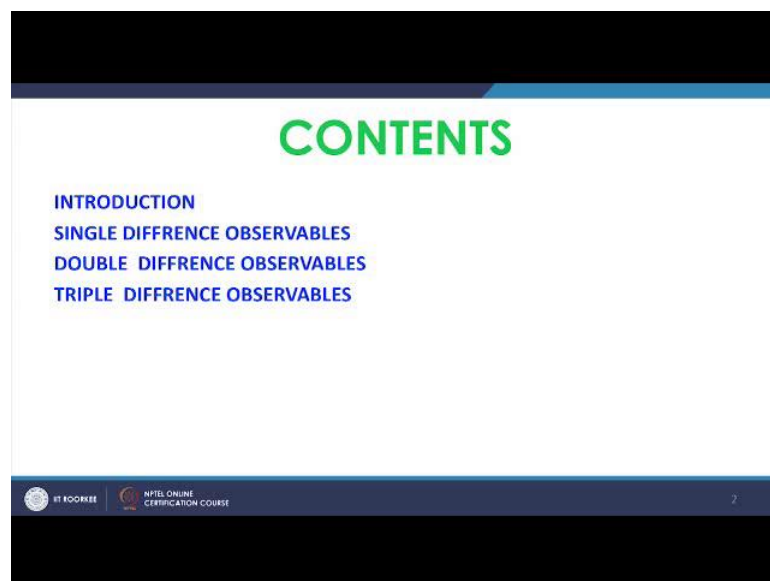


GPS Surveying
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Lecture – 11
GPS Data Pre- Processing-1 (Differencing)

Welcome friends. Today I am going to discuss on GPS Data Pre-Processing, this is the first part of the lesson. I will be talking on Differencing.

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As you know GPS observables are fraught with errors.



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INTRODUCTION

- Observables along with associated errors are being expressed through undifferenced observable equations.
- Un-differenced observable equations get processed to obtain position of the receiver (antenna phase centre).
- Accuracy of estimated positions depend on the quality of GPS observables.
- To improve accuracy, the errors are required to be eliminated /reduced by carrying out pre-processing of GPS data.
- Of the different pre-processing methods applied to eliminate/reduce errors present in the observables, methods involving transformation of observed data are widely prevalent.
- These methods generate new observables by combining different observables.
- Transformation methods which are based on differences of the same frequency/code observables are known as differencing methods.
- The differencing techniques widely applied at present are: single differencing, double differencing and triple differencing.

$$PR_r^i = \rho_r^i + c \delta t_r - c \delta t^i + I_r^i - I^i - T_r^i - d_r + d_r^i + e_r^i$$

$$L_r^i = \rho_r^i + \lambda_r N_r^i(1) + c \delta t_r - c \delta t^i - I_r^i - T_r^i - d_r + d_r^i + e_r^i$$



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And there are two types of GPS observables one is code pseudo range PR code pseudo range from satellite r to receiver r at any time t.

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$$PR_r^i(t) = \rho_r^i(t) + c \delta t_r + \underbrace{I_r^i(t) + T_r^i(t)}_{\text{minimised}} - c \delta t^i + d_r^i + dt_r^i + e_r^i$$

Pseudo-range Observable
Eqⁿ

$$L_r^i(t) = \rho_r^i(t) + \lambda_r N_r^i(1) + c \delta t_r - c \delta t^i - I_r^i(t) + T_r^i(t) + dt_r^i + dt^i + dt_r^i + e_r^i$$

① Two stations - at the same epoch & at the same satellite
 Two satellites - one receiver & at the same epoch
 At two epoch → same receiver for same satellite

And the phase pseudo range can be written like this and these pseudo range can be written with an observable equation like this - that is the geometric range of the receiver from the satellite are at on t. Then the receiver clock errors, then ionospheric error, tropospheric error, then satellite clock error, then our having that receiver hardware error,

satellite hardware error, multi path error and the random error. So, similarly this is called the code pseudo range observable equation, pseudo range observable equation.

This is also we have learnt in the last class. Similarly we can write the phase pseudo range equation like this that genetic range plus integer ambiguity, then receiver clock error, satellite clock error, ionospheric error, tropospheric error, receiver hardware error, satellite hardware error, multi path error and random error. Now in these both the equation though I have used similar symbol, but the actual amount of error that will be present in code pseudo range will be different from phase pseudo range.

