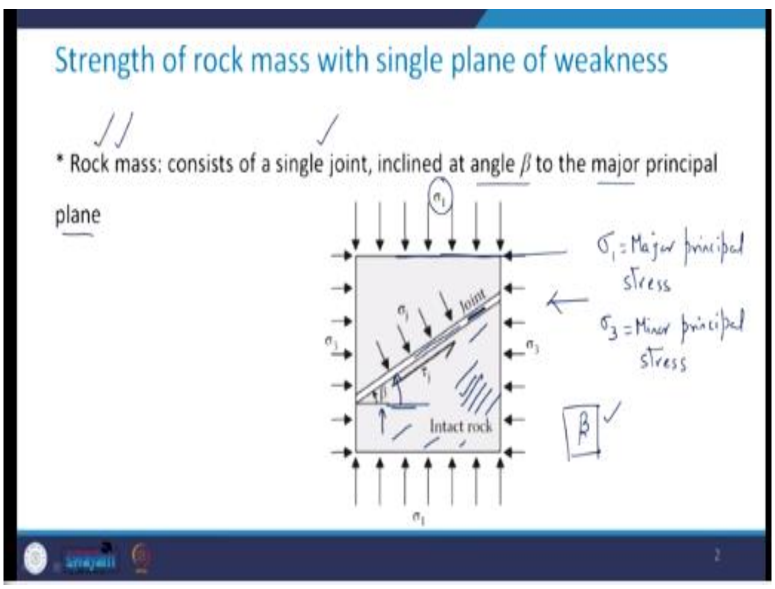
Rock Engineering
Prof. Priti Maheshwari
Department of Civil Engineering
Indian Institute of Technology-Roorkee

Lecture-34
Failure Criteria for Rock Mass

Hello everyone. In the previous class we discussed about the parameters of Mohr Coulomb failure criterion and Hoek and Brown failure criterion. We saw that how the parameters of Hoek and Brown criterion can be used for the determination of Mohr Coulomb parameters for the rock mass under effective stress conditions. Then we had some discussion on the deformation modulus along with the disturbance factor.

Today, we will learn about few failure criteria which are applicable in case of the rock masses. The first one includes the failure criteria for the rock mass, when the rock mass has a single plane of weakness, this is also called as single plane of weakness theory. Then we will discuss about the Barton's theory to determine the rock wall strength. So, to start with, let us have a look at this slide.

(Refer Slide Time: 01:48)

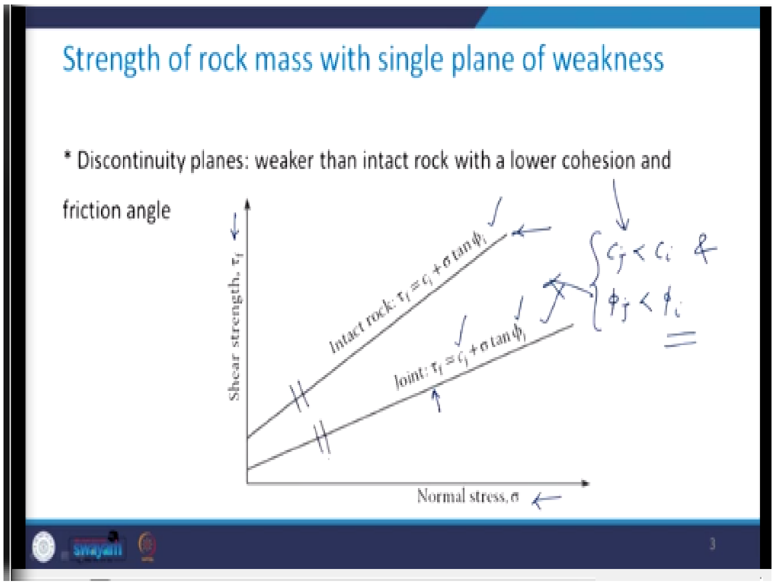


Where you have a figure, which is representing the rock mass having a single joint which is inclined at an angle β to the major principal plane, and here the σ_1 is the major principal stress and σ_3 is the minor principal stress. So, on this plane your major principal stress is acting, and from this plane only we are taking the orientation of this discontinuity which we are representing by beta.

I want you to remember this fact that how the orientation of this joint, which is this joint is being measured because all our plots, all our result will be depending upon this value of beta and how it is measured. So, this part represents the intact rock portion and this is what is the joint and you know that when there is a presence of joint in the intact rock, we call it as rock mass. The condition here is that you have only one plane of weakness.

