Underground Space Technology
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Module No # 07
Lecture No # 29
Elasto-Plastic Analysis of Tunnels: Tresca Yield Criterion-02

Hello everyone, in the previous class we discussed about elastoplastic analysis of circular tunnels using Tresca yield criterion. I mentioned to you that how the equations of equilibrium, compatibility equations, they can be solved with the help of appropriate boundary conditions. So, today we will continue our discussion with respect to Tresca yield criterion for the elastic-plastic analysis of circular tunnels, and we will try to see how the stress distribution takes place.

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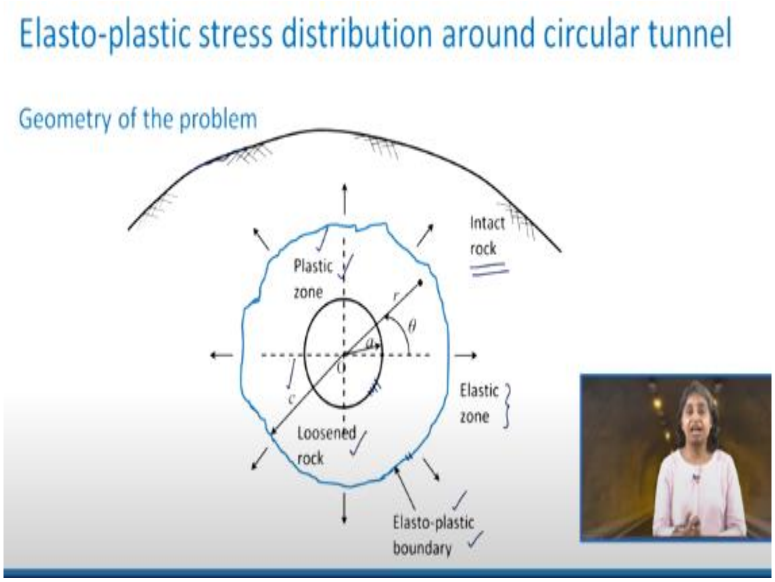

So, in order to have the continuity, first let us understand, (refer time: 01:08) once again that, what was the geometry of the problem, so here we had the ground surface, and we considered the rock medium to be intact rock. And upon the excavation of this circular tunnel of radius a, there was the formation of the plastic zone in the surrounding area of this circular excavation, which we were defining by a boundary called elastoplastic boundary.

Why we are calling it an elastoplastic boundary means, that the region between this excavation and the elastoplastic boundary is the loosened rock, and we are calling this as a plastic zone, and beyond this elastoplastic boundary we have the elastic zone. Then we considered that since the loading condition, boundary conditions, they all are symmetric, along with the symmetric geometry. So, we can very well assess that the elastoplastic boundary, will also be circular in shape with the radius c, so this, this was the geometry, and then we derived various expressions for stresses.

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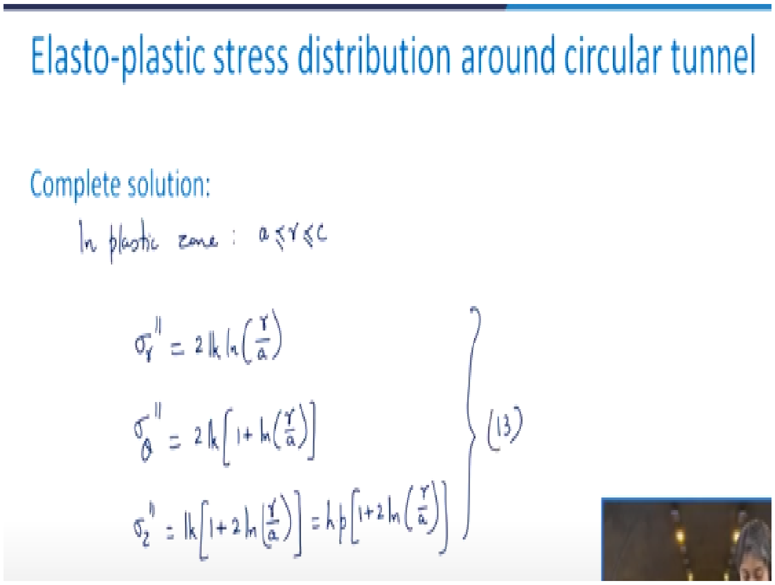

