Underground Space Technology
Prof. Priti Maheshwari
Department of Civil Engineering
Indian Institute of Technology - Roorkee

Module No # 05
Lecture No # 21
Stress Distribution around Non- Circular Openings in Elastic Ground Conditions: General Guidelines for Design

Hello everyone, in the previous class we learnt about the green span method using which we can find out the stress distribution around non-circular openings in elastic ground conditions. We dealt with various non-circular shapes of the opening, such as elliptical, then rectangular, and ovaloidal. And with the help of a few figures, I showed you that, how the stress distribution around such openings in elastic ground conditions look like.

So, today we will discuss those figures in detail, and we will try to obtain some of the general guidelines for design of such non-circular openings.

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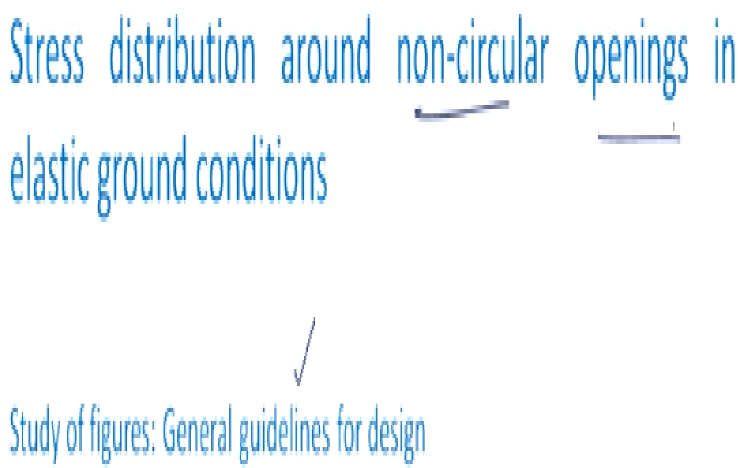

So, as I just now mentioned that, we will study those figures in detail and try to obtain the general guidelines for design of non-circular openings. And, you should remember that, as of now we are considering the ground condition to be elastic.

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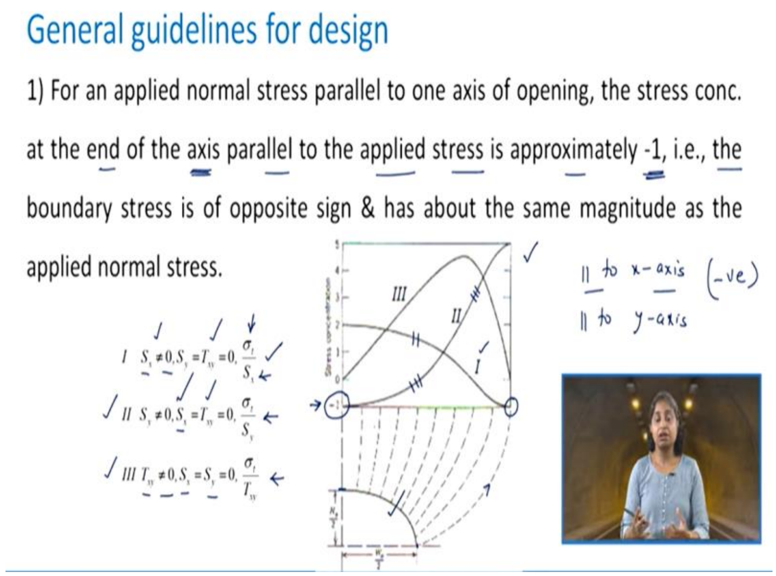

So, first of all, this was a picture that I showed you in the previous class, so, let us take a look that, what is that general guideline that one can obtain? So, let us say that you have the applied normal stress which is parallel to one axis of the opening. So, you had 3 stress conditions, first stress condition was defined by these equations or expressions. In that case, the load was applied only in x direction, that is $S_x \neq 0$, and S_y, and $T_{xy} = 0$.

And, hence the stress concentration factor = σ_t/S_x, Whereas the second case, we had the applied stress in the direction parallel to the y direction, that means, $S_y \neq 0$, and there was no stress in x-direction. That is $S_x = 0$ and $T_{xy} = 0$, and accordingly, the stress concentration factor = σ_t/S_y.

