Underground Space Technology
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Module No # 04
Lecture No # 18
Elastic Analysis of Circular Tunnels-Displacements

Hello everyone, in the last class, we saw that how we can determine the stress distribution using the elastic analysis of the circular tunnels. Today we will learn about deriving the expression for displacements, radial displacement and the displacement perpendicular to the direction of this radial displacement. So let us start the derivation again. We start with the theory of elasticity using plane stress as well as the plane strain condition both, we will take up.

And then, we will try to compare that in which situation the displacements are more or less, or they are equal.

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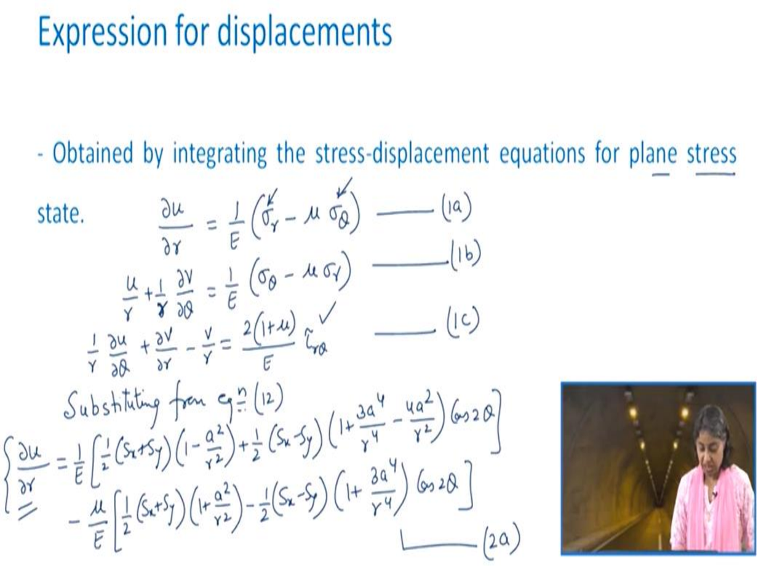

So here we obtain the displacement from the stress displacement equation for plane stress state and integrating these. So, what are these stress displacement equation for plane stress situation. So first, we will take a plane stress situation and then we will also discuss about the plane strain state. So here, we have these equations in the polar coordinate form in this manner that is:

$$\frac{\partial u}{\partial r} = \frac{1}{E}\left(\sigma_r - \mu\sigma_\theta\right) \quad (1a)$$

This is equation to 1a. Then another one is:

$$\frac{u}{r} + \frac{1}{r}\frac{\partial v}{\partial \theta} = \frac{1}{E}(\sigma_\theta - \mu\sigma_r) \qquad (1b)$$

Then the third equation is:

$$\frac{1}{r}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial r} - \frac{v}{r} = \frac{2(1+\mu)}{E}\tau_{r\theta} \qquad (1c)$$

Now, we already have this equation number 12 for the state of stresses that we discussed in the previous class. So, we connect these 2 equations like we have the expression for σ_r, σ_θ and $\tau_{r\theta}$.

So, we substitute the expression for these in these equations from equation number 12, so I am connecting it with the previous lecture. So here, it is substituting from equation number 12, so what we get here? Just substitute expression for σ_r as well as expression for σ_θ, so that is going to be bit long an expression. But then still let us write it:

$$\frac{\partial u}{\partial r} = \frac{1}{E}\left[\frac{1}{2}(s_x + s_y)\left(1 - \frac{a^2}{r^2}\right) + \frac{1}{2}(s_x - s_y)\left(1 + \frac{3a^4}{r^4} - \frac{4a^2}{r^2}\right)cos2\theta\right]$$
$$- \frac{\mu}{E}\left[\frac{1}{2}(s_x + s_y)\left(1 + \frac{a^2}{r^2}\right) - \frac{1}{2}(s_x - s_y)\left(1 + \frac{3a^4}{r^4}\right)cos2\theta\right] \qquad (2a)$$

Make this equation as equation number 2a. Then what we can do? This is the differential operator. If I integrate this equation, we will be able to get the expression for u.

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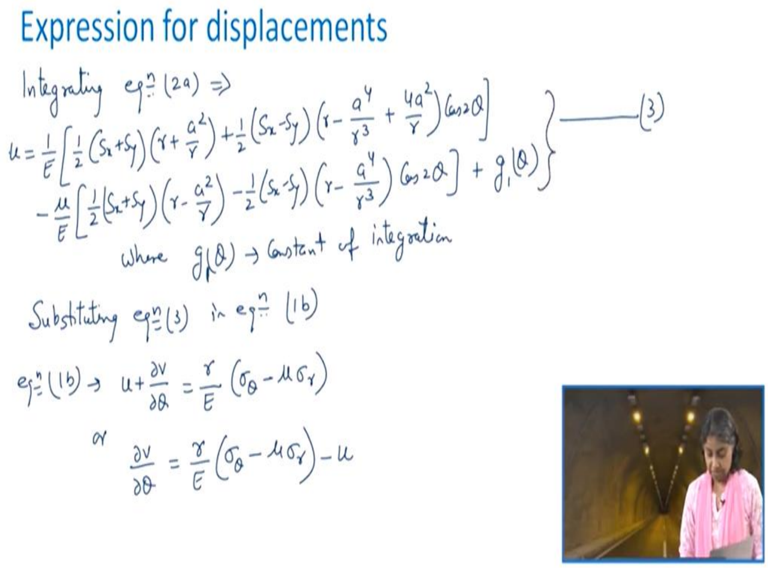

So let us try to do that so if I integrate it, what is that we are going to get is?