Let us try to write the complete solution, so we had here the complete solution as in plastic zone this means, that the radius is varying between a and c. Where a is the radius of the circular tunnel and c be the radius of the elastoplastic boundary. So, what we obtain as the radial stress remember in plastic zone, we were representing all these stresses with double prime. So, that is why we are writing here as,

$$\sigma_r^{''} = 2lk\ln\left(\frac{r}{a}\right)$$

then we had

$$\sigma_\theta^{''} = 2lk[1 + \ln\left(\frac{r}{a}\right)]$$

Then,

$$\sigma_z^{''} = lk\left[1 + 2\ln\left(\frac{r}{a}\right)\right] = hp[1 + 2ln\left(\frac{r}{a}\right)]$$

so, these equations let me represent by equation number 13.

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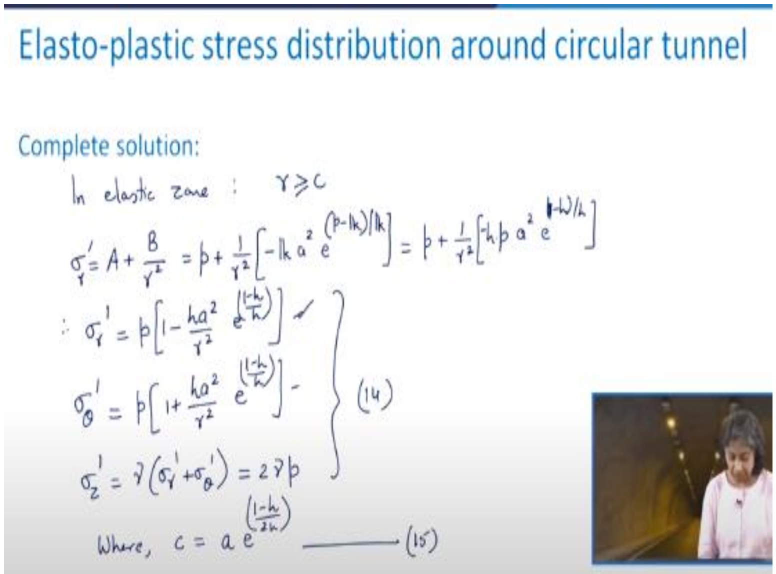

Now, what will happen in the elastic zone r will be greater than or equal to c, and we had

$\sigma_r^{'} = A + \frac{B}{r^2}$

and we found out the expression for these constants a and b so just substitute them here. So, what we will have is

$= p + \frac{1}{r^2}[-lk\ a^2 e^{(p-lk)/lk}]$

I will write

$= p + \frac{1}{r^2}[-hpa^2 e^{(1-h)/h}]$

So, we can write the

$\sigma_r^{'} = p[1 - \frac{ha^2}{r^2} e^{(\frac{1-h}{h})}]$

So, this is what is going to define the radial stress in the elastic zone.

Coming to

$\sigma_\theta^{'} = p[1 + \frac{ha^2}{r^2} e^{(\frac{1-h}{h})}]$

$\sigma_z^{'} = \gamma \left(\sigma_r^{'} + \sigma_\theta^{'}\right) = 2\gamma p$

And, this will be if you substitute this and this here, so, what you will get is

$= 2\gamma p$

So, I will make these equations as equation number 14. And, we found out the expression for

$c = ae^{(\frac{1-h}{2h})}$

now you see that here, we if we substitute at h equal to 1 what will happen?

