Underground Space Technology

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Module No # 05
Lecture No # 25
Openings in Laminated Rocks-02

Hello everyone, in the previous class we started our discussion on openings in laminated rocks. I explained to you what we exactly mean by laminated rocks, and the concept of the immediate roof and main roof. There, we saw that some of the layers upon excavation, they take part in the deformation, and few other layers they remain as it is. So, accordingly, we define the immediate roof and the main roof.

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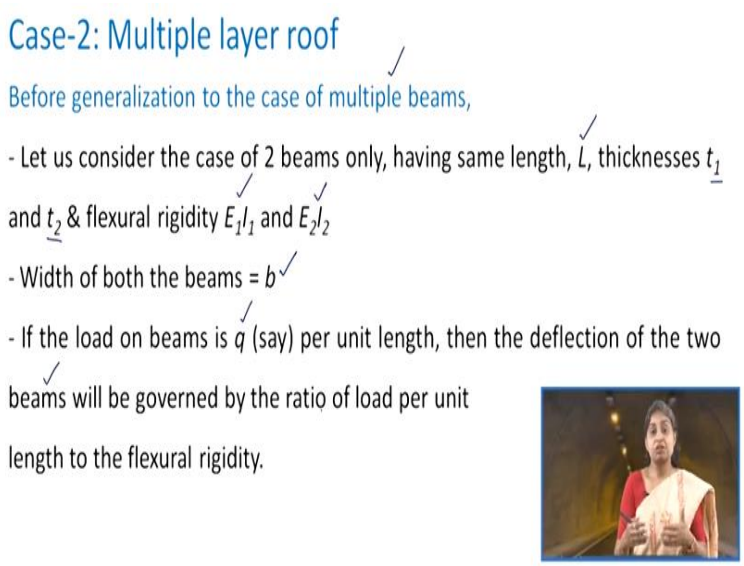

So, today we will see, how the multiple-layer roof works as far as the excavation in laminated rocks is concerned. Now, before we generalize the situation to the case of multiple beams. First, we will consider the case of only 2 beams that have the same length, and thicknesses as, t_1 and t_2 and the corresponding flexural rigidities as even E_1I_1, and E_2I_2. The width of both beams will be taken as the same and represented by b.

Now, if the load on the beams is, say q per unit length of the beam, then the deflection of the 2 beams will be governed by the ratio of load per unit length to the flexural rigidity. Now, depending upon which beam it is, the first beam or the second beam. The load on that will be its corresponding weight. So, we can say it as q_1 or q_2.

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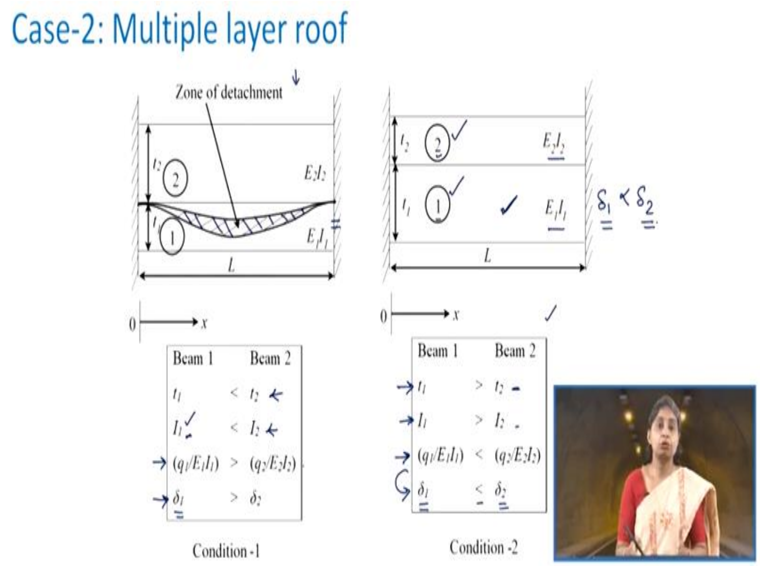

Now, here is a figure with the help of which, I will try to explain to you that, there can be the occurrence of 2 conditions in the case of 2 beams. Please remember that, I will always mark the lower beam as the first beam, which is represented by this number 1, and the upper beam would be given as beam number 2. And, corresponding to beams 1 and 2, you will have the thickness as t_1 and t_2 respectively and flexural rigidity as E_1I_1, and E_2I_2 respectively.