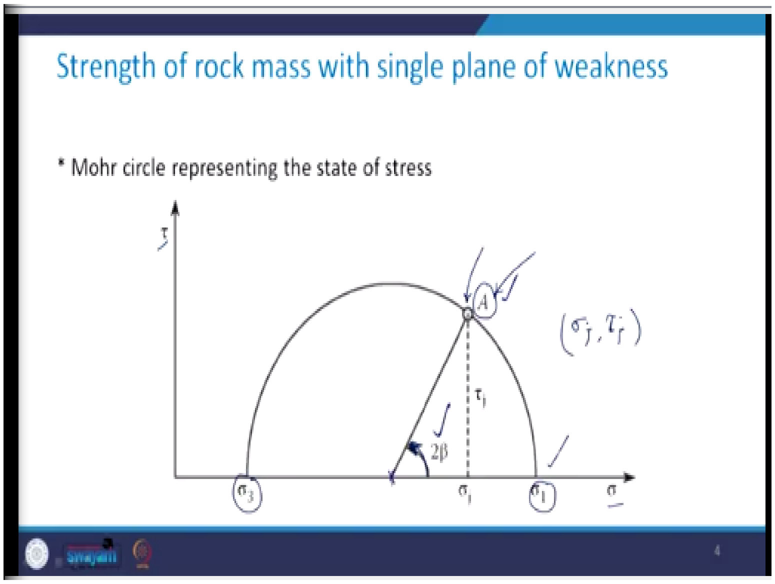
So, the theory that we are going to discuss is also called as single plane of weakness theory. What happens when there is an introduction of a discontinuity plane in the intact rock? Overall strength characteristic of the rock mass reduces as compared to the intact rock.

(Refer Slide Time: 04:21)



So, here in this figure, the plot for the intact rock and the joint has been shown in the space shear strength τ_f and normal stress σ, I do not need to explain you this. Now that obviously, intact rock will have much larger strength as compared to that of the joint, and this would result in the lower value of cohesion and the friction angle in case of the joints. What does that mean is that c_j is less than c_i and ϕ_j is less than ϕ_i as it is evident also from these two straight lines.

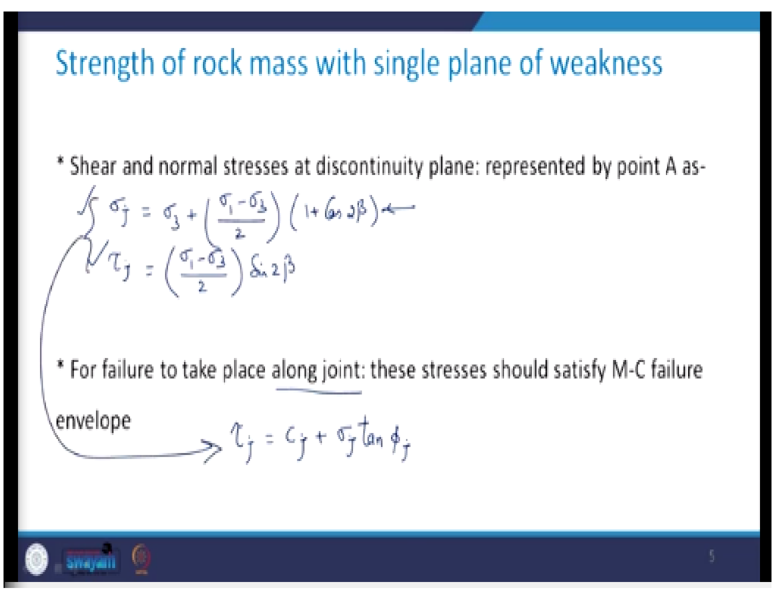
(Refer Slide Time: 05:29)



In case if we have a state of stress, you know that that can be represented with the help of Mohr circle. So, let us say that there is a point or the state of a stress which has been represented by this point A in τ-σ space, then this will have it is coordinate as σ_j, τ_j where σ_1 and σ_3 these are major and minor principal stress because here this is giving us the state of stress at failure.

So, you all know that by the property of the Mohr circle if we join this with the center, the angle that it subtends gives us that plane of the discontinuity that is half of this angle is going to give us the orientation of the plane of discontinuity with respect to the direction of this σ_1. That is direction of the plane on which this σ_1 is acting, which was horizontal in this case.

(Refer Slide Time: 07:04)



Now shear and normal stresses at discontinuity planes, which are represented by point A can be given as

$$\sigma_j = \sigma_3 + \left(\frac{\sigma_1 - \sigma_3}{2}\right)(1 + \cos 2\beta)$$

We have already discussed this or derived this with respect to the Coulomb-Navier failure criterion. But the same way you can use these trigonometry principles and can derive these, so I am not repeating it here.