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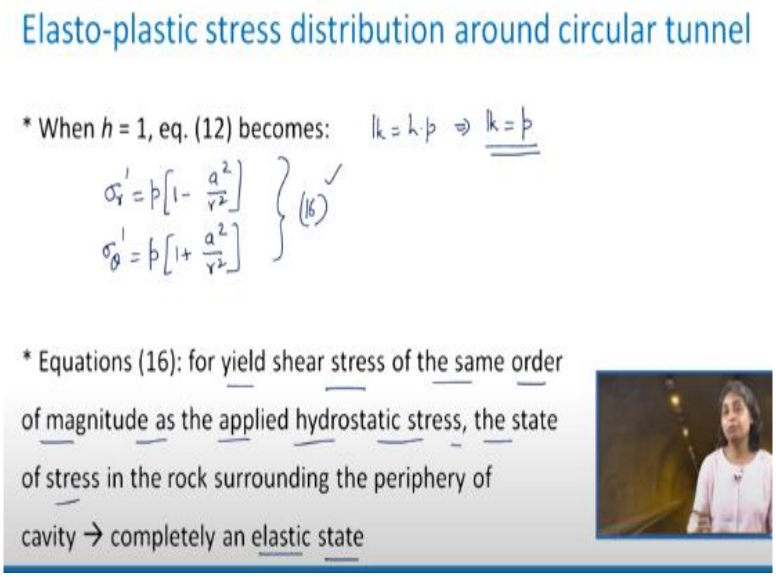

So, this equation 12 which was

$lk = h.p$

So, this will become

$lk = p$ for h =1

So, what we will have is

$$\sigma_r' = p[1 - \frac{a^2}{r^2}]$$

And, we will get

$$\sigma_\theta' = p[1 + \frac{a^2}{r^2}]$$

I will mark these equations as equations number 16. Now, these equations 16, in case if you have the yield stress of the same order of the magnitude as the applied hydrostatic stress that means, if this condition is satisfied that $lk = p$.

In this case, you see what will happen is that there is going to be the state of stress, which will be completely elastic which can be seen from this equation number 16. And, if you compare this with the results of the elastic analysis of the circular tunnel you will see that there also we got the same expression. So, what does that mean, that in case if the yield stress is of the same order of the applied hydrostatic stress, in that case, it is going to be the completely elastic state, and there will not be any formation of plastic zone.

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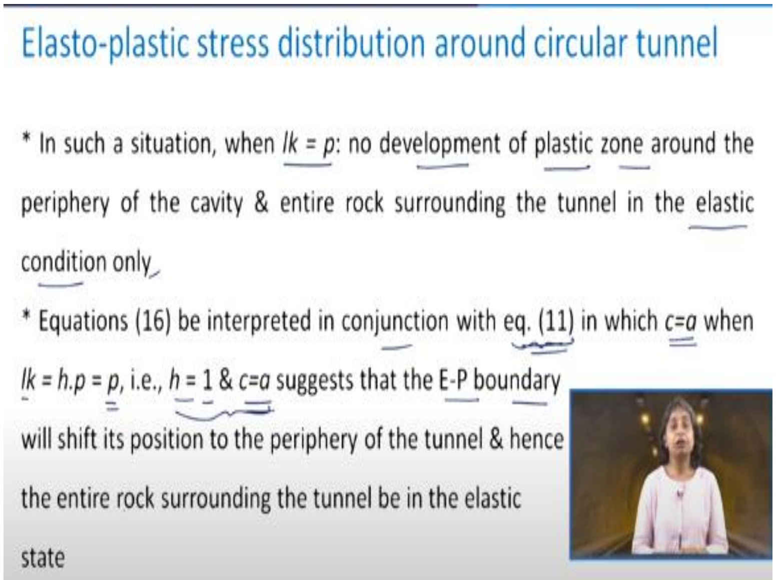

So, when we have the situation, where $lk = p$, there is going to be no development of the plastic zone around the periphery of the opening or the cavity. And, hence the entire rock surrounding the tunnel will be in the elastic condition only, So, this equation 16 can be interpreted in conjunction with equation 11. Now, when I say this equation 11, you need to refer back to the previous lecture, because this is the continuation of that, where we had c = a when kappa becomes equal to p or h = 1 and c = a.

So, these 2 conditions, these suggest that the elastoplastic boundary will shift its position to the periphery of the tunnel, and therefore the entire rock surrounding the tunnel will be in the elastic state.

