GPS Surveying
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Lecture – 13
GPS Data Processing- I (Point Positioning)

Welcome friends, today's lecture is on GPS data processing. We know, from GPS we get GPS observables, which are being used to process, towards finding of unknown position of points. Now finding out the position of points, we do make use of GPS observables. Now, these observables may be, code observables, or phase observables. Again there may be number different types of code observables, as well as different of the phase observables. We may take either the code observables, single code observables, or multiple code observables, or single code, or multiple code observables, along with single or multiple phase observables. However, these observables may also be, from an autonomous mode, and also from relative mode.

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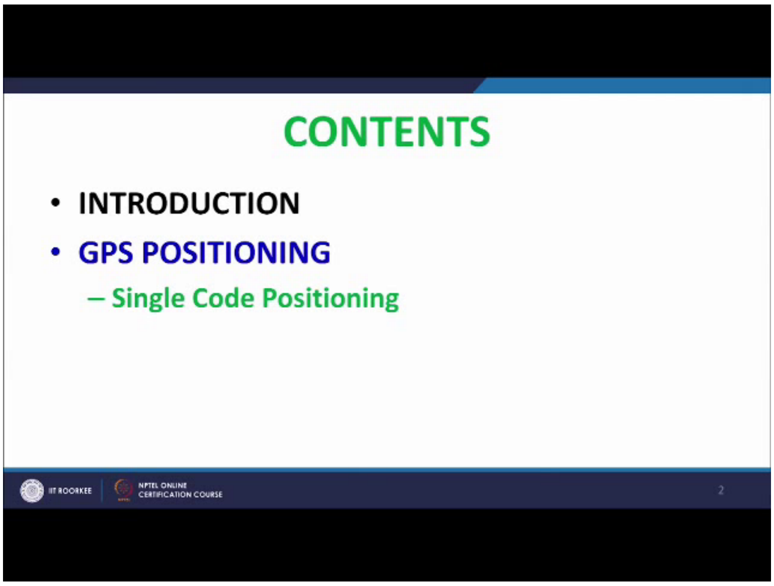

Now, todays class, I will be discussing on, GPS point positioning. That means, we will be considering, the GPS observables, from a single receiver, and of single code observables, which is the CA code observables. As I told you, GPS positioning means, determining the position of a GPS receiver, is using the GPS observable. It may be determined, using code, or carrier phase observables, and this may, there may be, autonomous positioning, or relative positioning.

Now, in case of single point positioning, we go for autonomous positioning, in which we place a single receiver at any point, and it may be in a static condition, or in kinematic condition, depending upon the type of GPS surveying, we have to carry out. However, one point has to be noted, that, whatever is the positioning, whatever is the method of positioning, or whatever is the number of observables, we do take in processing the GPS data, it is always imperative to process, the GPS CA code observables. So, I am be taking up, the fundamentals of GPS (Refer Time: 03:46) positioning using a CA code GPS observable.

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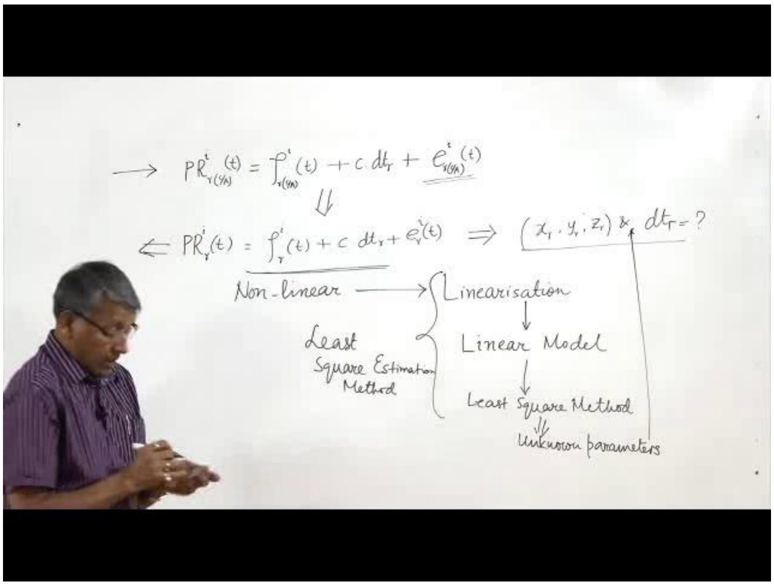