$$u = \frac{1}{E}\left[\frac{1}{2}(s_x + s_y)\left(r + \frac{a^2}{r}\right) + \frac{1}{2}(s_x - s_y)\left(r - \frac{a^4}{r^3} + \frac{4a^2}{r}\right)cos2\theta\right]$$
$$- \frac{\mu}{E}\left[\frac{1}{2}(s_x + s_y)\left(r - \frac{a^2}{r}\right) - \frac{1}{2}(s_x - s_y)\left(r - \frac{a^4}{r^3}\right)cos2\theta\right] + g_1(\theta) \qquad (3)$$

We will have a constant of integration which will be function of theta only because here, the integration was done with reference to r. So here, this $g_1(\theta)$ I am calling as constant of integration. Now, I make this equation as equation number 3. So, what I do is? I substitute this equation 3 to this equation 1b and see what we get. So, substituting this equation number 3 in equation 1b that we had in the previous slide.

So, your equation 1b, I can write as:

$$u + \frac{\partial v}{\partial \theta} = \frac{r}{E}(\sigma_\theta - \mu\sigma_r)$$

Or I can write it as:

$$\frac{\partial v}{\partial \theta} = \frac{r}{E}(\sigma_\theta - \mu\sigma_r) - u$$

Now, I just substitute this expression u in this; let us see what we get.

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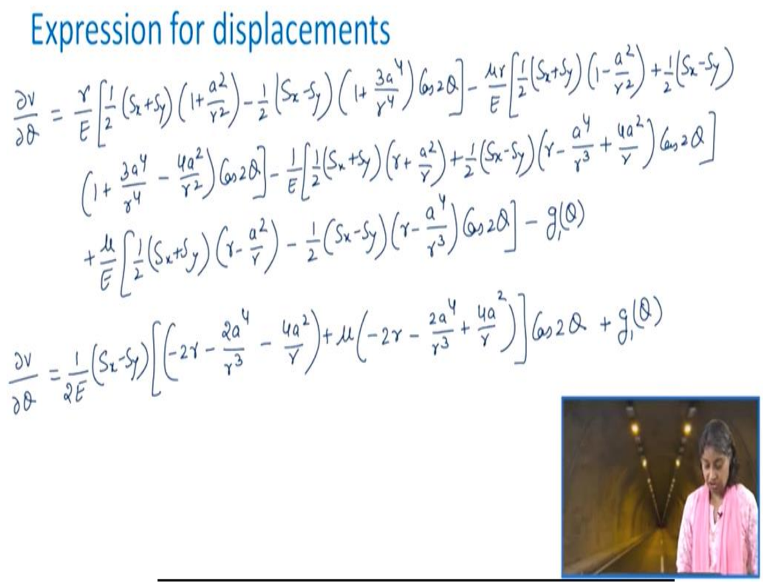

$$\frac{\partial v}{\partial \theta} = \frac{r}{E}\left[\frac{1}{2}(s_x + s_y)\left(1 + \frac{a^2}{r^2}\right) - \frac{1}{2}(s_x - s_y)\left(1 + \frac{3a^4}{r^4}\right)cos2\theta\right]$$
$$- \frac{\mu r}{E}\left[\frac{1}{2}(s_x + s_y)\left(1 - \frac{a^2}{r^2}\right) - \frac{1}{2}(s_x - s_y)\left(1 + \frac{3a^4}{r^4} - \frac{4a^2}{r^2}\right)cos2\theta\right]$$
$$- \frac{1}{E}\left[\frac{1}{2}(s_x + s_y)\left(r + \frac{a^2}{r}\right) + \frac{1}{2}(s_x - s_y)\left(r - \frac{a^4}{r^3} + \frac{4a^2}{r}\right)cos2\theta\right]$$
$$+ \frac{\mu}{E}\left[\frac{1}{2}(s_x + s_y)\left(r - \frac{a^2}{r}\right) - \frac{1}{2}(s_x - s_y)\left(r - \frac{a^4}{r^3}\right)cos2\theta\right] - g_1(\theta)$$

And then I have constant of integration as well, so I will write it like this so just further simplify this many terms will get cancel out and ultimately, what you will get is:

$$\frac{\partial v}{\partial \theta} = \frac{1}{2E}\left(s_x - s_y\right)\left[\left(-2r - \frac{2a^4}{r^3} - \frac{4a^2}{r}\right) + \mu\left(-2r - \frac{2a^4}{r^3} + \frac{4a^2}{r}\right)\right] cos2\theta + g_1(\theta)$$

It is the expression that you are going to get.

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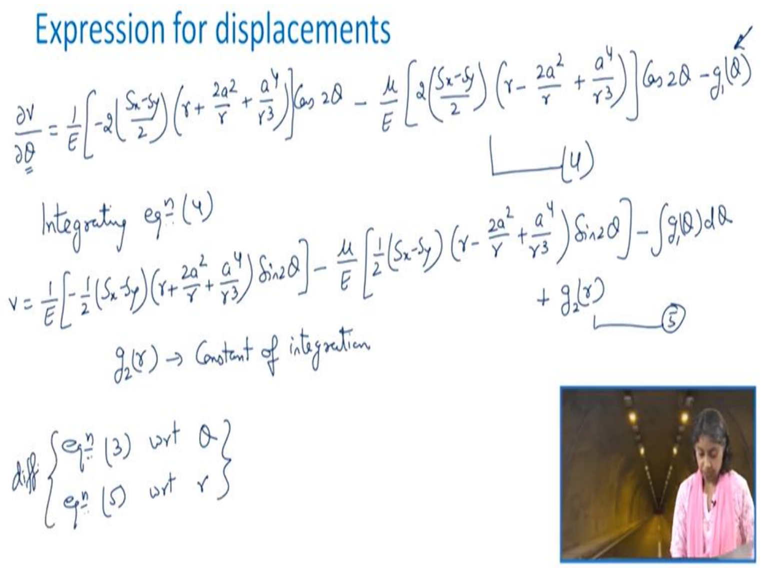

