

**Underground Space Technology**  
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**Module No # 06**  
**Lecture No # 26**  
**Open in Laminated Rocks - 03**

Hello everyone, in the previous class, we learnt about multiple layers of the beams or the layers when we excavate in laminated rocks. So, we first saw that how the 2 layers or 2 beams can be analyzed, and then we extended or made it more generalized situation. In case if you have, say n number of layers which are participating in the deflection. So, in that case, we took the theory of beams to analyze such type of underground excavations in laminated rocks.

I mentioned to you that based upon the aspect ratio of the opening either you have to use the theory of beams or theory of plates. So today, we are going to discuss those cases where there is going to be biaxial bending in the roof portion, and therefore one has to go for the use of theory of plates for the analysis of opening in laminated rocks.

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### Rectangular roof

\* Case of Caverns:  $\frac{W}{B}$  or  $\frac{L}{B} \leq 2:1$

\* Immediate roof: acts as a rectangular plate subjected to bi-axial bending.

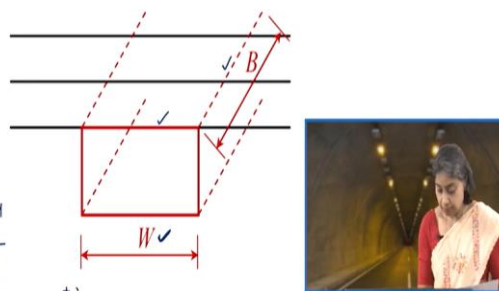
\* 3-D situation ✓

• Maximum deflection of immediate roof

under bi-axial bending

$$\delta_{max} = \frac{\alpha W^4}{Et^2}$$

W: shorter lateral dimension of excavation



So let us try to take a look that the cases of caverns where you have W by B or L by B is less than or equal to 2 is to 1. We all know that B is the dimension perpendicular to the plane of the screen, and W is the width of the opening. So, in case if the aspect ratio is such that W by B is less than or equal to 2 is to 1, which is in case of caverns. In that case, the immediate roof acts as the rectangular plate which is subjected to biaxial bending.

That means that the bending will happen in both directions in this direction as well as this direction. And therefore, one cannot use the theory of beams for the analysis because it becomes a 3-D situation. So, one needs to go for the theory of plates while analyzing the opening; having this W by B less than or equal to 2 is to 1. And in this case, the maximum deflection of the immediate roof in the condition of biaxial bending would be given by:

$$\delta_{max} = \frac{\alpha\gamma W^4}{Et^2}$$

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## Rectangular roof

\* Maximum stress:  $\sigma_{max} = \frac{6\beta\gamma W^2}{t}$  ← At centre of larger dimension

t: thickness of immediate roof

$$\alpha, \beta = f\left(\frac{B}{W}\right)$$

Where this W be the shorter lateral dimension of the excavation and maximum stress that can be given by:

$$\sigma_{max} = \frac{6\beta\gamma W^2}{t}$$

And this is going to occur at the center of larger dimension where this t be the thickness of the immediate roof and  $\alpha$  and  $\beta$  these are the function of B by W. So, this is how the maximum deflection and the maximum stress can be obtained. The question comes what should be the value of this  $\alpha$  and  $\beta$ ?

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## Rectangular roof

\* Coefficients for rectangular roof carrying udl -

$B/W$	1.0	1.25	1.50	1.75	2.0	$\infty$
$\alpha$	0.0138	0.0199	0.0240	0.0264	0.0277	0.0284
$\beta$	0.0513	0.0665	0.0757	0.0806	0.0829	0.0833

\* If  $R_o$ : modulus of rupture (outer fibre tensile strength)

$$\sigma_{max} = \frac{R_o}{F_s}$$

$$\therefore \text{Span width, } W = \sqrt{\frac{R_o t}{6 \beta \gamma F_s}}$$



So the, for the rectangular roof which is carrying the uniformly distributed load for various ratios of the B by W. Value of  $\alpha$  and  $\beta$ , they are tabulated here so in case if you have as the ratio as 1 then,  $\alpha$  is 0.0138 and  $\beta$  is 0.0513 and this  $\beta$  increase to 0.0833 when B by W tends to infinity. Similarly, this  $\alpha$  increases to 0.0284 for the ratio B by W tending to infinity. Now, if we define this  $R_o$  as the modulus of rupture, which is the outer fiber tensile strength.

