Course Name: An Introduction to Climate Dynamics, Variability and Monitoring

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Week-9

Lecture 52

INDIRECT FEEDBACK: WATER VAPOUR FEEDBACK AND SPECTRAL OUTGOING LONGWAVE RADIATION

Good morning class and welcome to our continuing lectures on climate dynamics, climate modelling and climate variability. In the previous class we derived the expression for water vapour feedback parameter alpha H2O and what we saw was that it is composed of two terms Fdel omega sg that is the partial derivative of the net incoming flux of the troposphere with respect to the water vapor mass mixing ratio at the sea level into d omega s g dT, which is the total derivative of the water vapor mass mixing ratio at sea level with respect to the climate temperature. And we saw that these two terms together are negative. Because d omega sg dT is positive. As temperature increases, the mass mixing ratio, saturation mass mixing ratio of water vapor is increasing. And del F, del omega sg is also positive. The equation and text in the image can be written as follows:

$$\frac{\partial \Psi}{\partial \omega_s^g} = -\frac{\partial F}{\partial \omega_s^g} > 0$$

$$\alpha_{H_2O} = -\frac{\partial \Psi}{\partial \omega_s^g} \cdot \frac{d\omega_s^g}{d\tau} < 0$$

That is, as water vapor, saturation water vapor content increases, there is a net increase. There is an increase in the net incoming flux because the outgoing long wave radiation flux decreases. Because of this, alpha H2O is negative. As a result, it means that the net incoming energy flux into the climate system increases due to increased quantity of water vapor in the air.

which is in turn caused by an increase in the climate temperature. So, as climate temperature increases above its steady state level, the saturation mass mixing ratio of water vapor in the ground increases which in turn means that the net incoming flux at the tropopause level is increasing since the outgoing long wave radiation decreases and the climate heats up further. So, an increase in temperature cause a further heating because of the water vapor effect. Hence, water vapor concentration effect is a positive feedback which increases the temperature perturbation and moves it further away from equilibrium. We can do a detailed modeling.

We have already found the exact expression of d omega sg dT term as this expression here. And we can also find an exact expression of del F del omega sg and we will look at the method when we discuss how the flux, net incoming flux can be, how to quantify the change in the net incoming flux for due to the effect of greenhouse gases and water vapor is a greenhouse gas. So, we can evaluate this value and a full evaluation will show that alpha H2O is approximately minus of 0.4 times alpha naught, where the alpha not is the flank black body, flank feedback. Remember del F del P, the first term. Alpha 0 we have derived as 3.2 watt per meter square Kelvin. So, alpha H2O based on this expression is around minus 1.28 watt per meter square Kelvin which means that an increase in 1 Kelvin for the climate system temperature results in a decrease in the net temperature. results in an increase in the net incoming flux by 1.28 watt per meter square. So, just as the temperature feedback, the direct temperature feedback, what we saw is 1 Kelvin rise resulted in a 3.2 watt per meter square decrease in the net incoming flux. Here, 1 Kelvin rise causes a 1.28 watt per meter square increase in the net incoming flux. All right.

$$\alpha_{H_2O} = -0.4\alpha_0$$

$$\alpha_0 = 3.2\,\mathrm{W/m^2K} \quad \Rightarrow \quad \alpha_{H_2O} = -1.28\,\mathrm{W/m^2K}$$

So, together then the total feedback is alpha not plus alpha H2O, which is 3.2 minus 1.28, which is around 1.92 watt per meter square Kelvin.

$$\alpha = \alpha_0 + \alpha_{H_2O}$$

$$= 3.2 - 1.28$$

$$= 1.92\,\mathrm{W/m^2K}$$

So, overall the climate system still remains stable because alpha is still positive, though the amount of positive value of alpha is lower. So, when we consider both temperature feedback and water vapor feedback, 1 Kelvin increase in temperature of the climate system causes an effective decrease in the net incoming radiation by 1.92 watt per meter square. It is taken by putting these two together. Similarly, we can look at other aspects like cloud cover, albedo effects, etc.

And the total alpha will be the summation of all of those alpha values. Here also, we can understand what we mean by a runaway greenhouse effect, alright. So, for example, in Venus, the alpha H2O is much larger than alpha naught because the climate is almost entirely made up of water vapor. A significant amount of water vapor is present, alright. Because of the presence of water vapor, you have a very large value of alpha H2O, which makes the total alpha negative, which is causing a runaway greenhouse effect condition in Venus.

