Underground Space Technology
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Module No # 05
Lecture No # 22
Stress Distribution under Different in-situ Stress Condition: Design Principles

Hello everyone, in the previous class we discussed about stress distribution all around the boundary of various non-circular openings, such as elliptical, ovaloidal, square, and, rectangular shapes. And then, I mentioned to you that, how the stress distribution looks like under different in-situ stress condition, that is, uniaxial, biaxial, and, hydrostatic state of stresses. So, today, based on those figures that I discussed in the previous class, we will discuss the design principles that can be derived from those figures.

So, we will discuss case by case, that is, for the 3 different stress conditions, we will take them separately, and then we try to understand that, where the maximum stress concentration is going to be, whether it is at the end of the horizontal axis, or the vertical axis, or at some other place, so, all those aspects, we are going to discuss today.

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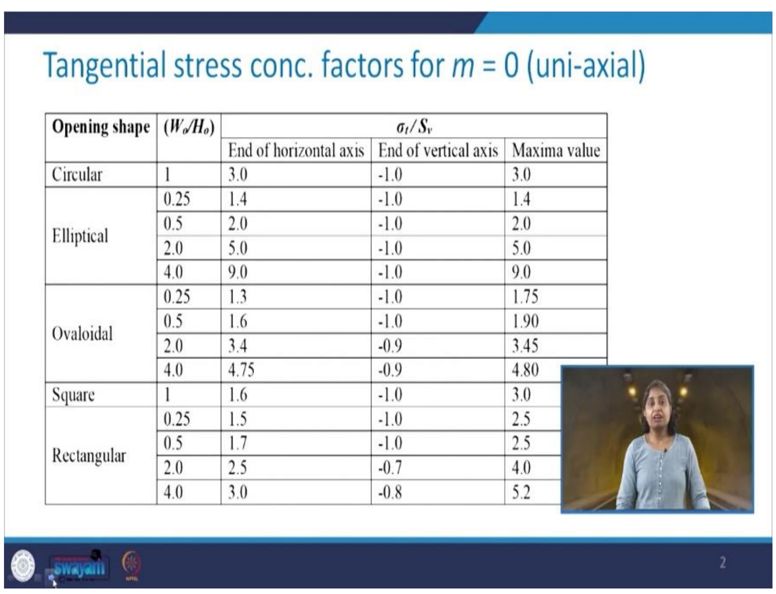

Opening shape	(W_o/H_o)	σ_t/S_v		
		End of horizontal axis	End of vertical axis	Maxima value
Circular	1	3.0	-1.0	3.0
Elliptical	0.25	1.4	-1.0	1.4
	0.5	2.0	-1.0	2.0
	2.0	5.0	-1.0	5.0
	4.0	9.0	-1.0	9.0
Ovaloidal	0.25	1.3	-1.0	1.75
	0.5	1.6	-1.0	1.90
	2.0	3.4	-0.9	3.45
	4.0	4.75	-0.9	4.80
Square	1	1.6	-1.0	3.0
Rectangular	0.25	1.5	-1.0	2.5
	0.5	1.7	-1.0	2.5
	2.0	2.5	-0.7	4.0
	4.0	3.0	-0.8	5.2

So, here I am considering the first state of stress, which was represented by m to be equal to 0, or we can say that, it is the uniaxial state of stress. Please remember that, $m = \sigma_h / \sigma_V$, and when I say that m = 0, this means your sigma h value = 0. So, here the first column we have various opening shapes, varying from circular to the rectangular, and we in between, we have elliptical, ovaloidal, and the square shape.

Now for the elliptical shape, we have considered 4 values of W_o/H_o, please recall W_o be the width of the opening, and H_o be the height of the opening, this is what is W_o this is H_o. So, for the elliptical shape of the opening, we are considering 4 values of W_o/H_o, which are 0.25, 0.5, 2 and 4. Similarly, for ovaloidal and rectangular shape, we have the 4 values of W_o/H_o as 0.25, 0.5, 2 and 4.

Then as I mentioned to you in the previous class that we are going to focus on 3 values of the stress concentration that is at the end of the horizontal axis then at the end of the vertical axis and of course its maxima value. So, the same thing has been presented here for the circular case these values are 3 at the end of horizontal axis -1 at the end of vertical axis and the maxima value was equal to 3.

Then, as I mentioned to you in the previous class that, we are going to focus on 3 values of the stress concentration that is at the end of the horizontal axis, then at the end of the vertical axis, and of course its maxima value. So, the same thing has been presented here for the circular case, these values are 3 at the end of horizontal axis, -1 at the end of vertical axis, and the maxima value was equal to 3.

Likewise, for the elliptical opening, say I consider W_o/H_o =2, then at the end of the horizontal axis the stress concentration factor is 5, at the end of the vertical axis it is -1, and the maxima value is 5. Similarly, for other shapes, it is given now from where these values are coming, you remember I showed you various figures in the previous class. So, all I have done is picked the numerical values, at the end of horizontal axis, vertical axis and maximum value and compiled it, in the form of this particular table for m = 0 value.

Now, based upon these values what we did, here for elliptical, ovaloidal, and, rectangular shape where W_o/H_o is varying, so corresponding to this, I have the maxima value of the stress concentration.