Now, we process this observables equation to determine the position of the receiver. Now the quality of the position will depend upon how what will be the quality of these observable equations; that means, these observable equations should be free from all these errors. So, to make these observable free from the errors we need to pre process the GPS data and to pre process the GPS data we adopt many methods one of the simplest and easiest and widely prevalent method is called transformation of GPS observables in which we make use of this observables to arrive at a different type of observables where these errors is reduced or removed and this one of the simplest method is called method of differencing.

So, today I will discuss on differencing. Today's class will be containing these three topic - like single difference observables, double difference observables, triple difference observables. That means, from these undefined observables we will go for a new type of observable that is called single difference observable. By taking single difference of this observables the double difference observables, by taking the double difference means difference of the single difference observables and triple difference observables which will be the observables taken as a difference of double difference observables.

Now, there are infinite ways how these can be carried out, but however, we will do it in some particular way which has been research and found that those are the best way to do to remove many of these errors or to minimize many of these errors. Now let me start with the single difference GPS observables.

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SINGLE DIFFERENCE OBSERVABLES

May be obtained by taking difference of original un-differenced observables (UD)

From two different stations
From two different satellites
At two different epochs

OBSERVABLES FROM TWO DIFFERENT STATIONS

single difference observables are derived by taking difference of the un-differenced observables from the same satellite observed simultaneously at two different stations and at the same epoch of observation.

these differenced observables are free from errors arises out of space segment (the satellite clock error (δt^s) and satellite hardware error (δt^h), as these errors get cancelled during differencing.

Atmospheric other errors get minimized depending on the separation between receivers. Even though the nominal reception time is the same for the observables, their emission times (from satellite) differ depending on the distance between the stations and thus, the amount in reduction of other errors.

these may be used to estimate only relative position.

$$PR_1(t) = \rho_1^s(t) + c\delta t_{e_1} + c\delta t_{h_1} + I_1^s(t) + T_1^s(t) + \delta t_{e_1} + \delta t_{h_1} + \epsilon_{1,1}^s(t)$$

$$PR_2(t) = \rho_2^s(t) + c\delta t_{e_2} + c\delta t_{h_2} + I_2^s(t) + T_2^s(t) + \delta t_{e_2} + \delta t_{h_2} + \epsilon_{2,1}^s(t)$$

$$\Delta PR_{1,2}(t) = PR_1(t) - PR_2(t) = (\rho_1^s(t) - \rho_2^s(t)) + (c\delta t_{e_1} - c\delta t_{e_2}) + (c\delta t_{h_1} - c\delta t_{h_2}) + (I_1^s(t) - I_2^s(t)) + (T_1^s(t) - T_2^s(t)) + (\delta t_{e_1} - \delta t_{e_2})$$

$$= (\rho_1^s(t) - \rho_2^s(t)) + (c\delta t_{e_1} - c\delta t_{e_2}) + (c\delta t_{h_1} - c\delta t_{h_2}) + (I_1^s(t) - I_2^s(t)) + (T_1^s(t) - T_2^s(t)) + (\delta t_{e_1} - \delta t_{e_2})$$

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Now see by the name itself we can understand that, in case of single difference observable we will take the one time difference of these observables, but how you will take? There are three ways how it is to be done; one is the in first case we will take the difference of observable between two different stations taken from a single satellite and the same epoch observation. So, in that case as we can see now if the observables are from the same satellite and the observables are taken by the receivers at two different locations, so if we take the difference of those observables then the errors that will be arises out from the space segment will be removed or minimized.

So, from two stations at the same epoch and at the same satellite this is the first case. So, when the observables are from the same satellites. So, it is expected that the satellite clock error, then the satellite hardware error, these two will be identical in both the cases and this will be minimized or removed. Now for other errors like your tropospheric ionospheric error, tropospheric error these two specifically ionospheric means atmospheric error we will be minimized or removed depending upon the distance between the receiver. If the distance between the receivers are small; that means, 1 kilometer or 2 kilometer then it is expected that the signals will morph through identical type of atmosphere.

So, the ionospheric error, tropospheric error is expected to be same and one will take the difference then they will cancel out. Now this is a method generally we will use for

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SINGLE DIFFERENCE OBSERVABLES...