So, different planetary systems based on the various values of alpha will either have a stable climate or a runaway climate. Next, we are looking at the radiative forcing term. We have C dT prime dT plus alpha T prime equals to RFt. We have seen how alpha can be calculated for two cases.

$$C\frac{dT'}{d\tau} + \alpha T' = R_F(K)$$

Now, we will do the radiative forcing term for CO2 increase.

So, CO2 is the main greenhouse gas. So, how do we calculate the RF contribution for CO2? So, RF CO2 is change in the net incoming flux with respect to change in the mass mixing ratio of CO2 due to multiplied by the change in the CO2 mass mixing ratio X prime CO2. So, del F del x CO2 into x prime CO2, the gradient of flux with CO2 concentration into the change in concentration of CO2. Similarly, we can do for methane and the other greenhouse gases. So, we will do the CO2 case Suppose we assume that the CO2 concentration increases by a factor beta compared to the steady state concentration.

$$R_{F_{CO_2}} = \frac{\partial \Psi}{\partial X_{CO_2}} X'_{CO_2}$$

So, the original mass fraction of CO2, yCO2 was the original mass fraction under steady state condition and mu increased CO2 mass fraction is yCO2 mu. Remember mass fraction is defined as mass of CO2 by the mass of total air. This ratio yCO2 new by yCO2 is given as the term beta, the factor beta by which CO2 has increased.

$$\frac{y_{CO_2}^{new}}{y_{CO_2}} = \beta$$

So, if CO2 concentration has doubled, beta becomes 2, etc. Now, we will show through modeling methods that the radiative forcing of CO2 is of the order is approximately 5.3 into log of beta.

$$R_{F_{CO_2}} \approx 5.3 \ln \beta \, \mathrm{W/m^2}$$

So, if beta is the ratio of the new concentration to old concentration, then 5.3 log beta is the radiative forcing due to the increase in the CO2 concentration and this value is watt per meter square. Note that when beta is less than 1, log beta becomes negative. So, radiative forcing is negative, which means that the net incoming flux is decreasing. Whereas when beta is positive, the radiative forcing is positive and the net incoming flux is increasing.

And we will try to look at a very simple model to see how this functional relationship is obtained. Let us assume this is the tropopause, this is the troposphere and this is the ground. We can, for this very simple model, we take an average temperature of troposphere as T suffix small t. This is the average tropospheric temperature and we are assuming that the troposphere temperature is constant between the ground level and the top of the troposphere. Of course, this is not true and a more detailed model will actually give us, look at the temperature, the falloff ratio of temperature, falloff of temperature with altitude and we will use that exact terminology.

And we assume that the ground temperature is Tg, which is also the mean temperature of the ground. So, we take the mean troposphere temperature as Tt and the mean ground temperature as Tg. And we also assume that the transmittance for any frequency of radiation with a frequency nu is tau nu. So, for a given frequency nu of radiation emitted by the ground, the transmittance of the troposphere is gamma nu. This is capital gamma, capital gamma nu.

We also assume the troposphere temperature is lower than the ground temperature, which is reasonable. The spectral transmittance of the troposphere is gamma nu and of course, transmittance is between 0 and 1. So, if all the radiation is being absorbed, then the transmittance is 0. If all is being emitted, is transmitted to the top of the troposphere, then transmittance is 1. And the transmittance can be defined as, and we have defined transmittance before in previous weeks, as gamma nu as exponential of minus mod of 1.66 integral 0 to H, H is the height of the atmosphere, the mass absorption coefficient for the greenhouse gas J, for our case it is CO2, partial density of greenhouse gas J into dZ. Again this 1.66 is the correction factor for the between the plane parallel for the gray atmosphere case for the plane parallel approximation versus the curved surface. Alright. So exponential minus modulus of 1.66 integral 0 to H mass absorption coefficient of the greenhouse gas partial density of greenhouse gas d z. This is the spectral transmittance for any frequency mu. And remember this is spectral mass absorption coefficient. The partial density is basically the mass fraction of the greenhouse gas yj into the density of air at that altitude. So rho j is yj into rho z. Mass fraction of the greenhouse gas into rho z.

$$\Gamma_S = \exp\left[-1.66 \int_0^H K^j_{\nu\,\text{abs}} \rho_j \, dz\right]$$

And here we are assuming that the greenhouse gas mass fraction is not a function of z. So we have a well-mixed troposphere where CO2 or CH4 concentration is not changing with altitude at least in the tropopause and which is a reasonable approximation. absorptance is 1 minus gamma nu, 1 minus the spectral transmittance. By the Kirchhoff's law of radiation, we know that for an atmosphere in thermal equilibrium, the spectral emittance E nu is equal to the absorptance A nu which is 1 minus gamma nu.