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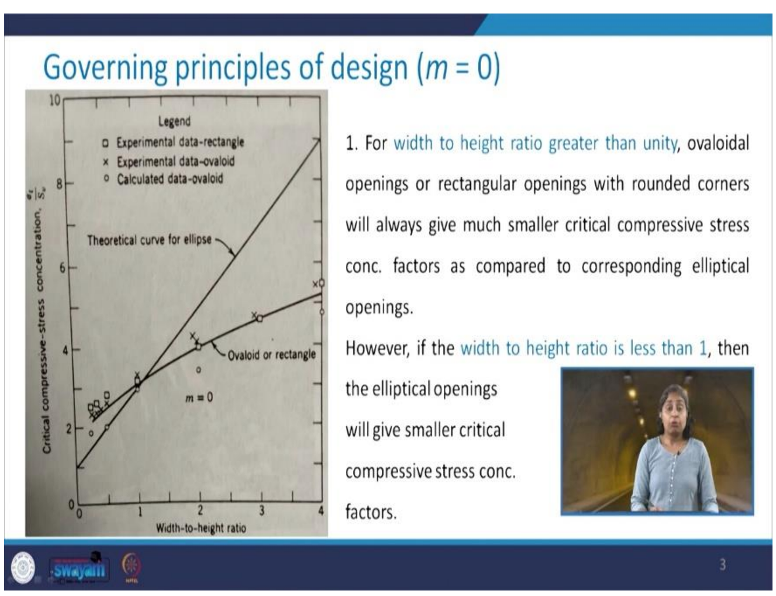

So, this is what that have plotted here, so on x axis it is width to height ratio, and in our case, it was varying as, 0.25, 0.5, 2 and 4 and on y axis. It is the critical value of the stress concentration, that we are taking as whatever is the maximum value of the stress concentration, that is what is the critical value. So, the one value that we obtain from that table which I showed you in the previous slide plus, we have the experimental values also with reference to the rectangular opening, and ovaloidal is opening.

So, this circle legend is showing us the calculated data for ovaloidal shape, while this square shape legend showing us the calculated data for overloidal shape, while this square shape legend is corresponding to experimental data with respect to rectangle, and, cross is showing for the ovaloid. And, this is what is the theoretical curve for ellipse, so this theoretical curve I am getting from the value of the values, which were there in the table in previous slide. So, what is the governing principle, that we can deduce from here, that when you have width to height ratio greater than unity, this means this.

So, towards this side of the figure you have $W_o/H_o > 1$, ovaloidal opening, or rectangular openings with rounded corners, they will always give much smaller value of the critical compressive stress concentration factor. So, why is it so? You take a look here in this figure, see this is the theoretical curve for the ellipse. And, if you just try to draw a smooth curve, with respect to these experimental data, then you will see that you take any value of width to height ratio let us say.

Then, corresponding to this the stress concentration factor for ovaloid, or rectangle shape, is this and for ellipse, it is much larger than this particular value. So, therefore we can say that, when this $W_o/H_o > 1$, it is the ovaloidal or the rectangular shape of the opening, under uniaxial state of stress, will give you lesser value of critical compressive stress concentration as compared to the elliptical shape of the opening, having the same width to height ratio, so that is what you need to keep in mind.

Now, if we have the other side of the W_o/H_o, let us say this side it was > 1. But on this side, this is W_o/H_o is < 1, so what happens here, that elliptical openings are giving us the smaller critical compressive stress concentration factor. So, take a look you take any value, and then you see this line is for ellipse, so which is giving me the lesser value of the critical concentration factor as compared to the 1, that I will obtain for ovaloidal, or, the rectangular shape.

So, depending upon width to height ratio, you can decide that, which type or which shape of the opening is going to give you less value of the critical compressive stress concentration. And, accordingly you can pick the appropriate shape of the opening, so this is the first governing principle of design for the uniaxial state of stress.

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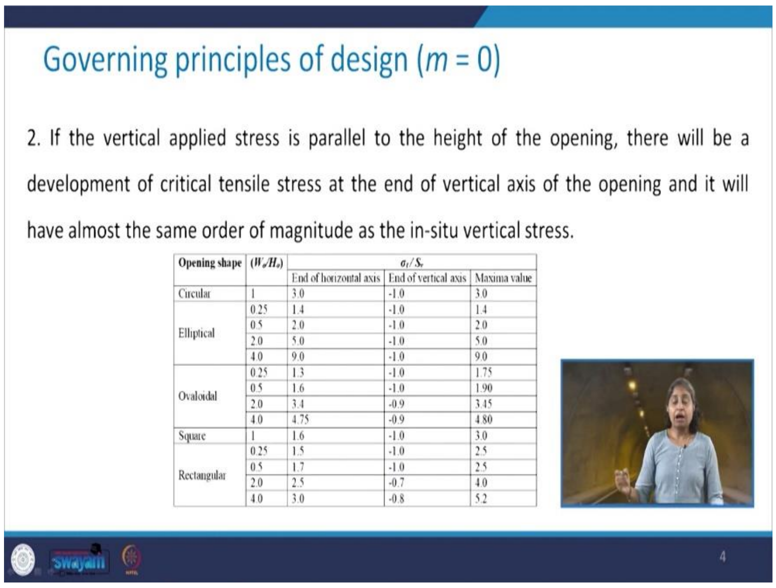