This will be true for both conditions. Now take a look here, in the first portion of this figure, where we have the thickness of the lower layer which is, $t_1 < t_2$. So, t_1 which is less than the thickness of the t_2 now, if t_1 is less than t_2 and we have assumed that the width of the beam is going to be the same in both cases. So, we can say that the value of I_1 the moment of inertia, is going to be less for the first beam, as compared to that for the second beam.

Accordingly, if I_1 is less so we can say that more or less if, this is the situation that, $q_1/ E_1I_1 > q_2/ E_2I_2$. Then, in that case, we can have δ_1 which will be $> \delta_2$ What does this mean physically is that, upon the action of their own self-weight, their own weight, the first beam will deflect more as compared to the upper beam. So, what will happen? Because of that, there is going to be the development of this zone of detachment.

However, what happens in the second case? We have the thickness of the lower beam, which is this beam is more than the thickness of the upper beam, and accordingly, the I_1 value will also be $> I_2$. Now, accordingly, if $q_1/ E_1I_1 < q_2/ E_2I_2$, this would result in a situation, where the lower beam will deform less as compared to the upper beam. That means, the displacement deflection of the lower beam, which is $\delta_1 < \delta_2$. So, what will happen in this case?

In this case, there will not be any zone of detachment and both beams will deflect together, because $\delta_1 < \delta_2$. Once again, I will repeat that, please remember, I am marking t first beam as the lower beam and the second beam as the upper beam, and these 2 types of situations can occur in case there are 2 layers in the roof that are participating in the displacement.

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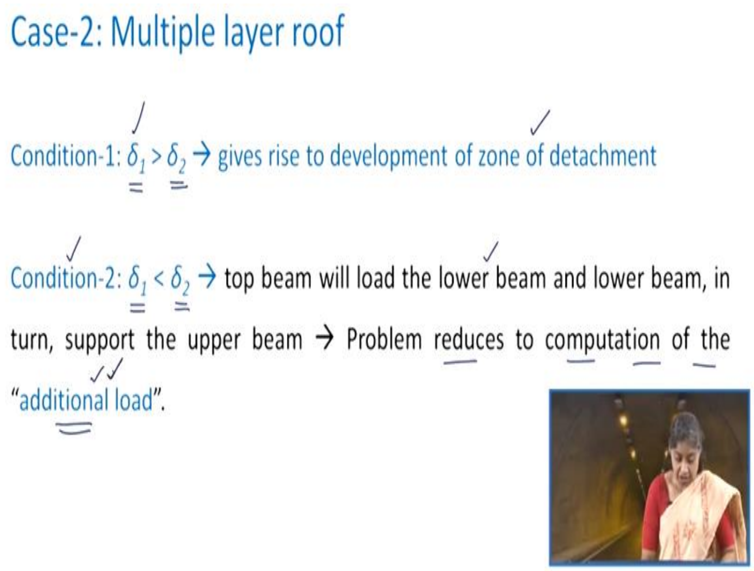

So, we have condition one, as I explained to you, where the deflection of the upper beam is less than the deflection of the lower beam. This gives rise to the, development of a zone of detachment and in condition 2, your deflection of the lower beam is less than the deflection of the upper beam. Therefore, what will happen in this case, is that the top beam will load the lower beam, and lower beam in turn, will support the upper beam.

So, in this case, the problem will reduce the computation of the additional load. This additional load will be because of these phenomena since $\delta_1 < \delta_2$. So, what will happen is that the top beam will load the upper beam apart from its self-weight and, the lower beam in turn will support the upper beam. So, therefore this additional load will come into the picture.