So, if it is absorbing of frequency with the absorptance of A nu, then the emittance is also same as the absorption, absorptance and that is equals to 1 minus the transmittance. We assume the ground to be a blackbody and it is emitting radiation in the upward direction at a frequency nu, whose magnitude is given by the blackbody radiation. So the ground is a blackbody which is emitting radiation in the upward direction at a frequency nu and its magnitude is given by the blackbody radiation intensity B nu g. So B nu g is the spectral blackbody radiation intensity at frequency nu as emitted by the ground which is 2h nu cube by c square, c is the speed of light, h is the Planck's constant. looked at this expression before, into 1 by e to the power h nu by k B T, where T is the ground temperature, which is Tg minus 1, which is watt per meter square star radians hertz.

$$B_{\nu g} = \frac{2h\nu^3}{c^2} \cdot \frac{1}{e^{\frac{h\nu}{k_B T}} - 1} \frac{\text{W}}{\text{m}^2 \cdot \text{sr} \cdot \text{Hz}}$$

And the total upward radiation over the entire hemisphere, if we integrate over the entire upper hemisphere, then it becomes pi B nu g watt per meter square hertz. So, pi times this, so we are assuming gray, like it is not dependent on the direction of radiation as it is true for a black body. So, it becomes pi into d nu g is the total upward radiation, spectral radiation flux density. The fraction of this ground emitted radiation that reaches the top of the troposphere is the transmittance into this flux. The transmittance is gamma nu and the total upward hemispheric flux density is pi into b nu g.

So, gamma nu into pi b nu g. So, this is the radiation that is being emitted from the surface that is reaching the top of the troposphere. Along with this, the troposphere is also absorbed emitting radiation at its own temperature Tt. And here our simplifying assumption helps us to create an analytical solution, otherwise we have to look at multiple layers of the atmosphere and integrate over the entire atmospheric upward radiation for different temperatures. This will of course be done in a more detailed model, but here we are assuming an isothermal troposphere at a mean temperature Tt. Then the total emission under such condition is of course, the emittance of the troposphere E nu into the pi B nu t, where B nu t is the spectral blackbody radiation intensity direction dependent, which is basically 2 h nu cube by c square by 1 by e to the power h nu Kb into the temperature of the troposphere minus 1. where emittance is of course 1 minus, 1 minus the transmittance term. So, phi b nu t is the radiation flux emitted towards the upper hemisphere by the troposphere as a whole and emittance gives you that total value. So, this is the flux at the top of the troposphere, upward directed radiation flux at the top of the tropophore. At the top of the troposphere, this is the tropopause coming from the ground. And this is the upward directed radiation flux at tropopause coming from the atmosphere below.

Alright. So, the total spectral outgoing long wave radiation, SOLR. Okay. To the top. Outgoing longwave radiation is OLR, which is integration over all frequencies. If you look at a specific frequency, then SOLR, spectral outgoing longwave radiation, is given by this F upward for a given frequency OLR, which is the part coming from the ground and the part coming from the atmosphere.

So, pi gamma nu B nu G into pi emittance, which is 1 minus gamma nu B nu T watt per meter square hertz. Now, we have two limiting cases. The atmosphere is totally opaque to radiation coming from the ground. So, transmittance is 0. So, this we put 0 here and it just becomes pi d mu t, that emission from the atmosphere itself.

$$SOLR = F^{\uparrow}_{\nu\, OLR} = \Pi\Gamma_\nu B_{\nu g} + \Pi(1-\Gamma_\nu)B_{\nu t}$$

$$\mathrm{W/m^2 \cdot Hz}$$

Or the atmosphere is totally transparent, where so transmittance is 1. So, the upward going is only from the ground, pi d mu g. These are the two extreme cases. Now let us assume that the partial density rho i of the absorbing gas CO2 changes. So now we are looking at the perturbation effect.

So here till now we have looked at the net emission at the, net outgo emission at the tropopause level. Now we are changing the concentration rho i of the absorbing gas. then we want to find how much the radiative forcing will be due to this change. Alright, now how does this change affect? It affects by changing the transmitter. Clearly, because transmittance is defined, if you remember, by this term here.

So clearly, if the concentration of CO2 is changing, rho j is changing, so transmittance is going to change. As soon as transmittance changes, In this expression the new transmittance value will have to be put in and the outgoing spectral radiation is going to change. So, that is the effect, alright. So, we want to find out how much radiative forcing will be due to this change assuming all other climate system variables are held constant. The radiative force RF will be given by the change in SOLR caused by change in rho i when integrated over all frequencies.