Opening shape	(W_o/H_o)	σ_t/S_v		
		End of horizontal axis	End of vertical axis	Maxima value
Circular	1	3.0	-1.0	3.0
Elliptical	0.25	1.4	-1.0	1.4
	0.5	2.0	-1.0	2.0
	2.0	5.0	-1.0	5.0
	4.0	9.0	-1.0	9.0
Ovaloidal	0.25	1.3	-1.0	1.75
	0.5	1.6	-1.0	1.90
	2.0	3.4	-0.9	3.45
	4.0	4.75	-0.9	4.80
Square	1	1.6	-1.0	3.0
Rectangular	0.25	1.5	-1.0	2.5
	0.5	1.7	-1.0	2.5
	2.0	2.5	-0.7	4.0
	4.0	3.0	-0.8	5.2

Now, coming to the second one, so here I have put this table for your reference, is the same table that we had seen earlier. In this case, if the vertical applied stress is parallel to the height of the opening, then there will be the development of the critical tensile stress at the end of the vertical axis of the opening, and it will have almost the same order of the magnitude, as that of the in-situ vertical stress. Take a look here, at the end of the vertical axis, is this column and you can see that everywhere it is near about -1, either it is exactly -1, or may be 0.7 or 0.8 or 0.9.

So, this is what is another governing principle for the design under uniaxial state of stress. If you recall, when we discussed about the circular tunnels, then I mentioned that it is the uniaxial state of stress which is critical because at the crown portion of the tunnel, there is going to be the occurrence of the tensile stresses. And, therefore, if the tensile strength of the material is less than the tensile stress which is being, which this opening is being subjected to, there is going to be the occurrence of the tension cracks, tensile cracks, and one needs to be careful as far as the provision of the support systems are concerned in the crown portion of the tunnel.

So, similar observation is seen here that at the end of the vertical axis of the opening, you have the critical tensile stress because here it is all -1, the stress concentration factor is all with a negative sign. And, they have almost the same order of the magnitude as that of the in-situ vertical stress. So, please remember, this is, this forms the second governing principle of design under uniaxial state of stress. I am emphasizing again, that right now we are discussing corresponding to m = 0 which signifies the uniaxial state of stress.

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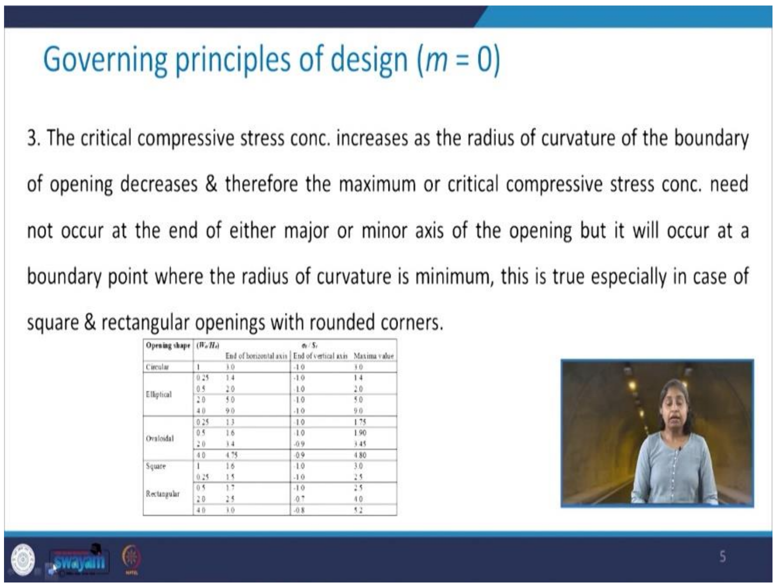

Then the critical compressive stress concentration, it increases as the radius of the curvature of the boundary of the opening reduces. And, therefore the maximum or the critical compressive stress concentration needs not to occur at the end of, either major or the minor axis of the opening. But it will occur at a point on the boundary, where you have the radius of curvature as the minimum, that is where the minimum radius of curvature is there, you will have the maximum stress concentration. And this is very striking especially in case of the square, or the rectangular openings with rounded corners, although it is also evident from this table.

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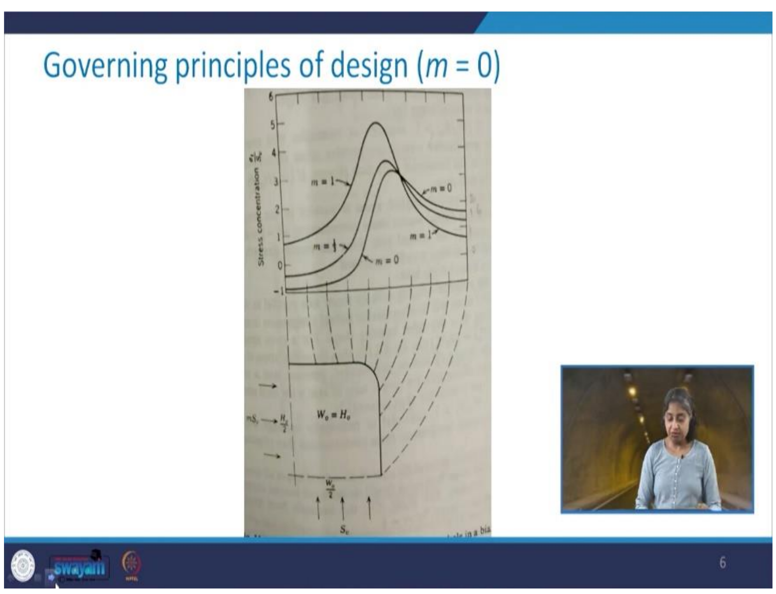