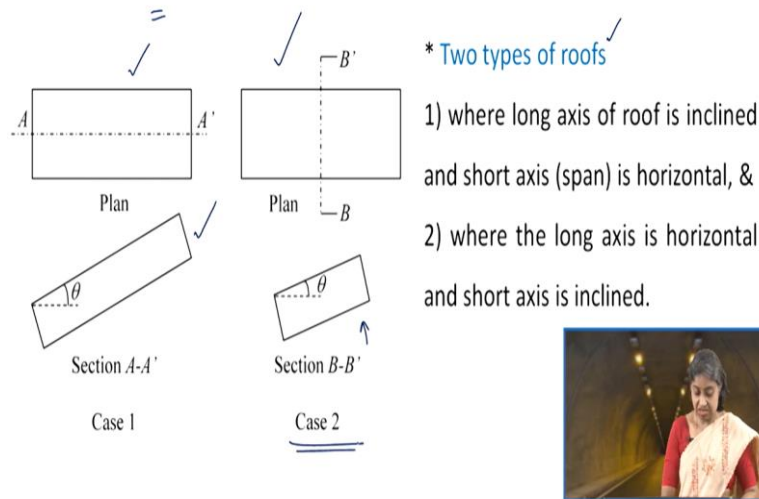
Then your  $\sigma_{max}$  is equal to  $R_o$  upon the factor of safety ( $f_s$ ), and therefore the span width W can be written as:

$$W = \sqrt{\frac{R_o t}{6 \beta \gamma f_s}}$$

So, this is how the span width is obtained in case of the roof acting as a plate or experiencing biaxial bending.

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## Inclined roof



Now as of now, we have discussed only the horizontal layers as a part of roof. But then there can always be the occurrence of the inclined roof. So let us take a look how can we handle such situations. There can be 2 types of roofs. Where we can have 2 situations: first is that the long axis, if the roof is inclined and short axis is horizontal. So, take a look let us see if this long axis and this is what is the short axis.

So, the situation is going to be like this so this short axis which is horizontal, but the long axis is inclined. So, this is how it looks like in plan, and in the section A-A', if you see this is how it will look like where this theta gives me the inclination of the long axis with the horizontal. However, there can be another situation where the long axis is horizontal, and the short axis is inclined.

This means it is like this so you the long axis is horizontal; it is not inclined, but the short axis is inclined. So, this is how it looks in plan, and if you just take a section B-B' it will look like this as part of case 2.

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## Inclined roof

\* Each type of roof: angle measured between roof and the horizontal is designated by  $\theta$

\* Assumption: stress in roof layer is due to flexure caused by gravity.

\* For either type, the maximum stress is given by

$$\sigma_{max} = \frac{\gamma W^2}{2t} \cos \theta \quad \text{for } \left(\frac{W}{B}\right) > 2:1 \quad \rightarrow \text{beam theory}$$

\* Design span width:

$$W = \sqrt{\frac{2t}{\gamma \cos \theta} \sigma_{max}} = \sqrt{\frac{2t R_o}{\gamma f_s \cos \theta}}$$



Now, each type of roof, whether it is type 1 or type 2 the angle that is measured between roof and the horizontal we are designating it by theta, and the assumption that is involved here is that the stress in the roof layer is due to the flexure which is caused by gravity. There are no external forces except for the force by gravity. So, for either type of the situation, the maximum stress is given by:

$$\sigma_{max} = \frac{\gamma W^2}{2t} \cos \theta$$

Now, this is there for W by B greater than 2 is to and since this is the situation, what we are going to use is beam theory. And hence the design span width can be given as:

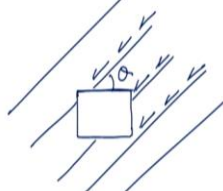
$$W = \sqrt{\frac{2t}{\gamma \cos \theta} \sigma_{max}} = \sqrt{\frac{2t R_o}{\gamma f_s \cos \theta}}$$

So, this is how we are going to obtain the design span width in case if we have the inclined roof and this is for the situation when you have W by B more than 2 is to 1.

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## Inclined roof

\* As  $\theta$  increases  $\rightarrow$  behavior of roof will create problem depending on mobilization of shear along plane of laminae.



\* Bond strength in across direction  $\rightarrow$  failure: tensile in nature.