Now just, we will rearrange these terms and see what we get as:

$$\frac{\partial v}{\partial \theta} = \frac{1}{E}\left[-2\left(\frac{s_x - s_y}{2}\right)\left(r + \frac{2a^2}{r} + \frac{a^4}{r^3}\right)\right] cos2\theta - \frac{\mu}{E}\left[2\left(\frac{s_x - s_y}{2}\right)\left(r - \frac{2a^2}{r} + \frac{a^4}{r^3}\right)\right] cos2\theta - g_1(\theta) \quad (4)$$

Please do not make mistake. I understand this is a long-expression but then ultimately, what we need to get is the distribution of the displacement all along the periphery.

So do not make mistake and do not miss any of these steps, so mark this equation as equation number 4. Now, if you just integrate this equation, what we will get is? We integrating this equation number 4, so what we get here as v now here we have to integrate with reference to theta. Because then only you will get the expression for v, so this is going to be:

$$v = \frac{1}{E}\left[-\frac{1}{2}\left(s_x - s_y\right)\left(r + \frac{2a^2}{r} + \frac{a^4}{r^3}\right)\right] sin2\theta - \frac{\mu}{E}\left[\frac{1}{2}\left(s_x - s_y\right)\left(r - \frac{2a^2}{r} + \frac{a^4}{r^3}\right)\right] sin2\theta - \int g_1(\theta) d\theta + g_2(r) \quad (5)$$

Make this equation as equation number 5. In this case, here we have your $g_2(r)$ as the again the constant of integration.

Now what we do is we differentiate this equation number 3 with respect to θ. So, equation number 3 with respect to θ and equation number 5 with respect to r differentiation.

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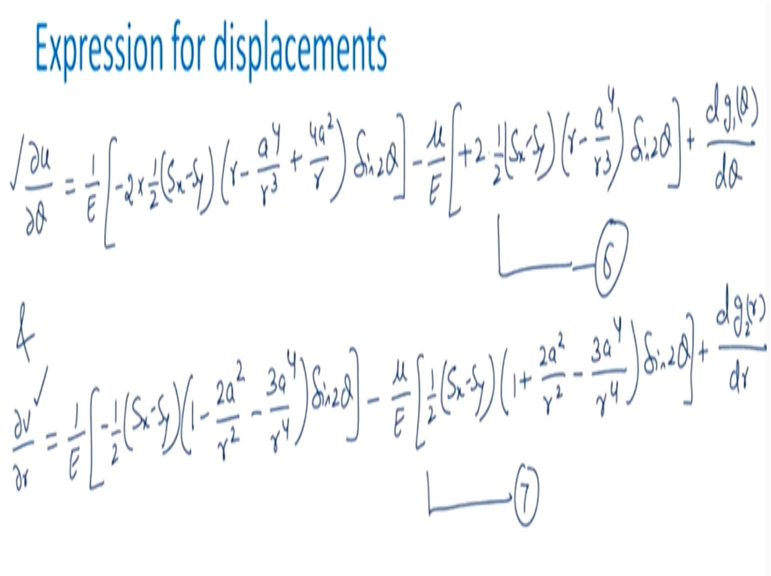

So, what are we going to get? So here, we have:

$$\frac{\partial u}{\partial \theta} = \frac{1}{E}\left[-2\times\frac{1}{2}(s_x - s_y)\left(r - \frac{a^4}{r^3} + \frac{4a^2}{r}\right)sin2\theta\right] - \frac{\mu}{E}\left[2\times\frac{1}{2}(s_x - s_y)\left(r - \frac{a^4}{r^3}\right)sin2\theta\right] + \frac{dg_1(\theta)}{d\theta} \tag{6}$$

This is going to be my equation number 6. And the next one we will have as:

$$\frac{\partial v}{\partial r} = \frac{1}{E}\left[-\frac{1}{2}(s_x - s_y)\left(1 - \frac{2a^2}{r^2} - \frac{3a^4}{r^4}\right)sin2\theta\right] - \frac{\mu}{E}\left[\frac{1}{2}(s_x - s_y)\left(1 + \frac{2a^2}{r^2} - \frac{3a^4}{r^4}\right)sin2\theta\right] + \frac{dg_2(r)}{dr} \tag{7}$$

I will have this equation as equation number 7. So basically, what I need to is now that I will substitute this ∂u/∂θ and ∂v/∂r in equation number 1 see that we obtained from the theory of elasticity.