But then, I will have a figure also with respect to the square opening, where you $W_o/H_o = 1$ and the sides of these square they are parallel to the axis, and y axis as you can see from here. Now, you see at the ends of the vertical and the horizontal axis the radius of curvature is almost infinite. But in this zone, where it is minimum, so you see that the maximum stress concentration is occurring in this zone, that is in this case m = 1, it is here for m = 1/3, it is here and for m = 0, it is here.

So, this is, these are the locations, these correspond to, approximately wherever you have the least radius of curvature, which happens to be in this zone. So, this makes the next governing principle for design under state of stress.

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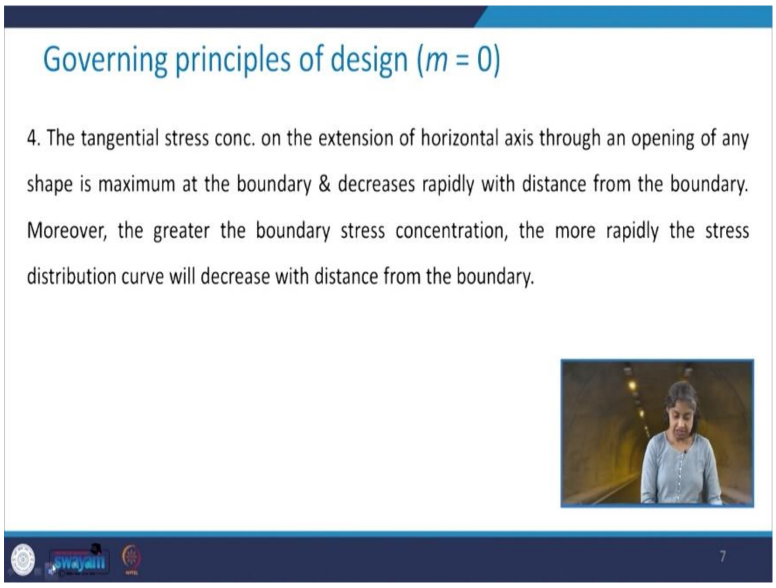

Then, the Tangential stress concentration on the extension of horizontal axis, take a note, it is on the extension of the horizontal axis through any opening of any shape is maximum at the boundary, and it reduces rapidly with its distance from the boundary. Then, another thing which you have to note here, that more the boundary stress concentration factor, and more rapid is going to be the reduction of this stress concentration with distance from the boundary.

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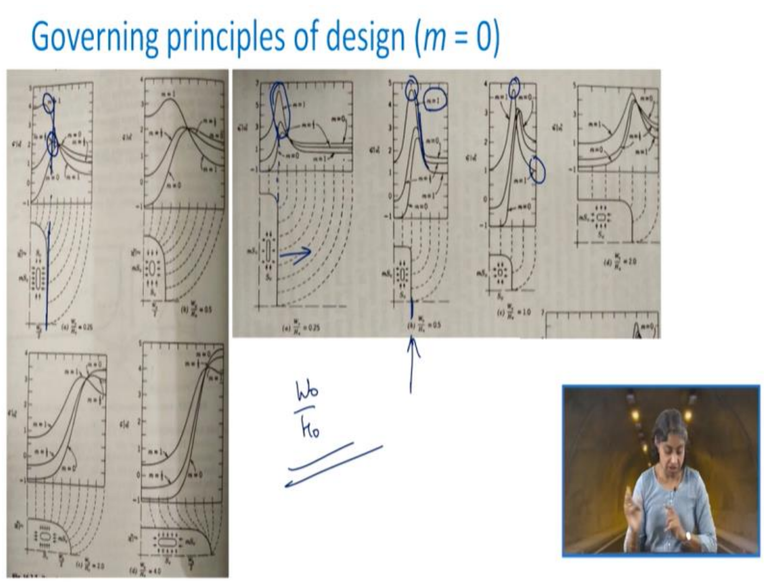

Let us take this figure, and try to understand this, so what I said was, that extension of the horizontal axis. So, I extend here is the end, so I extend this horizontal axis, and you see that here near in this region only you are getting the maximum value. You take any figure, and approximately on those zones, you will have the maximum value. And, you see that as you move away from this horizontal boundary, see how the stress concentration factor is reducing.