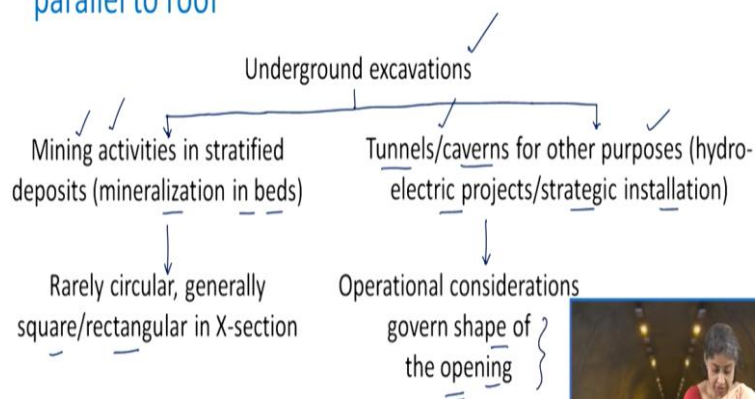


Now, as this theta increases that means the inclination of the layers of the beams, they increase the behavior of the roof will create problem depending upon the mobilization of shear along the lamina plane. See how it will look like the situation is going to be something like this, so here you have the opening, and there are the planes of weakness. So, there is going to be the mobilization of shear along these planes; this angle is theta.

So just imagine what will happen as a theta increase. This is going to have the problem the roof is going to have the problem. Since the bond strength in across the direction, it will cause the failure because of the tensile in nature.

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## Openings in rocks containing planes of weakness not parallel to roof



Now, coming to the opening which have the planes of weakness which are not parallel to roof. So, we can have the underground excavation divided into 2 parts or 2 categories. I should say the first one corresponds to the mining activities in stratified deposits. There can be

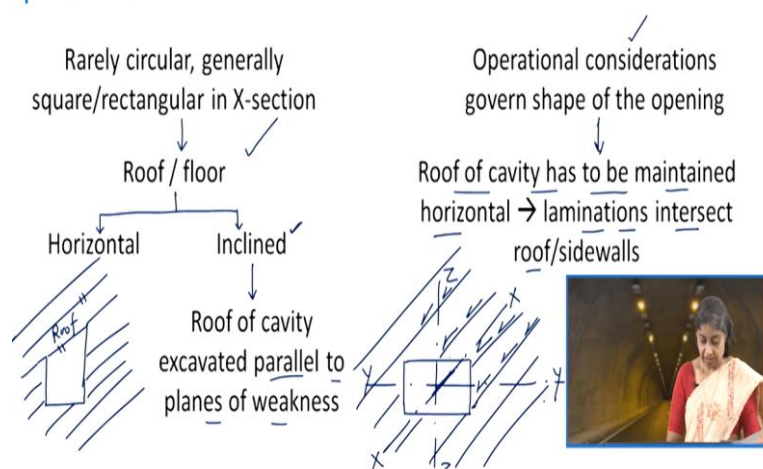
mineralization in beds. And other category belongs to the tunnels or caverns for other purposes, such as hydroelectric projects strategic installation.

Now in case, if we have the first situation, that is mining activities in stratified deposits, these are rarely circular and generally square or rectangular in cross-section. However, in the case of the tunnels and caverns, which belong to the other category of the underground excavation, we have the operational considerations which govern the shape of the opening. So, please remember that these 2 are the distinct main categories as far as underground excavations are concerned.

In case if you have the mining activities in stratified deposits, they are rarely circular usually, they are square and rectangle in cross-section. However, in case of tunnels or caverns which are used for the hydroelectric projects there the operational considerations are the ones which governs the shape of the opening, whether circular, elliptical, ovaloid, or square, or rectangular, or any other shape so that is what is the difference.

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### Openings in rocks containing planes of weakness not parallel to roof



Now, in case if you have the mining activity so I mentioned that these are rarely circular but generally square and rectangle in cross-section. So, there will have the roof or the floor, and these can be either horizontal or inclined. Now in case if they are inclined, the roof of the cavity excavated parallel to the plane of weakness. Take a look, how so you see here this is inclined, so what we do is we have the excavation in such a manner that the roof is parallel to the planes of weakness.

So, this is how we go for the opening in rocks which have the planes of weakness not parallel to roof, especially in case of the mining consideration. But in case if you have the situation

where operational considerations are governing the shape of the opening in that case the roof of the cavity has to be maintained horizontal. Because it is operational consideration so, in such cases, what will happen is that the laminations will intersect the roof or the side wall see how it looks like.