So, the steady state concentration we are calling at rho i naught z which is y i naught into rho z. The mass fraction under steady state concentration of CO2 into the density change with height. Let us now assume that the absorber density increases by a factor B. So, yi nu by yi naught is beta.

This is what we have done for CO2. And this can be done for methane and also can be done for water vapor. So, whenever we are trying to find the del F, del change of the water vapor mass mixing ratio, that term also can be evaluated this way. So, we will show you how. So, y i nu by y i not is beta, this factor. So, y i nu into rho z, rho z is not changing by y i not into rho z becomes rho i nu by rho i not.

$$\frac{y_i^{\text{new}}}{y_i^o} = \beta \quad \Rightarrow \quad \frac{\rho_i^{\text{new}}(z)}{\rho_i^o(z)} = \beta$$

So, the partial, new partial density by the old partial density is given by this factor beta which is independent of z because it is a well-mixed atmosphere, ok. The spectral transmittance then becomes rho i nu is beta into rho i naught z and this beta into rho i naught z goes into the new spectral transmittance value expressed here. The beta can be taken out of the expression because beta does not depend on z. So, you get this expression here. we take a log of this log gamma gamma nu nu is exponent exponential term goes out.

$$\Gamma_\nu^{\text{new}} = \exp\left[-1.66\int_0^H K^i_{\nu\,\text{abs}}\beta\rho_i^o(z)\,dz\right]$$

So, it becomes this term here and the old transmittance is tau nu naught equals to exponential minus 1.66 this term without the beta term. So, log of tau nu naught is this expression here. So, the only difference between log of tau nu new by log of tau nu not is this terminology beta. 1 by beta into log tau nu nu equals to log tau nu naught, clear? So, tau nu naught, so if you take the antilog, the nu gamma nu, the nu transmittance is equals to the old transmittance to the power beta.

$$\Gamma_\nu^{\text{new}} = \exp\left[-1.66\beta \int_0^H K_{\nu\,\text{abs}}^i \rho_i^o(z)\,dz\right]$$

$$\ln \Gamma_\nu^{\text{new}} = -1.66\beta \int_0^H K_{\nu\,\text{abs}}^i \rho_i^o(z)\,dz$$

$$\Gamma_\nu^o = \exp\left[-1.66 \int_0^H K_{\nu\,\text{abs}}^i \rho_i^o(z)\,dz\right]$$

$$\ln \Gamma_\nu^o = -1.66 \int_0^H K_{\nu\,\text{abs}}^i \rho_i^o(z)\,dz$$

So, if CO2 fraction is doubling, then the, then what, the nu transmittance of the atmosphere is the old transmittance square, okay, for CO2. So that is the relationship here, that for any factor change, the new transmittance is the old transmittance to the power of that factor. Now, transmittance value is always between 0 and 1. So suppose beta is 2 and the original transmittance was 0.5. So the new transmittance is 0.5 square equal to 0.25.

$$\frac{1}{\beta}\ln \Gamma_\nu^{\text{new}} = \ln \Gamma_\nu^o$$

$$\Gamma_\nu^{\text{new}} = (\Gamma_\nu^o)^\beta$$

$$0 < \Gamma_\nu^o < 1$$

Suppose $\beta = 2$ and $\Gamma_\nu^o = 0.5$:

$$\Gamma_\nu^{\text{new}} = (0.5)^2 = 0.25$$

So because the transmittance is less than 1, if beta is greater than 1, the new transmittance will be lower than the old transmittance. Other way around is also, if beta is, so if beta greater than 1, new transmittance is lower than the original transmittance. If beta is less than 1, that is the CO2 concentration is decreasing, the new transmittance will be greater than the old transmittance, as we would expect. So, if the density of the absorbing gas changes by a factor beta, such that rho i nu equals to beta into rho i naught. Then the spectral transmittance at frequency nu changes by tau nu naught beta to the power beta. So, the steady state spectral outgoing long wave radiation is the original value of transmittance pi tau nu naught into the black body spectral intensity of the ground into pi 1 minus gamma nu naught black body spectral intensity of the troposphere. The new SOLR is pi into gamma nu naught to the power beta blackbody spectral radiation intensity of the ground into pi into 1 minus gamma nu naught to the power beta.

$$SOLR_{\text{new}} = F_{\nu\,OLR}^{\uparrow\,\text{new}} = \Pi\,(\Gamma_\nu^o)^\beta\,B_{\nu g} + \Pi\left[1-(\Gamma_\nu^o)^\beta\right]B_{\nu t}$$

So, these are the nu transmittance values into the blackbody spectral radiation intensity of the troposphere. So, this term changes. So, the change in the outgoing long wave radiation Sol R new minus Sol R naught is the difference between this term and this term. Once we put that, we get pi into the difference between the blackbody radiation intensity of the ground minus the blackbody radiation intensity of the atmosphere into the difference between the new transmittance value minus the old transmittance. This you can see yourself if you So, pi into difference between the spectral radiation intensity between the ground and the troposphere into the difference between the new transmittance and old transmittance.