So, you can see that it is almost true in all these situations of any shape of the opening, having any value of W_o/H_o. Now another aspect which was there, that, more the stress concentration and rapid is going to be the, reduction with the distance from the boundary. So, take a look, a example, at this figure here you see corresponding to m = 1, this is the maximum stress concentration factor, and you see when you, away from the boundary of the opening. So, you see that this m = 1, see how it is going here the reduction is so rapid.

So, similar is the situation which seen, you can see here m = 1, maximum stress concentration, but the reduction is rapid, when you had more stress concentration when you move away from the boundary. So, this makes the next governing principle of the design under uniaxial state of stress, that at the extension of the horizontal axis, you are going to have the maximum stress concentration factor, which reduces with the distance from the boundary, then larger this stress concentration factor, rapid is the reduction of the stress concentration factor.

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Tangential stress conc. factors for $m = 1/3$ (bi-axial)

Opening shape	(W_o/H_o)	σ_t/S_v		
		End of horizontal axis	End of vertical axis	Maxima value
Circular	1	2.67	0	2.67
Elliptical	0.25	1.1	2.0	2.0
	0.5	1.65	0.7	1.65
	2.0	4.7	-0.35	4.70
	4.0	8.5	-0.50	8.50
Ovaloidal	0.25	1.0	0.70	2.0
	0.5	1.3	0.25	2.0
	2.0	3.1	-0.40	3.1
	4.0	4.5	-0.50	4.5
Square	1	1.25	-0.4	3.5 (at corner)
Rectangular	0.25	1.1	0.0	3.0 (at corner)
	0.5	1.5	-0.2	3.2
	2.0	2.1	-0.25	4.1
	4.0	2.5	-0.3	5.4

$m = \frac{1}{3} = \frac{\sigma_h}{\sigma_v} \Rightarrow \sigma_h = \frac{1}{3}\sigma_v$

Now, coming to the next state of stress, that is bi-axial, now, when I say bi-axial that means that you have these stress applied, these stresses in horizontal as well as, in the vertical direction. So here we took a typical value of m as 1/3, which is equal to σ_h / σ_V, and this gives me, $\sigma_h = \sigma_V/3$. So, it is not that this is the only bi-axial state of stress you can have $\sigma_h = \sigma_V/2$ or maybe $\sigma_V/4$ or let us say any other factor.

Here, typically I am considering this m to be equal to 1/3 and I am calling this as a bi-axial state of stress. So, like we had in the uniaxial state of stress condition here also we had the stress distribution plots which I showed you in the previous class. So, from those plots only we picked the numerical values at the end of the horizontal axis, at the end of vertical axis, and the maxima value all at the boundary of the opening.

And, we have presented it here, in a form of a table, so for this circular opening shape at the end of the horizontal axis, the value of these stress concentration factor was found to be 2.67. At the end of the vertical axis, it was 0 and the maxima value was also 2.67 for elliptical, and ovaloidal and rectangular shapes. We consider 4 values of W_o/H_o =which were 0.25, 0.5, 2 and 4. And accordingly, we had the respective value of the stress concentration factor at the end of horizontal axis, at the end of the vertical axis, and the maximum value.

Now come to the square opening, there you can see that the maximum value is occurring at the corner. Similarly, here in case of the rectangular one, this maximum value is occurring at the corner. You remember, I mentioned to you that, the maximum value will be there, wherever you have the minimum radius of curvature.

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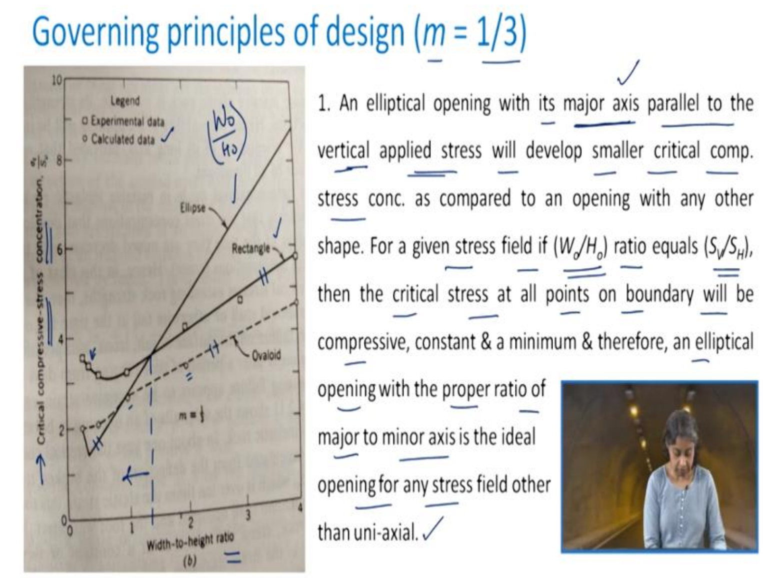

And, therefore we can derive some of the governing principles with respect to the bi-axial state of stress when m assumes a values of 1/3. So, from that table, that I showed you in the previous slide again, here as we did in case of the uniaxial state of stress, we try to plot the value of the critical compressive stress concentration with reference to width to height ratio here. So, there are 3 plots, as you can see one is for ellipse, this is for rectangle, and this dotted one is for ovaloidal shape of the opening.