And the third case was when you have $T_{xy} \neq 0$, and $S_x = S_y = 0$, and the stress concentration factor= σ_t/T_{xy}. So, please remember that, for all the cases that we are going to discuss or that I already showed you in the previous class, these three conditions they are going to be same. That means, the case one will deal with when you have the applied stress in x-direction, only case 2 when the loading is in y direction, and in case 3 when applied stresses in x and y direction, they are equal to 0 but, $T_{xy} \neq 0$.

So, here I have picked typically one shape, but these general guidelines are true for all the shapes, that are why we are calling these as general guidelines to be adopted for design. So, in case if you have the normal stress parallel to one excess of the opening this means that either it can be case one, or it can be case 2, it cannot be case 3. Then in that case, the stress concentration at the end of the axis parallel to the applied stress is approximately -1.

This means, that the boundary stress is of opposite sign and has about equal or same magnitude as for the applied normal stress. Now, take a look if I consider the case 1, so, this is the curve for case one this is the stress distribution for case one, for the elliptical type of the opening. Now, here I have taken only one shape of the opening but then this guideline is true for all the shapes. Now what it says? If I take case 1 this means that loading is parallel to x-axis.

So, what is this guideline tells me is that, the stress concentration at the end of the axis parallel to the applied stress. So, the stress is applied parallel to x-axis, so at the end of the x-axis we have to look. So, you see here it is the end of the x-axis and, if I go follow this and take a look here this stress concentration factor here is -1. Now I take, case 2 where the loading is parallel to y axis. So, at the end of y-axis, let us take the look, so, this is the end of the y-axis, so at y-axis, see again here, if I follow this curve 2 here you see this is again it is -1.

So, this becomes one of the general guidelines that in case, if the applied normal stress is parallel to one axis of the opening, the stress concentration at the end of the axis parallel to the applied stress is approximately equal to -1. Why we are putting that negative sign because the magnitude of the boundary stress is of the opposite nature as compared to the applied normal stress.

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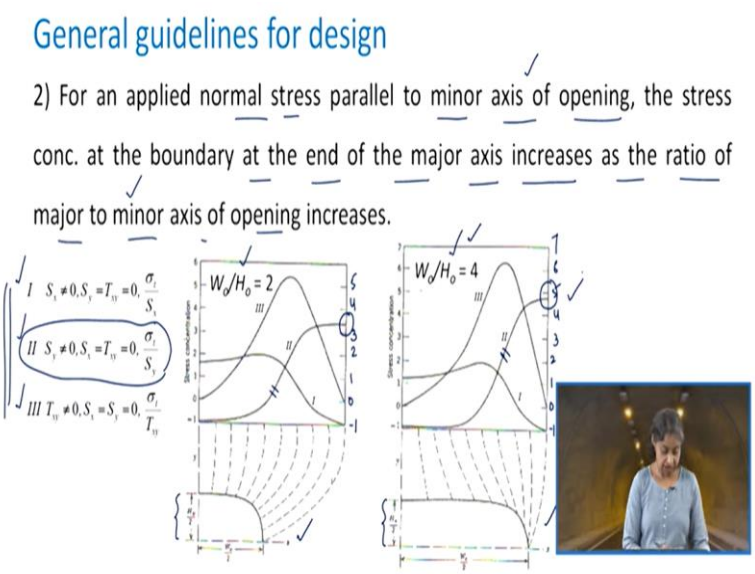

Now coming to the second guideline, what it says? That for an applied normal stress, which is parallel to minor axis of the opening the stress concentration at the boundary at the end of the major axis increases, as the ratio of major to minor axis of the opening increases. So, here I have taken the shape of an ovaloid, take a look here, in this case the major to minor axis ratio is 2, and in this case, it is 4. Again, I need not to explain you, the three cases of the loading case 1, case 2, and case 3.

And for your convenience, I have pasted these 3 cases on every slide, so that you are able to connect that, which case corresponds to which type of applied stress condition. Now, what is that here? That for an applied normal stress parallel to minor axis of the opening. So here if you take a look at these 2 figures which one is the minor axis? It is the y-axis which is the minor axis. This means which case where you have the normal stress parallel y-axis.

So, we have our case as case 2, so I will focus on case 2 curve in both the situation, which is this, which I have marked so the stress concentration at the boundary at the end of the major axis. Now, the applied stress is parallel to the minor axis but we have to look at the end of the major axis, this means the major axis is x-axis. So, I go here this is what is the position similarly here, this is what is the position?

Now, read what is its value. So, it is somewhere, see, this is, -1 0 1 2, this is 3, this is 4. So, this is somewhere in between 3 and 4, while when you have $W_o/H_o = 4$, that means, major to minor axis ratio of the opening is increasing. Then, in that case, what you have is see this is, -1, 0, 1, 2, 3, 4 and this is 5.