Observables at two different epochs

- difference observables by taking difference of the observables from the same satellite observed at any station but at different epochs of observations.
- satellite as well as receiver clock errors and satellite as well as receiver hardware errors get cancelled
- Other errors also get reduced depending on the epoch of observations and interval
- in case of carrier phase observables, single difference between epochs also helps in eliminating the cycle ambiguities as it does not change with time.

$$PRN_i(t_1) = \rho_i(t_1) + c\delta t_{rcv} - c\delta t_{sat} - I_i(t_1) + T_i(t_1) + d_{rcv,i} - d_{sat,i} + d_{e,i}(t_1)$$

$$PRN_i(t_2) = \rho_i(t_2) + c\delta t_{rcv} - c\delta t_{sat} - I_i(t_2) + T_i(t_2) + d_{rcv,i} - d_{sat,i} + d_{e,i}(t_2)$$

$$\Delta PRN_i(t_1, t_2) = PRN_i(t_2) - PRN_i(t_1) = \rho_i(t_2) - \rho_i(t_1) + I_i(t_2) - I_i(t_1) + T_i(t_2) - T_i(t_1) + d_{e,i}(t_2) - d_{e,i}(t_1)$$

Now the next type the third one is that we do take the observation at two different epoch, but the signal is coming to same receiver, same receiver from same satellite. So, as the signals are coming to same receiver and from the same satellites it is expected that the errors arising out of space segment as well as receiver segment both are removed or minimized and other error also will be minimized depending upon the time interval between two epoch. And this is a very important type of difference so it is called delta range and it has many other users' applications.

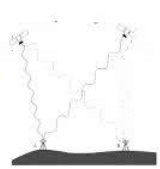
Now, with this we can say that single difference using single difference observables we can reduce or minimize many of the errors that we present that we get or that are present in GPS observables and by reality positioning we go for this single differencing.

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DOUBLE DIFFERENCE OBSERVABLES

DOUBLE DIFFERENCE OBSERVABLES

- Observables from differences between two single differences
- Two broad categories of double differencing.
- First category using single differenced observables of two different stations from two different satellites. There may be two types.
 - First type consists of taking differences of the single differenced observables obtained from a satellite observed simultaneously by a pair of receivers at any epoch of observation;
 - Second type, differences may be considered between the single differenced observables obtained from observing same pair of satellites simultaneously by a receiver at same epochs of observation
- Double difference observables are free from satellite and receiver clock errors
- Other errors in observables get reduced.
- Double difference observables are mostly used for further processing to obtain desired parameters and to carry out ambiguity resolution.

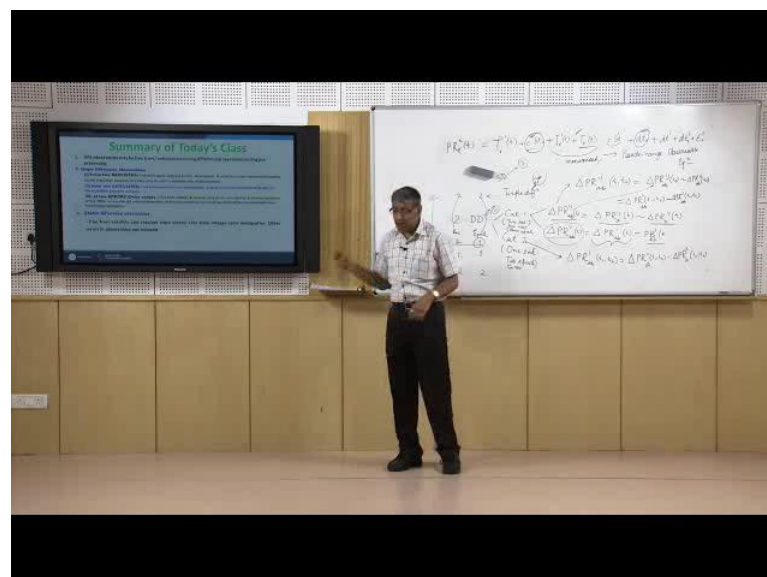


$$\begin{aligned} \Delta PR_{i,j}^k(t) &= \Delta PR_{i,j}^k(t) - \Delta PR_{i,j}^k(t) \\ \Delta PR_{i,j}^k(t) &= \Delta PR_{i,j}^k(t) - \Delta PR_{i,j}^k(t) \end{aligned}$$

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Now next variety is that double difference. Actually in double difference as you can see in this figure, in case of double difference we do take the difference of single difference observables. Now, there are three ways how we can go the single difference. Similarly in case of double difference we have two out of three cases, we can get two types of double difference, two categories - category 1 and category 2.

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Again under category 1 there may be two types. Now in category 1 actually category 1 we do we will consider signals or observables from two different satellites - two

satellites and two receivers. Now under category 2 we consider as you can see in the next figure it is from one satellite and two epochs. So, in both cases it will be two receiver here one or same epoch.