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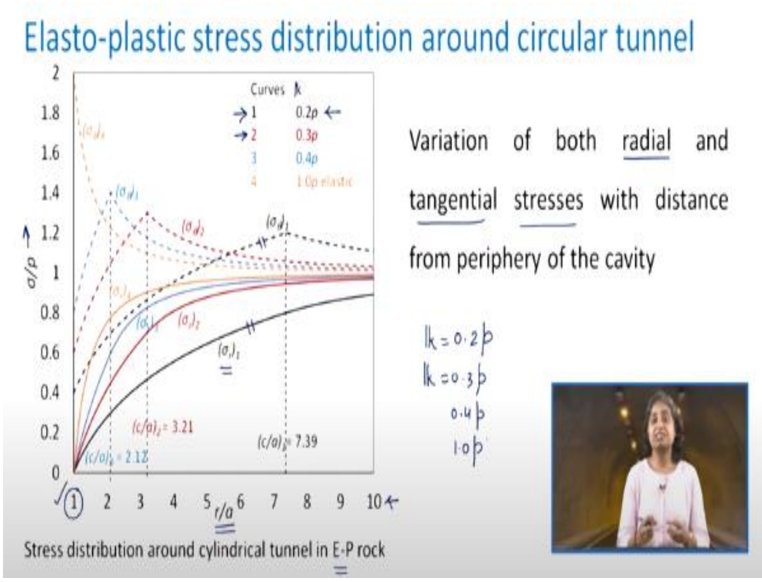

Now, here is the stress distribution around the cylindrical tunnel in the elastoplastic rock where this rock is following the Tresca yield criteria. So, in this figure the variation of both radial as well as tangential stresses, with distance from the periphery of the cavity has been shown. So, you can see that on x-axis we have plotted r/a and that is starting from, so 1 corresponds to the location at the periphery of the tunnel because there r becomes equal to a, and r/a ratio will become equal to 1 and then we have taken up to 10 and on y-axis we have sigma upon p.

Now, this sigma can be radial stress or it can be tangential stress, so what the various curves correspond to is that various values of this kappa and different curves have been drawn. Kindly note that I have color coordinated here in this figure for example, the first curve which corresponds to point two times p of the yield stress, so that means here in this case

$$lk = 0.2p$$

So, all the figure, all the plots with black color they correspond to this first case, which is this plot, and this plot then the solid lines they are representing the radial stresses, and the dotted ones they are representing the tangential stresses. Similarly, this red color is for the yield stress as 0.3p, blue is for 0.4 p, and the orange one is for 1.0p, that means when the yield stress becomes equal to the in-situ stress. Let us try to discuss these cases one by one in detail.

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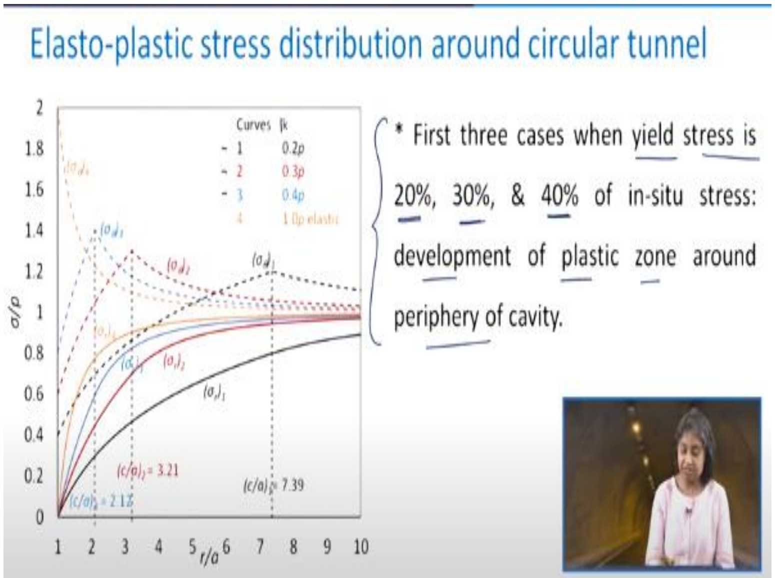

So, the first three cases 1, 2, 3 these correspond to when the yield stress is 20%, 30% and 40% of the in-situ stress respectively. And, in all the three cases there is going to be the development of plastic zone around the periphery of the cavity. I explained you that when the yield stress becomes equal to the in-situ stress, in that case there will not be any development of the plastic zone around the cavity.