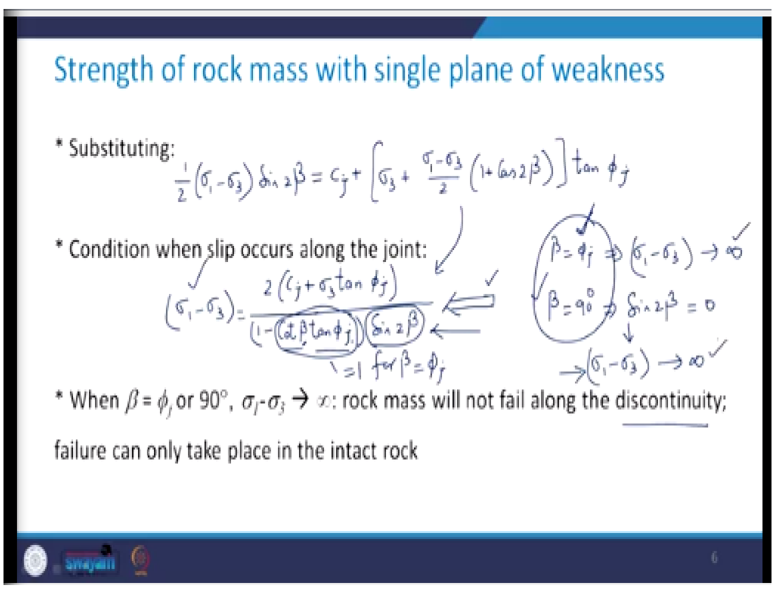
$$\tau_j = \left(\frac{\sigma_1 - \sigma_3}{2}\right)\sin 2\beta$$

So, this is how a state of stress which was represented by the point A can be given in terms of σ_1, σ_3 and β. For the failure to take place along the joint these 2 stresses should satisfy the Mohr coulomb failure envelope which is given as

$$\tau_j = c_j + \sigma_j \tan \phi_j$$

We just saw it in the previous slide. Now what I will do is I will just substitute these two values in this expression, and see how it looks like.

(Refer Slide Time: 08:49)



So, that is going to give me this expression which is

$$\frac{1}{2}(\sigma_1 - \sigma_3)\sin 2\beta = c_j + \left[\sigma_3 + \left(\frac{\sigma_1 - \sigma_3}{2}\right)(1 + \cos 2\beta)\right]\tan \phi_j$$

Just simply substitute the expression for τ_j and σ_j in the Mohr Coulomb envelope for the joints. If you solve this or simplify this condition when the slip occurs along the joint you will get as

$$(\sigma_1 - \sigma_3) = \frac{2(c_j + \sigma_3 \tan \phi_j)}{(1 - \cot \beta \tan \phi_j) \sin 2\beta}$$

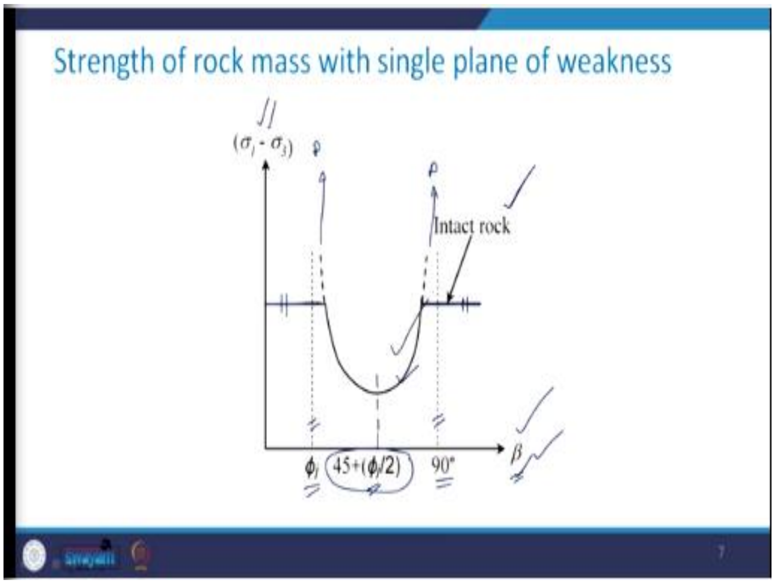
So, if the slip has to occur along the joint, this condition should be satisfied. Now take a look at this expression and just substitute $\beta = \phi_j$ in this case. So, say if I substitute $\beta = \phi_j$ in this case, what will happen? See here this is $cot\beta$ and this is $\tan \phi_j$, if this $\beta = \phi_j$ what will happen? This quantity will become equal to 1, will be equal to 1 for $\beta = \phi_j$.

Once this becomes equal to 1, this denominator here will become equal to 0 and that would result into this $(\sigma_1 - \sigma_3) \rightarrow \propto$. Then you take another value of β which is equal to say 90°. Then just substitute that β= 90°, so you take a look at this quantity, which is $\sin 2\beta$ this will become equal to 0. So, again in this case your $(\sigma_1 - \sigma_3) \rightarrow \propto$, what does this signify physically?

Rock mass will not fail along the discontinuity either when this $\beta = \phi_j$ or β= 90°, because in these two conditions the strength of the discontinuity which is given by the $(\sigma_1 - \sigma_3) \rightarrow \propto$. So, the failure can take place only in the intact rock. So, it is not that, if in the rock mass intact rock as well as discontinuities are there, then the rock mass will fail always along the discontinuity.

