Air Pollution and Control
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Lecture 17
Gaussian Dispersion Model for Line Source and Area Source

Hello friends, you may recall last time we discussed about Gaussian dispersion model for point source. So, today we will extend that concept to model the air pollution dispersion from the line source and from the area source. So, to do that, basically, first of all we will see the line source Gaussian dispersion model.

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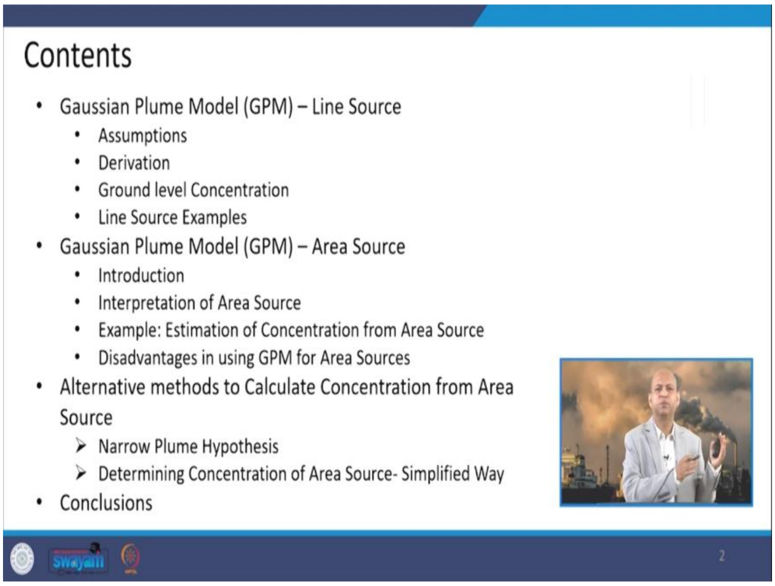

So, basic assumptions which we use for derivation of the line source dispersion model. And some examples, so, that the clarity is emerged by this concept of the line source dispersion model. Then we will go for area source dispersion model. So, basically, as I said during a point source Gaussian dispersion model that it can be extended to line source, in the sense, because line is nothing but a series of points.

So, we can integrate and develop a line source model or you can reiterate many times, iteration of many times for the point source and then line source modeling can be done, but it is very time expensive, time consuming and very resource consuming. So, we go for some empirical derivations for line source modeling.

Similarly, for area source modeling, we again either apply this point source model by extending it to some distance in the wind direction and assuming that it is a virtual source which emit the complete emission which is coming out of in area. Or another way is that area source can be divided into several strips, for lines, line sources, several lines sources. Then you can also integrate. So, there are multiple ways. So, let us see how these different methodologies do justice for this line source modeling and area source modeling.

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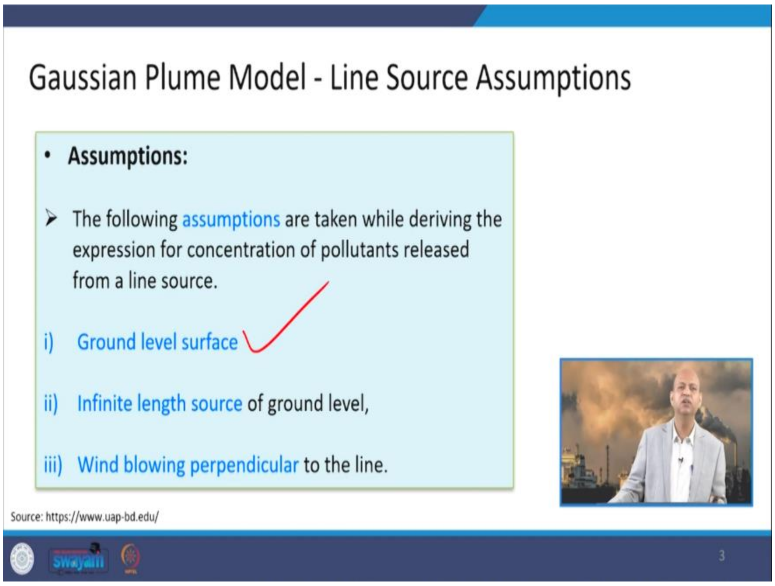

So, first of all we have certain assumptions for line source modeling of the this plume dispersion, like in this particular example, illustration which we are considering, we assume that the source is at the ground level that means, H is effective height is 0. But we can also derive relationship with the effective height of the plume at the H.

And we will go through that only basically. But when we are going for like transport emissions, tailpipe emissions from the vehicles which are moving on the road, so, we considered that it is from the ground source. It is not much above the ground, so, we do not consider the H or effective stack height which is basically the parameter of the stack or chimney.

So, you can also consider if there are industries in the row like in an industrial area or near like bank of the river, in this series of industries are there and several points sources are there then you can use this line source dispersion modeling assuming certain stack height, effective stack height.

But, if you are calculating emissions from vehicular tailpipe emissions and then you want to know the concentration at a certain distance in the perpendicular direction of the highway, then you can assume that this is the sources or the ground level. Then in finite length source at the ground level we assume. Because it is not that it is certain stretch, we assume that this is kind of infinite source, so, that the emissions at the line is highest and then it disperses in the vertical direction.

We assume that the wind is at the perpendicular of that highway or the line source. So, the concentration we will calculate at the perpendicular or at that 90° of the line source dispersion. Then we also assume that wind blowing perpendicular to the line, as I said, and the uniform emissions etc. Which are standard assumptions, which we have discussed in the point source modeling.

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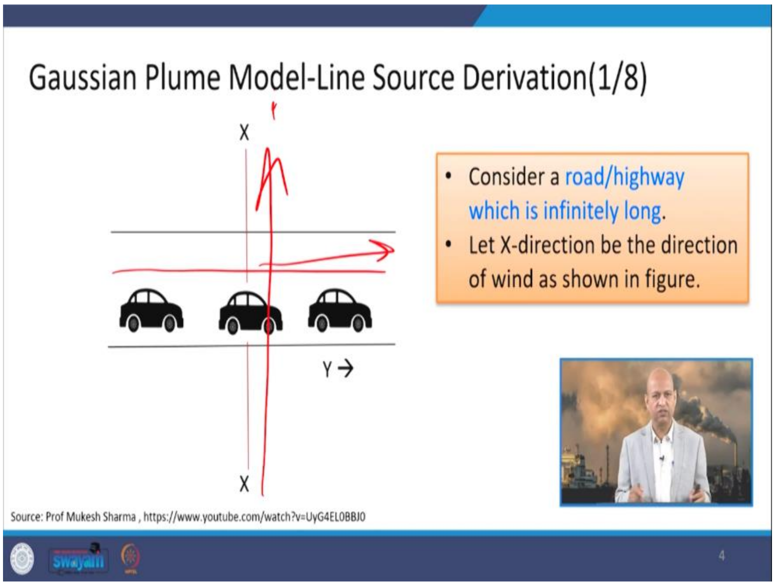

So, let us see for example, this is highway and vehicles are moving in this direction. This is the direction of vehicular movement. So, the line source is this one and the wind direction is perpendicular to this. So, maybe we can get concentration of wind is blowing in that direction. So, concentration at a certain distance at the perpendicular of this line source. So, this X direction is, basically, the direction of the wind and Y direction which horizontal direction we are taking, this is the direction of the vehicular movement.

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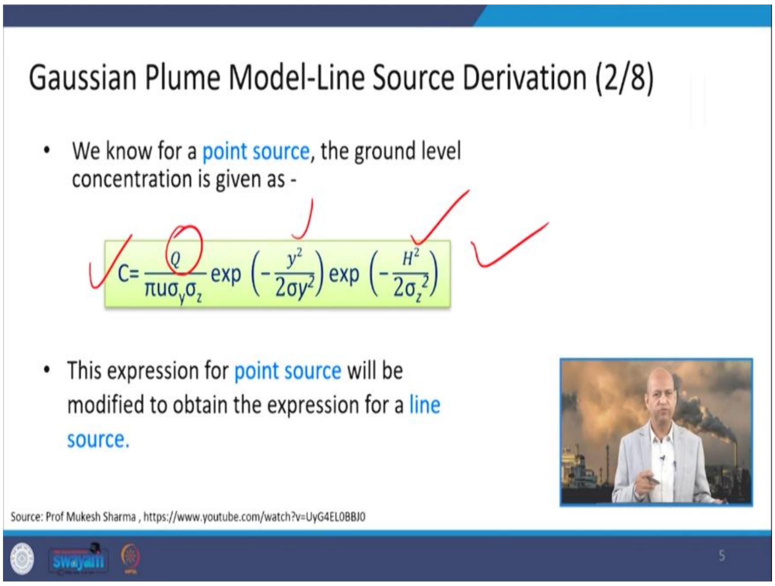

Now, you know, like this was the standard equation if you remember, for the point source dispersion modeling for the ground level concentration. So, this was like effectively stack height parameter and then this concentration, Q is the emission, y is that lateral distance. So, the expression for the point source is modified for the line source now, we go for the line source basically.

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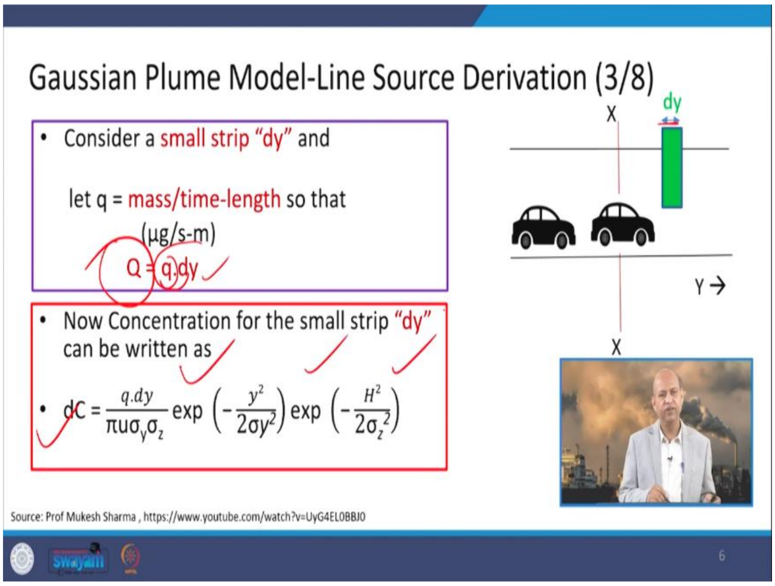

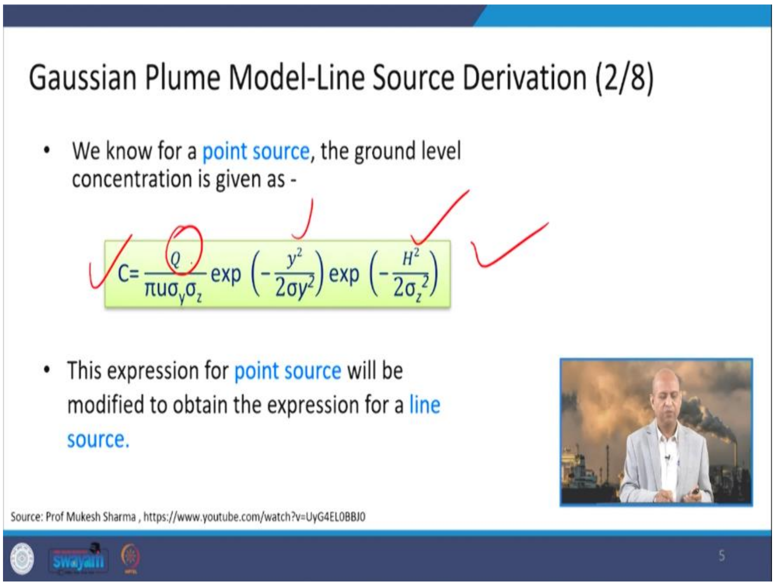

So, what we do? We assume that there is a very this narrow strip of dy distance. For that we assume that the emission in terms of mass per unit of time per length, otherwise, the emission we are calculating in this like Q is mass per unit of time. Here, we are taking a small q, which is mass per unit of length and per unit of time, per second meter, like microgram per second meters something like that.