Now, the, from GPS signal, we do get the GPS pseudo range code. From say satellite I to receiver R, CA code type, at any epoch of observation. So, this is the notation I am using, for GPS observable, of CA code that is being received by, receiver R from satellite I, at the epoch of observation T. Now we know that, GPS observable B pole going for positioning, we do give for P processing operation. Now this has got the P processed. So, we will get this, we may write the GPS observable, pre-processed, GPS pseudo range, observable, may be written like this. This represents the geometric range of the satellite I, to receiver R at an epoch T and that is, with respect to CA code observable, multiplied by the along with this we have, the velocity of propagation of the signal, multiplied by the error in receiver clock, and the error associated with the observable, and this error contains the residual error, that has been present even after pre-processing as well as random error.

So, this is the pseudo range observable from CA code, but if we generalize the symbol pseudo code observable, I can write it like this, to make it simplified. Now these observable has to be processed, to determine, from this we have to determine, the position of the receiver. Let us say, the position of the receiver is XR, YR, ZR, and the error in receiver clock error is DTR. So, these are the 4 unknown, which we need to determine, by processing code pseudo range observables.

Now, how do you do it? We can do it by, there are different methods, by which this can be processed, of this Least Square Estimation method is one of the method, which will apply to, show you how this is lead to GPS position. Now in Least Square method of analysis, this pseudo range observable, which is a non-linear, this is non-linear actually. So, first, we need to change or modify, this non-linear relation to linear, or it is called linearization. Once the non-linear pseudo range observable has been liberalised, then we will be need to convert it to a linear model.

Now, after getting it the linear model, we have to use Least Square method of analysis, to provide us the unknown parameters, which are nothing but, this, this, the unknown parameters. So, these are the 3 main steps that have to be taken, to convert this code pseudo range observable, to unknown determination, by Least Square estimation method.

Now, as I told you this is the actual, or position of the receiver, which we need to determine, and this is the error in receiver clock, which need to determine.

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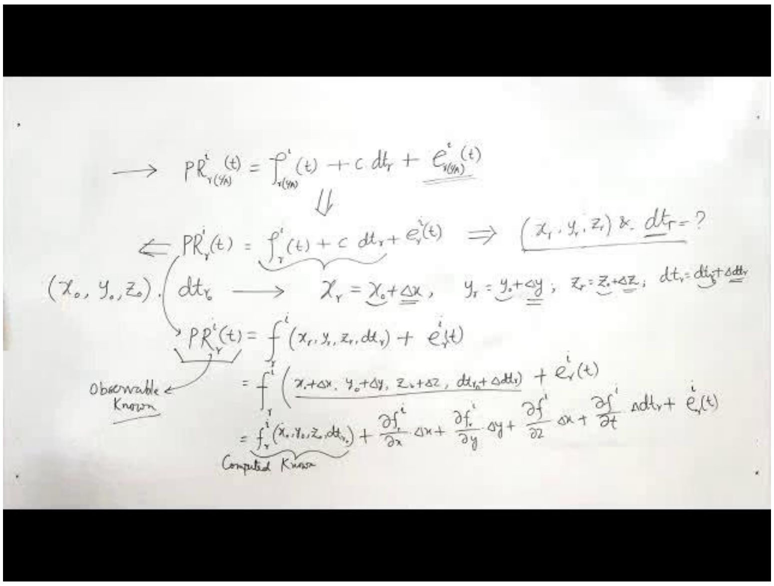

But, initially, we can assume that some value, X0, Y0, Z0, which is the position of the receiver that is assumed position of the receiver, and DTR0 is the error in receiver clock. Now suppose this is the assumed value, and this value differ from the actual value. So, let us say, XR equal to X0 plus del X, YR is equal to Y0 plus del Y, then ZR is equal to Z0 plus del Z, then DTR is equal to DTR0 plus del DTR. Now you can see that, if we assume these values, as we have assumed these values, now our unknown is del X, del Y, del Z, del TR. Now, this pseudo range equation, we can write like this, is equal to, instead of this, we can write it, a function, which is FR XR YR ZR GYR which are the actual these values, correct values, plus error.