It will also depend upon the orientation of that discontinuity, rock mass may not fail along the discontinuity, but the failure can take place in the intact rock, so that possibility can be there. So, the single plane of weakness theory gives us the idea about the range of the orientation of the discontinuities along which there will be the failure along the discontinuity.

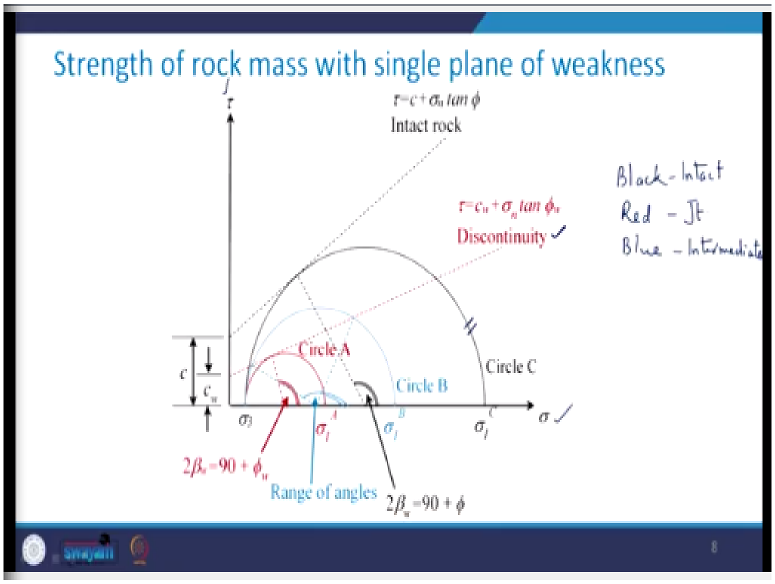
(Refer Slide Time: 13:43)



Take a look here that it is the space $(\sigma_1 - \sigma_3)$ versus β. So, if we just substitute the values of β in the previous expression which we have derived. We have seen that when $\beta = \phi_j$, this tends to infinity and for $\beta = 90°$, again this is tending to infinity. This horizontal line is giving us the strength characteristic for the intact rock. So, if β is lying between ϕ_j and 90°, the failure will take place along the discontinuity.

Otherwise, the failure will be taking place along the intact rock. If you try to plot for the various values of beta, $(\sigma_1 - \sigma_3)$ versus β, you will see that you will get a minimum value of this strength $(\sigma_1 - \sigma_3)$ at this angle, which is $(45 + \phi_j/2)$. This can be proved graphically as well as it can be proved numerically also. Like you take the differentiation of that expression $(\sigma_1 - \sigma_3)$differentiate it with respect to β and make it equal to 0. And you will see that for $\beta = (45 + \phi_j/2)$ you will get minima of that equation or this curve.

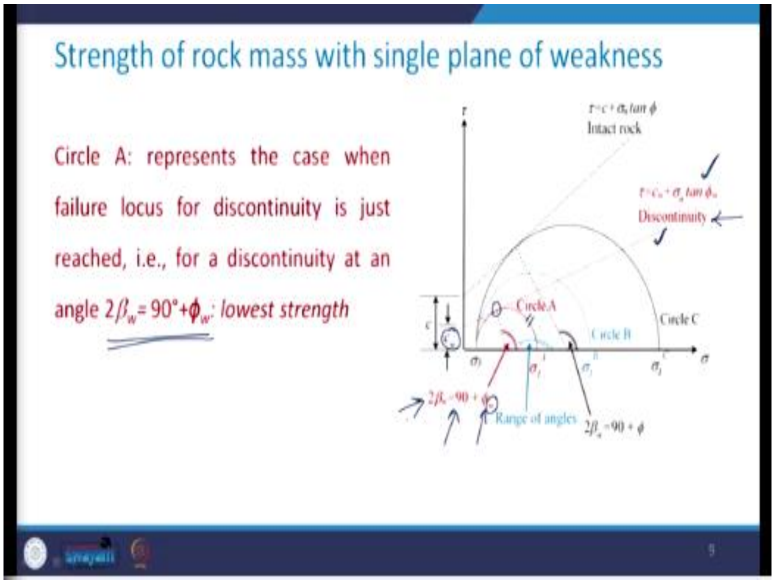
(Refer Slide Time: 15:45)



Now let me explain you this all over again with the help of Mohr circle representation. So, here in this figure there is a space of τ versus σ. Because you know that we have been Mohr envelope in this space in the form of $\tau = c + \sigma \tan\phi$. And here you can see that there are three circles in three different colors. One is for the intact rock and that has been given as the black color that is for the intact rock.

The other one is red color for the joint which is this, and the third one is blue which is somewhere in between, so that is intermediate condition Why do we have all these three and how these are going to help us in interpreting the single plane of weakness theory? Let us have a look. So, first you focus on this red circle which is represented by circle A.