So, that way this Q, is multiplication of this small q, which is a microgram per second meter and dy is the meter that the length of the, which we have assumed the small length, small segment of that line source. Now, if we want to calculate the concentration for the small stripe this dy then basically dC we want to calculate. dC. So, dC is in this way, you can see.

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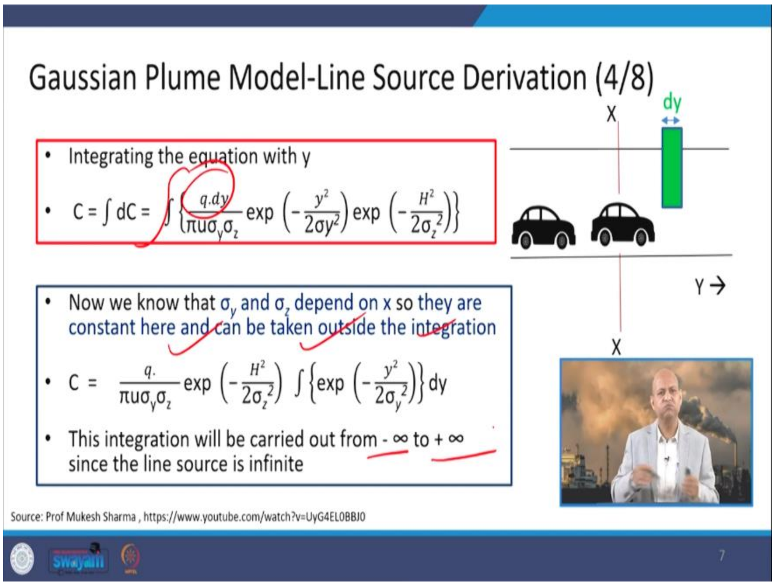

Now, if you integrate the concentration, so, for all the stripes, when we you extend this concept for the line source then there are numerous dy basically, so, you how to integrate. So, integration of this dC which is pertaining to the dy, so, the integration happens this way, this is the integration of all the dy, all the stripes, several stripes, many stripes. So, ultimately we get this kind of equation. So, this is for the line source then when we go for like integration minus infinity to plus infinity for infinite source.

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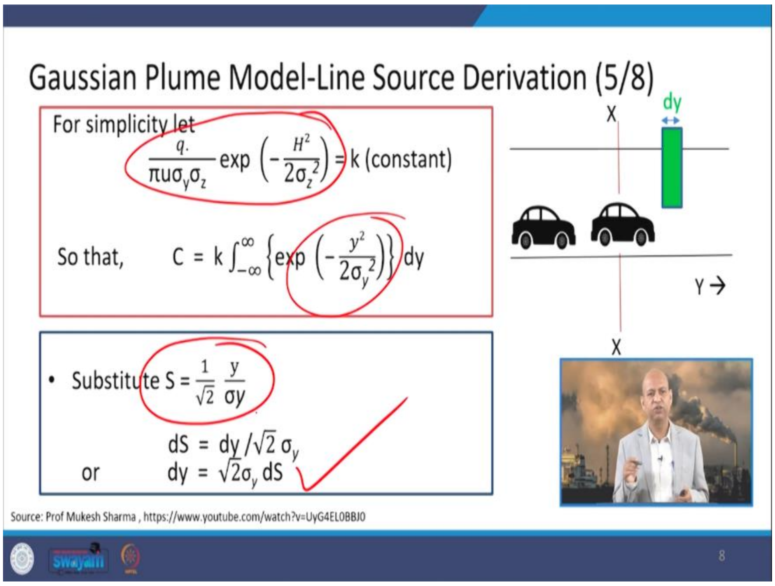

$$\frac{q.}{\pi u \sigma_y \sigma_z} \exp\left(-\frac{H^2}{2\sigma_z^2}\right) = k \text{ (constant)}$$

So, that simplifies this basically, we take this particular parameter as the k constant. This H stack height and σz in the vertical dispersion coefficient and this q divided by π u qy qz. So, in that case this concentration becomes the k into this again integration for this particular parameter. So, this gives ultimately this kind of relationship, when we substitute S as the one of the square root of 2 y upon σy. This S parameter we take basically.

$$C = k \int_{-\infty}^{\infty} \left\{\exp\left(-\frac{y^2}{2\sigma_y^2}\right)\right\} dy$$

$$S = \frac{1}{\sqrt{2}} \frac{y}{\sigma y}$$

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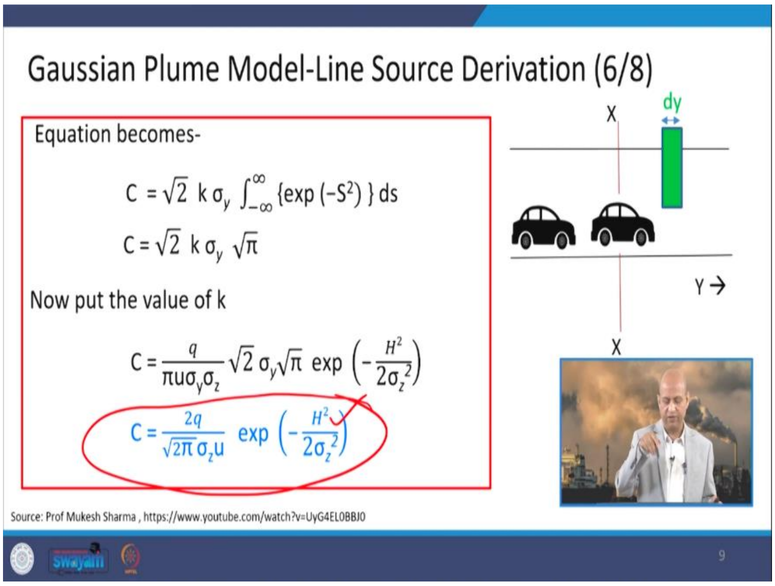

So, then we ultimately when we put all those parameters into the standard equation, we get this kind of relationship. This is the concentration at the perpendicular certain distance x and h is here, this effective stack height we are considering at present. When we go for ground level then H will be zero. So, this was for the perpendicular distance, perpendicular to the highway.

$$C = \frac{2q}{\sqrt{2\pi}\, \sigma_z u} \exp\left(-\frac{H^2}{2\sigma_z^2}\right)$$

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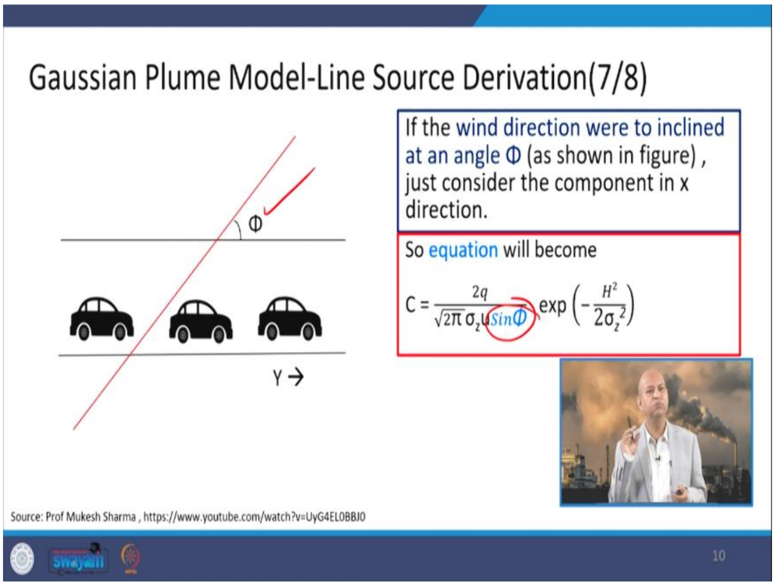

But, if you have certain angle like phi then this sin phi parameter is in the denominator that gives the concentration. When sin phi is 90° then basically this becomes 1, so, it goes away and it is 0 then it becomes infinite, we will see. So that we do not use for that.

$$C = \frac{2q}{\sqrt{2\pi}\,\sigma_z u Sin\Phi} \exp\left(-\frac{H^2}{2\sigma_z^2}\right)$$

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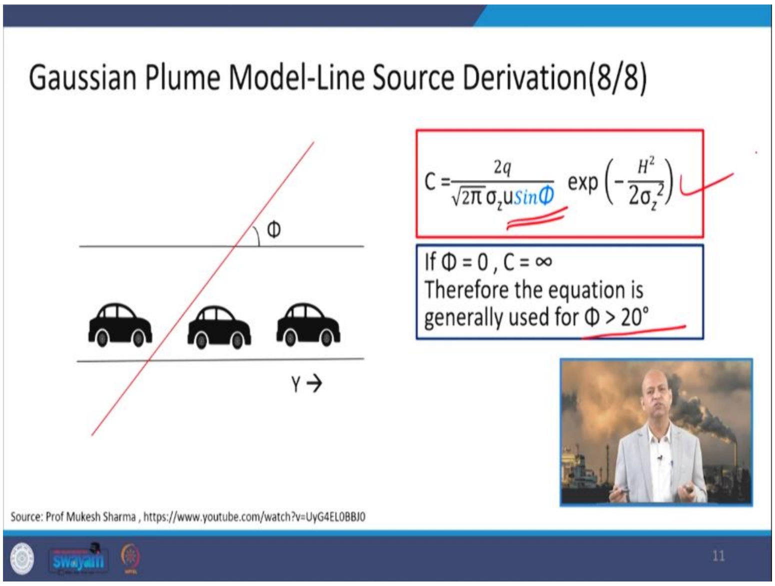

That means certain minimum angle must be there and that is here like phi (ϕ) should be greater than 20°, otherwise, calculation will be erroneous. So, only beyond 20°, we use this particular equation. Otherwise, when it is assumption based like perpendicular then this parameter goes away because sin 90 is 1 and that this becomes 2 to q upon under root of 2π σz u and this this particular parameter is there, which also goes away, because we consider the ground level emissions.

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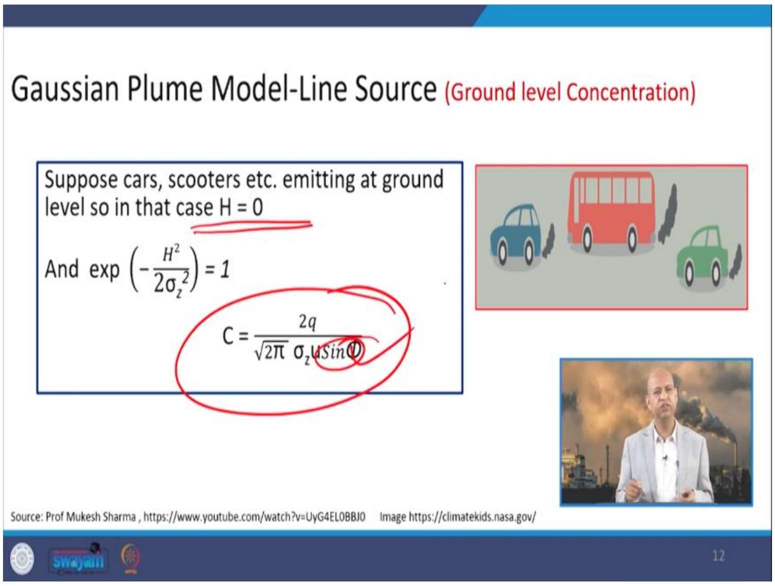

So, this is, basically, exponential of the ground level emissions, when H = 0. So, this becomes the ultimate equation for line source dispersion modeling for vehicular emissions and this goes away when we consider 90°.

$$C = \frac{2q}{\sqrt{2\pi}\,\sigma_z u Sin\Phi}$$

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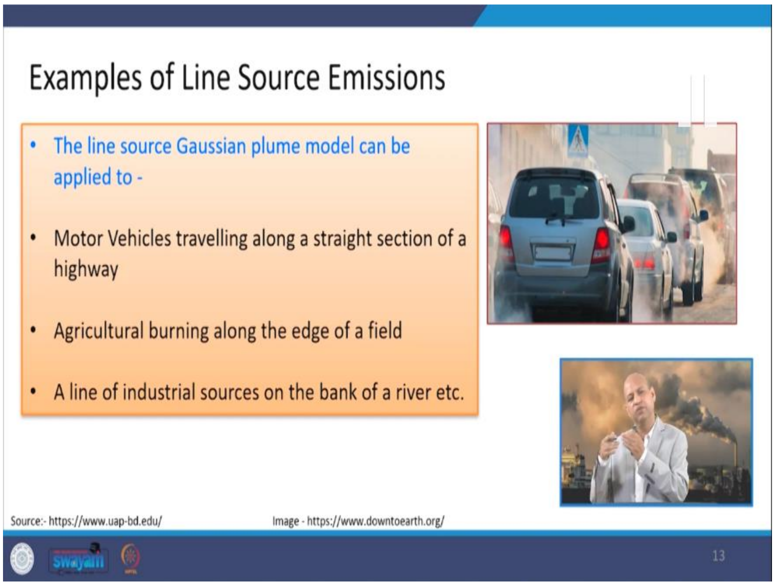

So, certain examples of these lines source Gaussian model, as I said in introduction also, this could be like motor vehicle movement on the highways, some agricultural burning in along the edge of the field, so that can also be considered as the line source emissions. A line of industrial sources on the bank of a river or stream that can also be like the line source.