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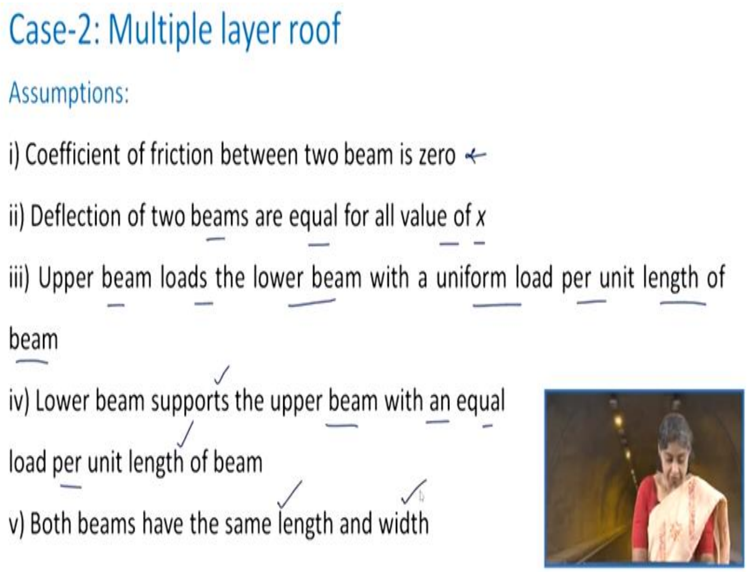

Now, before I go to the analysis of this 2-layer beam, using the theory of beams. We have to make a few assumptions, such as the coefficient of friction between the 2 beams having to be equal to 0. Then, the deflection of both the beams is going to be equal for all the values of x. So, starting from one end of the beam to the other end of the beam, at all the points, or at all the locations, or at all values of x.

The deflection of both beams is going to be equal. Then, the upper beam loads the lower beam with the uniform load per unit length of the beam. So, this means that the loading that we are considering is always, the uniformly distributed load. Further, the lower beam supports the upper beam with an equal load, which is also per unit length of the beam. Both the beams are considered to have the same length and the width.

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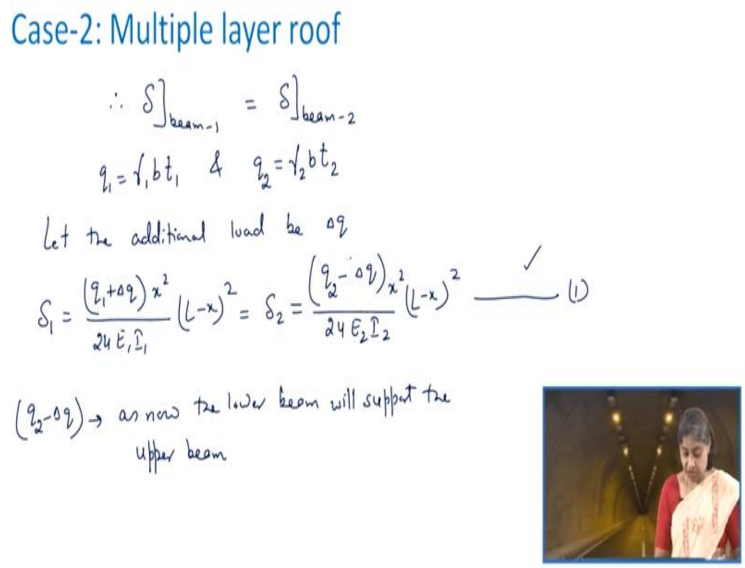

let us try to derive now, that how is going to be the expression for deflection, as well as for the bending moment, and the shear force. So, as for the assumption we had, the deflection of both beams is going to be the same. So, here we have

$\delta_{beam-1} = \delta_{beam-2}$

Now, we have

$q_1 = \gamma_1 bt_1$, and we have q_2 which is the self-weight of the upper beam, $q_2 = \gamma_2 bt_2$

They may have, different thicknesses and different unit weights. But the width is going to be the same as per the assumption. Now, let us say that the additional load is Δq. So, let the additional load be delta q. So, in that case, our,

$$\delta_1 = \frac{(q_1 + \Delta q)x^2}{24E_1I_1}(L-x)^2 = \delta_2 = \frac{(q_2 - \Delta q)x^2}{24E_2I_2}(L-x)^2$$

See, these expressions we saw in the previous class, that is how we can derive these with reference to the single beam here. Now, this equation, I will mark as equation number 1. So, in this case now for the second beam why we are taking this (q_2 - Δq) is that now the lower beam will support the other beam. So, this is how we get equation 1.