Now, say here I have the opening like this, so it is roof is to be horizontal, and you have the planes of weakness like this and say if you have, say this third direction. So maybe this is x-x, and this is z-z, and you have y-y axis. Here, there is going to be the mobilization of the shear stresses all along the plane of weakness like this.

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### Roof not parallel to laminations

\* Behavior of stratified / laminated rock in roof governs the design.

\* Function of ✓

- i) Attitude of planes of weakness (dip) ✓
- ii) In-situ shear strength of stratified / laminated / foliated rock along plane of weakness ✓
- iii) Bond strength across plane of weakness ✓



So, how to handle such situation so in this case, the behavior of these stratified or laminated rock in roof governs the design, and it is the function of attitude of the planes of weakness or also called as the dip. Then the in-situ shear strength of the stratified or laminated or foliated rock along the plane of weakness is another governing factor plus the bond strength across the plane of weakness.

So, these are basically the 3 parameters which govern the design of the excavation where the roof is not parallel to the laminations.

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## Roof not parallel to laminations

\* Bond strength in tension:

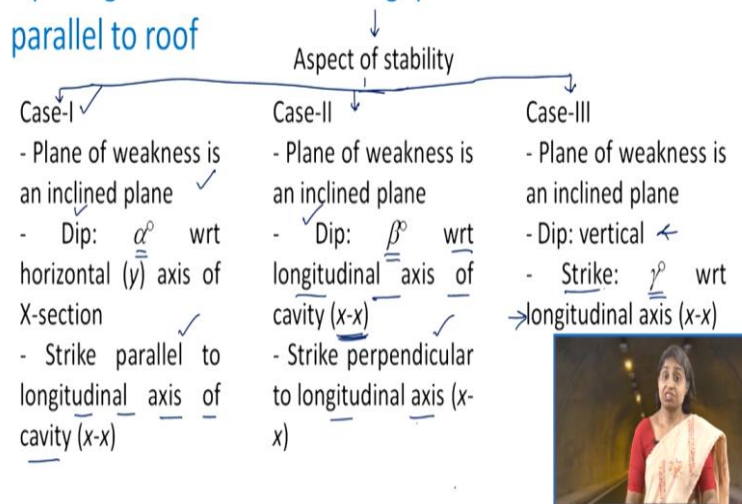
- i) zero, especially if soft gouge material is present along planes of weakness
- ii) Very high, if there exists a cementitious product along planes of weakness → could be even greater than strength of non-laminated rocks



Now, the bond strength in tension is zero, especially if there is a soft gouge material along the planes of weakness. But this can be very high if there is the existence of a cementation product along the planes of weakness, and it can be as high as that it can become even greater than the strength of the non-laminated rocks, so one needs to be careful here as well.

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## Openings in rocks containing planes of weakness not parallel to roof



Coming to the aspect of stability for the openings in rock containing planes of weakness are not parallel to the roof, you can have the 3 different cases. So, we have the first case, then case 2, and you have case 3. So here I have elaborated each of these cases, but then in subsequent slides, we will learn each case in detail one by one. So as far as the case 1 is concerned here of course, the plane of weakness is an inclined plane.

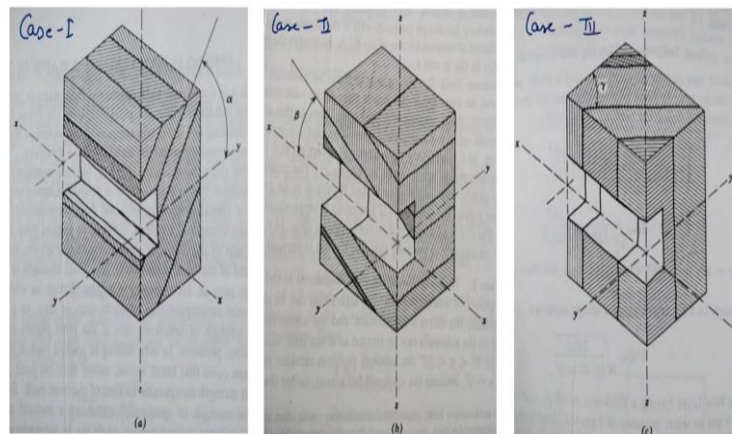
Having the characteristic as that its dip is going to be alpha degree with respect to horizontal, which is the y-axis of the cross-section, as I just now mentioned in the previous figure. Then

the strike of these planes they are parallel to the longitudinal axis of the cavity, that is the x-axis. In case 2, again the plane of weakness is an inclined plane. So basically, here we are dealing with all the 3 cases where the plane of weakness always an inclined plane.