So, we can see there are three variability's - one is called satellite, another is called receiver, another is called epoch. So, under category 1 it is same satellite two satellite two receivers, but under category 2 it is the epoch which will change, but it will be the two receivers and one satellite. So, this is the two changes; that means, either the satellite can be one or epoch can be one, in both cases we will take the signals from two different stations or two different receivers. Now under category 1; that means, if the satellites are two satellites, two receiver there may be again two different types in one case it is that as we can see in your previous one is that difference from thus difference satellite at the same time, but different receiver that is del.

So, here we can see that the signal is coming from two satellites to a single receiver - a first receiver and single receiver b; that means it is the category of first difference. So, this is the first difference this was the thing. So, we have to take the difference of this first difference is this one and then this one this is our receiver a and this is receiver b. So, this one and the difference of this will give us that this difference. Now another variety is that it is also written as pseudo range ij ab t , but here the single this will be different PR ab i t minus difference PB .

So that means, what is this? This is the single difference variety this one. So, we have seen the single difference while we have taken this signal coming from a single satellite by receiver. So, this is what is that and when another satellite we are taking in this case are like this one geo satellite from there we have taking it. So, this is the case this one. So, we can take apparently it looks like that both are same, but it is not the same because here you will see the implications are different.

And the third variety which we tell in the category 2 that is the pseudo range where. So, we are taking the difference in single difference observables. So, single satellite, two receivers, two epochs; single satellite, two receiver and two epoch. So, that is the variety as we have seen it this one. So, this is taken this is the first one and then when I am taking for the same instant this will be coming here. So, this one the difference of these two will provide us the double difference.


Now double difference observables - they remove both the most of the receiver as well as satellite errors. Double difference, because this atmospheric errors are not modeled, there will be very the status of the atmospheric errors will be very unbalanced. Moreover multi path error is also will not be taken care of in double difference. However these double difference observables are the best kind of observables that we should as a pre processing and generally we do go for double differencing before we go for processing of the GPS signal.

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TRIPLE DIFFERENCE OBSERVABLES

TRIPLE DIFFERENCE OBSERVABLES

- involves differencing of two double differences at two different epochs
- only one type of triple difference observables.
- Obtained from TWO receivers from same TWO satellites at the same TWO epochs.
- errors eliminated in single- and double differences thus get eliminated also during triple differencing.
- carrier phase pseudo-ranges, integer ambiguity gets cancelled
- cycle slips get mapped as individual outliers thus helps in their elimination during further processing.
- introduces correlations between observations and reduces the data weights.
- triple difference observables lose some geometrical strength and thus, lead to degraded precision.
- primarily useful as preprocessing technique to get good approximate positions for further processing of DD observables.



$$\Delta PR_{ab}^{ij}(t_1, t_2) = \Delta PR_{ab}^{ij}(t_1) - \Delta PR_{ab}^{ij}(t_2)$$



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And the last one that we will consider as the triple difference, in case of triple difference we will take the difference of double difference, so if we take this $\Delta PR_{ab}^{ij}(t_1)$ and $\Delta PR_{ab}^{ij}(t_2)$ that means, we are taking two receiver, two satellites and two epoch of observation.

So, two satellites, two receivers, two epoch of observation we will get for triple difference observable, like this - $\Delta PR_{ab}^{ij}(t_1)$ minus $\Delta PR_{ab}^{ij}(t_2)$. So, this is one of the varieties; that means, this is, I can take here this one or I can take there this variety or I can take there this variety. So, in that way we can; that means, indeed if you take this one than; that means, $\Delta PR_{ab}^{ij}(t_1)$ minus $\Delta PR_{ab}^{ij}(t_2)$, my difference $\Delta PR_{ab}^{ij}(t_1)$ minus $\Delta PR_{ab}^{ij}(t_2)$. So, there are three varieties of double difference I can take this one for the t_1 and this one for t_2 I can take this one for or this one.