So, some of the points that you are seeing as a rectangular legend, these are the experimental data and, remaining this, round or circular region is the calculative data. So, in this case you take look at this figure, and then read the text which is given here, and then try to understand what is the governing principle of design, that we can have from here. An elliptical opening with its major axis parallel to the vertical applied stress will develop smaller critical stress concentration.

Now, you see, when I say an elliptical opening with its major axis parallel to the vertical applied stress. This will give you the idea about the ratio of W_o/H_o, now for a given stress field if this $W_o/H_o = S_V/S_H$, then the critical stress at all the points on the boundary, they are compressive constant and a minimum. And, hence the minimum elliptical opening with the proper ratio of major to minor axis becomes, the ideal opening for any stress field other than the uniaxial one.

So, you see here that, if I have W_o/H_o like this, where you have the major axis parallel to the vertical applied stress, this will always develop the low value of the critical compressive stress concentration in this zone.

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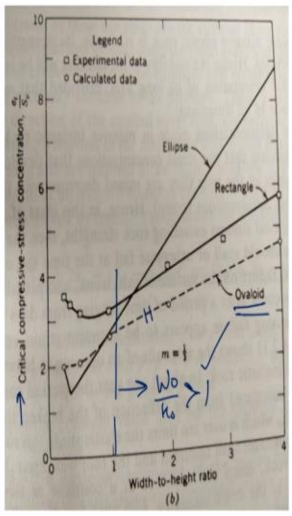
Governing principles of design ($m = 1/3$)



2. When $(W_o/H_o) > 1$, the ovaloidal openings will induce the least critical compressive stress conc. as compared to openings of any other shape.

Then, when you have $W_o/H_o > 1$, so this is what is the zone. where $W_o/H_o > 1$, it will be the ovaloidal shape, you see this which will give you the least value of the stress concentration factor. When we have this type of the size of the opening, then it is better to go for the ovaloidal shape, then it gives you the minimum value of the critical compressive stress concentration factor, as compared to the opening of any other shape.

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Governing principles of design ($m = 1/3$)

3. Tensile stress at end of vertical axis of all openings in a uni-axial stress field (m=0) will decrease & become compressive as the mobilization factor m increases. For $(W_o/H_o) \leq 1$, this transition from tension to compression occurs at m = 1/3.

$m = \frac{1}{3}$

Opening shape	(W_o/H_o)	σ_t/S_v		
		End of horizontal axis	End of vertical axis	Maxima value
Circular	1	3.0	-1.0	3.0
Elliptical	0.25	1.4	-1.0	1.4
	0.5	2.0	-1.0	2.0
	2.0	5.0	-1.0	5.0
	4.0	9.0	-1.0	9.0
Ovaloidal	0.25	1.3	-1.0	1.75
	0.5	1.6	-1.0	1.90
	2.0	3.4	-0.9	3.45
	4.0	4.75	-0.9	4.80
Square	1	1.6	-1.0	3.0
Rectangular	0.25	1.5	-1.0	2.5
	0.5	1.7	-1.0	2.5
	2.0	2.5	-0.7	4.0
	4.0	3.0	-0.8	5.2

m=0

Opening shape	(W_o/H_o)	σ_t/S_v		
		End of horizontal axis	End of vertical axis	Maxima value
Circular	1	2.67	0	2.67
Elliptical	0.25	1.1	2.0	2.0
	0.5	1.65	0.7	1.65
	2.0	4.7	-0.35	4.70
	4.0	8.5	-0.50	8.50
Ovaloidal	0.25	1.0	0.70	2.0
	0.5	1.3	0.25	2.0
	2.0	3.1	-0.40	3.1
	4.0	4.5	-0.50	4.5
Square	1	1.25	-0.4	3.5 (at corner)
Rectangular	0.25	1.1	0.0	3.0 (at corner)
	0.5	1.5	-0.2	3.2
	2.0	2.1	-0.25	4.1
	4.0	2.5	-0.3	5.4

m=1/3

Now, the tensile stresses at the end of the vertical axis, of all the opening in the uniaxial stress field will reduce, and it becomes compressive, when you increase this mobilization factor m from 0 to 1/3. For $W_o/H_o \leq 1$, this transition from tension to compression, it occurs at 1/3 value of m. So, you see that, both the tables I have pasted here, this corresponds that m = 0, this corresponds to m = 1/3.

So, take a look at the end of the vertical axis here, and this is here, so here we saw that these values are negative. However, in this case most of these values now are positive, except for the elliptical case here, and the ovaloidal case here, square and the rectangular shape here. So, most of the values, they are transitioning from the negative to compressive stresses. So, this is what is the next principle of design for the biaxial state of stress having the value of m as 1/3.