But, when it has 20%, 30% and 40% of the in-situ stress then in that case there is going to be the development of the plastic zone, so, you should keep these things in mind, so here this is kappa.

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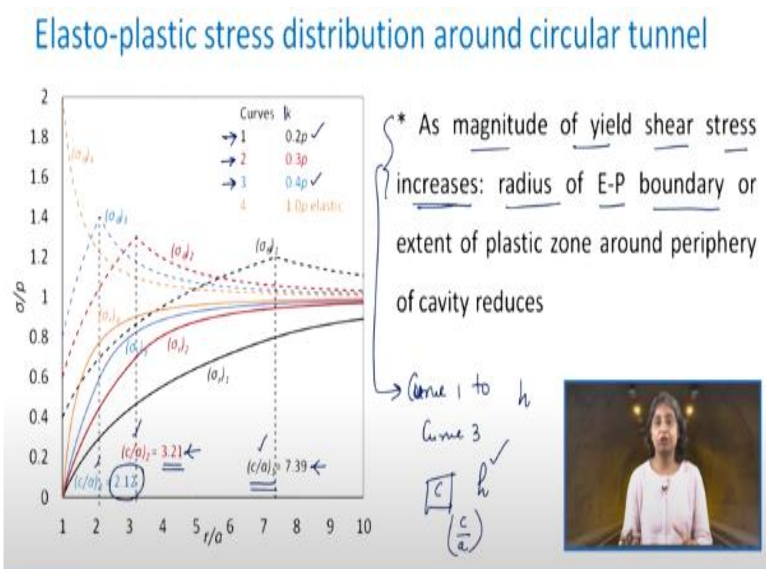

Now, as the magnitude of yield stress increases this means, what as we move from curve 1 to curve 3 see this is increasing from 0.2p to 0.4p. What happens to the radius of elastoplastic boundary, this around the periphery, it reduces or the extent of the plastic zone around the periphery of the cavity reduces. And, this is very obvious because when we have h to be equal to 1 this elastoplastic boundary it coincides with the tunnel periphery.

So, lower is the value of the yield stress, bigger is going to be the radius of the elastoplastic boundary, or larger will be the extent of plastic zone around the periphery of the cavity. Now, you know what is the expression for c, I just now explained you, so, we can substitute different values of h in that expression and find out the corresponding value of c/a. So, see in case of the first one where you are getting c/a as 7.39 while, when this yield stress increases from 20% to 30% of the in-situ stress that is the case 2, in that case it is reducing and it is becoming 3.21.

Any, further increase say to 40% of the in situ stress, and you see, here the radius of the elastoplastic boundary or the extent of the plastic zone, it reduces to 2.12. So, this is what you can get from those expressions you just simply substitute various values of h, and you will be able to obtain this c/a values, for any value of h. So, this is how we can have this general statement that, with the increase in the magnitude of yield shear stress the extent of the plastic zone around the periphery of the cavity reduces.

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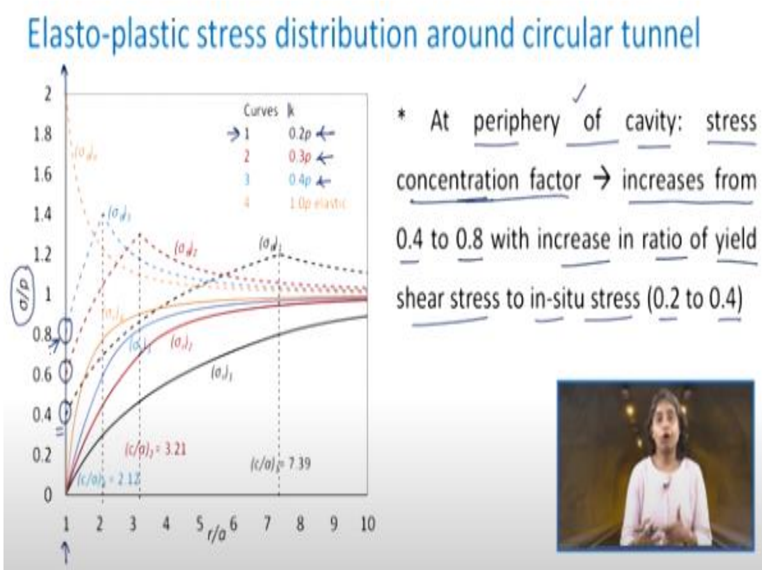