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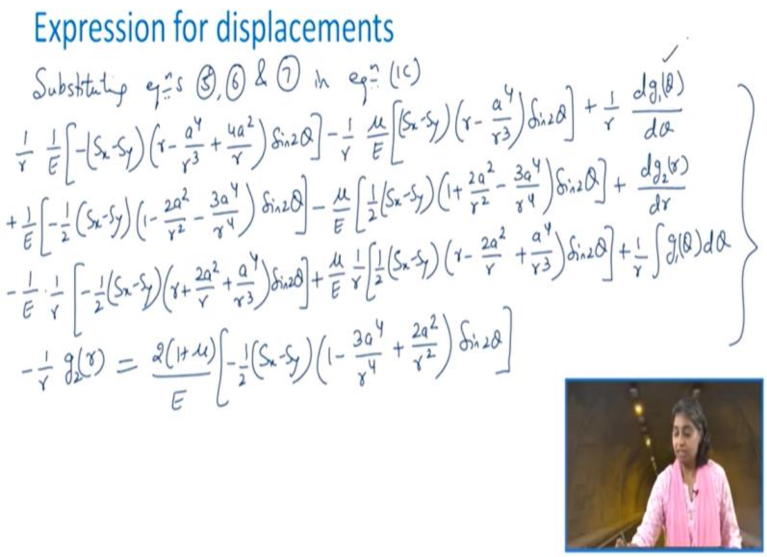

So, what we do is that we substitute equations 5, 6 and 7 in equation 1c. So ultimately, again, we are going to get very long-expression and also, we will substitute the expression for $\tau_{r\theta}$ from equation number 12c. That we obtain in the previous class the expression for the stresses, so what we are going to get is:

$$\frac{1}{r}\frac{1}{E}\left[-(s_x-s_y)\left(r-\frac{a^4}{r^3}+\frac{4a^2}{r}\right)sin2\theta\right]-\frac{1}{r}\frac{\mu}{E}\left[(s_x-s_y)\left(r-\frac{a^4}{r^3}\right)sin2\theta\right]+\frac{1}{r}\frac{dg_1(\theta)}{d\theta}$$
$$+\frac{1}{E}\left[-\frac{1}{2}(s_x-s_y)\left(1-\frac{2a^2}{r^2}-\frac{3a^4}{r^4}\right)sin2\theta\right]$$
$$-\frac{\mu}{E}\left[\frac{1}{2}(s_x-s_y)\left(1+\frac{2a^2}{r^2}-\frac{3a^4}{r^4}\right)sin2\theta\right]+\frac{dg_2(r)}{dr}$$
$$-\frac{1}{r}\frac{1}{E}\left[-\frac{1}{2}(s_x-s_y)\left(r+\frac{2a^2}{r}+\frac{a^4}{r^3}\right)sin2\theta\right]$$
$$+\frac{1}{r}\frac{\mu}{E}\left[\frac{1}{2}(s_x-s_y)\left(r-\frac{2a^2}{r}+\frac{a^4}{r^3}\right)sin2\theta\right]+\frac{1}{r}\int g_1(\theta)d\theta-\frac{1}{r}g_2(r)$$
$$=\frac{2(1+\mu)}{E}\left[-\frac{1}{2}(s_x-s_y)\left(1-\frac{3a^4}{r^4}+\frac{2a^2}{r^2}\right)sin2\theta\right]$$

So, although it is a very long-expression but no need to worry. Here, what we are going to do is. We are going to compare the terms on the both sides of equation because we need to find out constants of integration $g_1(\theta)$ and $g_2(r)$.

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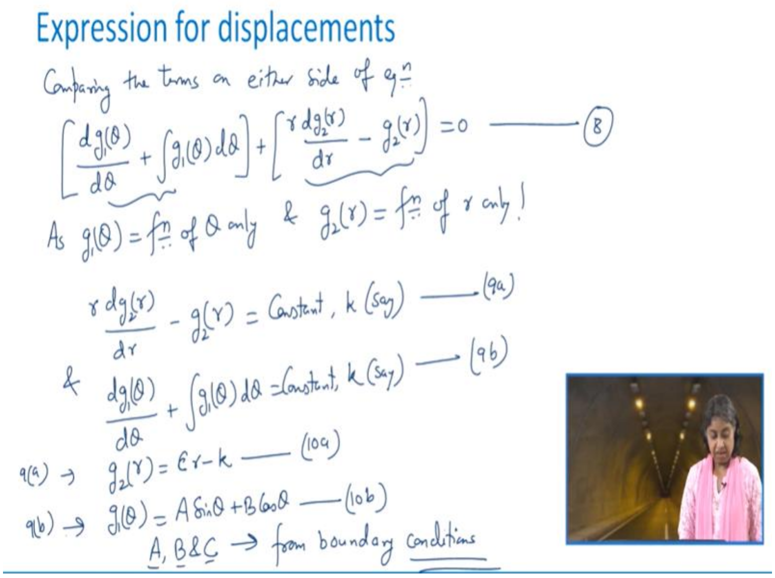

So, what we do is; we comparing the terms on either side of the equation, so you will see what we get is:

$$\left[\frac{dg_1(\theta)}{d\theta} + \int g_1(\theta)d\theta\right] + \left[r\frac{dg_2(r)}{dr} - g_2(r)\right] = 0 \qquad (8)$$

Make this equation as equation number 8. Now, we know that this $g_1(\theta)$ is only the function of θ and not the function of r and $g_2(r)$ is the function of r only here.

So basically, if this equation is to have some meaning, both of these terms, that is this term and this term individually. They should be as:

$$r\frac{dg_2(r)}{dr} - g_2(r) = constant\ , k\ (say) \qquad (9a)$$

$$\frac{dg_1(\theta)}{d\theta} + \int g_1(\theta)d\theta = constant, k\ (say) \qquad (9b)$$

Say I make that constant as k, and of course, the other term also should be a constant. Now, the solutions of these differential equations so mark it as 9a, and this as 9b. So, the solutions is going to be for the 9a it is going to be say:

$$g_2(r) = Cr - k \qquad (10a)$$

$$g_1(\theta) = Asin\theta + Bcos\theta \qquad (10b)$$

Now, these constants A, B and C, these are to be determined from the boundary conditions, as I already mentioned to you. So, we will apply these, and we will try to get these constants A, B and C.