Now, we write these things like this F function, I2R, instead of XR, I can write it X0 plus del X, Y0 plus del Y, Z0 plus del Z, DTR0 plus del DTR, plus. This is the error. Now this is the function, of a function, having this. So, we can write it like this, F of IR, X0, Y0, Z0, and DTR0. Now we can see this is a func, this is a function, which is known to us, because these are known. So, if we know the function by putting the values of this, we can get this. So, this is known. Now that is the computed value. Computed, computed known value, computed known.

And now, I can see this is a function, which may be expanded by using Taylor series. So, I will get DF (Refer Time: 13:00) FI by del X plus del FI. I am, DY del Y, plus first derivative of the function, with respect to Z, first derivative of the function with respect to time, plus. Now you can see that, this is the value which is also available from observable. This is observable, that is also known, and this is the value which is computed known. So, this, minus this, we can do, that is known as residual observable.

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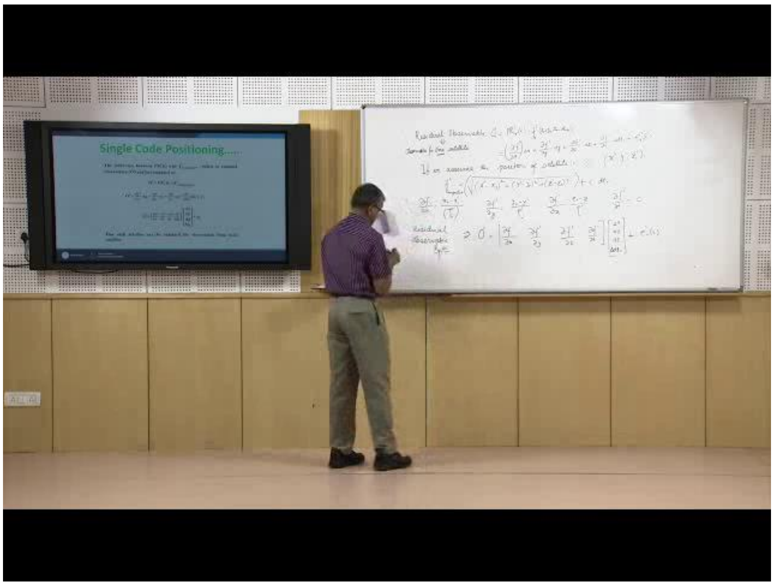

So, actually, we know we call it residual observable, which is denoted by suppose; OI is equal to PIRIT minus function 0. So, this is the residual observable, now this is equal to, we know, DF, first derivative of the function with respect to X, del X, plus first derivative of the function del Y, plus first derivative of the function with respect to Z, del Z, plus first derivative of the function with respect to time, plus, error - residual error in the observable.

Now, if we assume, we assume, the position of satellites, of the satellite I, is XIYIZI, then the computed function, we can say, will be FI, will be, square root of XI minus X0 whole square, YI minus Y0, whole square, ZI minus Z0, whole square, plus C into DTI. So, this is the function we know. So, this we can that is what is computed, this is the computed function. So, now, if we take the partial derivative of the function, with respect to X, then we will get X0 minus XI divided by rho I. Now this rho I is nothing but, this part is nothing, but this part. So, similarly the partial derivative of the function with respect to Y, will be equal to Y0 minus YI rho YR, and first function partial derivative of the function with respect to Z will be equal to Z0 ZI rho IR and the partial derivative of the function with respect to time, will be equal to C, this is the time.

Now, we can see that, the, this residual observable OI, here this part is this one, this part is this one, this part is this one, and we can get this thing. So, we get a observable, residual observable equation, and this equation is from one satellite, from one satellite, observable, observable from, observable from one satellite. Residual observable from one satellite, so if we have observable from M satellites, then we can write like this of course, this again now, one more thing, this observable OI, OI can be written in matrix form like this, DFI by del X, del Y, del Z, DTI, del TR, plus our EIRT. So, this is what is the final, residual observable equation.