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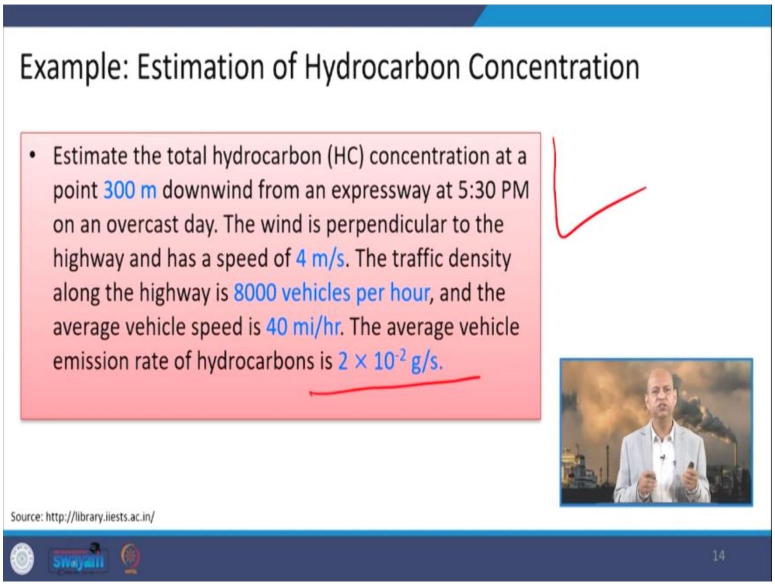

So, that way we can assume all these assumptions and we can calculate using that equation which we just derived. Now, for a better illustration, better visibility, we go for, estimation of hydrocarbon concentrations for taking this example for line source dispersion. So, this gives estimated the total hydrocarbon concentration at a point 300 meter downwind from the expressway at 5:30pm on an overcast day. So, that gives hint which kind of atmospheric stability will be there.

The wind is perpendicular to the highway, which we assume and has a speed of 4 meter per second and the traffic density along the highway is 8000 vehicles per hour that is the kind of flow rate you can say. And the average vehicle speed is 40 mile per hour. We can convert it into kilometer, meter. The average vehicle emission rate of hydrocarbons is per vehicle 2 into 10 to the power minus gram per second. So, these are the input values which we will be using for deriving this concentration at the particular distance of 300 meters at the perpendicular of the highway.

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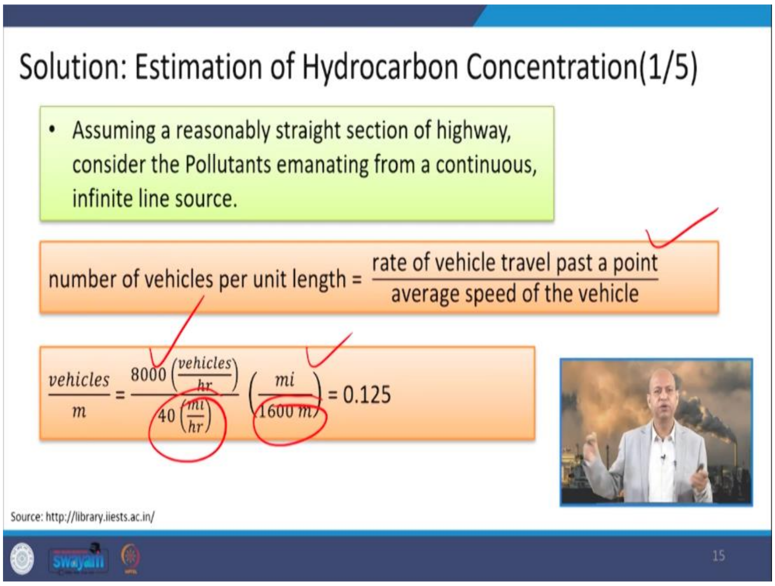

So, number of vehicles per unit length is basically rate of vehicles travel past a point divided by the average speed of the vehicle. So, this is given 8000 vehicles per hour, 40 miles per hour and miles you can convert into meter. So, this has been done, here value has been put. So, ultimately we get 0.125 vehicles per meter per unit of the length. Although vehicle is never in fraction, but for calculation purpose we use it, otherwise, there will be some number of the vehicles in a particular stretch.

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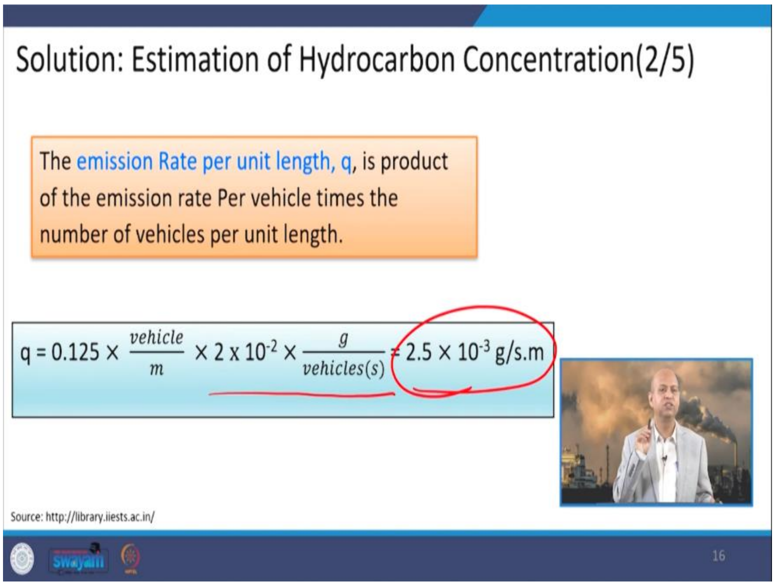

Then the emission rate, q, which is we estimate like mass per unit of length per unit of time. So, we calculate here. This is, basically, vehicle per meter and then we get this multiply by that we that emission rate which is given per vehicle. So, we multiply that so, we get this one. 2.5

into 10 to the power minus 3 gram per second per meter. So, that is the small q, which will be used in the equation which we have just derived.

$$q = 0.125 \times \frac{vehicle}{m} \times 2 \text{ x } 10^{-2} \times \frac{g}{vehicles(s)} = 2.5 \times 10^{-3} \text{ g/s.m}$$

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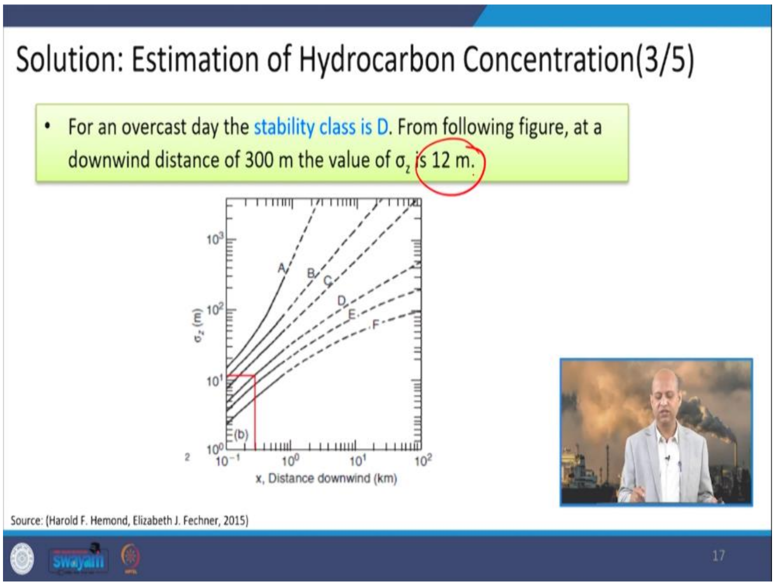

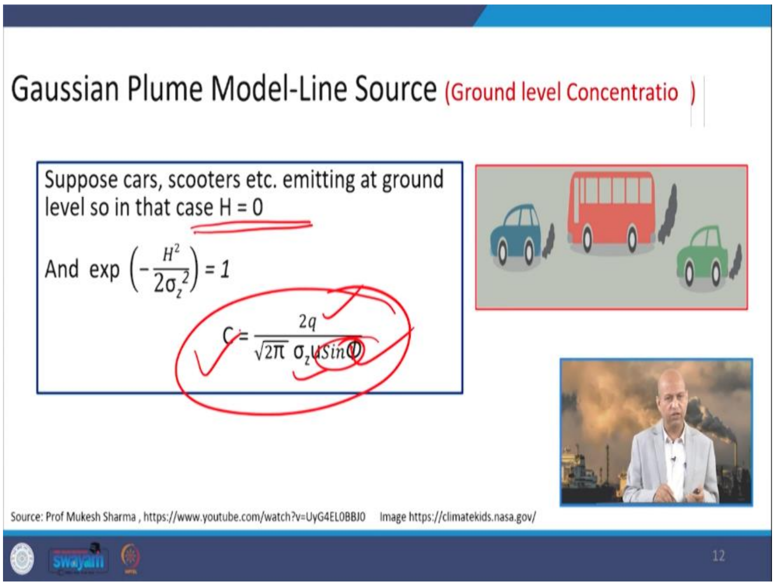

Then we need σz, because in this equation, we need σz, we need small q, small q we have calculated. Sin phi is not there, because 90°, we are calculating concentration we have to calculate π value we know. So, σz we need to know. How to calculate σz?

We know the downwind distance 300 meter. So, for 300 meter, we go like this and at this point we get σz values 12 meter. This is in kilometers, so, you have to be careful, when you go for meter. 300 meter the value of σ_z is derived 12 meter from this estimated chart.

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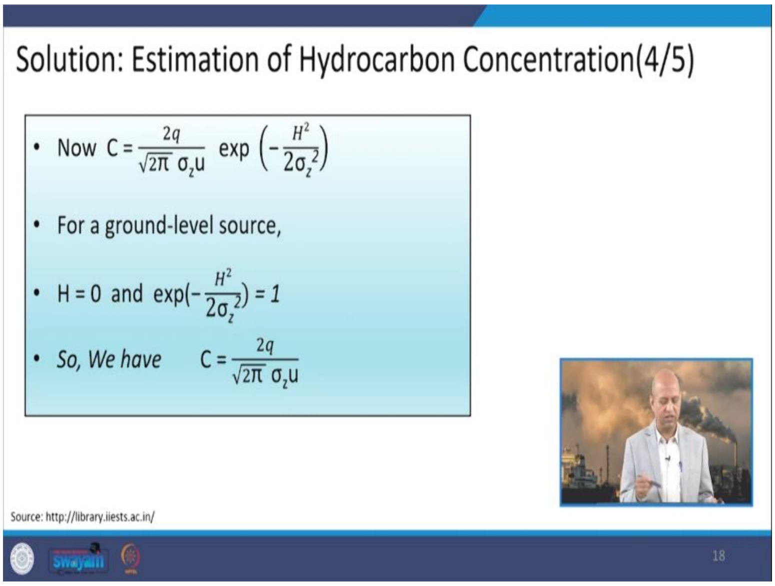

Then we put those values σz and q. h is a 0 because ground level concentration as you know.