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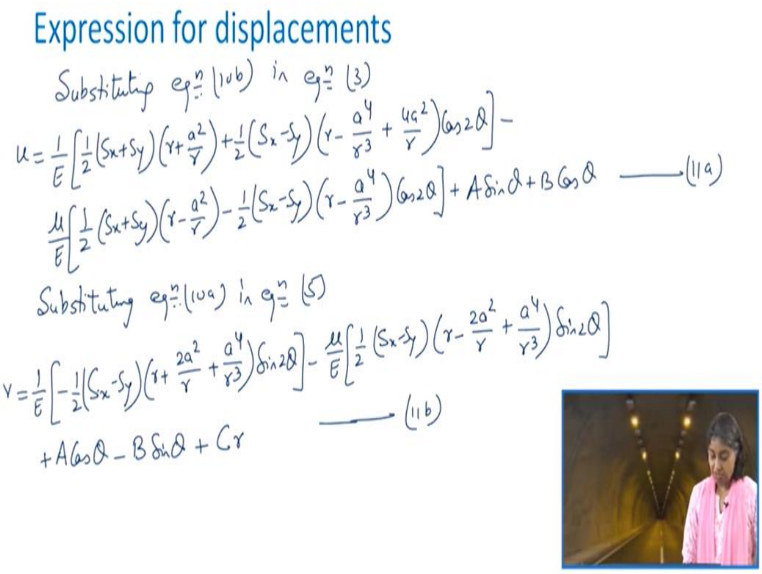

But before that, we substitute equation 10b in our equation number 3 so, we will get the expression for u now. So that is going to be:

$$u = \frac{1}{E}\left[\frac{1}{2}(s_x + s_y)\left(r + \frac{a^2}{r}\right) + \frac{1}{2}(s_x - s_y)\left(r - \frac{a^4}{r^3} + \frac{4a^2}{r}\right)cos2\theta\right] - \frac{\mu}{E}\left[\frac{1}{2}(s_x + s_y)\left(r - \frac{a^2}{r}\right) - \frac{1}{2}(s_x - s_y)\left(r - \frac{a^4}{r^3}\right)cos2\theta\right] + Asin\theta + Bcos\theta \quad (11a)$$

So, I mark this as 11a, and if I just substitute equation in 10a in equation number 5. So what we get as the expression as v as:

$$v = \frac{1}{E}\left[-\frac{1}{2}(s_x - s_y)\left(r + \frac{2a^2}{r} + \frac{a^4}{r^3}\right)sin2\theta\right] - \frac{\mu}{E}\left[\frac{1}{2}(s_x - s_y)\left(r - \frac{2a^2}{r} + \frac{a^4}{r^3}\right)sin2\theta\right] + Acos\theta - Bsin\theta + Cr \quad (11b)$$

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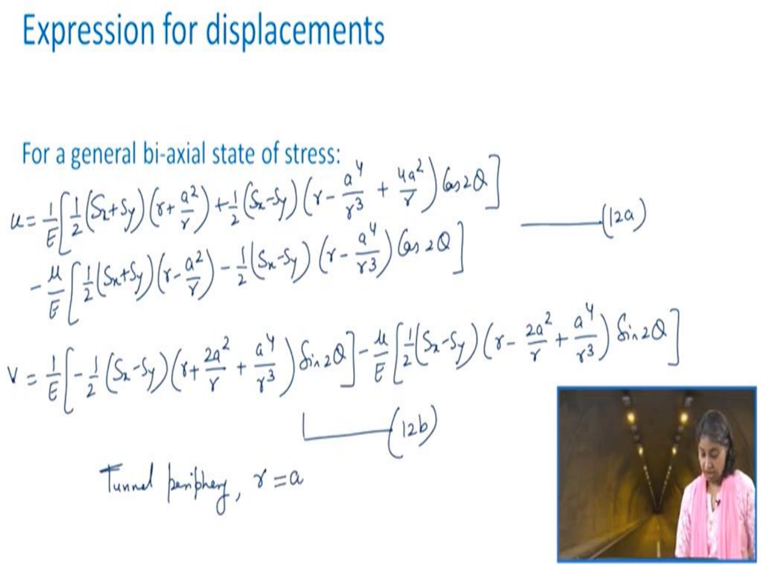

So, this is what is going to be the equation for v, makes it as 11b. Now, let us try to apply the boundary condition to obtain these constants. So, the first boundary condition here is going to be that displacement v will be equal to 0, when θ = 0 or π by 2 for all values of r. This is due to symmetry so let us substitute C to be equal to 0 at θ = 0. What we get is?

$$v = 0 \; at \; \theta = 0 \quad \Rightarrow A + Cr = 0$$

$$v = 0 \; at \; \theta = \frac{\pi}{2} \quad \Rightarrow -B + Cr = 0$$

And the solutions for these 2 will be only that is A = B = C will be equal 0.

So, for the general biaxial state of stress, how can we write the expression for u and v. Let us see, u will equal to:

$$u = \frac{1}{E}\left[\frac{1}{2}(s_x + s_y)\left(r + \frac{a^2}{r}\right) + \frac{1}{2}(s_x - s_y)\left(r - \frac{a^4}{r^3} + \frac{4a^2}{r}\right)cos2\theta\right] - \frac{\mu}{E}\left[\frac{1}{2}(s_x + s_y)\left(r - \frac{a^2}{r}\right) - \frac{1}{2}(s_x - s_y)\left(r - \frac{a^4}{r^3}\right)cos2\theta\right] \qquad (12a)$$

That is going to be say 12a and similarly we can write this expression for v as:

$$v = \frac{1}{E}\left[-\frac{1}{2}(s_x - s_y)\left(r + \frac{2a^2}{r} + \frac{a^4}{r^3}\right)sin2\theta\right] - \frac{\mu}{E}\left[\frac{1}{2}(s_x - s_y)\left(r - \frac{2a^2}{r} + \frac{a^4}{r^3}\right)sin2\theta\right] \qquad (12b)$$

This as 12b. Now, what happens at tunnel periphery? What is the value of r, which is equal to a.