So, now this is somewhere in between 4 to 5 or you can say that it is close to 5. So, this means that when this ratio, W_o/H_o increases, now here you have to compare for the same shape. It is not that, one shape you take $W_o/H_o = 2$, and you take one shape, and for $W_o/H_o = 4$, you take another shape, no is the same shape but only the difference is the ratio of major to minor axis of the opening. So, as this W_o/H_o for a particular shape it increases, the boundary stress at the end of major axis also increases. So, please remember, this is what is the second general guideline for the design of non-circular opening in elastic ground condition.

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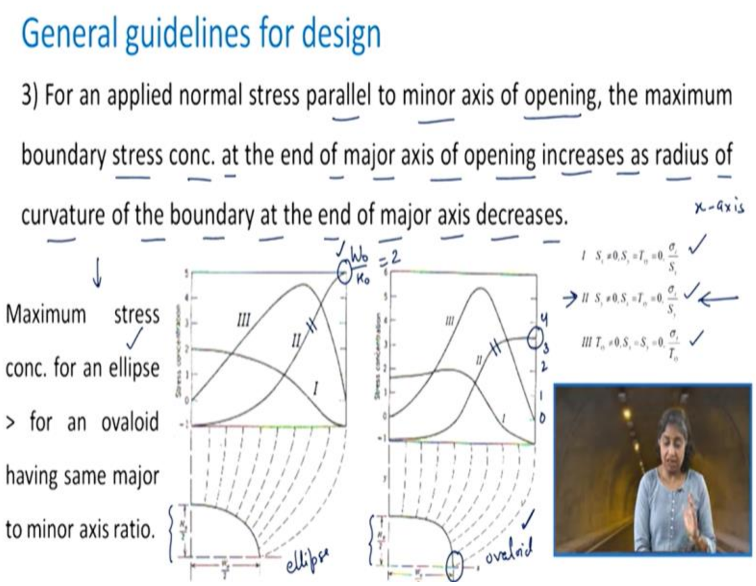

We come to the third case, so again here we have all the 3 loading conditions 1, 2, 3. Now, in this case, the applied normal stresses parallel to minor axis of the opening, so, here you see which one is the minor axis, here it is the y-axis, that is the minor axis, so which can condition, that I have to focus on it is the second one. The maximum boundary stress concentration at the end of the major axis of the opening.

This means, in our case it is going to be the x-axis, it increases as the radius of curvature of the boundary at the end of the major axis reduces. So, you see here, what we have taken is 2 different shapes having W_o/H_o as same. So, in this case, what we have taken here this is ovaloid, and this is ellipse, and this is ovaloidal shape. For both of these $W_o/H_o = 2$ here.

Now, in this case the loading is this second case, so, we focus on this, and what it says that the boundary stress concentration at the end of the major axis, so, I mark this point and I mark this point. Now, please tell me in which case it is going to be the more radius of curvature, that is going to be in this case because it is, more flat. So, you know that for a vertical plane, what is the radius of curvature, it is infinity, so, here the radius of curvature is more in this case, and therefore you see the stress concentration, it is somewhere between 3 and 4.

So, 0, 1, 2 here it is 3 and 4 but then here it is 5 and, therefore you can say that the maximum stress concentration at that location, where it is the end of the major axis for ellipse is greater than for an ovaloid having the same major to minor axis ratio. Now, if you take, let us say W_o/H_o

= 4, and again compare these 2, you are going to get the similar observation. So, that is how we frame the third general guideline for the design.

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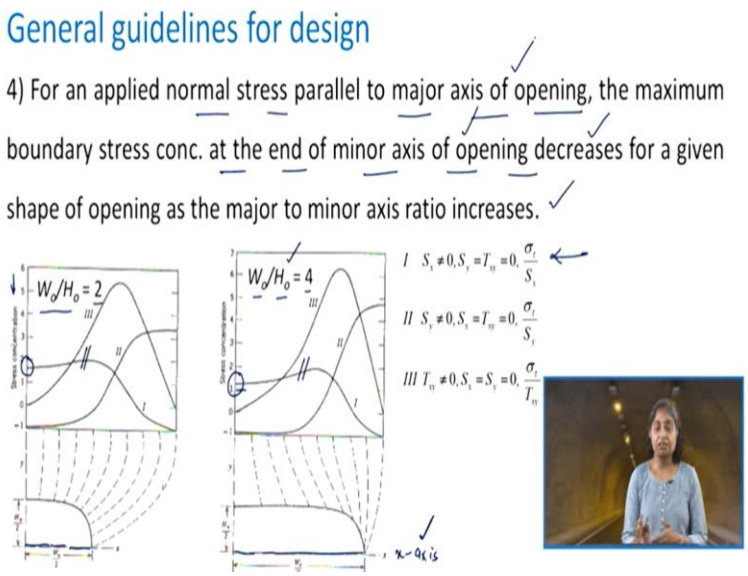