Then, the next thing which should be noted here is that, at the periphery of cavity, what happens to the stress concentration factor. So, when I say periphery of the cavity this means what? r/a = 1, so I will now focus on this axis where r/a = 1. This stress concentration factor which has been plotted as sigma by p on y axis it increases from 0.4 to 0.8 with increase in the ratio of yield shear stress to in situ stress from 0.2 to 0.4.

Take a look here, this corresponds to 0.2p situation, and this corresponds to 0.3 p situation, and this corresponds to 0.4 p situation. So, see here at r/a, equal to 1, corresponding to the first case this value is 0.4 and for the third case this value is 0.8. So, we can say that as the ratio of yield shear stress to in situ stress it increases from 0.2 to 0.4, the corresponding increase in the stress concentration factor at the periphery of the cavity increases 0.4 to 0.8.

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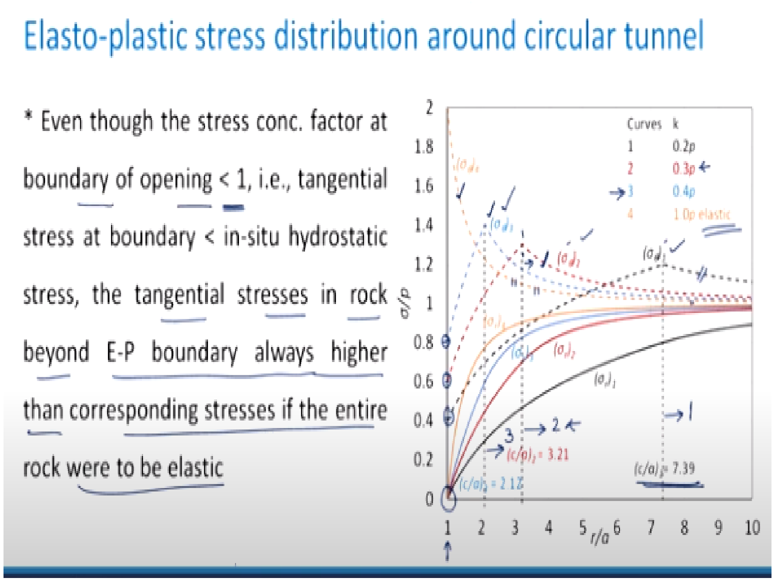

Although the stress concentration factor at the boundary of the opening is less than 1 so boundary of the opening is this r/a = 1. So, you see that for all, these stress concentration factor these are all less than 1 and of course sigma r will be equal to 0 because it is the stress-free boundary and the radial stresses are going to be equal to 0 that is what, was our bound 1 of the boundary condition.

So, even though the stress concentration factors at the boundary they are less than 1, or in other words we can say that the tangential stresses both, sigma theta 1, sigma theta 2, and sigma theta 3 all are less than 1. The tangential stress in the rock beyond the elastoplastic boundary is always higher than the corresponding stresses, as if the entire rock were to be elastic. Take a look here when the complete rock is elastic, what is that case? it is this one, this orange color one.

Now, take a look of the situation beyond the elastoplastic boundary in each of the respective case that means in the first case, I need to take a look beyond this, in the second case beyond this, and the third case beyond this, this is the first case, second case, and then the third case. Now, you see here if you take the third case, see this orange curve is always below this blue curve which corresponds to the third situation.

So, in this case it is true, that the tangential stress beyond the elastoplastic boundary is higher than the situation, if the entire rock mass were to be elastic. Similar is the situation if you see the case 2 here, this is what is the elastoplastic boundary situation and see beyond this again, the red curve is above the orange curve corresponding the situation. That at any location the stress concentration factor for the second case will be larger than the elastic situation.

And, the same situation is there in the first case, where you have the yield shear stress as 20 % of the in-situ stress, take a look here this black dotted line is above this orange line, which is corresponding to the elastic case. So, this is again another conclusion which you can derive from the study of this stress distribution, around the circular tunnel when the rock is following elastoplastic condition.