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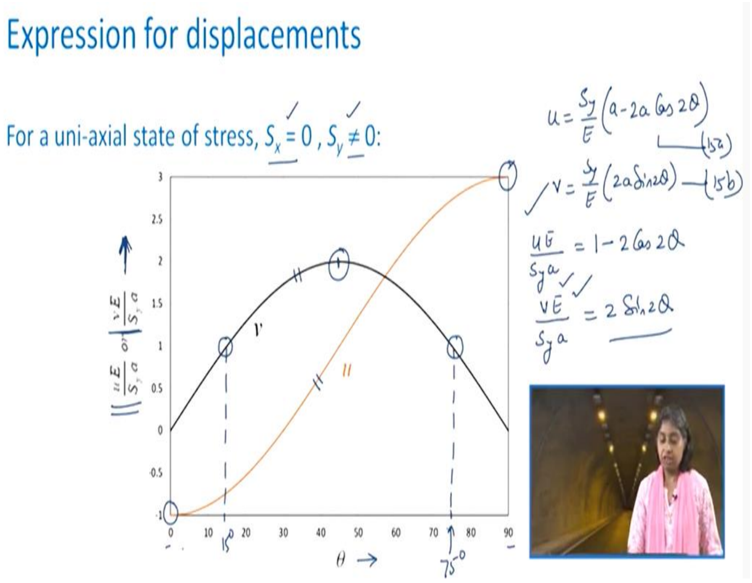

So, let us try to take a look that what happens at the tunnel periphery at r = a to this displacement. So, we get u as:

$$u = \frac{1}{E}\left[(s_x + s_y)a + 2(s_x - s_y)acos2\theta\right] \quad (13a)$$

And expression for v will be:

$$v = -\frac{1}{E}\left[2(s_x - s_y)asin2\theta\right] \quad (13b)$$

So, you see how from such long expressions and at the tunnel periphery, we get much more simplified expressions for this displacement.

So, any point on this periphery will be defined by the θ. Now, we take typically the uniaxial state of stress and hydrostatic state of stress. And then try to see that what are this displacement and how we can draw their variation. So, for a typical uniaxial state of stress, that is when you have S_x to be equal to 0 and S_y is not equal to 0. We have the variation as has been shown in this particular figure.

But before I discuss this figure, let us go back and first try to see that what happens to their expression first. So, typical hydrostatic state of stress what we have is $S_x – S_y$ = -p, which is compressive in nature. So, what we have here as u equal to so just substitute, so this term will become equal to 0. So, all you will have is:

$$u = -\frac{2pa}{E} \quad \& \quad v = 0 \quad (14)$$

So, this is what is going to be for the hydrostatic of stress and for the uniaxial state of stress where you the stress only in the y-direction or the vertical direction.

And no stress in the x-direction. So, this is going to be the variation but then if we just substitute the value of S_x and S_y in those 2 expressions which were given by equation number 13. Let us see that what all are the equations that we get. So, expression that we will get here will be:

$$u = \frac{S_y}{E}(a - 2acos2\theta) \quad (15a)$$

$$v = \frac{S_y}{E}(2asin2\theta) \quad (15b)$$

So, we will represent it in the non-dimensional forms displacement so that can be written as:

$$\frac{uE}{S_y a} = 1 - 2cos2\theta$$

$$\frac{vE}{S_y a} = 2sin2\theta$$

Now, you see the variation we have plotted here. That is on x-axis you have various values of theta varying from 0 to 90 degree. And on y-axis you have this non-dimensional displacement in terms of uE upon S_y into a or vE upon S_y into a.

So, this orange colour curve is showing you the variation of u and the black colour one is giving you the expression for v. So, you take a typical value may be let us say you take $\theta = 45$ degree and just substitute it here in this expression, see you will get vE upon S_y into a as 2. So, this is what is the one that you are getting from on this figure. As well similarly for, let us say 75 degree of θ say here, this is 75 degree.

Or you can take here as, I mean $\theta = 15$ degree so just substitute the value and see what you will get is this vE upon S_y into a as 1, these two values. For the 90 degree, you have seen that the displacements how it is going to be here it is u is going to be non-dimensional one is going to be 3, and here it is going to be -1 at $\theta = 0$. So, this is how we can find out the variation of the displacement for the uniaxial state of stress.

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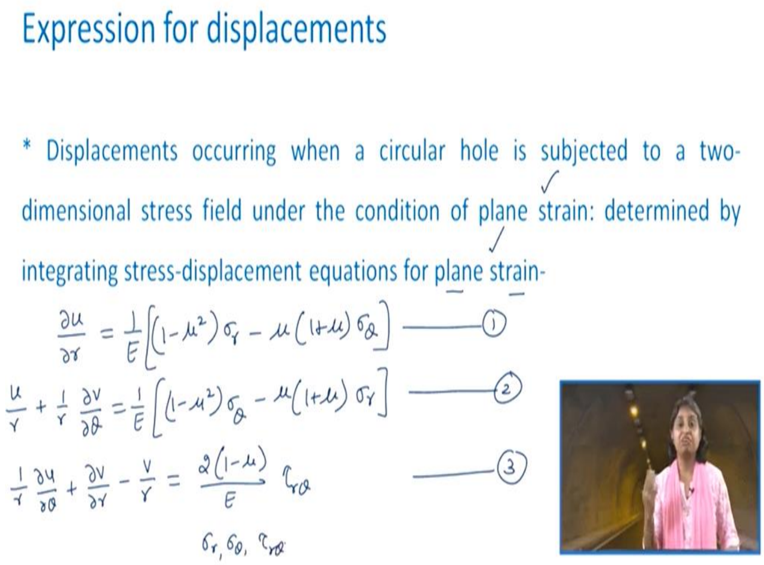

Now the displacements which occur when the circular hole is subjected dimensional stress field under the condition of the plane strain. As of now, we were discussing about the plane stress condition. In case, if we consider it to be a plane strain condition. So then, in that case, we have to consider the stress displacement equation for plane strain condition and then carry out the integration.