Now, coming to the fourth one in case, if you have the normal stress parallel to major axis of the opening. So, take a look on this figure, so, major axis is x-axis, which is this one in both the cases, and here we are considering the ovaloidal shape. So, what it says according to this loading condition, our case is going to be the first case, so, focus on this first one this one and this one. The maximum boundary stress concentration at the end of minor axis, of the opening at the end of minor axis, means at the end of the y axis.

So, here so, just mark this point and you mark this point. This boundary stress concentration, at the end of minor axis of opening, it reduces for a given shape of opening as the major to minor axis ratio increases. Now, take a look, when you had W_o/H_o = 2, this ratio is somewhere in between 1 and 2, or maybe you can call it as, 1.5 or 1.6, by looking at this figure. But then, when this increases to 4, that are W_o/H_o becomes equal to 4, in that case you see, it is still between 1 and 2, but take a look it is close to 1.

So, when W_o/H_o ratio increases, this value of the stress concentration, at the end of the minor axis of opening it reduces. In case, if you have the applied normal stress, parallel to the major axis of the opening and similar observation, you will see for the other shape of the opening.

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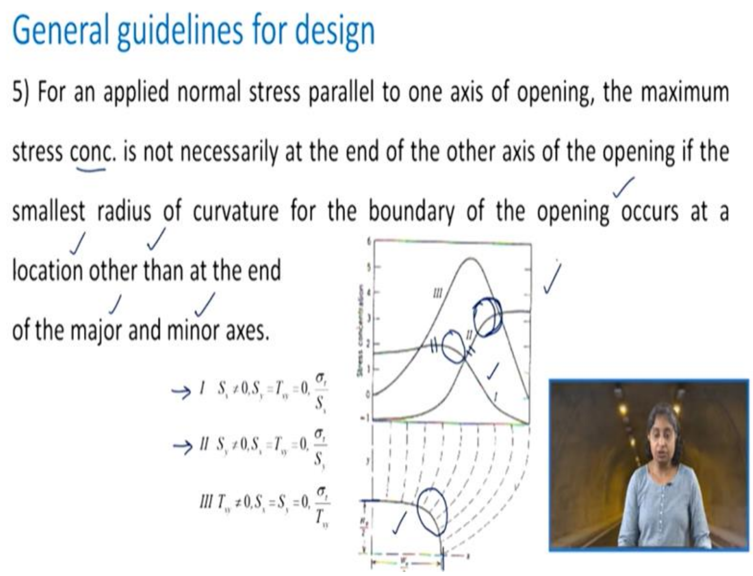

Now for an applied normal stress, parallel to one axis of the opening, so what all load conditions can be there? It can be either, load case 1 or load case 2, so, let me take this as case 2 and then load case 1. These 2 cases, both the loading conditions, the maximum stress concentration does not necessarily appear at the end of the other axis of the opening. If the smallest radius of curvature for the boundary of the opening occurs at a location, other than at the end of major to minor axis please understand.

Take a look at this figure, this is the location where the radius of curvature is less as compared to this location, or this location. So, you see, if you take a look either the first loading condition, or the second loading condition. See for the first loading condition, it is somewhere here that you are getting the maximum stress concentration, and for the second one, it is somewhere here you are getting the kind of maximum stress concentration value.

So, you see that these are not essentially occurring at the end of either of the axis, but these occur wherever you have the smallest radius of curvature, for the boundary of the opening. So, this is what is very important and you should keep this in mind.