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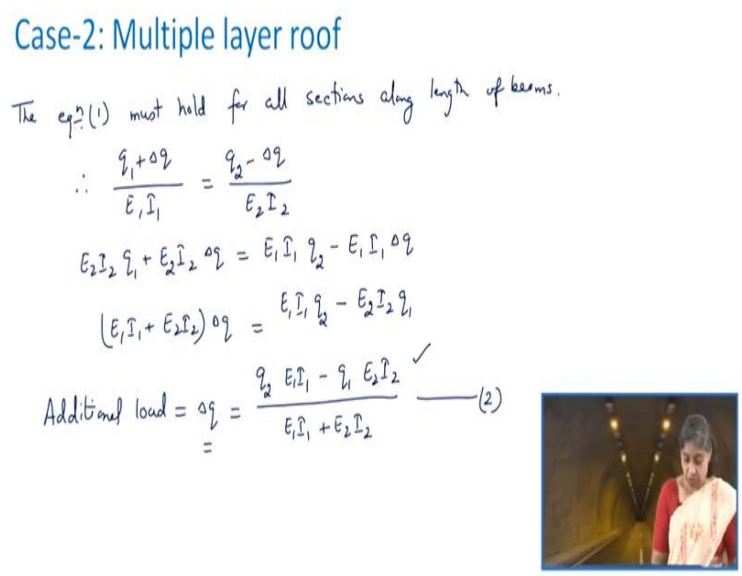

Now, equation 1 will hold good for all the values of x. So, we have the equation 1 must hold for all the sections, along the length of the beams. So, therefore what you will get is that

$$\frac{(q_1 + \Delta q)}{E_1 I_1} = \frac{(q_2 - \Delta q)}{E_2 I_2}$$

Now, we can solve this for Δq, so let us see, how it is done. So, we have here

$$E_2 I_2 q_1 + E_2 I_2 \Delta q = E_1 I_1 q_2 - E_1 I_1 \Delta q$$

Now, collect the terms for Δq together. So, I will have here as

$$(E_1 I_1 + E_2 I_2)\Delta q = E_1 I_1 q_2 - E_2 I_2 q_1$$

So, this additional load which is equal to Δq can be written as

$$\Delta q = \frac{q_2 E_1 I_1 - q_1 E_2 I_2}{E_1 I_1 + E_2 I_2}.$$

So, this is how we will get equation number 2, and the expression for q in terms of weight and the flexural rigidity of the beams lower as well as the upper beams. Now, I can substitute this expression for Δq in the earlier expression that is equation number 1.

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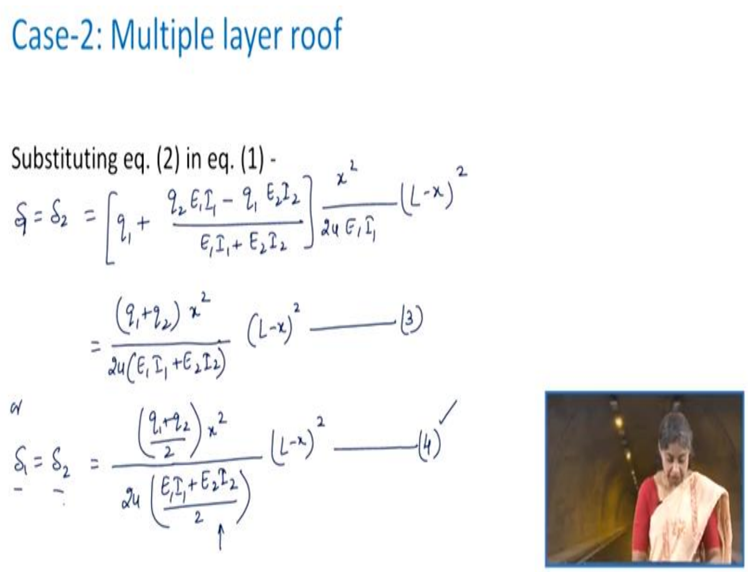

So, we can obtain the expression for the respective deflection. So here, if I just substitute this, what we will get is $\delta_1 = \delta_2$.