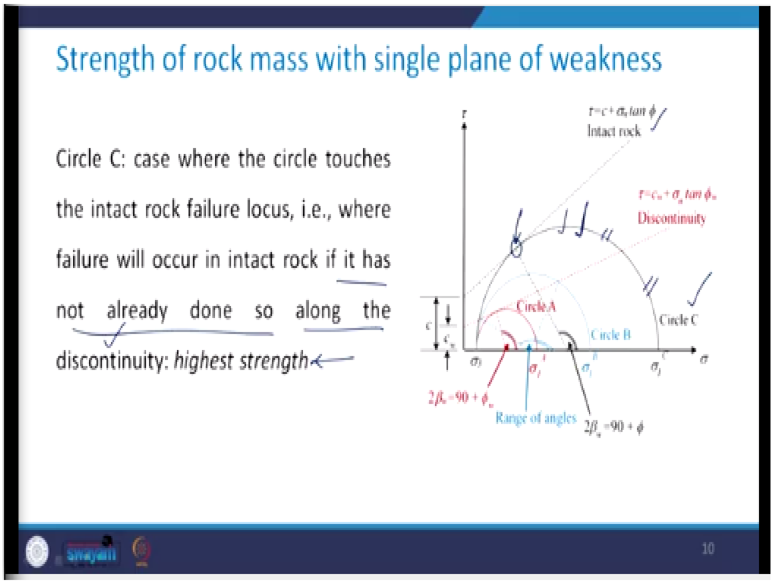
(Refer Slide Time: 17:20)



So, you see this is your circle A. So, this is representing where you have the failure envelope for the discontinuity. And you know that for the discontinuity you are going to get the lowest strength. And that you will get at this angle that is when you have this $2\beta_w = 90° + \phi_w$ by 2. So, here I have replaced that ϕ which we were discussing earlier with respect to the discontinuity, I am writing it as *w* that is for wall, that I am writing here as ϕ_w.

Now in this case you have the cohesion intercept as c_w. So, this is going to give us the failure locus for the condition when the state of stress just reaches the discontinuity. Now you can see here that this is a point which is tangent to this equation for the discontinuity. And the angles you can see that from this point if you add it to the center of the circle, this angle would work out to be $2\beta_w = 90° + \phi_w$. So, before I go to the intermediate case, let us see the other extreme which is there in terms of the intact rock.

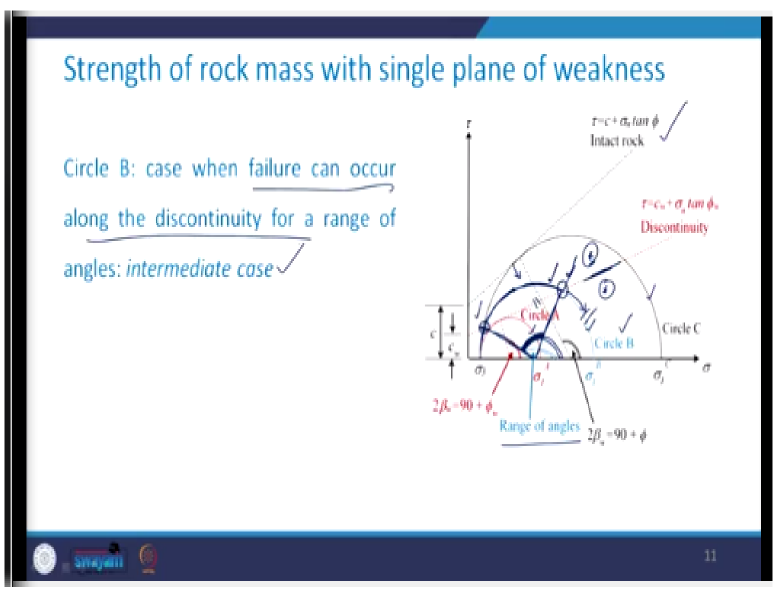
(Refer Slide Time: 19:24)



So, this circle for the intact rock has been represented by this circle C, and this case the circle touches the intact rock failure locus. See, this circle is touching this envelope for intact rock at this point. So, if the rock mass has not failed along the discontinuity, then there are chances that it can fail with respect to the intact rock. So, in that case to represent that state of stress, this Mohr circle will be helpful.

And in that case, you are going to get the highest strength. And that is why here it is written that corresponding to the circle, the failure will occur in the intact rock, if it has not already done so, along the discontinuity.

(Refer Slide Time: 20:39)



Now take a look at the last circle, which is the circle B which is the intermediate case what will happen? So, first we had two extreme cases that is the lowest strength in case of the discontinuities and the highest strength in case if the failure is taking place in the intact rock. What happens if the state of stress is somewhere in between two extremes which is represented by circle B in this case which is of the blue color here?

Please understand this carefully, focus on this figure, this is the circle corresponding to any state of stress, which is the intermediate case between circle A and circle C. Now from the center, we draw two lines, one joining this intersection of circle B with the discontinuity envelope at this location, and another at this location, so I have these two lines. Now you know that any state of a stress which is lying above this will signify the state of fail.