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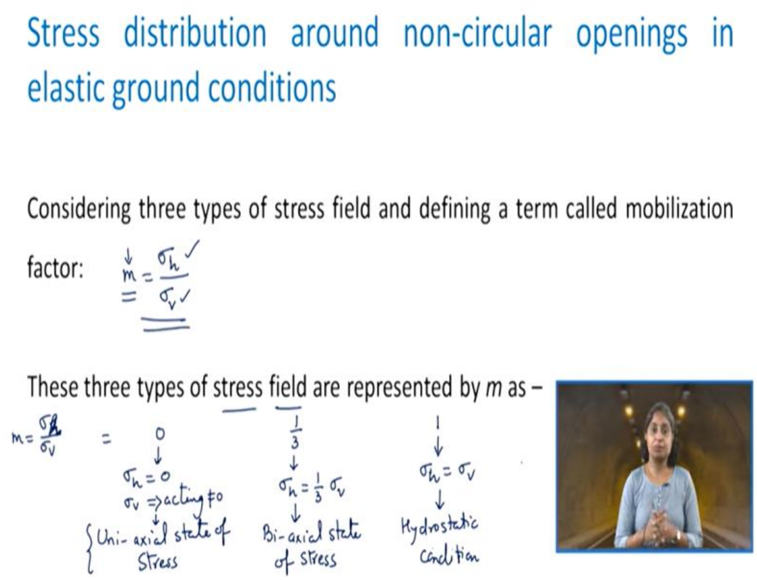

Now coming to the various conditions, so, here, what you have seen was the general state of stress and the 3 conditions for different type of loading. That, we assumed and with reference to those 3 different cases case 1, 2 and 3, we try to obtain the boundary stress distribution. But then to be more specific, can we have 3 types of these stress field and we define a term which is called as the mobilization factor.

That, I define as m = σ_h / σ_V, that means, the horizontal stress divided by the vertical stress. Now these 3 types of these stress fields, we represent them by m and I assign three different values to this factor m, and accordingly I define respective 3 types of stress fields. So, let us say I have m = σ_h / σ_V, this is equal to 0, 1 by 3 and 1. I am taking these 3 values, for example.

Now, in this case, what does this mean that $\sigma_h = 0$ and $\sigma_V \neq 0$. So, this in a sense we can say that it is uniaxial state of stress, so, this is, by assigning the value 0, to this factor m we can say that it is representing the uniaxial state of stress. Now, come to the second one what does this mean that $\sigma_h = (\sigma_V / 3)$, this means, σ_h and σ_V both are non-zero and therefore, we call this as, Bi-axial state of stress.

And, when you have m =1, this means that $\sigma_h = \sigma_V$, see this is coming from the definition of m. So, what happens in this case that the horizontal and the vertical stresses, they are same or equal to each other. So, we can call this as the hydrostatic condition or we can say that it is the hydrostatic state of stress.

So, with reference to these three typical values of m, we can represent the uniaxial state of stress, the biaxial state of stress, and the hydrostatic state of stress. Now, considering these values of m, the stress distribution for different shapes of the opening in elastic ground conditions they were plotted.

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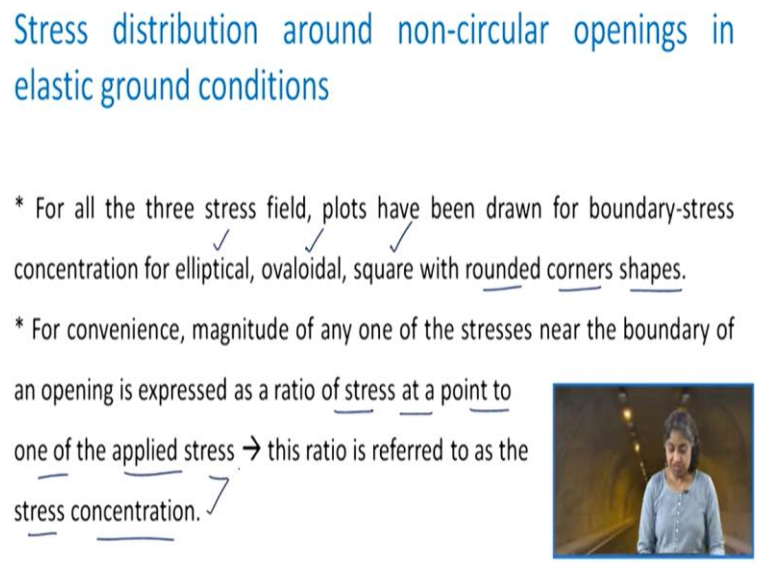

So, basically shapes which were considered these, were elliptical, ovaloidal, and square with rounded corner shapes. I told you, in the previous class that, why we should consider the rounded corner shapes it is done, to avoid the infinite stress concentration at the corners. Because, that much of the stress concentration cannot be handled by any material. So, when we go for the construction, we just make the corners of the square or rectangular type of the opening as the rounded corners.