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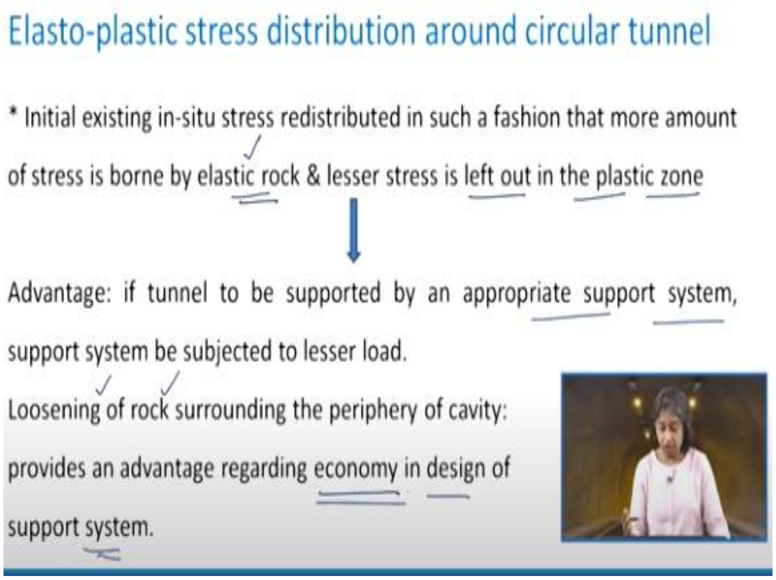

So, what we can have from here is that the initial existing in-situ stress it gets redistributed in such a fashion that more amount of stress is born by the elastic rock and the lesser stress is left out in the plastic zone. The advantage of this is that if the tunnel is to be supported by an appropriate support system, those support systems will be subjected to lesser load because larger portion of the stresses loads, they are being taken by the elastic portion of the rock.

Now, what happens here is that the loosening of the rock which happens around the periphery of the cavity, or surrounding the periphery of the cavity, it provides an advantage regarding economy in the design of the support system. This, we will learn in detail when we study about the rock mass tunnel support interaction analysis. So, if you allow the deformation to take place of course these stresses on to the support system, they are going to be low and therefore we can achieve the more, larger economy in the design of the support system.

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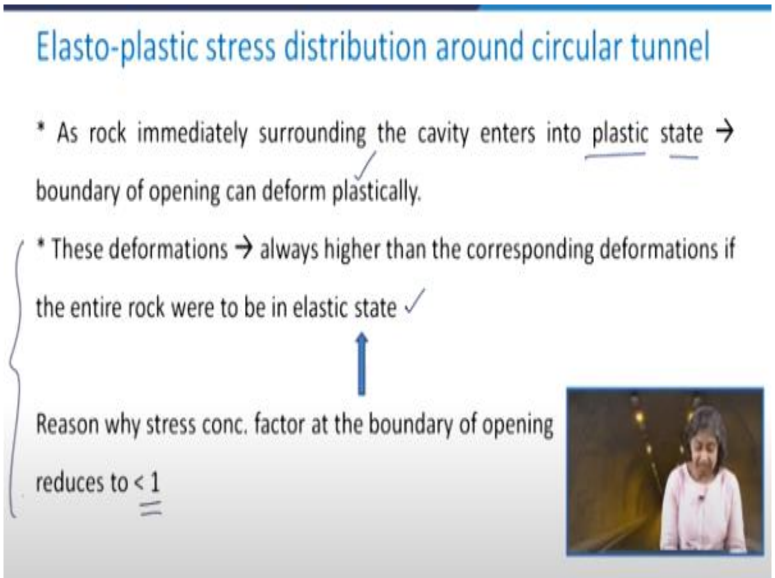

So, what happens is that, as the rock immediately surrounding the cavity enters into the plastic state, what happens to the boundary of the opening it can deform plastically. Now, these deformations they are going to be always higher than the corresponding deformations, if the rock were to be in the elastic state. And that is the reason, why stress concentration factor at the boundary of the opening they are less than 1. Please remember this is an extremely important observation.