That will be equal to say I write here as,

$$\left[q_1 + \frac{q_2 E_1 I_1 - q_1 E_2 I_2}{E_1 I_1 + E_2 I_2}\right] \frac{x^2}{24 E_1 I_1} (L-x)^2 \quad = \frac{(q_1 + q_2)x^2}{24(E_1 I_1 + E_2 I_2)} (L-x)^2$$

make this equation as, equation number 3. Or you can write

$$\delta_1 = \delta_2 = \frac{\left(\frac{q_1 + q_2}{2}\right)x^2}{24\left(\frac{E_1 I_1 + E_2 I_2}{2}\right)} (L-x)^2$$

Now, if you just compare this equation number 4 with the equation of a single beam. So, then you will realize that these 2 beams are acting as 1 beam, except for the case that their weight we can take as the average of their individual weight, and the flexural rigidity, can also be taken as the average of their individual flexural rigidity. So, this is a simple form where you can obtain the expression for the deflection of both beams.

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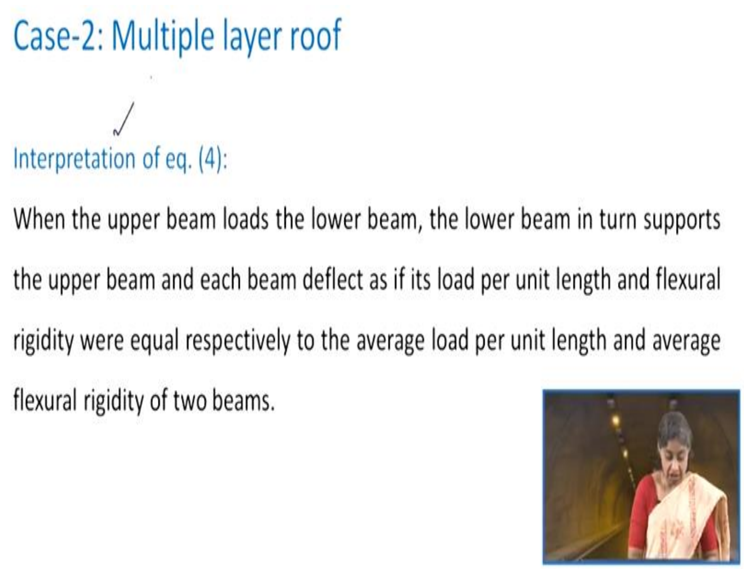

Now, the interpretation of this equation is going to be in this particular manner, when the upper beam loads the lower beam. Then the lower beam, in turn, it supports the upper beam, and each beam deflects as if its load per unit length, and the flexural rigidity were equal to the respective average load per unit length, and average flexural rigidity for the 2. As I explained to you, in this case, this q was taken to be $(q_1 + q_2)/2$, and flexural rigidity EI was taken as $(E_1I_1 + E_2I_2)/2$. So, it is kind of an average load per unit length of the beam and average flexural rigidity of both beams. So, this is how this equation 4 can be interpreted

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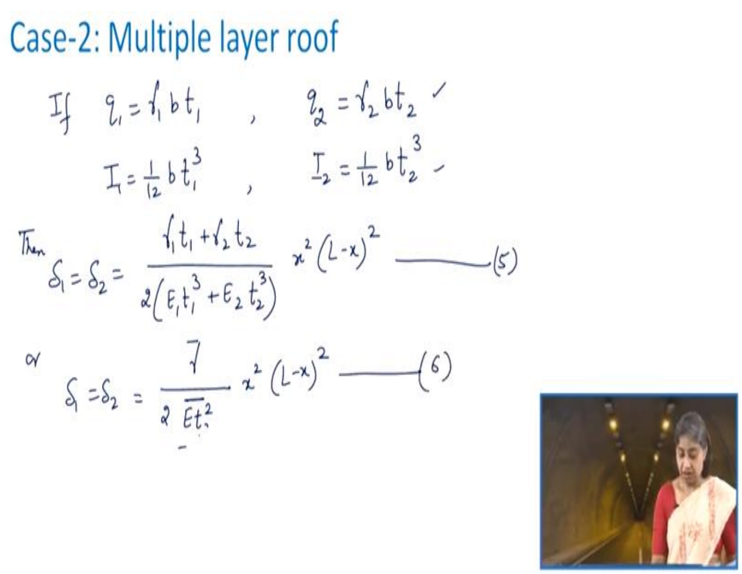

Now, if we have this

$$q_1 = \gamma_1 b t_1 \ , \ q_2 = \gamma_2 b t_2$$

$I_1 = \frac{1}{12} b{t_1}^3$, $I_2 = \frac{1}{12} b{t_2}^3$.