So, let us take a look that what all are those equations for the plane strain situation, so these are given as,

$$\frac{\partial u}{\partial r} = \frac{1}{E}[(1-\mu^2)\sigma_r - \mu(1+\mu)\sigma_\theta] \tag{1}$$

$$\frac{u}{r} + \frac{1}{r}\frac{\partial v}{\partial \theta} = \frac{1}{E}[(1-\mu^2)\sigma_\theta - \mu(1+\mu)\sigma_r\ ] \tag{2}$$

$$\frac{1}{r}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial r} - \frac{v}{r} = \frac{2(1-\mu)}{E}\tau_{r\theta} \tag{3}$$

So, these are the 3 equations providing stress displacement relationship under plane strain condition. So, if we just substitute the expressions for σ_r, σ_θ, and $\tau_{r\theta}$ from the previous lecture that, we derived all these expressions.

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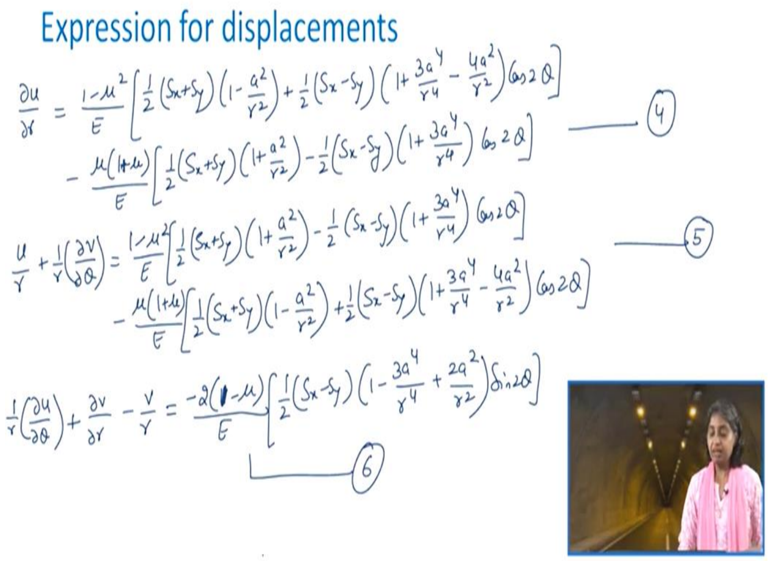

So, if we just substitute what we are going to get is:

$$\frac{\partial u}{\partial r} = \frac{1-\mu^2}{E}\left[\frac{1}{2}(s_x+s_y)\left(1-\frac{a^2}{r^2}\right)+\frac{1}{2}(s_x-s_y)\left(1+\frac{3a^4}{r^4}-\frac{4a^2}{r^2}\right)cos2\theta\right]$$
$$-\frac{\mu(1+\mu)}{E}\left[\frac{1}{2}(s_x+s_y)\left(1+\frac{a^2}{r^2}\right)-\frac{1}{2}(s_x-s_y)\left(1+\frac{3a^4}{r^4}\right)cos2\theta\right] \quad (4)$$

I make this as equation number 4, and I will have another equation that is:

$$\frac{u}{r}+\frac{1}{r}\frac{\partial v}{\partial \theta} = \frac{1-\mu^2}{E}\left[\frac{1}{2}(s_x+s_y)\left(1+\frac{a^2}{r^2}\right)-\frac{1}{2}(s_x-s_y)\left(1+\frac{3a^4}{r^4}\right)cos2\theta\right]$$
$$-\frac{\mu(1+\mu)}{E}\left[\frac{1}{2}(s_x+s_y)\left(1-\frac{a^2}{r^2}\right)+\frac{1}{2}(s_x-s_y)\left(1+\frac{3a^4}{r^4}-\frac{4a^2}{r^2}\right)cos2\theta\right]$$

(5)

This will be equation number 5 and then last equation that is:

$$\frac{1}{r}\frac{\partial u}{\partial \theta}+\frac{\partial v}{\partial r}-\frac{v}{r} = \frac{-2(1-\mu)}{E}\left[\frac{1}{2}(s_x-s_y)\left(1-\frac{3a^4}{r^4}+\frac{2a^2}{r^2}\right)sin2\theta\right] \quad (6)$$

Now, if we integrate these three equations: 4, 5 and 6. We will be able to get the expression for u and v as we have done in the previous case of the plane stress condition.