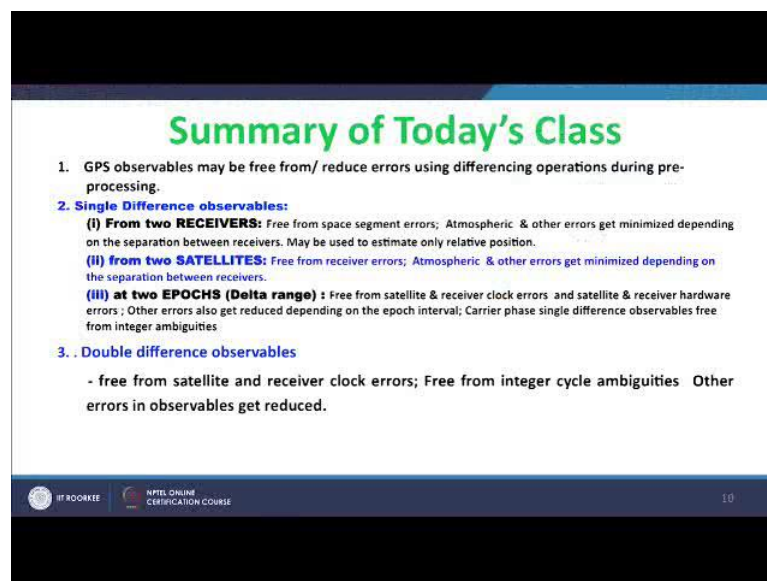
So, in that we ultimately we will end up with the same, so only one output only one type of triple difference. So, double difference you will get two types of double difference

whereas, single difference we will get for three types of observables. So, triple difference observables are the observable well which we will be able to identify the presence of cycle slip. So, this is very important to as a pre processing steps, as well as we can minimize many of the errors.

And triple difference observables are processed to get the initial position of the receiver and then from that position as the starting point we do process the data double defines data as the position from triple difference data as the input to the double difference processing then we end up with a good solution. So, this is the process how you will end up with a better GPS position.

With these I will conclude to this class, but however, we can summarize today's class like this - that the GPS observables are fraught with errors.

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The slide is titled "Summary of Today's Class" in green text. It contains three main points:

1. GPS observables may be free from/ reduce errors using differencing operations during pre-processing.
2. **Single Difference observables:**
 - (i) **From two RECEIVERS:** Free from space segment errors; Atmospheric & other errors get minimized depending on the separation between receivers. May be used to estimate only relative position.
 - (ii) **from two SATELLITES:** Free from receiver errors; Atmospheric & other errors get minimized depending on the separation between receivers.
 - (iii) **at two EPOCHS (Delta range) :** Free from satellite & receiver clock errors and satellite & receiver hardware errors ; Other errors also get reduced depending on the epoch interval; Carrier phase single difference observables free from integer ambiguities
3. **Double difference observables**
 - free from satellite and receiver clock errors; Free from integer cycle ambiguities Other errors in observables get reduced.

At the bottom of the slide, there are logos for IIT ROORKEE and NPTEL ONLINE CERTIFICATION COURSE, along with the number 10.

The quality of the GPS observable will depend upon how much errors can be minimized or removed and that is we can do very easily, quite simplified way by taking the different difference methods.

Now, in differencing we do take the similar type of observables. Now, fundamentally there may be infinite times, but basically as a practical point of view we have found that there are three fundamental ways how it is differencing operation can be done. The first one is the first differencing in which we do take the difference of observables from either

from two satellites coming to the same receiver and at the same epoch or the observables that is coming from a single satellite, but at two receivers at the same epoch or these observables we do take coming from same satellites at different epoch to the same receiver. So, in that way and we have found that for different conditions the errors may be removed either for the space segment or for the receiver segment or both.

Now the next variety is the double difference, which is again it is the difference of the single difference observables and there may be two categories. In the first category there will be two satellites observables coming from two satellites to two receivers, but we will consider observables at the same epoch observation and that case we will be able to minimize the error from the receiver as well as satellites.

In the second category we will be taking, we may take the observables from two receivers coming to form one satellite, but at two different epoch in that case also, but we may also reduce the error due to receiver as well as satellite; however, this double difference errors also contains the error due to multi path and also the (Refer Time: 25:55) as well as the atmospheric errors causes some erratic behavior. So, we need to further process it. And however, second or double difference observables are the best observables which we should go for processing for precise positioning after finding out the approximate position by making use of triple difference observables.

Now, triple difference observable is the difference of observable of double difference and in that case in triple difference observables provides us the cycle slips nicely. So, the cycle slip error can be indicated and that subsequently removed by triple difference observables.

So, with this I would like to end. So, triple difference is a very good thing to be done pre processing before further processing is to be done. So, with this I want to conclude today's class and in the next class we will be taking up again GPS pre processing, but it is the second part.

Thank you, see you again.