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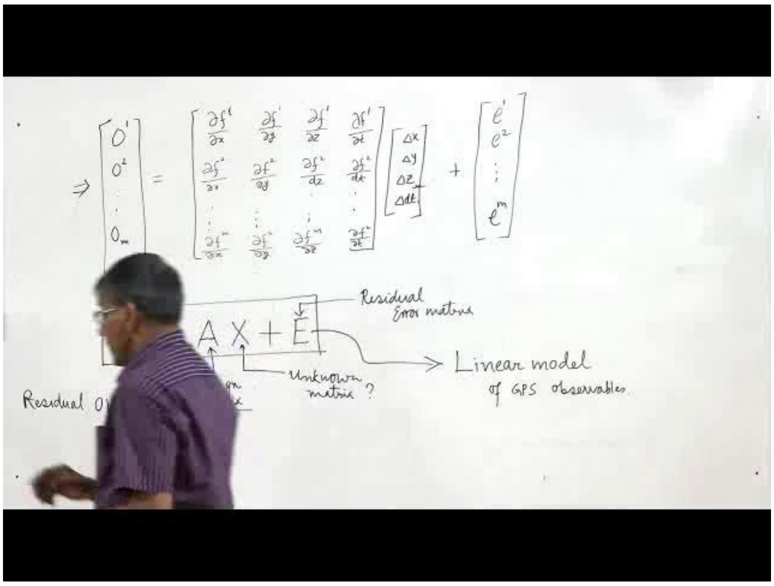

Now, this observable equation is from only one satellite, I. If we observe M satellites, like one, two, M satellites, then we will get a matrix, observable matrix, like this, will get DF1, DXDF1, DYDZ, and we know that DFI by DT is equal to C. So, we can write C also here, or we can write also DF1 by DT, and here del X, del Y, del Z, del TR, DF2, DXD2, DY like that DFM, DXDFM, DY DT and we will have the error matrix. So, we will have E1, E2, and EM.

So, finally, this is the observable equations from M satellites. Big thing again, we can write in matrix form, O is equal to, this is called A matrix, X unknown matrix, and error matrix. So, this is the observable matrix, or residual observable, residual observable, observable matrix. This is the called design matrix, design matrix, because it depends on the partial derivative of the function, which we are taking for measurement, and it will depend upon the construction of the satellites, as well as position of the satellites, and the assumed position of the receiver. So, it is called design matrix. This is the unknown matrix, which we need to; we want to find out the unknown parameters, are involved in this matrix. E is the error matrix or the residual error. We have, in the observables. Now this is the, this is the linearised form. This is the, actually this is the linear model. So, this is the linearised form, and this is the linear model, of GPS pseudo range observables.

Now, this linear model may be solved by using the Least Square method of analysis, normal equation method. There are many ways how this can be solved.

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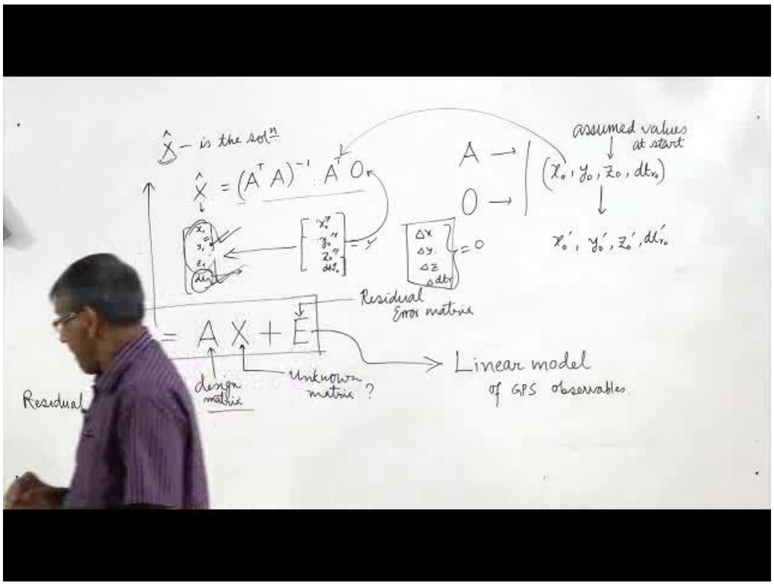

So, one of the ways is the, normal equation method of Least Square analysis. Now suppose, X bar is the solution, is the solution. That means, this, this is the right solution, then it is we get it from, transpose A, inverse A, transpose O. So, from the matrix O already we have seen, this is known, A matrix already I have seen, the matrix and this is by multi, this is the multi by multiplying this in this way, matrix multiplication we get the solution of the unknown matrix, means we get the value of XR,YR, ZR, DTR by using this.