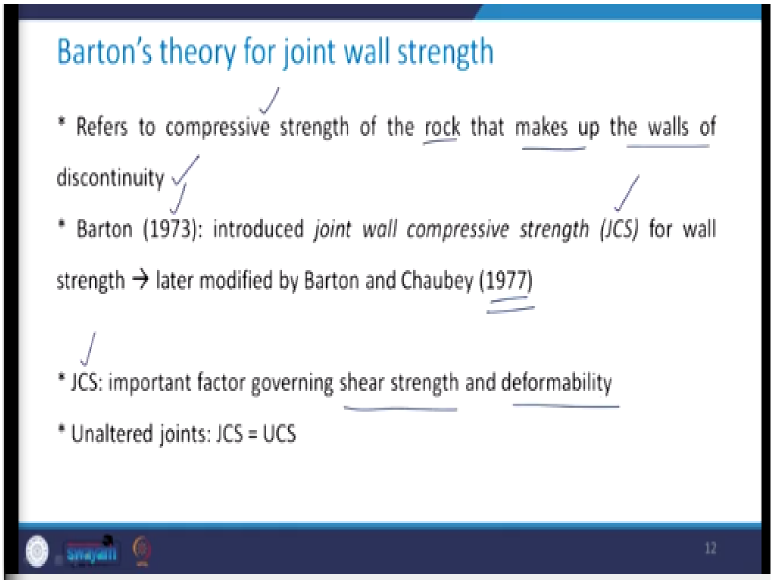
If it is lying below this that means still the rock mass has not failed along the discontinuity. Now you will see this part of the circle, this is lying above the discontinuity envelope but well below the intact rock envelope, what does that signify? That the failure is not going to take place in the intact rock, because still the state of stress is much below the failure envelope of the intact rock, but the failure will occur along the discontinuity in this zone if the state of stress is here.

Now, so this is going to give me the range of angles along which the failure can take place through the discontinuity. In case if you have this line, you will have this angle, in case if you have this line, you will have this complete angle. Now the range of this angle from here to here, which I am drawing with a solid thick line, this is the range of the angle if the discontinuity is in this range, then the failure can occur along the discontinuity.

So, we had two extreme cases where one is the lowest strength, another one is the highest strength that is corresponding to intact rock and the lowest strength is corresponding to the strength of the discontinuity. Now in case if you have any state of stress in between these two extreme cases, then it is not that for all the orientation of the discontinuity you are going to observe the failure along it.

If you have the portion from this to this let us say, and from this portion to this portion no failure is going to occur. But for these ranges of the angles failure can occur along the discontinuity for this intermediate case. So, this is what that I wanted to discuss with you the single plane of weakness theory.

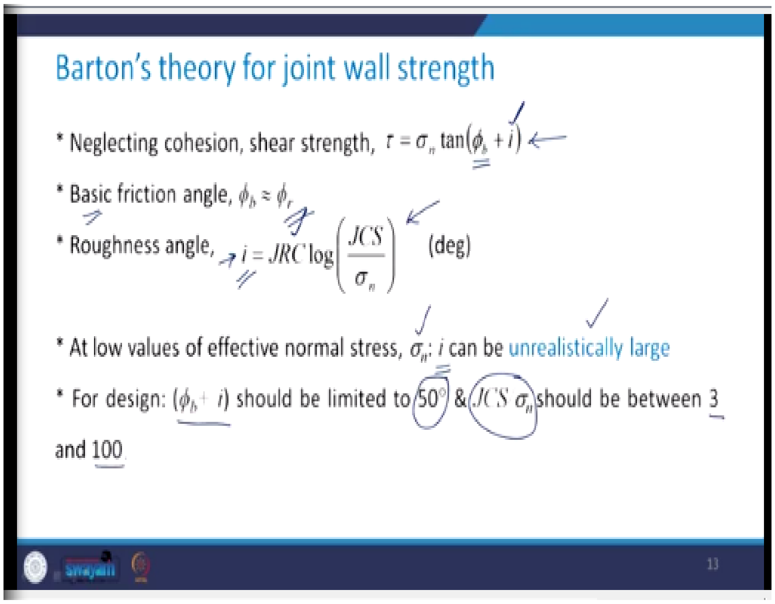
(Refer Slide Time: 25:30)



Coming to the next theory, which is the Barton's theory for joint wall strength. We have had some brief discussion about this when we were discussing about the classification of the rock mass. But for the sake of completeness and some additional input, I have put this again here. So, when I talk about the joint wall strength, this refers to the compressive strength of the rock that makes up the walls of the discontinuity.

Barton in 1973 introduced a term called joint wall compressive strength which was given by JCS for the wall strength, and this was later modified by Barton and Chaubey in 1977. This JCS is an important factor governing the shear strength and the deformability. In case of the unaltered joints, this JCS is equal to the UCS of the rock. Now if we neglect the cohesion, the shear strength for the joint wall can be given by this expression.