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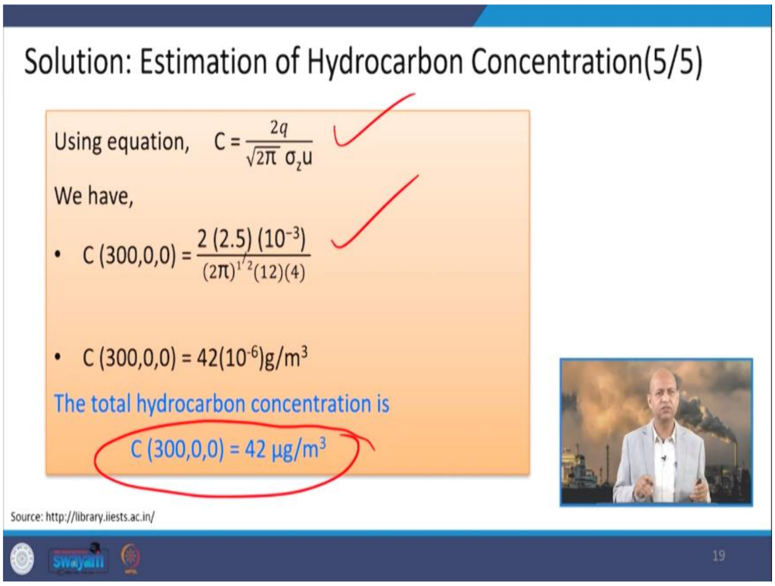

So, ultimately we are using this equation, we are having those values in this equation and we get this concentration at the 300 meter perpendicular to the highway. (300, 0, 0) means y = 0 and this H = 0, z = 0. And ground level concentration. So, this is 42 micrograms per cubic meter. Then we can compare according to the situation. This is just an example.

At that point if receptor is susceptible to certain concentration and there may be some background concentration, we can add into that. And then we get the summation of the concentration that we compare with National Ambient Air Quality Standard and then we can learn whether it is exceeding that or not. So, those kinds of things are possible with these calculations.

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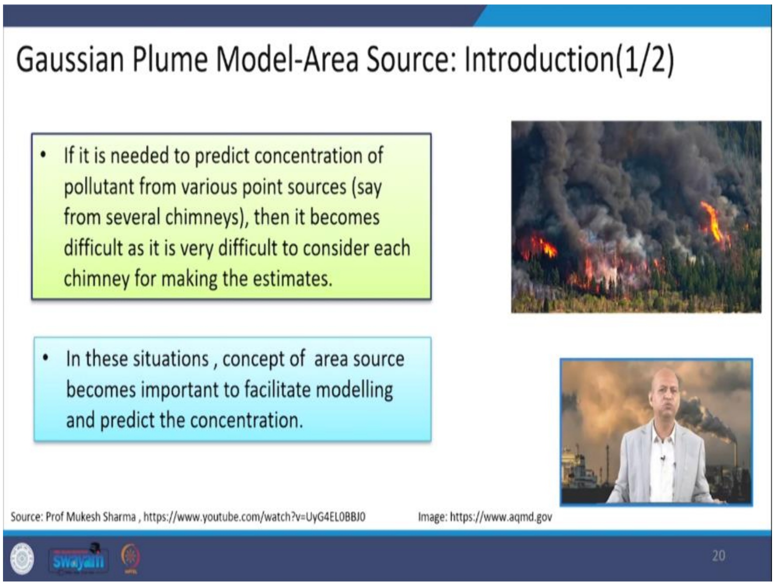

Now, we go for area source this Gaussian plume model for area source dispersion. So, basically, area source can be several industries in a particular locality or you know, this kind of fire in the forest, many many kinds of situations may be there.

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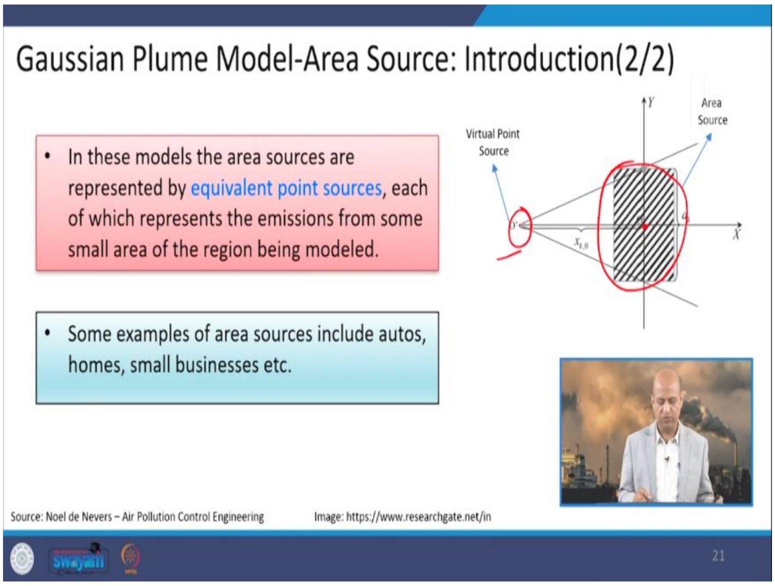

And then we want to apply the point source dispersion model. So, we need to assume that there is a virtual point, where this whole emission is coming out of that point. So, we assume in the upwind direction one point. This is X_0 distance from this center point of that area, we consider this is the area. And this whole emission which is coming out of this area is basically being omitted from this, this we assume. This is the virtual source; this is not the exact actual source.

Actual sources in this particular area, but when we want to apply the point source dispersion modeling, so, we assume that this is, this whole mission is coming from the one point, the complete emission, total emission, which is in this area is coming from a point source. This is the basic assumption we need to know.

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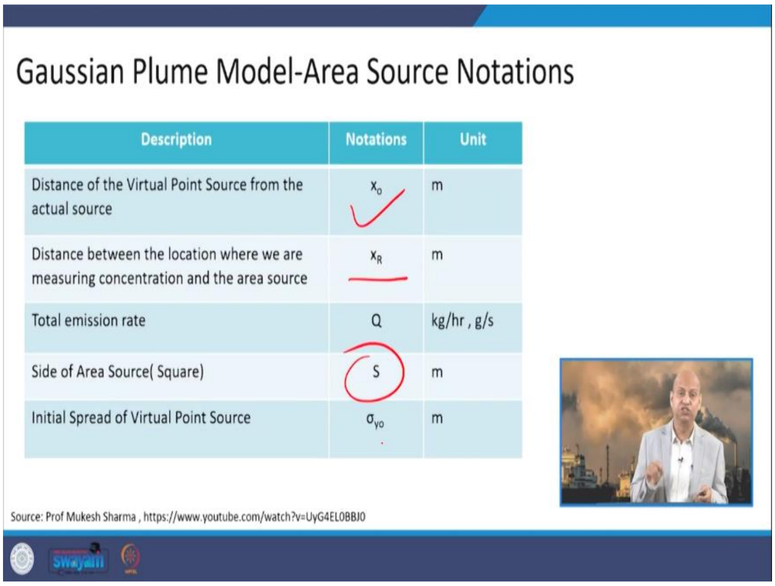
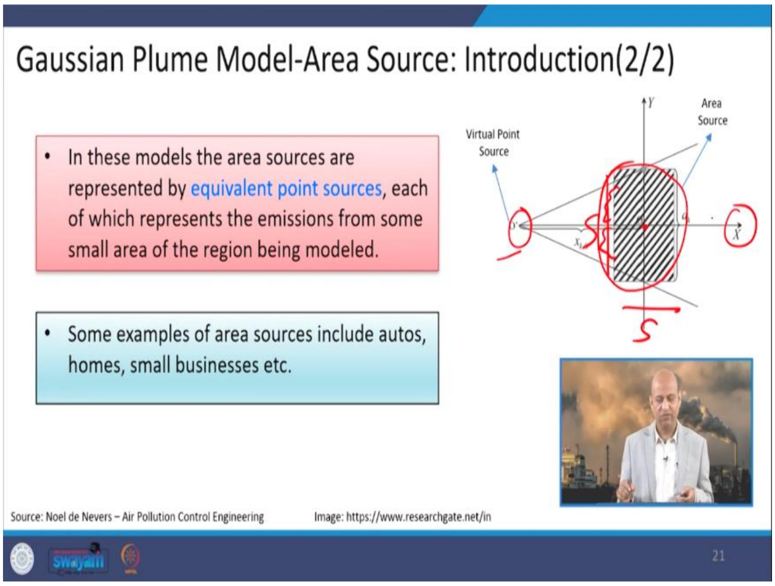


Description	Notations	Unit
Distance of the Virtual Point Source from the actual source	x_0	m
Distance between the location where we are measuring concentration and the area source	x_R	m
Total emission rate	Q	kg/hr , g/s
Side of Area Source(Square)	S	m
Initial Spread of Virtual Point Source	σ_{y0}	m



Now, we go for you know, certain nomenclatures or these parameters like distance of the virtual point source from the you know, this actual source of the area source is X_0, we assume. This is in meter. Then distance between the location where we are measuring concentration at the area source, like this is the receptor. So, we assumed this as X_R.

You can do it, X_t, whatever you want to know, you want to have, you can, you are free, means these are just notations. Then total emission rate Q, capital Q, kilogram per hour or gram per second that kind of, that is the total emission which is coming out of the area and we assume this whole Q is coming from a particular point, which is virtual point, not the actual but assumed point. Side of the area of the source is square is S.

So, we assume this is the square source, width and length are equal. So, S is the distance. This is S. This is S, like that. So, this is S in meter. Then is initial spread of virtual point source is σ_{y0} in the lateral direction, so, spread will be like this. So, this will be σy from the central line. So σ_{y0} is the initial spread.

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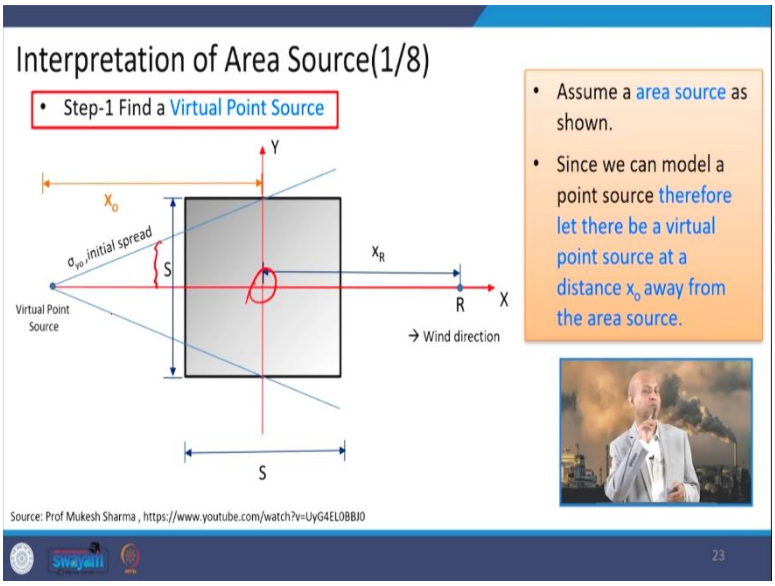

Now, how to calculate? You can see here this is the X_0 distance of the virtual point from the area. So, central point of the area we measure the distance and σ_{y0} is basically this kind of distance σ_{y0} and this X_R is the distance from this area source to the receptor. So, the total distance X_R plus X_0 will be the distance of the receptor from the virtual point which we have assumed.