But its inclination is different in all the 3 cases, and accordingly, we have defined these cases, so the dip is going to be beta degree with respect to the longitudinal axis of the cavity, which is the xx-axis. And the strike is perpendicular to longitudinal axis. When I show these cases with the help of figures, it will become more clear. In case 3, the dip is in vertical direction and the strike gamma degree with respect to the longitudinal axis, which is xx-axis.

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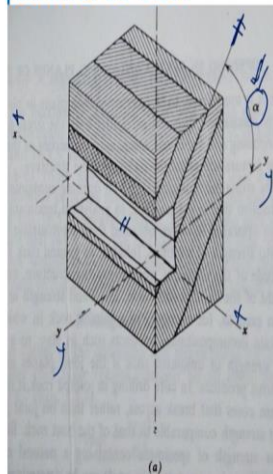
### Openings in rocks containing planes of weakness not parallel to roof



Now in the next slide, I have put the 3 figures showing various 3 cases, so this is what is your case 1, this is case 2, and this is the situation case 3. So, you see that as I explained the case 1 dip is alpha degree, in case of case 2 it was beta, in case of the third situation, the dip is vertical and you have this gamma degree here, which is coming into picture.

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## Openings in rocks containing planes of weakness not parallel to roof



Case-I

- Plane of weakness is an inclined plane
- Dip:  $\alpha^\circ$  wrt horizontal (y) axis of X-section
- Strike parallel to longitudinal axis of cavity

(x-x)

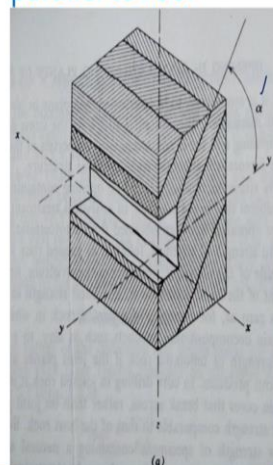


Let us understand these one by one with reference to first case again, I have reproduced the same thing but with the help of the figure. Let us try to understand of course, the planes of weakness they are inclined no more horizontal with respect to roof; of course, dip is alpha degree with respect to the horizontal axis of the cross-section. So, you see that this is what the horizontal axis which is yy, and with respect to this the dip is alpha degree. The strike is parallel to the longitudinal axis of the cavity, so the longitudinal at axis of the cavities is what here it is xx.

Now the strike direction is what where the plane cuts the horizontal plane, so you see that if you just take the look, then this will become parallel to this longitudinal axis. Now there can be various situations based upon the range of the value of this angle alpha. Accordingly, you will have to take care of the opening and the corresponding support system.

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## Openings in rocks containing planes of weakness not parallel to roof



Case-I

i) If  $\alpha = 0^\circ \rightarrow$  horizontal planes of weakness:  
underground excavations designed by using beam / plate theory

- Stress conc. & comp. strength of rock in sidewalls can be treated as if rock were isotropic.



Let us state the first case, that is, when  $\alpha$  equal to 0 degree, we have the horizontal plane of weakness. You see that if this  $\alpha$  becomes equal to 0, what we will have is the horizontal plane of weakness. And in that case, we will go back and use the beam or plate theory for the analysis of underground excavations and accordingly once we know the maximum reflection bending moment and shear force.

Accordingly, we can design the underground excavation stress concentration, and compressive strength of the rock in the side walls can be treated as if the rock were isotropic. In case, if you have the situation when  $\alpha$  is equal to 0 degree now, this  $\alpha$  can take other values like more than 0. So, we have divided these in 3 different ranges like when  $\alpha$  equal to 0,  $\alpha$  varying between 20 to 70 degrees, and  $\alpha$  being more than 70 degrees.

So, depending upon the value of  $\alpha$  accordingly, you will have to decide that how should we go about the analysis of the opening and subsequent design. Further, not only for the roof but for the side walls, also we have to see depending upon the inclination of these planes of weakness, we have to assign the support system or design the support system. So, remaining different values of  $\alpha$  for case 1, along with case 2 and case 3 that will learn in the next class. Thank you very much.