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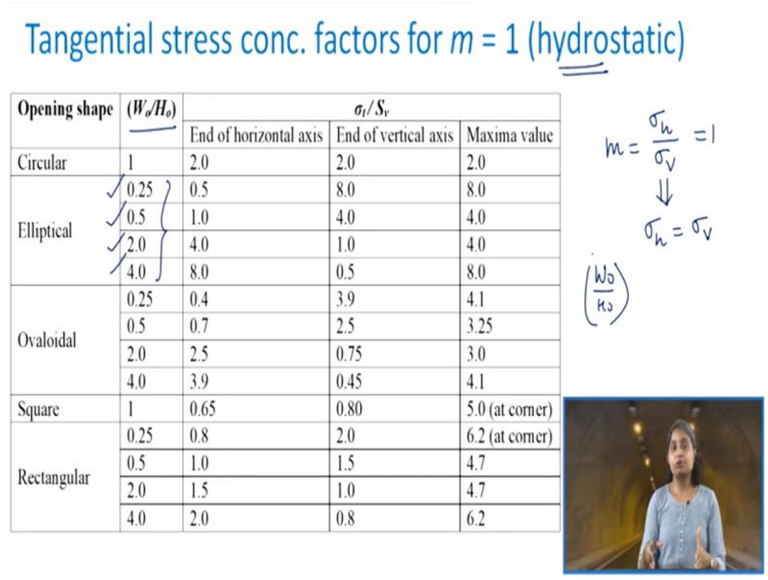

Tangential stress conc. factors for $m = 1$ (hydrostatic)

Opening shape	(W_o/H_o)	σ_t/S_v		
		End of horizontal axis	End of vertical axis	Maxima value
Circular	1	2.0	2.0	2.0
Elliptical	0.25	0.5	8.0	8.0
	0.5	1.0	4.0	4.0
	2.0	4.0	1.0	4.0
	4.0	8.0	0.5	8.0
Ovaloidal	0.25	0.4	3.9	4.1
	0.5	0.7	2.5	3.25
	2.0	2.5	0.75	3.0
	4.0	3.9	0.45	4.1
Square	1	0.65	0.80	5.0 (at corner)
Rectangular	0.25	0.8	2.0	6.2 (at corner)
	0.5	1.0	1.5	4.7
	2.0	1.5	1.0	4.7
	4.0	2.0	0.8	6.2

Then, we come to the next state of stress which is hydrostatic state of stress that is $m = \sigma_h / \sigma_V = 1$. So, this gives me that $\sigma_h = \sigma_V$, now in this case, again we have developed similar table with respect to various opening shape, and different values of this width-to-height ratio of opening, which is represented by W_o/H_o. So, we have taken as usual in the earlier cases also.

Here, we have taken 4 values of W_o/H_o that is 0.25, 0.5, 2 and 4, and then we have obtained these values from the figures, that I showed you in the previous class corresponding to hydrostatic state of stress. And how, complied all these values at the end of horizontal axis, at the end of vertical axis, and also the maximum value. Now, like we did in case of the uniaxial and biaxial state of stress, here also we are going to plot these values, with reference to W_o/H_o and let us try to obtain some of the design principles.

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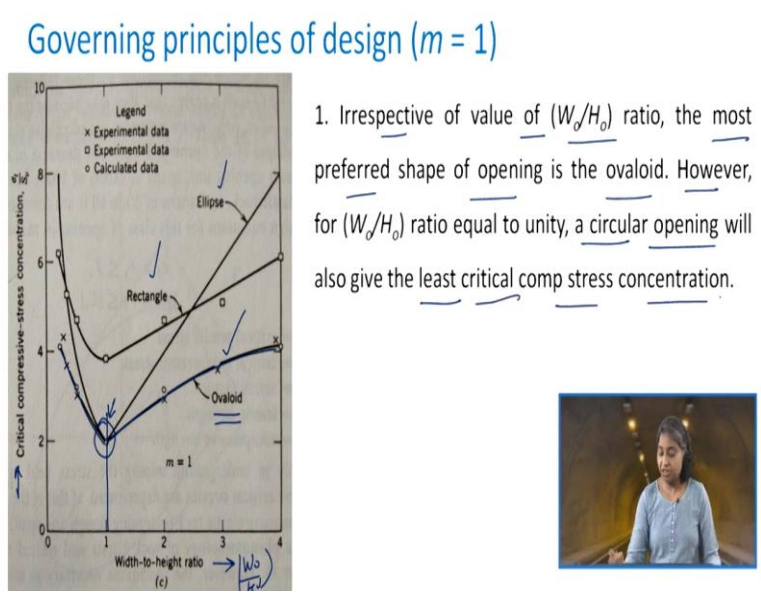

So, here on the x-axis, you have width to height ratio, which is W_o/H_o that theoretically we have varied it from 0.25 to 4. And, then on y axis we have the critical compressive stress condition, so in this case, you can see that, irrespective of the value of W_o/H_o ratio. The most preferred shape of the opening is ovaloid, so you see that this one this plot is for the ovaloid shape. And, whatever is the value of W_o/H_o, this will always give me the least value of critical compressive stress concentration.

So, this will always be the most preferred shape of the opening, as compared to ellipse and the rectangular opening. However, you see take a look here, at when $W_o/H_o = 1$, so here you see, you have the ovaloidal shape, you have the ellipse, and when this $W_o/H_o = 1$, this is also, corresponds to the circular opening, this will also get the least value of the critical compressive stress concentration.