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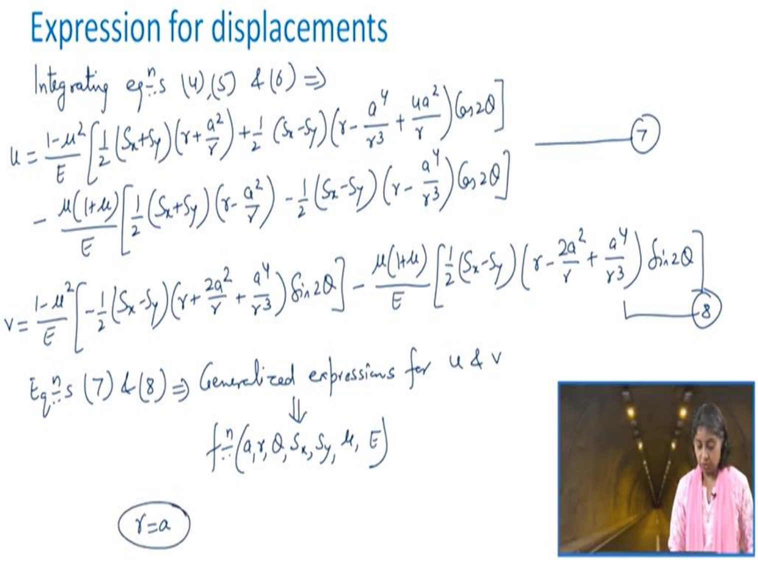


So, I (refer time: 45:39) integrate these equations 4, 5 and 6. So, what we get is:

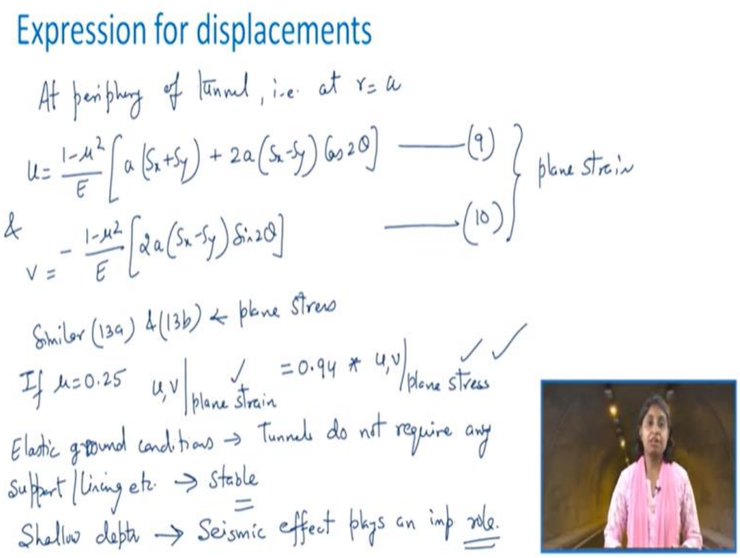
$$u = \frac{1-\mu^2}{E}\left[\frac{1}{2}(s_x+s_y)\left(r+\frac{a^2}{r}\right)+\frac{1}{2}(s_x-s_y)\left(r-\frac{a^4}{r^3}+\frac{4a^2}{r}\right)cos2\theta\right] - \frac{\mu(1+\mu)}{E}\left[\frac{1}{2}(s_x+s_y)\left(r-\frac{a^2}{r}\right)-\frac{1}{2}(s_x-s_y)\left(r-\frac{a^4}{r^3}\right)cos2\theta\right] \quad (7)$$

And we can also obtain the expression for v as:

$$v = \frac{1-\mu^2}{E}\left[-\frac{1}{2}(s_x-s_y)\left(r+\frac{2a^2}{r}+\frac{a^4}{r^3}\right)sin2\theta\right] - \frac{\mu(1+\mu)}{E}\left[\frac{1}{2}(s_x-s_y)\left(r-\frac{2a^2}{r}+\frac{a^4}{r^3}\right)sin2\theta\right] \quad (8)$$

This is equation number 8. So, you see here that these equations 7 and 8; these are the generalized expression for u and v. And these are the functions of a, r, θ, s_x, s_y, μ, and E. So, if we want to find out the displacement at the tunnel periphery, so what we do is. We need to substitute r to be equal to a.

(Refer Slide Time: 48:39)



So, let us see that what happens at the (refer time: 48:39) periphery of the tunnel that is at r = a, what we have u here as:

$$u = \frac{1-\mu^2}{E}\left[a\left(s_x + s_y\right) + 2a\left(s_x - s_y\right)cos2\theta\right] \tag{9}$$

And we have v as:

$$v = -\frac{1-\mu^2}{E}\left[2a\left(s_x - s_y\right)sin2\theta\right] \tag{10}$$

Now, if I compare the expression, and 9 and 10 which are for plane strain situation to the expressions which were similar for the plane strain situation, that is expression 13a and 13b. These were for the plane stress condition so, if we take μ to be equal to say 0.25.

Then, we will see that u or v for the plane strain situation will be equal to 0.94 times u, v of plane stress condition. So, since we are conducting the elastic analysis so, this is what is going to be the relationship between the displacements, if you follow the plane strain straight or if you follow plane stress state. So, in case of the elastic ground conditions, basically tunnels, they do not require much support or the lining.

So, most of the time, they are stable, and they stand on its own but then if the excavation is at the shallow depth, then what happens is? The seismic effect plays an important role, so one needs to be careful with reference to that. So, most of the time, in case, if it is elastic ground conditions, tunnels are more or less stable so this is how following the theory of elasticity, we can determine the expression for the displacement all around the circular tunnels.

So, this is what I wanted to discuss with you as far as the elastic analysis of circular tunnels is concerned. So, we learnt about the derivation for the expression of stress all the component of stresses. And both the components of the displacements now. Write now; we are not considering any lining or any support system for the tunnels. But in case if let us say you have a concrete lining, then how to carry out the analysis following the theory of elasticity yes, we have the close-form solution available for that and that we will learn in the next class. Thank you very much.