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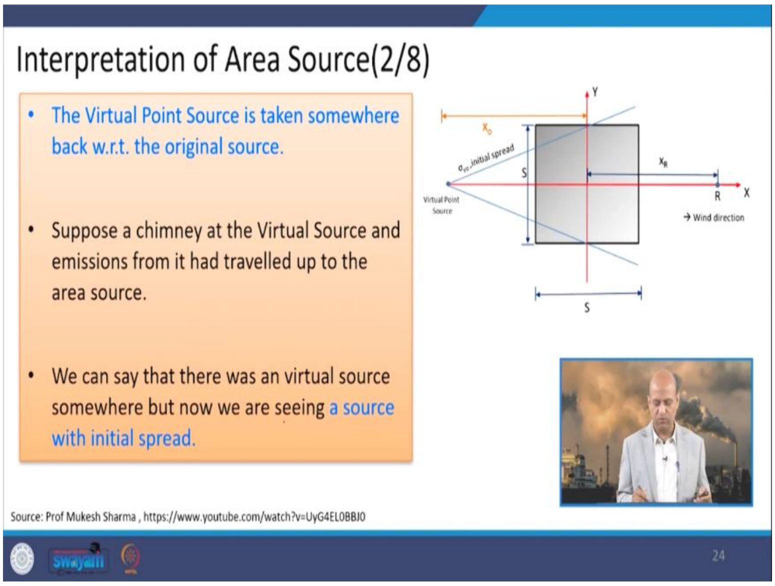

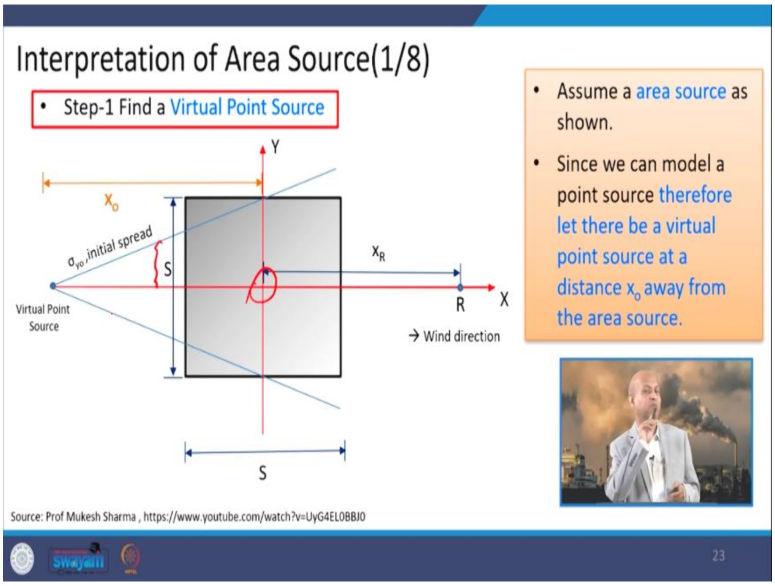

Here you can X_0 plus X_R. And you can see here, other distances is like S, which we have assumed.

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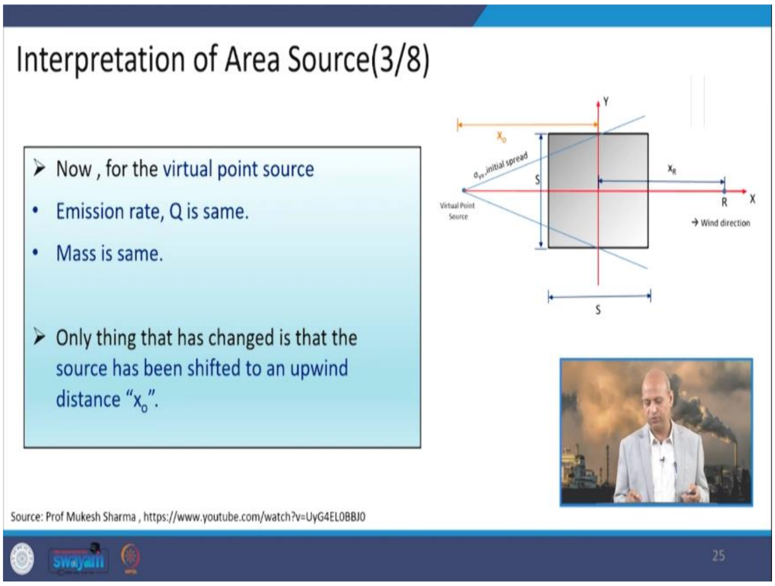

And then you see this other points like Q is the same, which is in the area source and mass is the same, X_0 is in upwind direction, as we have already seen.

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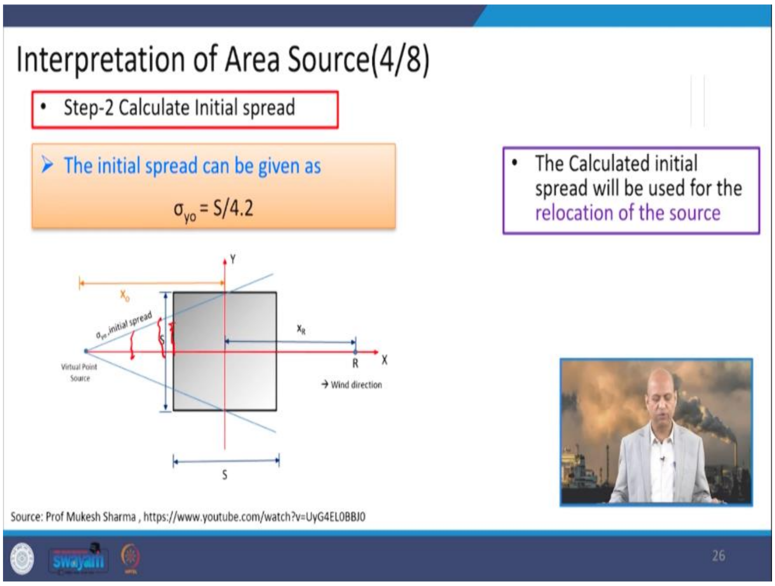

Now, how to calculate σ_{y0} which is dispersion like it can be here, it can be here, that way. So, σ_{y0}, we assume as S / 4.2. S is this distance. So, 4.2 may be something like here. 4.2. S by 4.2. This is σ_{y0}.

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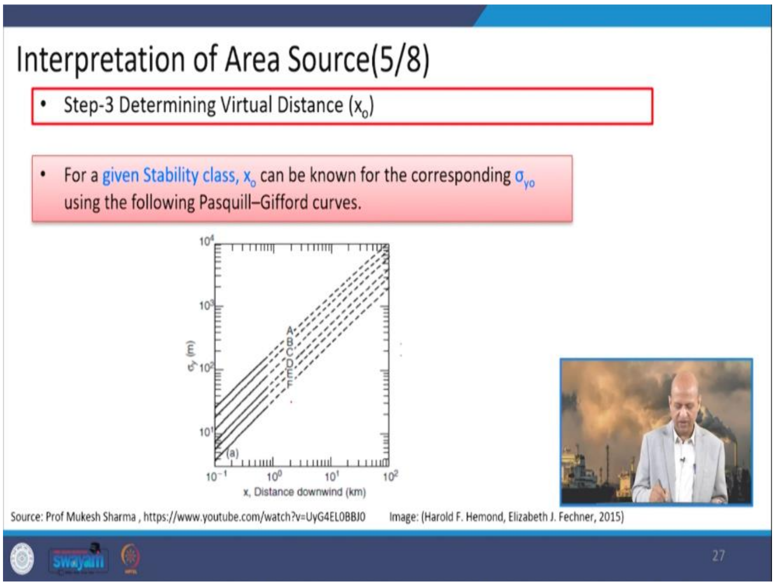

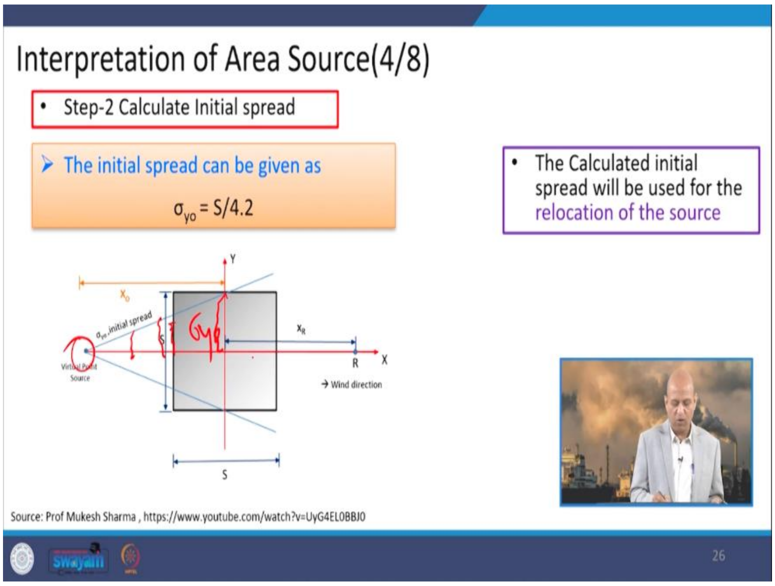

And from σ_{y0}, we can assume, we can calculate using this particular σy derivation chart, the distance, horizontal distance, which will give us X_0 distance basically. Because this is the x 0, so for if this is the point and at the X_0 point, this is the σ_{y0} So, how much this X_0 distance is there?

Which you can get from here. Because σy = 0, you can read on this particular axis. You go towards the right and see which particular atmospheric stability condition there A B C D is up to that go, go down. So, in this case, basically, that overcast is there. So, d neutral condition we assume, so up to d we will go and we calculate distance X_0.

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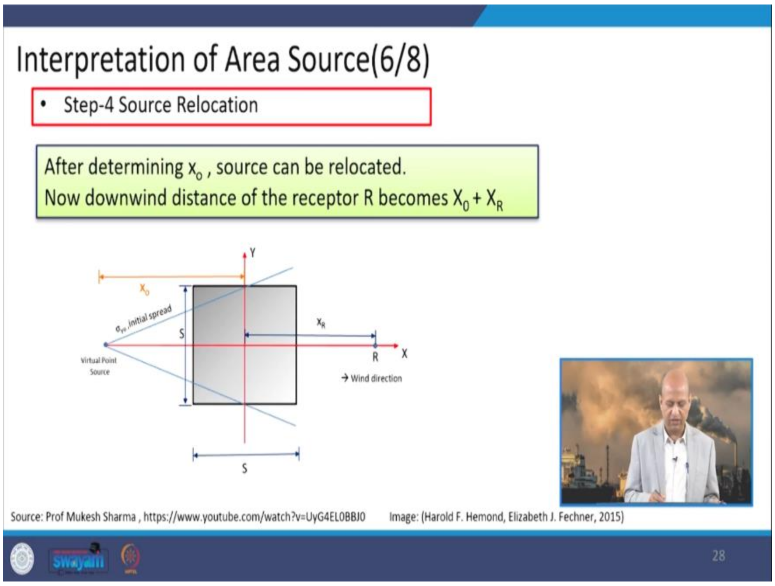

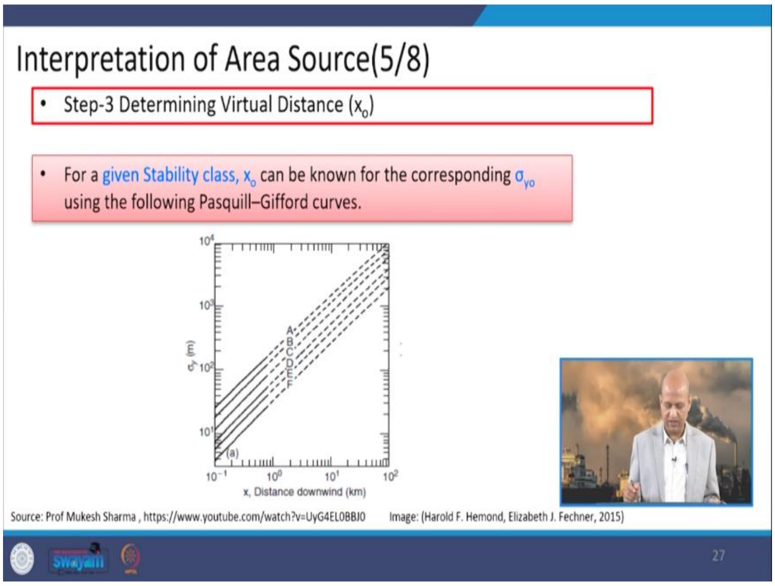

And that X_0 is calculated by with the help of this chart and we get certain value, then we can add into X_R.