So, in that case, now if I just substitute all these expressions in our equation number 4. What we get as

$$\delta_1 = \delta_2 = \frac{\gamma_1 t_1 + \gamma_2 t_2}{2(E_1 t_1^3 + E_2 t_2^3)} x^2 (L - x)^2$$

This is equation number 5 or we can say that

$$\delta_1 = \delta_2 = \frac{\overline{\gamma}}{2\overline{Et^2}} x^2 (L - x)^2$$

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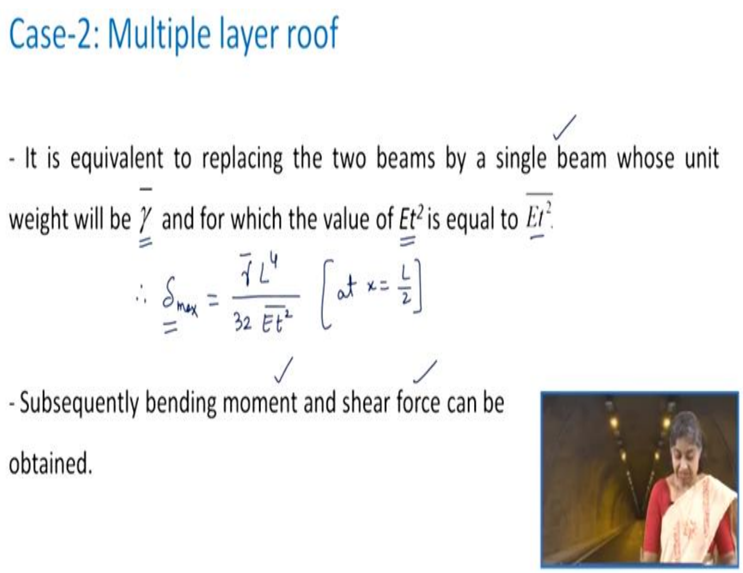

Now, what does this mean I have this $\overline{\gamma}$ as the average unit weight of 2 beams, or I can write it as

$$\overline{\gamma} = \frac{\gamma_1 t_1 + \gamma_2 t_2}{(t_1 + t_2)}$$

$$\overline{Et^2} = \frac{t_1 E_1 t_1^2 + t_2 E_2 t_2^2}{(t_1 + t_2)}$$

So, basically, this is a weighted average value of Et^2 for both beams. So basically, it is equivalent to replacing the 2 beams with a single beam, whose unit weight will be $\overline{\gamma}$ and for which the value of Et^2 will be equal to $\overline{Et^2}$.

So, that is how you can have the 2 beams replaced by a single beam, and therefore, in this case, you will have

$$\delta_{\max} = \frac{\overline{\gamma} L^4}{32\,\overline{Et^2}} \quad \left[\text{at } x = \frac{L}{2}\right]$$

that means at the middle point, and once you know this $\delta_{\max}$, subsequently you can obtain the bending moment, as well as the shear force, at any location x in the beam all along its length.

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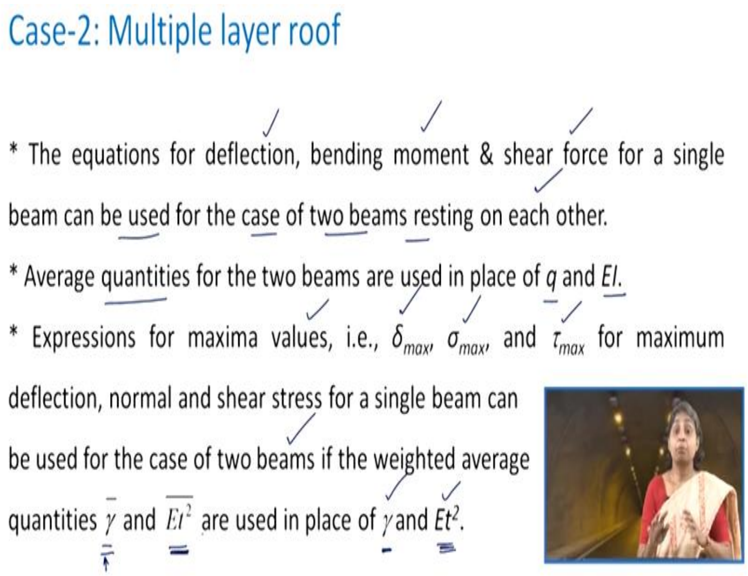