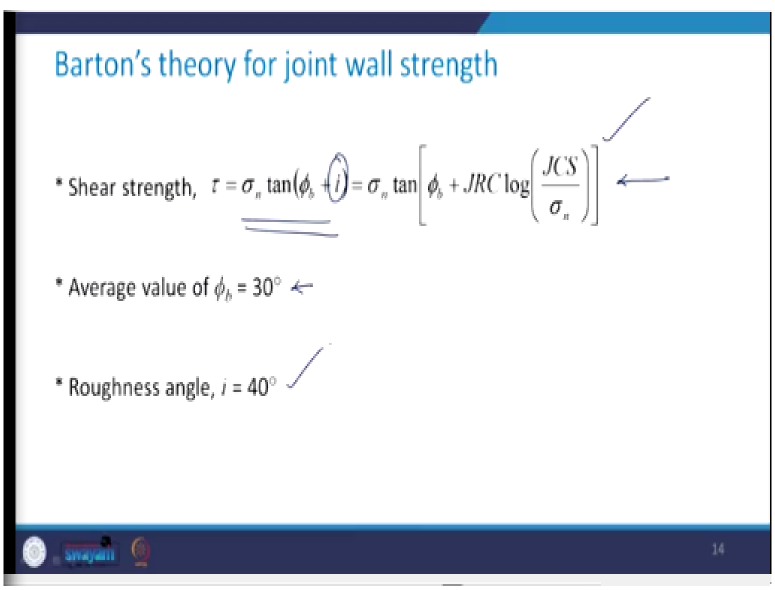
(Refer Slide Time: 27:03)



Where, you can see that this angle i which is the roughness angle, that also comes into picture. So, this term is there ϕ_b which is defined as a basic friction angle and is approximately equal to ϕ_r. And the roughness angle is represented by i and given by this expression. Where what is this JRC is the roughness coefficient. At low values of effective normal stresses of σ_n, this i can be unrealistically large.

So, for the design purpose this quantity that is ϕ_b+i, it should be limited to 50° and JCS upon σ_n should be between 3 and 100. Now the shear strength can be defined by this expression.

(Refer Slide Time: 28:12)

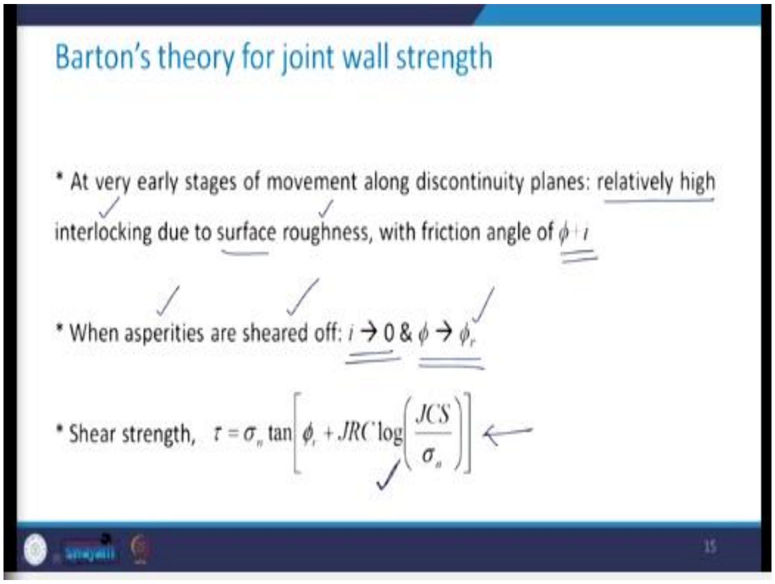


So, what we have done is, we have just substituted the expression for *i* here in this expression and the resulting equation is this.

$$\tau = \sigma_n \tan(\phi_b + i) = \sigma_n \tan\left[\phi_b + JRC \log\left(\frac{JCS}{\sigma_n}\right)\right]$$

Average value of ϕ for the rock is given as 30°, while the value of the roughness angle is 40°.

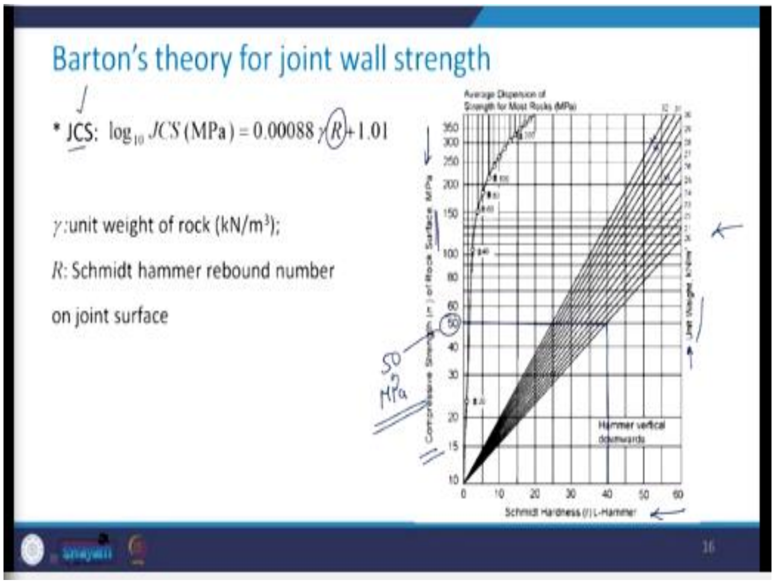
(Refer Slide Time: 28:35)