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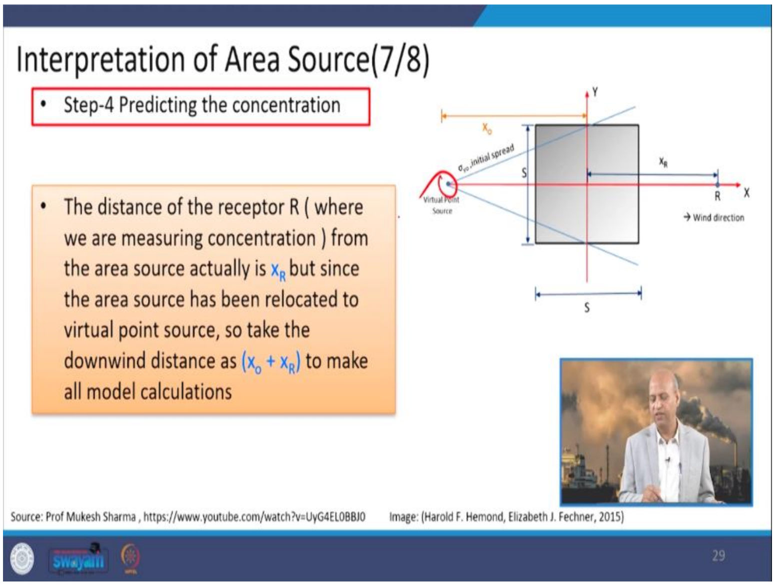

And we get the total distance. So, the X_0 plus X_R is the total distance from this virtual source to the receptor.

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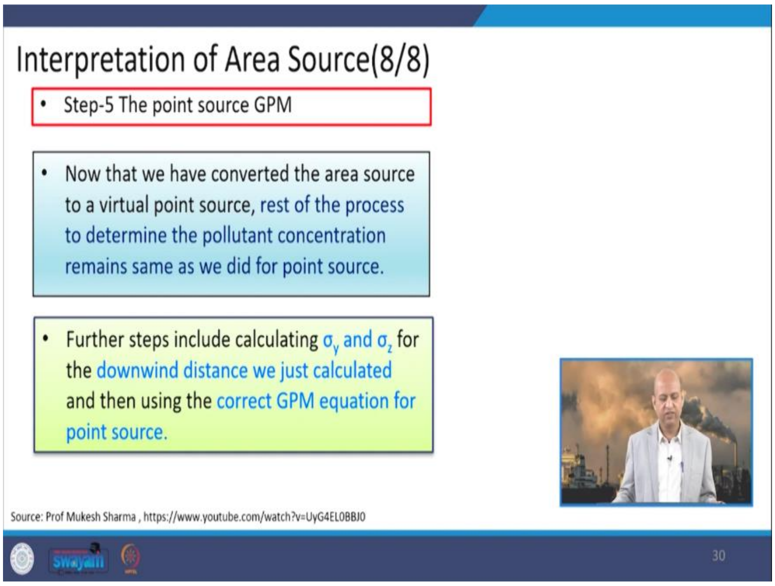

So, this is the step 5 for the point source modeling when we calculate σy and σz. Because, now, we know the distance and then we can calculate the concentration. Because point source emission modeling is very simple after that.

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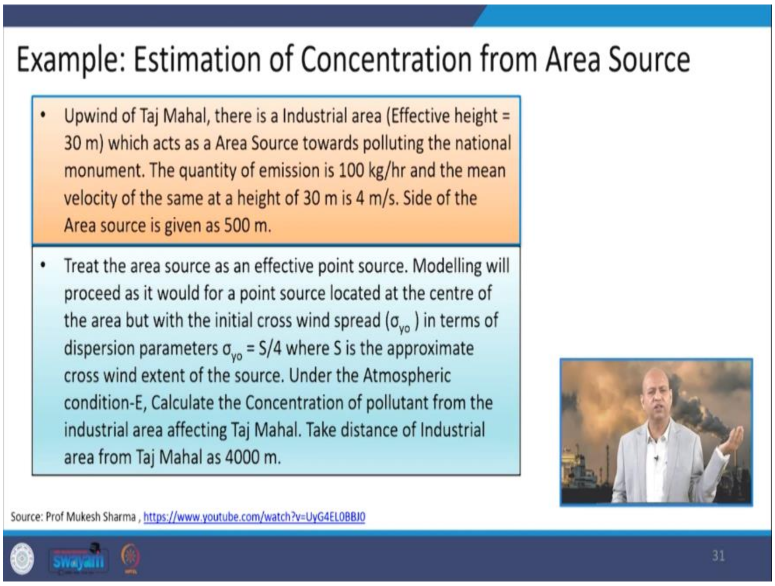

So, now, to illustrate, to understand this, let us have one more practical example. For example, we assume that the wind of the Taj Mahal there is a Taj Mahal at a particular point and upwind of that, there is an industrial area, effective height is 30 meter, capital H. here, effective height is given.

So, we can use that, which acts as an area source towards polluting the National Monument, which is Taj Mahal. The quantity of emission is 100 kilogram per hour. Q value is given. Capital Q. And the mean velocity of the same at the height of the 30 meter, which is effective it is 4 meter per second and side of the area source is 500 meter, so, capital S, this distance 500 meter, 500 meter.

Then other assumptions are like treat this area source as an effective point source that virtual point source and modeling will proceed as it would be a point source located that center of that area, but with the initial crosswind spread, which is σ_{y0}. That σ_{y0}, which we have seen, because S by 4.2, that value we know.

And approximate crosswind extent at the source, which will have S by 4, here S by 4, earlier we took S by 4.2, something like that under the atmospheric stability condition as E. We can calculate the concentration of the pollutant at the Taj Mahal, what will be there and the take the distance of industrial area from the Taj Mahal as 4000. So, that is X_R or X_T, you can say. So, in this we will add X_0. We will first calculate the x 0, then we will add into this, then the total distance from that virtual source point source will be calculated.

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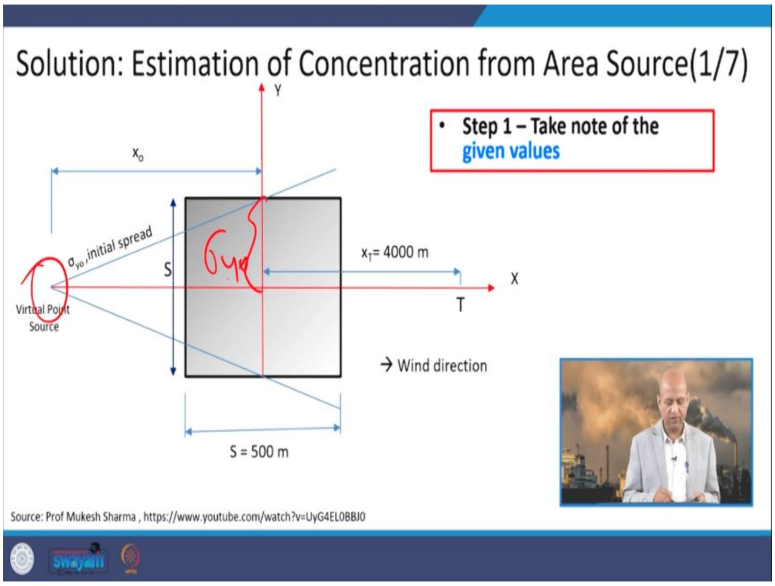

So, let us go. Here, X_0 is the virtual inner source distance, X_T is given 4000 meter. Just now, we have seen these values, as 500 meter. So, this can give us σ_{y0} this, this value σ_{y0}.

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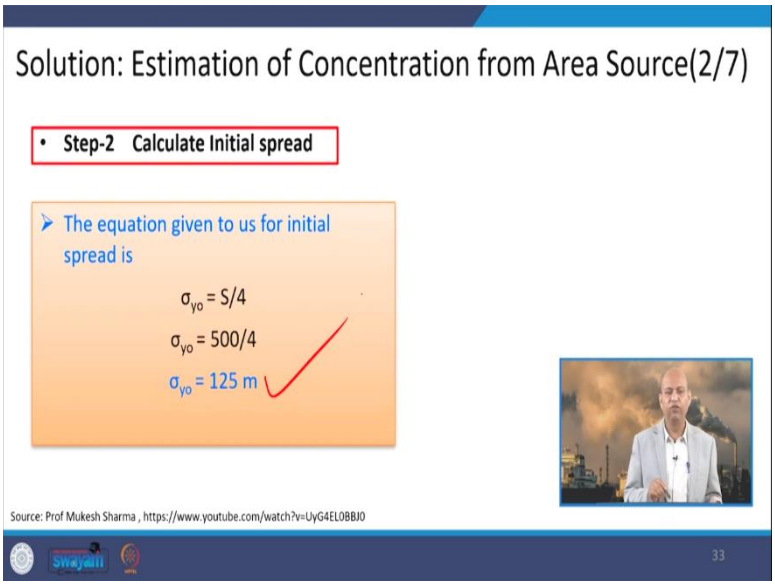

Which we calculate. σ_{y0} is S / 4, so, this is 125 meter, we calculate.

$\sigma_{yo} = S/4$

$\sigma_{yo} = 500/4$

$\sigma_{yo} = 125$ m

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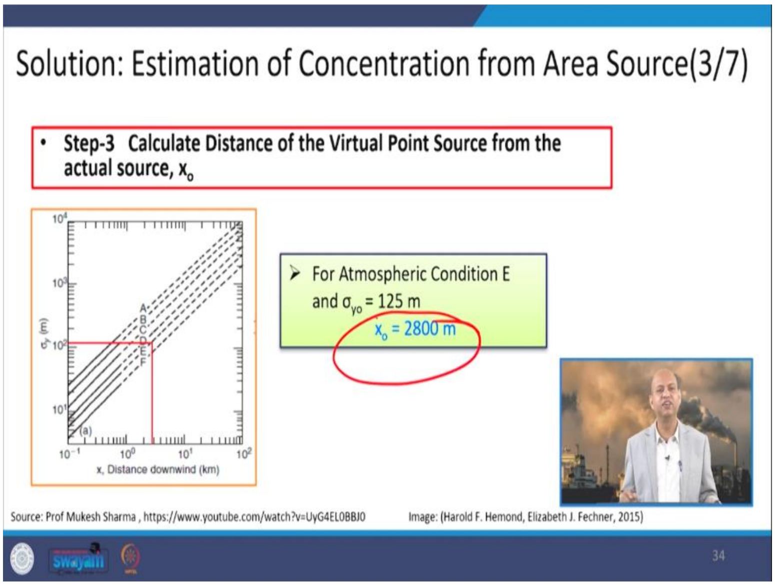

For 125 meter, for the E atmospheric stability, we go down and calculate this X_0 which is 2800 meter.