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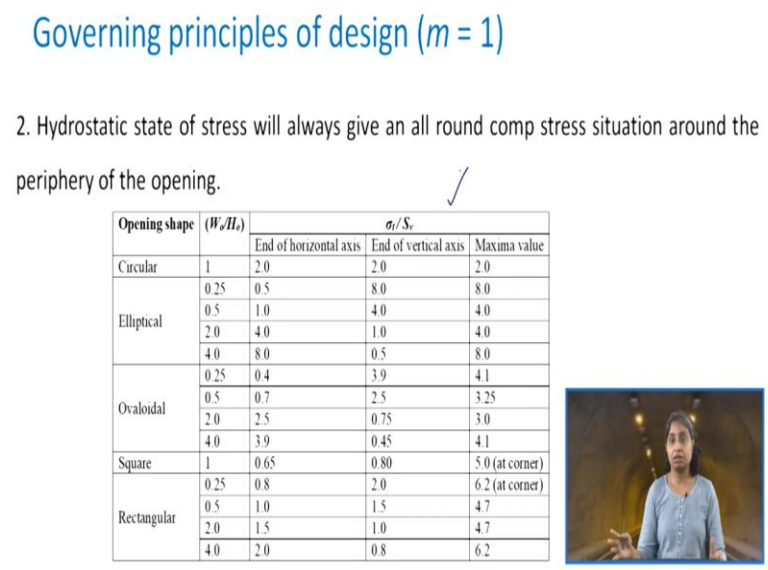

Opening shape	(W_o/H_o)	σ_t/S_v		
		End of horizontal axis	End of vertical axis	Maxima value
Circular	1	2.0	2.0	2.0
Elliptical	0.25	0.5	8.0	8.0
	0.5	1.0	4.0	4.0
	2.0	4.0	1.0	4.0
	4.0	8.0	0.5	8.0
Ovaloidal	0.25	0.4	3.9	4.1
	0.5	0.7	2.5	3.25
	2.0	2.5	0.75	3.0
	4.0	3.9	0.45	4.1
Square	1	0.65	0.80	5.0 (at corner)
Rectangular	0.25	0.8	2.0	6.2 (at corner)
	0.5	1.0	1.5	4.7
	2.0	1.5	1.0	4.7
	4.0	2.0	0.8	6.2

Then, the hydrostatic state of stress will always give, or it will always result in an all-around compressive stress situation, all around the periphery of the opening. As it is evidence from this table, you see not even a single value of these stress concentration factor is negative. So, this means that applied stresses we considered to be compressive, and since these all values or positive that means, that the stress which are getting generated, because of these applied stresses, they are also compressive in nature.

So, we can say that the hydrostatic state of stress, there will not be the occurrence of any tensile stresses around the periphery of the opening. So, one does not need to bother about the tensile strength of the material.

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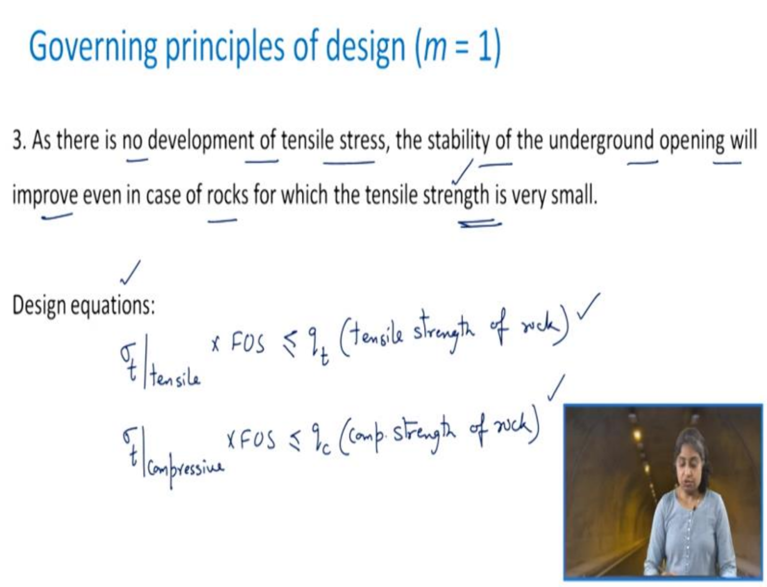

Now, because there is no development of the tensile stresses, this stability of the underground opening will improve even in case of the rocks, for which the tensile strength is very small. As I mentioned, that you really do not need to bother the tensile strength of the material. So, the design equations are going to be, this

σ_t (tensile) x FOS $\leq q_t$,

q_t is the tensile strength of the rock.

Then,

σ_c (compressive) x FOS $\leq q_c$

q_c is the compressive strength of the rock.

So, these are going to be the design equations, so this is how for different state of stress whether it is uniaxial bi-axial or hydrostatic state of stress. We can find out the stress distribution and therefore derive some governing principles for design, for various shapes of the opening such as elliptical ovaloidal, square, and rectangular shapes.

So, this was all about the stress distribution all around the boundary of the non-circular opening. Now, till now, we have considered only the single opening, but then you can have the situation, where you have to go for more than 1 opening. So, in the next class we will take up the study, which are relative to the multiple openings.

And, then we will see that how these stress concentration factors, they vary when you have the multiple openings, or what is the influence of one additional opening on to the stresses, which are there in case of the single opening, thank you very much.