zonal direction along a isopressure surface. So, temperature gradients have to be taken around the P equals to constant surface. So, the equation shows that the differential change in geostropic velocity which change in the log of the pressure along the vertical direction at a given altitude is equal to the temperature gradients of the constant pressure surface in the horizontal direction at that altitude. And in vector form again we can write this as a cross product. Gradient of the geostrophic velocity vector with the gradient of the log of the vertical component of pressure at a given altitude is equals to $-\frac{R}{f}$, f is the Fourier parameter into $\hat{k}$ cross gradient of temperature along a constant pressure surface.

$$\frac{\partial \vec{V}_G}{\partial \ln{(p_z)}} = -\frac{R}{f}\hat{k} \times \vec{\nabla}_p T$$

These equations are called thermal wind equations. And gives the gradients of the geostrophic wind with vertical change in pressure with respect to horizontal temperature gradients along isopressure contours. So why is this important? So we will close our lecture after understanding the physical sense of this. The gradient of temperature in the meridional direction is less than 0 in the mid to high latitudes in the northern hemisphere. Why? As you move from the equator towards high latitudes, clearly the temperature is falling.

So, $\frac{\partial T}{\partial y}$ along the isopressure contour is generally negative. So, what this means is? This term here is negative. Okay. Correlation force parameter is positive in the northern hemisphere. This term is negative in the northern hemisphere. This term in total is negative. Okay. Now, change in pressure with z as you move upward itself is negative. Right. So, $\frac{\partial u_G}{\partial \ln{(p_z)}}$ is less than 0. This is negative. Change in the zonal geostrophic velocity with respect to the change in pressure in the z direction, log of that is less than 0. However, as we move up in the z direction, the vertical component of pressure is falling. So, if you do a change in the Fourier system and try to do $\frac{\partial u_G}{\partial \text{z}}$, the change in the zonal geostrophic velocity with increasing altitude will be opposite in sign in terms of change in the zonal velocity with change in the vertical component of pressure.

Because $\frac{\Delta p_z}{\Delta z}$ itself is negative. Pressure is decreasing with altitude. So, if $\frac{\partial u_G}{\partial \ln{(p_z)}}$ is less than 0, $\frac{\partial u_G}{\partial z}$ is greater than 0. That is geostrophic velocity change with altitude will be positive in the northern hemisphere. What this means is the zonal component of geostrophic velocity with altitude is positive.

So the winds are going to accelerate in the eastern direction. The west to east moving winds will increase in magnitude as we move up from near the surface towards the upper troposphere. Thus, at these latitudes, as we move upwards, the wind accelerates in the west to east direction. Thus, strong west to east winds called westerlies tend to blow at high altitudes of the northern hemisphere as we move away from the equator due to the strong $\frac{\partial T}{\partial y}$ gradients. Now, the second point. $\frac{\partial T}{\partial y}$ equals to constant is greater than 0 in the northern hemisphere. As we move northwards in the, sorry, in the southern hemisphere, as we move northwards in the southern hemisphere, we are moving towards the equator. So, the temperature is rising. So, $\frac{\partial T}{\partial y}$ is greater than 0. However, note that F is negative.

Coriolis force parameter is negative in the southern hemisphere. So, once again, this term in total becomes negative. So, everything else remains the same, $\frac{\partial u_G}{\partial \ln{(p_z)}}$ which is less than 0 in the southern hemisphere. So, $\frac{\partial u_G}{\partial z}$ is greater than 0 in the southern hemisphere. So, in the southern hemisphere also, as we move up in altitude, you have strong west to east moving winds or westerlies.

So, powerful westerlies exist. in the mid-latitudes and the high-latitudes, especially in the regions where you have large gradients of south to north temperature. These westerlies are what we call jet streams in the mid-latitudes and the high-latitudes. And we will discuss these strong west, upper atmosphere westerly winds later in the class, but here it shows why these arise in the first place.

So, we will stop here today. We will continue our discussion in the next class. Thank you for listening.