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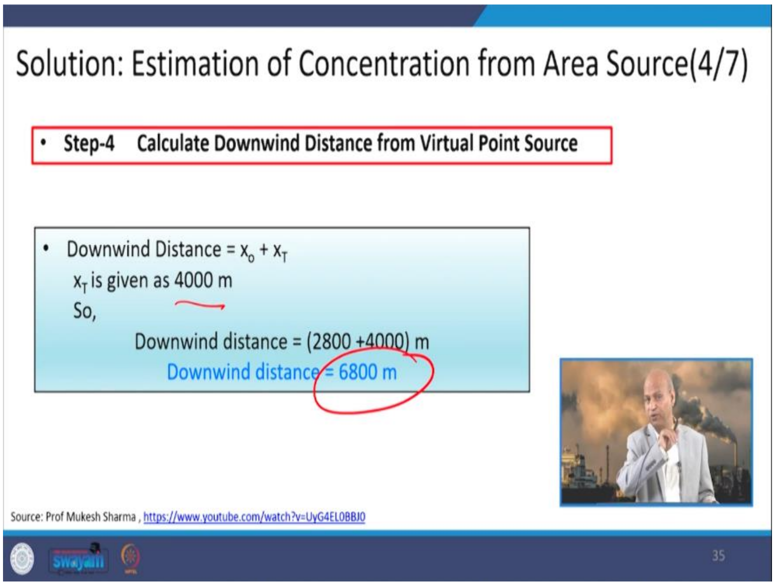

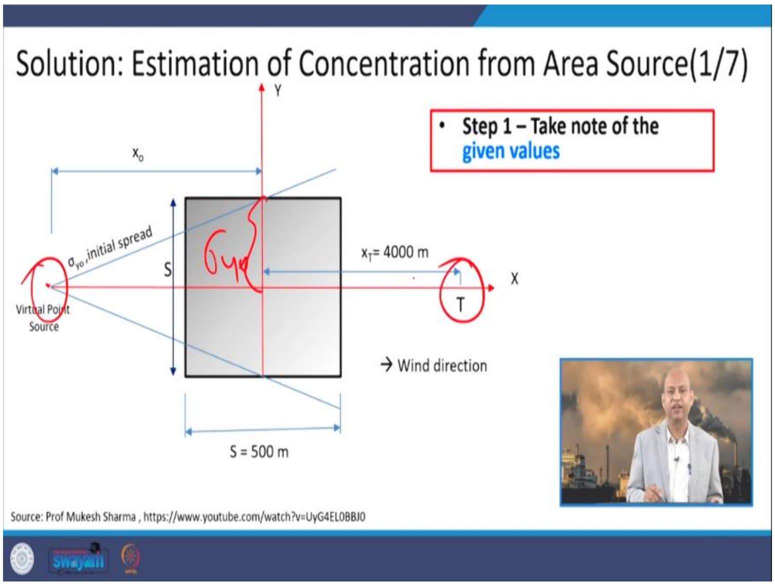

So, this 2800 meter we will add in to 4000 meter then we will get the total distance from this virtual source to the Taj Mahal X_0 plus X_T. So, X_0 is 2800 and X_T is 4000. So, 6800 meter is the total distance, which is now the downwind direction from the point source, virtual point source, basically.

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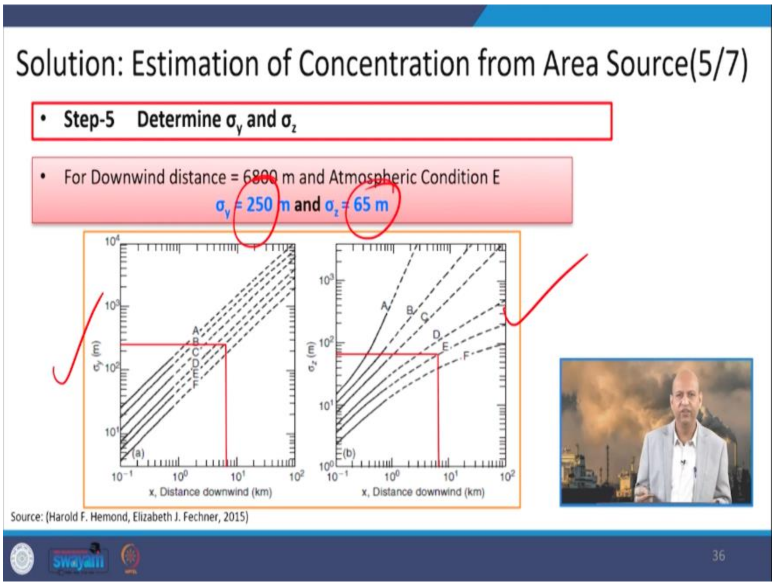

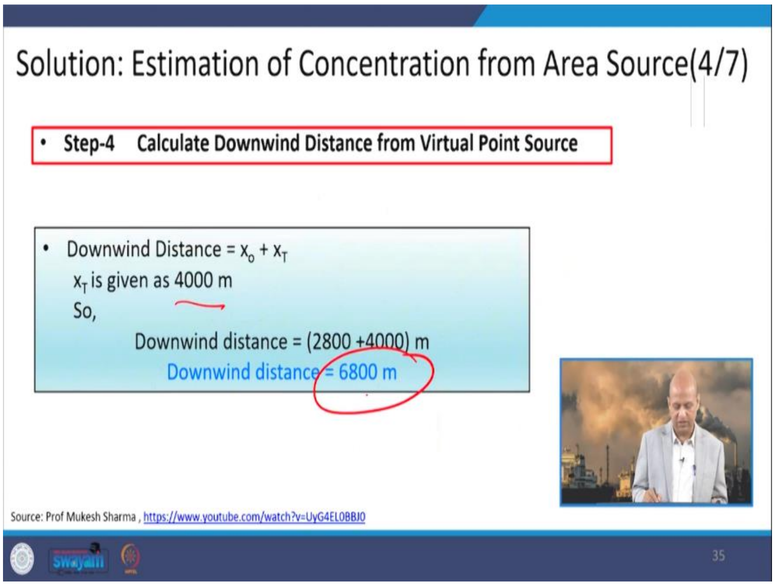

So, we will use that now, for that distance what is the σy and σz, we need to calculate. Again we use these charts which are given. So, E, atmospheric stability is given, the distance horizontal is now, how much, 6800 meter. Again, see, 6800 meters 6.8 kilometer, so, here 6.8 Kilometer, go up to E, go, horizontal and get the σy value, which is 250 meter. Similarly, from this chart, you get the σz value, which is 65 meter. So, those values now you have which are needed for that point source modeling.

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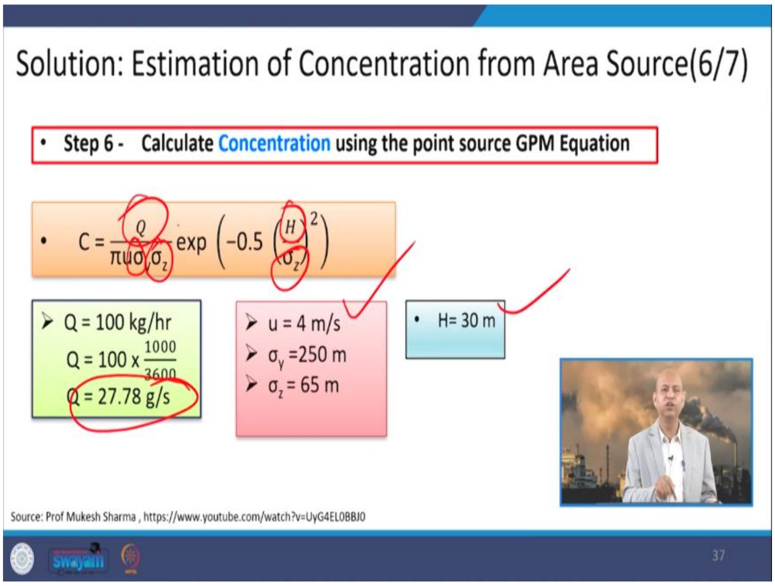

So, here is σy σz, you have calculated. H is given, 30 meter. Q is there you have got this 100 kilogram per hour, already given. So, you can calculate it in grams per second. Other values are given, u, that average velocity speed of the wind, 4 meter per second, σy, σz, you just calculated, 250 and 65. H is 30 meter given. All these values you put in the equation, you get the concentration.

$$C = \frac{27.78}{3.14 \text{ x } 4 \text{ x } 250 \text{ x } 65} \exp\left(-0.5\left(\frac{30}{65}\right)^2\right)$$

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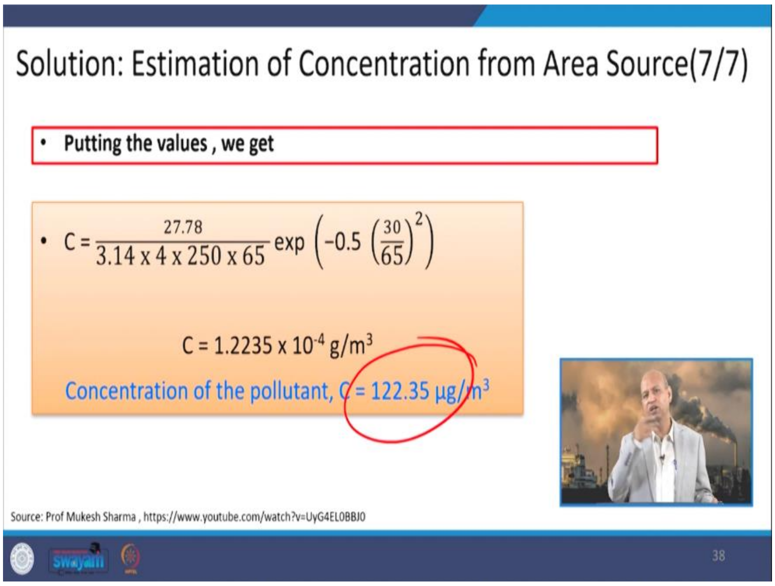

How much? Around 122 micrograms per cubic meter. So, you can compare it with again, you can add into a background concentration at the Taj Mahal, then come up with the total concentration. Then you can pass some judgment whether it is acceptable or not, whether it is exceeding the limit or not, all those kinds of things. So, that way you really use this in a particular way of assuming virtual point source for the area source and you can calculate the emissions from area source, how they are converted into concentration at a certain distance, distance for the particular that virtual point.

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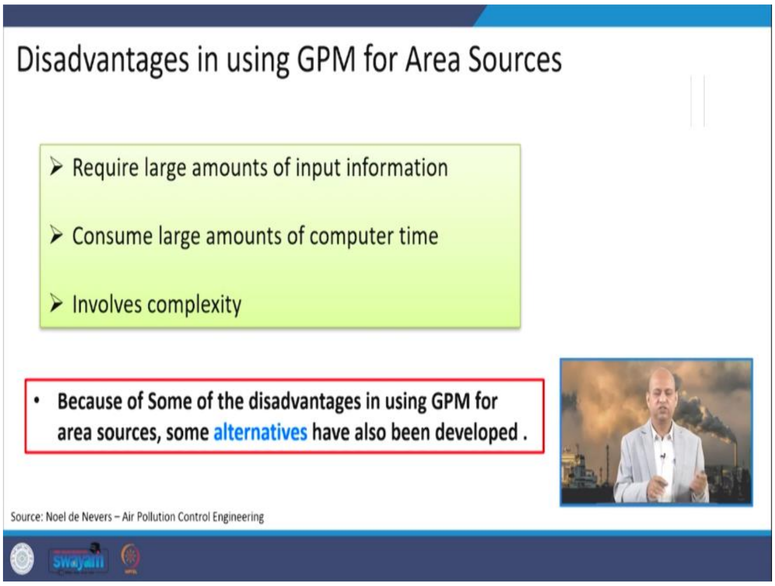

Then, there are certain disadvantages, basically, for this Gaussian plume model for area sources, like it requires large amounts of input information. And then it consumes large amount of computer time and involves complexity. So, those kind of limitations are there which need to be overcome. So, then some alternatives have been suggested by modelers that instead of this, we can go for simpler methods.

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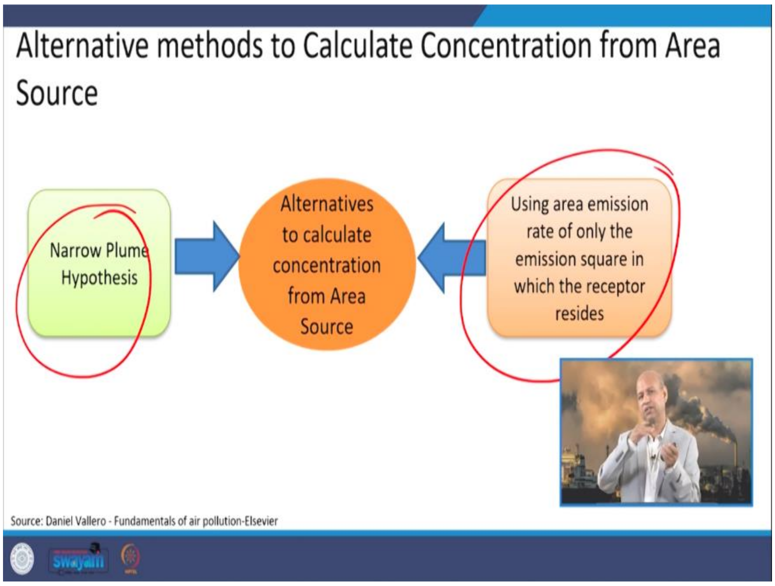

For example, we can go for Narrow plume Hypothesis, where we assume the line source modeling and we can use that. Or using area emission rate of only the emission square. And we assume that this is uniformly mixed there. And then how much will be this transported to the receptor and how much will be the concentration.