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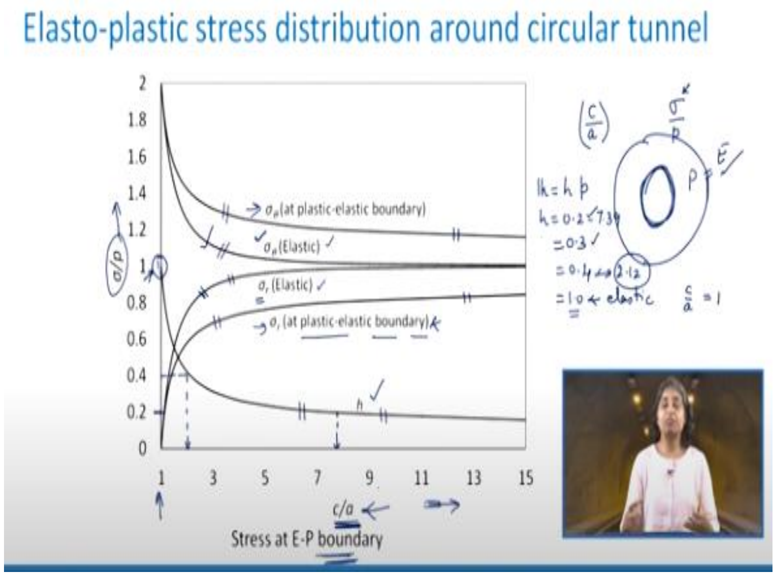

So, here is a figure, which gives us the idea about the stress distribution at the elastoplastic boundary. On the x-axis we have this c/a, where the c was the radius of the elastoplastic boundary and a, was the radius of the circular tunnel. So, basically on the x-axis we are presenting the non-dimensional form of the radius of elastoplastic boundary. And on the x-axis, we have this stress concentration factor, which is sigma upon p.

Now, this sigma can be sigma r or sigma theta that is, it can be the radial stress or the tangential stress and its ratio has been taken with respect to the in-situ stress. Now, take a look here, that as far as the elastic zone is concerned because, when you have the elastoplastic boundary. So, see if this is let us say your circular tunnel, so we had the elastoplastic boundary so at the boundary here we have the elastic situation, and here we have the plastic one.

So, this sigma r and sigma theta their variation in the elastic domain at this only will be something like this, and at the elastoplastic boundary, it is going to be obtained following these two plots. Now, we have also shown here the variation of the ratio of yield shear stress, to the in-situ stress. Remember, that the yield shear stress is $lk = h.p$, and if you recall in some of the earlier figure, we took different cases that means, when h was taken to be 0.2, h was taken to be 0.3 and to be taken as 0.4 and when we took it to be 1 it was completely elastic situation.

So, let us say that we have this elastic situation, so in that case, c/a will be equal to what? it is going to be equal to 1, so that is what is seen here. You see it is the value of h which is equal to 1 and here this c/a = 1, because in this case the radius of elastoplastic boundary becomes equal to the radius of the circular tunnel, this means that both concise. But then, we have seen that when h = 0.2, 0.3, and 0.4, if you just recollect for this we had maybe something like 7.39 and for this I think we had 2.12.

So, you see that here, if you take h to be equal to 0.2, and see what is going to be the value here it is somewhere here this will work out to be 7.4, similarly for 4.4 here you see this is going to be something like 2.12. So, this is how the variation of h will be there with respect, to the non-dimensional radius of the elastoplastic boundary. So, this is how we can find out the elastoplastic stress distribution around the circular tunnel, which is excavated in the rock following Tresca yield criterion.

Now, it is not necessary that always the rock will follow Tresca yield criterion, there are n number of other failure criterion, and some of these include Mohr-Coulomb failure criterion, hoek and brown criterion, and the list is long. So, what we will do is we will take up in the next class, another analysis where we will assume that the rock follows Mohr-Coulomb failure criterion.

Although the analysis is going to be on the similar way as we did for the Tresca yield criterion case, but we will get some interesting expressions for the stress distribution around the circular tunnel excavated in the rock, which is following more coulomb failure criterion, thank you very much.