Now for our convenience, we have done this earlier but then I am repeating it again here, that the magnitude of any one of these stress near the boundary of any opening, that we are expressing in terms of the stress concentration. And, this was defined as a ratio of stress at a point to one of the applied stress, this is not new to you, last one or 2 lectures, we have been using this term stress concentration.

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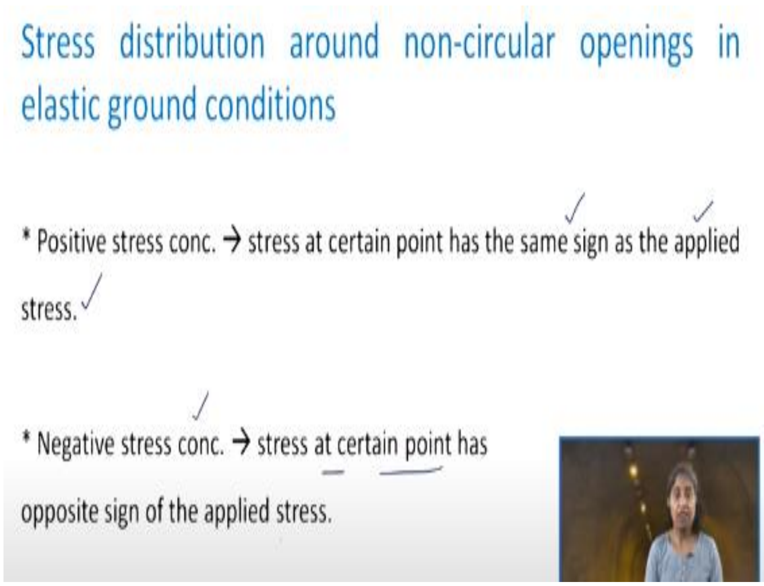

Now, when we have the positive stress concentration, this means that stress at a certain point has the same sign as that of the applied stress. In case, if there is the negative stress concentration, this signifies the presence of stress at certain point, as that of the opposite sign of the applied stress.

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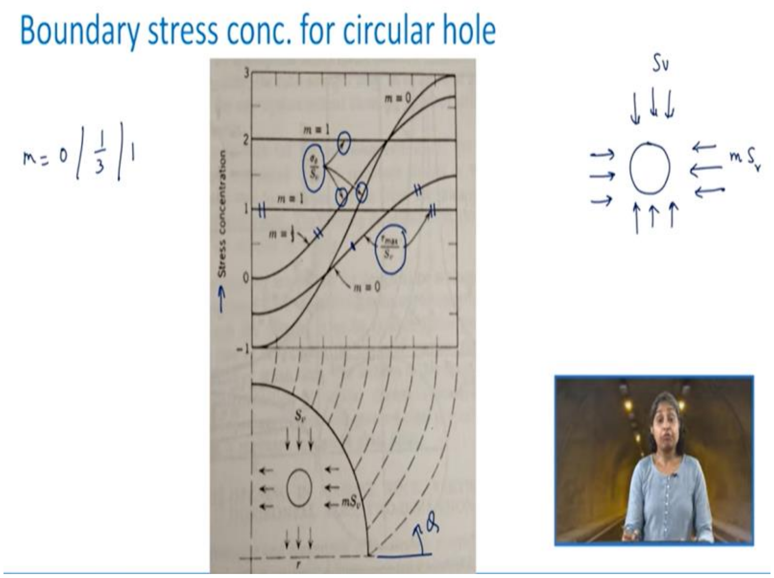

Now, here it is the distribution of the boundary stress concentration for a circular hole. So, see the most general form of the loading condition has been shown that is this is S_v and in the horizontal direction, it has been expressed in terms of m and this is going to be m times S_v. So, now depending upon what value this m assumes, whether m = 0 or 1 by 3 or 1, you will have the different type of the state of stress.

You know that, for m = 0, it is uniaxial, for 1/3 it is biaxial, and for 1 it is the hydrostatic state of stress. So, take a look here this plot is for m = 0, this is for m = 1/3, and this is for m = 1. So, you see that here the variations are shown for the stress concentration, and these three curves, you can see these arrows 1, 2, and 3 these are this is for m = 0, this is for m = 1, and this is for m = 1, by 3 these are for σ_θ / S_v.