Now, the equation for the deflection, bending moment, and shear force for a single beam can also be used in the case of 2 beams, which are resting on each other. See, there is no separation between the 2 beams. What you need to do is that, you have to consider the average quantities of the 2 beams in place of q and EI. Then, the expression for maximum values which is, $\delta_{\max}$, σ_{max}, and τ_{max}, for the maximum deflection, normal and the shear stress for a single beam.

It can be used for the case of 2 beams, as if you are taking the weighted average of the quantities which are represented by $\overline{\gamma}$ and $\overline{Et^2}$ in place of γ and Et^2. So, whatever was the expression for the single beam in terms of γ and Et^2, in the case of the 2 beams, you just replace, γ by $\overline{\gamma}$ and Et^2 by $\overline{Et^2}$, and in that case, we can simplify the analysis, by considering these 2 beams as an equivalent single b.

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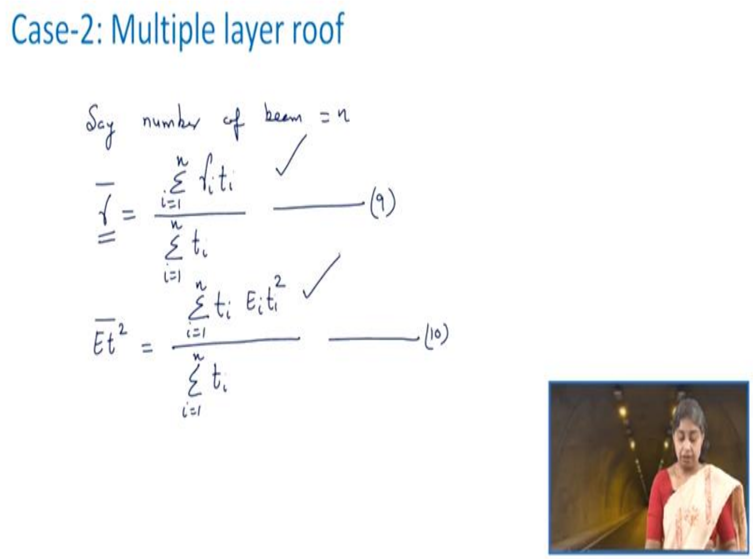

So, in case if we have N number of layers in the immediate roof then we can have, let us say the number of beams they are say N. So, in that case, we can write this

$$\overline{\gamma} = \frac{\sum_{i=1}^{n} \gamma_i t_i}{\sum_{i=1}^{n} t_i}$$

I will mark this as equation number 9 and then we have the

$$\overline{Et^2} = \frac{\sum_{i=1}^{n} t_i E_i t_i^2}{\sum_{i=1}^{n} t_i}$$

So, this is how we can handle any number of beams which are participating in the deformation, and we can find out the weighted average expression for $\overline{\gamma}$ and $\overline{Et^2}$.

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Case-2: Multiple layer roof

* As per the assumption: only deflections at any section, x are same for all the beams but maximum normal and shear stress for any one beam will depend upon the thickness of that beam.

So, accordingly as for the assumptions, the only deflections at any section x are the same for all the beams. But, normal and the shear force for each beam will depend upon the thickness of that beam. So, we cannot say that the maximum normal and the shear stress at any section x will be the same for all the beams. All we have assumed is that, only the deflections are going to be the same and not the maximum normal and the shear stresses.

So, this is how we can handle the situation of having multiple layers in the roof, which represent the analysis of excavation in laminated rocks. So, till now what we saw is that, we were using the theory of beams to analyze the excavation in laminated rocks. But, in the previous class, I explained to you that depending upon the aspect ratio of the opening, you have to either apply the theory of beams, or the theory of plates for the analysis.

Now, in case the situation is like that that the bending in the roof is happening both ways, that is not only in one direction, but in both the direction then, in that case, one has to use the theory of plates, how this is done what all are the various conditions, all these things, we will learn in the next class.