So, this how we do find the unknown position, of the receiver. This is the unknown position of the receiver, and this is the receiver clock error. Now as you can see this. As you have seen, in previous explanation, that the design matrix, and the observables, these 2 are computed, were started X0, Y0 Z0, DTR0. These are the assumed values at start.

That means, to start with computations, or processing, we need to assume some value for the position and the receiver clock error.

Now, and this has been assumed with the basis of on the basis of that, the design matrix value as well as residual observable value, we will get, and that will help in getting new values of the position. Now, after giving you the input, we will get a matrix, this will be something like X0 dust, Y0 dust, Z0 dust, and DTR0 dust. So, this is the, this is, this is the value we will get give this as the input to this.

Now, now, next, next time, we will give this as the input to this, and we will get some other value. So, and then again, I will give this value we will get some other value. And finally, we will come to; we will come the unknown value to this. So, it is, it (Refer Time: 28:04). First, we will assume the a value we will find out some unknown values, which is something, which will be given to this, that will give and this entirety process will go on, till, we get this value. When we will get this thing? When del X, del Y, del Z, del TR, all will be equal to 0. So, this is the converging criteria, that del X del Y del Z del TR is 0. In this way, we determine the position of the receiver. This is the position of the receiver, and the unknown clock parameter.

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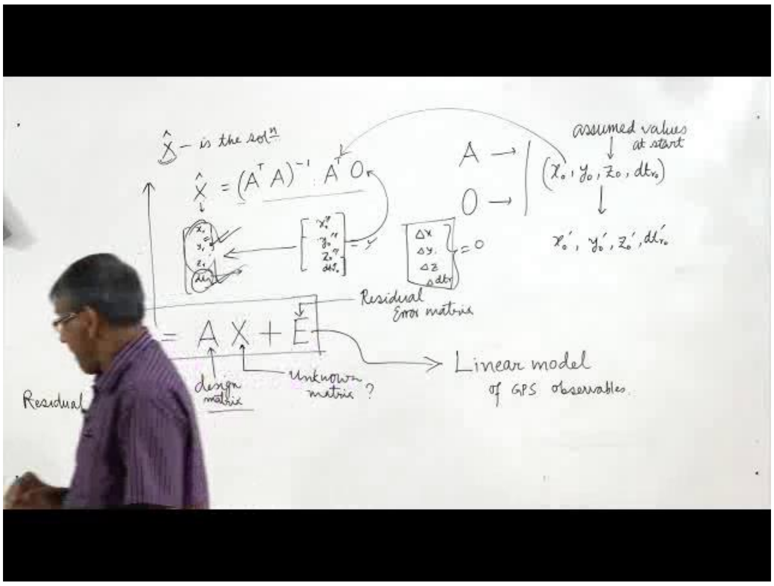

With this, I want to conclude today’s class. Before concluding, I will have to summarize the class, whatever we talked, that processing of GPS data is done to find out the unknown position of the point, at which the receiver has been placed. And to find out the position of the receiver, we may process, code observables, phase observables, either independently, or together. There may be single code observation there may be multiple code observation, observable which may be used to process the data, or it has to be like that, after processing the code we can take up the carrier phase.

Now, whatever is the observables we take, whatever is the positioning methods we apply, the minimum thing or the most, the first thing, we need to process the GPS CA code pseudo range observables. So, we need to find out the position of the receiver, using the CA code, pseudo range observables. There are many methods, by which we do process the GPS data, to find out the position of the receiver. Of this, one of the method is Least Square analysis, in which, we do first has to linearise the non-linear, GPS pseudo range observable, then from linear after linearization we have to develop the linear model, and that model will be solved by making use of the Least Square analysis.

During Least Square analysis, or solution, first we will assume a value, and from that value, we will find out, another approximate value and that will be iterated sequentially, leading to finally, to the actual value of the position. And to arrive at the final result of the position, we need to make the parameter del X, del Y, del Z and del TR to be 0. And since we have seen that there are 4 unknown, involved, in the in GPS observable solution, so, we need to have at least 4 observables. That means, observable form at least 4 satellites. And, if we have more observables, means satellite, observable for more satellites, it will be better.

With this, I want to conclude. See you in the next class, which will be again on GPS data processing. There I will like to, take up the GPS data processing, for relative positioning.

Thank you.