And, these 2 that is, this one which is for m = 0 and this one which is for m = 1, they are for T/S_V that means, T_{xy}/ S_v. So, likewise in this particular manner, with reference to θ which you are measuring in this direction. So, you have the variation corresponding to different state of stress, depending upon the value of m for the circular shape of the opening.

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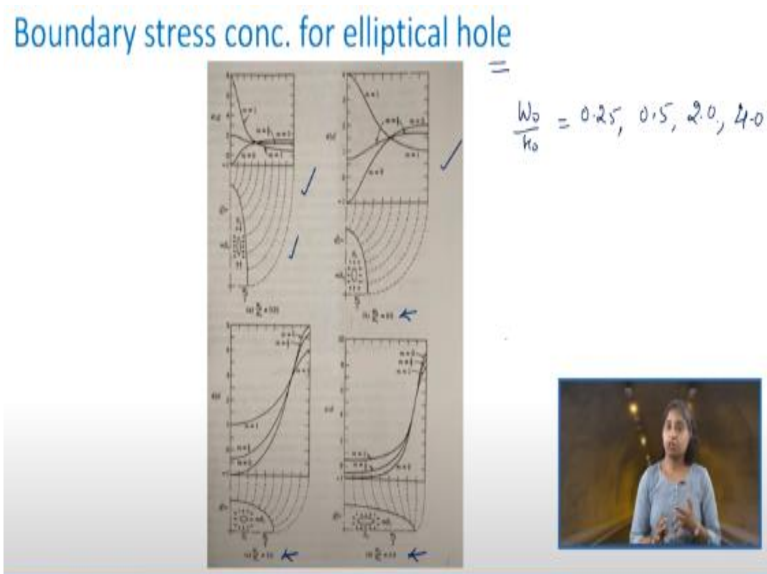

Similar type of variation, it is given here, in case if you have the elliptical hole. So, you can see that W_o/H_o has been varied from 0.25, then 0.5, then 2 to 4, so this is for 0.25, this is for 0.5, this is for 2, and this is for 4. And, accordingly, you can see that the 3 plots are there for each of these corresponding to the various state of stress whether uniaxial, biaxial or hydrostatic state of stress**.**

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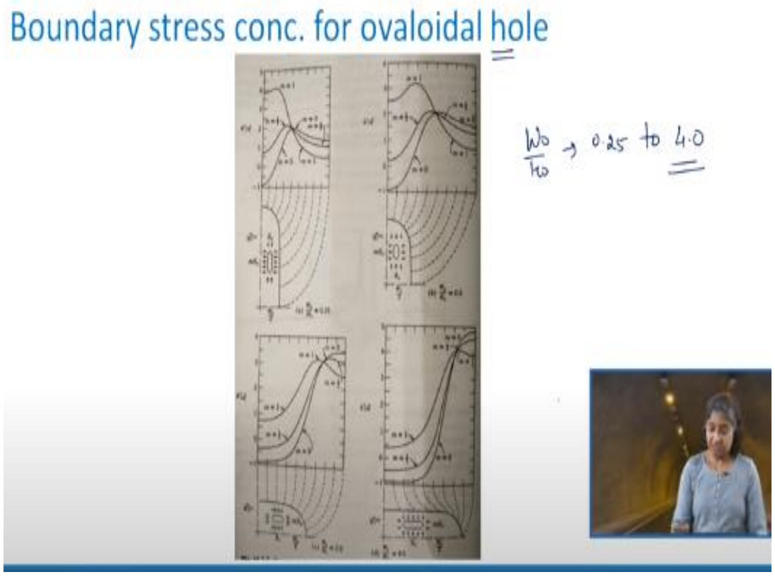

Similar type of variation for ovaloidal shape of the hole has been shown here, again here in this case W_o/H_o has been varied from 0.25 to 4. And, the various conditions, stress applied stress condition 3 plots are given for uniaxial, biaxial and the hydrostatic state of stress this is for ovaloidal shape of the opening.