Now at very early stages of movement along the discontinuity planes, there is relatively high interlocking due to surface roughness. So, it will have a friction angle of $\phi+i$, we have discussed this in detail that when the joint surface it will have teeth then what will happen and when you have the large value of the normal stress that is towards the end of the test with large value of the normal stress.

These asperities are sheared off, and in that case this *i* reduce to 0 and ϕ reduce to the residual friction angle which is ϕ_r. And therefore, the shear strength is given by this expression which is for the large value of σ_m.

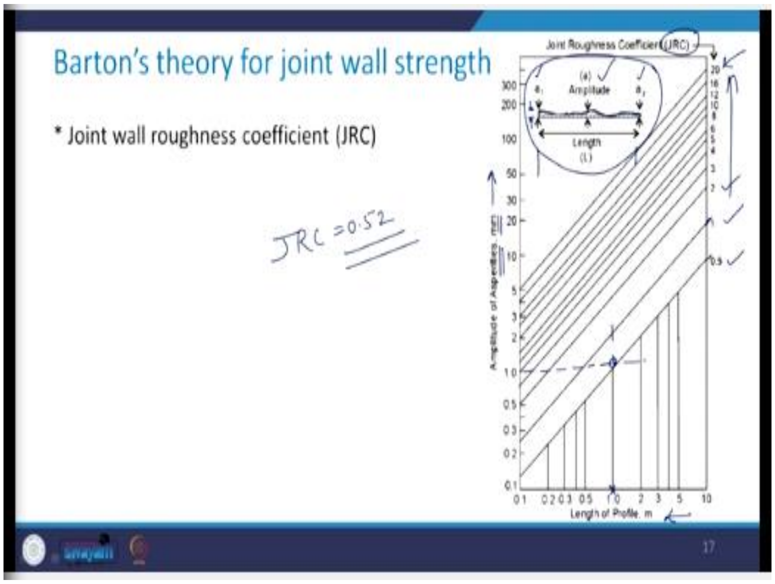
(Refer Slide Time: 29:39)



Now how to determine this JCS and JRC? See this JCS can be determined from the Schmidt hammer rebound number on the joint surface which is denoted as R, we are familiar with this figure. Here you have Schmidt hardness for L hammer, and here you have these lines are there, these correspond to various values of unit weight. And on this side that is on this scale, you have the compressive strength of the rock surface, so this is on the log scale.

You can pick a value of R, say it is 40, so you will have it here let us say it is 40 and then let us say that the gamma value is 20. So, you take it this side and you will get this as the value 50 MPa. Likewise, you can take I mean whatever is the value of this R and whatever is the value of this unit weight corresponding value of the compressive strength can be picked and that would take the value of UCS.

(Refer Slide Time: 31:19)



Coming to the joint wall roughness coefficient, this we did not discuss at that time. So, here this is how the joint wall roughness coefficient JRC can be determined. On this axis, you see you have the length of the profile in meter, and on this axis, you have the amplitude of asperities in millimeter, now what do we mean by these 2 things? Take a look at this small figure which is here.

This is the length and over this length whatever is the rock roughness profile you will just take it is amplitude at various locations. So, you see that here it is taken as a_1 here and a_2 here, and you can find out that what is the average amplitude of the asperities along that length. You have to choose the length in such a manner that more or less that amplitude is more or less same. In case if you have large variation in the amplitude then you divide it into some different lengths.

And corresponding to each length you obtain that average value of the amplitude. Now if the length of the profile is let us say 1m that is here, and say amplitude of the asperities is somewhere say it is 1mm. So, I will just take it like this and extend it, so it is intersecting somewhere here. So, you can see that here some numbers have been given which are representing the values of JRC starting from 0.512 and it goes up to 20.

So, this point is lying in between point 5 and 1, so you just interpolate and then you can find out that what is the value of JRC in this case. So, roughly if we take in this case maybe it is coming out to be say 0.52 or so. So, this is how that you can determine the value of JRC. Once you know

JCS, once you know JRC substitute in that expression for the shear strength and you will be able to obtain the joint wall strength as per Barton's theory.

So, just to summarize, we learnt about the single plane weakness theory and Barton's theory for joint wall strength. So, this is what that I wanted to discuss with you as per as our chapter on constitutive modeling of rocks and rock masses is concerned. So, in the next class, we will start the new chapter on the application area for these rock engineering. So, one of that or one of the major application areas is the tunneling. So, we will start our discussion with respect to tunneling from the next class. Thank you very much.