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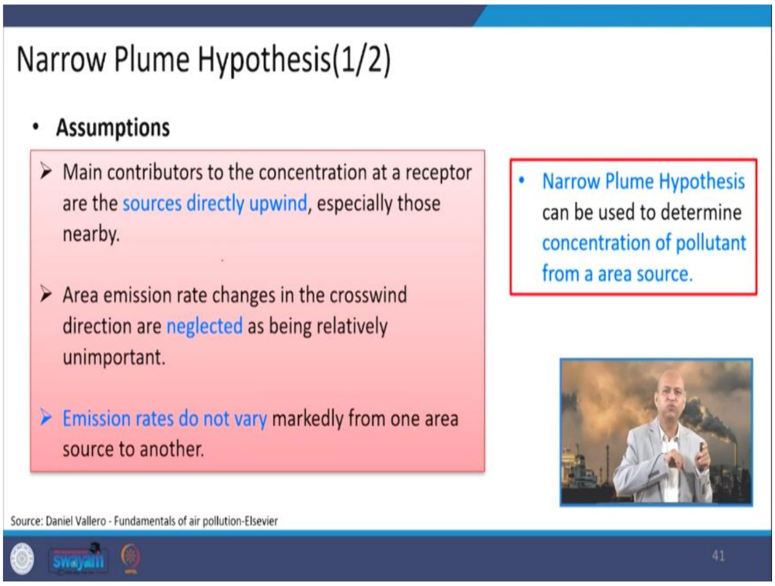

So, for Narrow plume hypothesis, basically, you have to see that receptor in the downwind direction and upwind, the source is not the point but the line source. So, for line source you know, you can calculate and then you can integrate.

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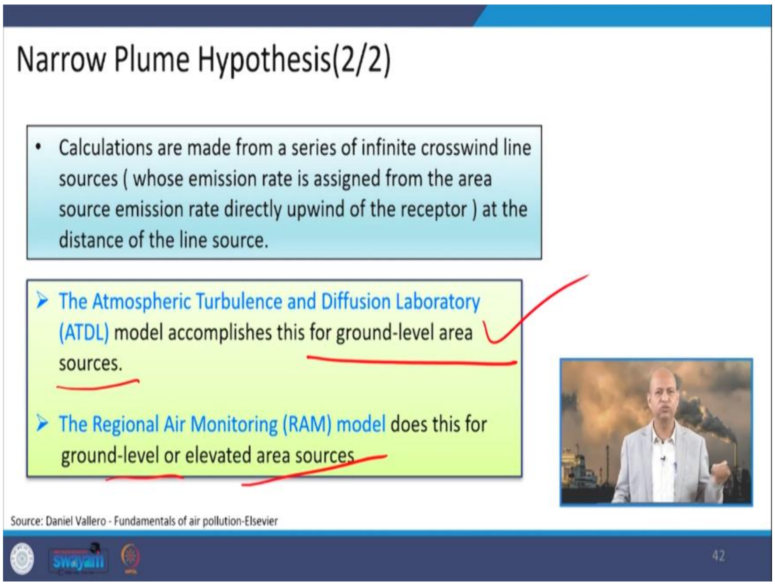

So, that is very simple thing, which we have already seen. And you know there are certain models like this Atmospheric Turbulence and Division Laboratory, ATDL model, which acts in accomplishes, ground level area sources, ground level area sources kind of computation is there. Regional Air Monitoring, RAM model, which calculate for ground level as well as elevated area source. So, those both is possible H is also possible, as we have done in case of Taj Mahal just we saw.

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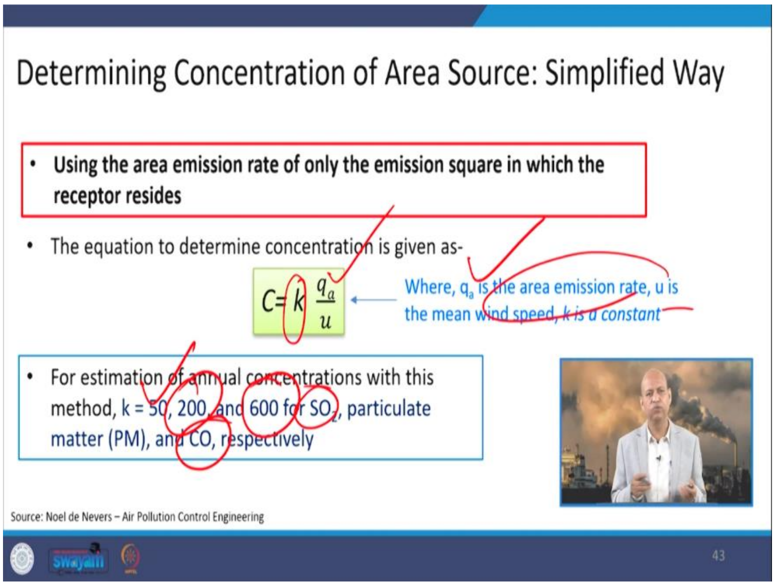

Then you know, there is another way simplified way, where as we have seen, we have just discussed that in area emission rate, q_a is the constant q_a and they u is the mean wind speed and the uniform concentration will be there and then this value constant is there for different pollutants like for a SO_2, it is 50, for particulate matter 200, for CO it is 600.

For other pollutants are also some empirical values have been derived from the experiments. So, that values you can use and you can quickly calculate the concentration. So, that is the concentration for that particular emission rate. But this is within the area source we assume. If you go for certain distance then again we have to go for either line source or virtual point source.

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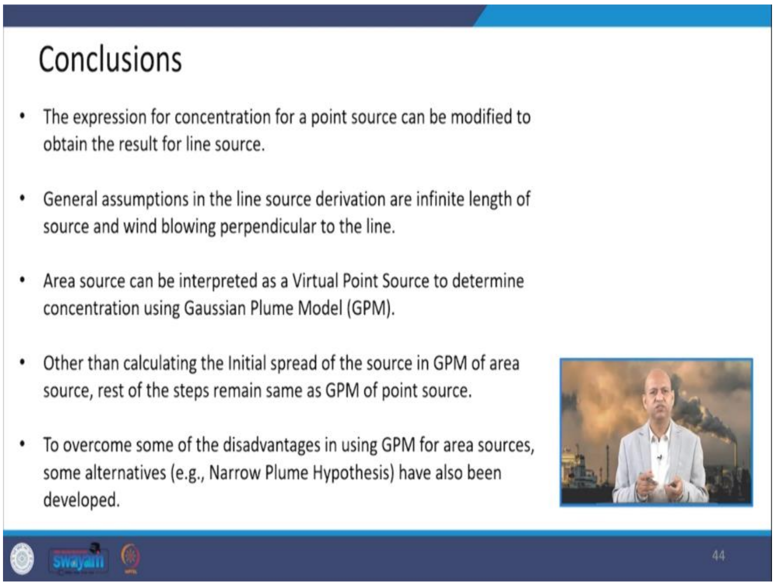

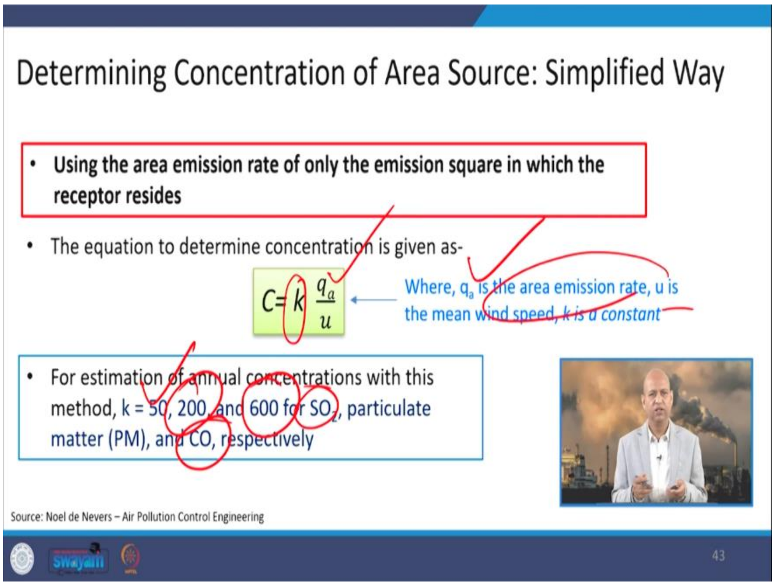

So, this is all for today as an introduction how do we extend the concept of Gaussian dispersion model from point source to the line source, from line source to the area source or area source again to the virtual point source. So, those kinds of things we have discussed. The expression of concentration for a point source can be modified in different ways and the results can be obtained accordingly.

General assumptions are there in the line source derivation like infinite length of the source, wind blowing in the perpendicular direction, those things are there, which we should keep in mind. Because, when line source is there, whole emission maximum, emission concentration at the line source and the concentration will decrease as we go away in the 90° are perpendicular to the line source, which is in finite according to our assumption.

In area source we can interpret as a virtual point in upwind direction. This is the area source we go in upwind direction at certain distance, which is, how do we calculate, we assume this area sources the square. And S / 4, 4.2, something, we get σ_{y0}, from σ_{y0} we can calculate the X_0, which is the distance from the area source to the virtual point.

And then we add into this the downwind distance of the receptor, where we want to calculate and simple points or dispersion modeling we apply. Other than calculating the initial spread of the source in this Gaussian plume model of the area source, rest of the steps as I said, it is only for point source dispersion modeling, which we have already discussed.

Now, to overcome certain limitations of this you know, this particular methodology of Gaussian Plume model, we go for some empirical relations like narrow plume hypotheses or just we have seen this particular way of calculating at the area source, but they also have certain limitations. So, this is you know, introduction for you for the line source modeling using this Gaussian dispersion plume and for area source.

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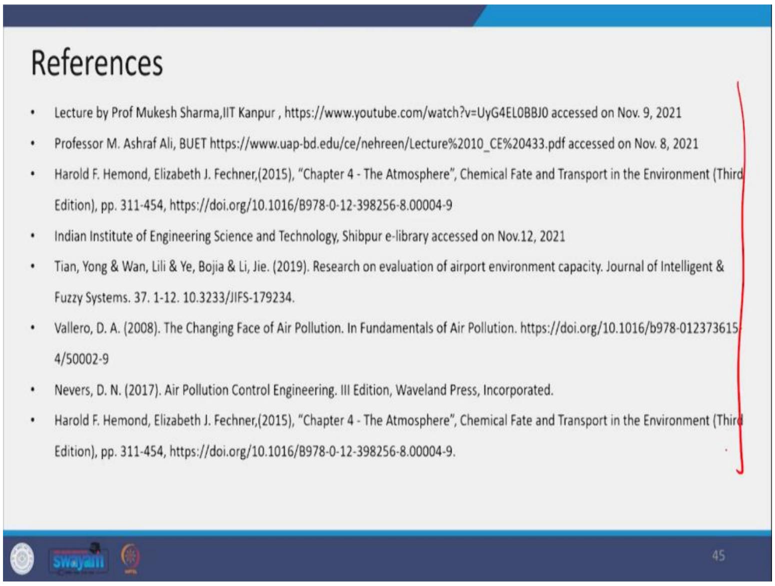

So, this is all for today for you, you can have additional information from these resources.. So, see you again in the next lecture, thank you for your kind attention. Thanks a lot.