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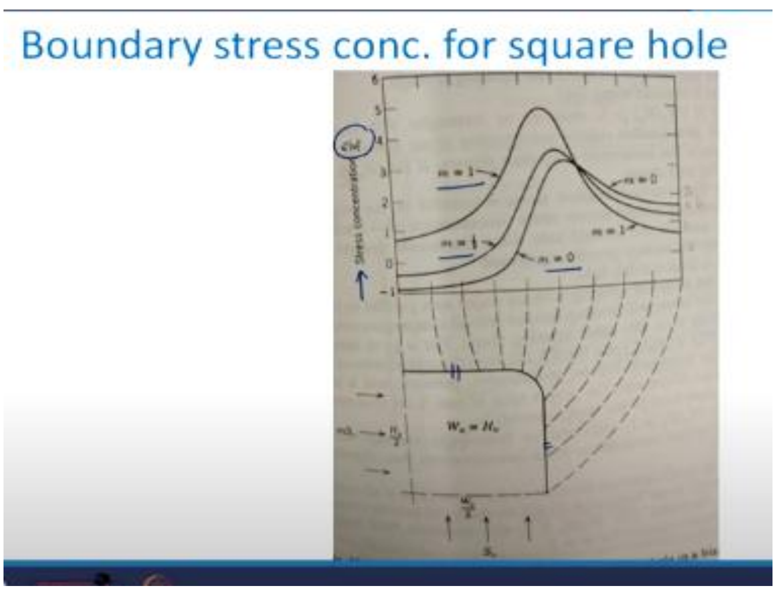

This is for square hole, where the sides of these squares, they are parallel to the axis. So, you can see that m = 0 m = 1/3 and m = 1, you have got the stress concentration which is σ_T/S_V.

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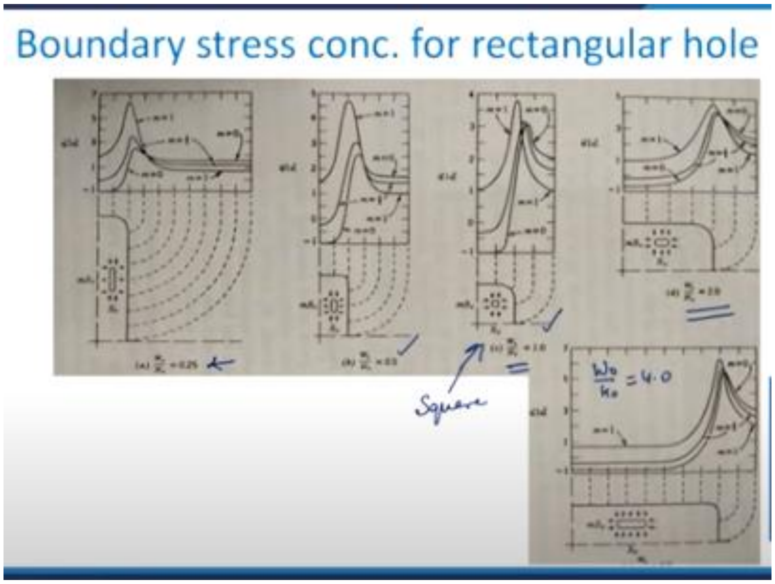

Then, this is for the rectangular shape of the opening in this case again, W_o/H_o = 0.25 for the first figure, this is 0.5, this is 1. And, of course this will correspond to the case of the square because, both the size they are equal. Now, this is W_o/H_o = 2, and this one is for W_o/H_o = 4. So, you see in all the 3 cases, how the stress concentration factor varies all along the boundary of the opening, for various values of W_o/H_o it has been shown.

Now, if we take the detailed look at all these figures, as a combined way what all things that we should take a note? We can take a note of the stress concentration factor, at the end of horizontal axis, at the end of vertical axis and also, we can get the maximum stress concentration factor corresponding to any state of stress, whether it is uniaxial, biaxial or hydrostatic. So, what we will do is that, we will try to present the data in the tabular form, where we will note down for a particular shape of the opening, what all are the different values of W_o/H_o ratio that are being considered.

And then, what are the corresponding values of the stress concentration factor at the end of vertical axis? at the end of horizontal axis? And, the maximum value of the stress concentration factor? So, today in this class, we discussed about the general guidelines based upon the boundary stress concentration variation corresponding to elliptical, ovaloidal and the square shape of the opening.

And, there we saw that, we had 5 or 6 general guidelines for the design of non-circular opening in the elastic ground condition. And, then in particular we took 3 state of stresses, uniaxial, biaxial and hydrostatic state of stresses. And, I showed you how the variation of the boundary stresses for different shape of opening they look like.

So, in the next class what we will do is? We will take each of this state of stress individually, then we try to get the numerical values at the end of horizontal axis, vertical axis and the maximum value of the stress concentration factor. And, based upon that, we will again try to obtain, some of the general guideline, with reference to different shapes of the non-circular opening, thank you very much.