If beta is greater than 1, then this value is negative. Because the new transmittance is lower, so this is negative. Also, because ground temperature is higher than the troposphere temperature, this term is positive. So, the total term becomes negative. So, the outgoing spectral long wave radiation value is decreasing.

$$\Delta F^{\uparrow}_{\nu OLR} = SOLR^{\text{new}} - SOLR^{o}$$
$$= \Pi(B_{\nu g} - B_{\nu t})\left((\Gamma^{o}_{\nu})^{\beta} - \Gamma^{o}_{\nu}\right)$$

If $\beta > 1$ (i.e., GHG concentration increases):

$$(\Gamma^{o}_{\nu})^{\beta} - \Gamma^{o}_{\nu} < 0$$

Hence:

$$\Delta F^{\uparrow}_{\nu OLR} < 0$$

It is becoming lower as we would expect. So, greenhouse concentration increases, outgoing long wave radiation decreases. And now we have the exact expression by which you can find this value for every frequency. And then you can integrate this to get the total change in the outgoing long wave radiation.

And this is what is done in numerical models. Alright. Let us define the difference between the old and the new transmittances D nu. Old transmittance minus new transmittance. And of course transmittance is between 0 and 1. So, the change in the outgoing long wave radiation spectral is minus pi D nu because the original case was this minus this.

So, this is minus D. So, minus pi D nu the ground radiation intensity minus the troposphere radiation intensity spectral value. What is the expression for this D nu with respect to beta? Because it is a function of spectral transmittance value actually and the beta value. So, here D nu has been plotted with respect to transmittance between 0 and 1 and for various values of beta.

$$D_{\nu} = \Gamma^{o}_{\nu} - (\Gamma^{o}_{\nu})^{\beta}$$

where $0 < \Gamma_{\nu} < 1$.

So,

$$\Delta F^{\uparrow}_{\nu OLR} = -\Pi D_{\nu}(B_{\nu g} - B_{\nu t})$$

Beta 1.4, beta is 2, beta 2.8, beta 4. And you can see here they are kind of a quadratic curve which has a peak and goes to 0 at either 0 or 1. So, if transmittance is 0, of course nothing changes. If transmittance is 1, again this is 0. So, both sides it goes to 0 and the middle there is a peak where the maximum difference is obtained for various values of beta. So, you can actually draw a curve along the maximum difference points for each of the values of beta.

If we do that, we will get a specific expression that we will discuss a little bit later. The value of gamma nu at each frequency mu will depend on the nature of the absorbing gas. So, clearly here the nature of the absorbing gas will be important. The what exact value of transmittance is there for a frequency nu, ok.

For CO2 you have a different transmittance spectrum with respect to frequency. For methane you have a different transmittance spectrum. For water vapor you have a different transmittance spectrum, right. So, the values will change, but with respect to beta, the d nu will be similar. If we know gamma nu versus nu plot for a given gas, then using this simple model, the delta SOR can be evaluated at all frequencies.

So, let us see how this itself can be done. So, this is the expression as we showed you, d and tau. We can show that for between, for beta values going from 1 to 4, these peak values follow a curve which can be approximated as D max equals to log beta by e. So, e to the power minus 1 log beta. So, the curve where the maximum D, the value of maximum D for each value of beta is given by this expression here.

And the peak is occurring between 0.3 and 0.7 values of transmittance, spectral transmittance. Alright. So, let us look at this is general. So, this we can use for water vapor, for methane, for CO2, etc. Only the transmittance expression has to be obtained for each individual gas to get the change in the outgoing long wave radiation.

Okay. So, this is the expression for CO2. Just one second. So, today we are running short of time. So, we will just finish this section in the next week's class and then we will start a new chap section on the development of the measuring ways how to measure different types of atmospheric and climatic variance. So we will just finish this section in the next week and start the new section. Thank you for listening and we will meet you in the next class.