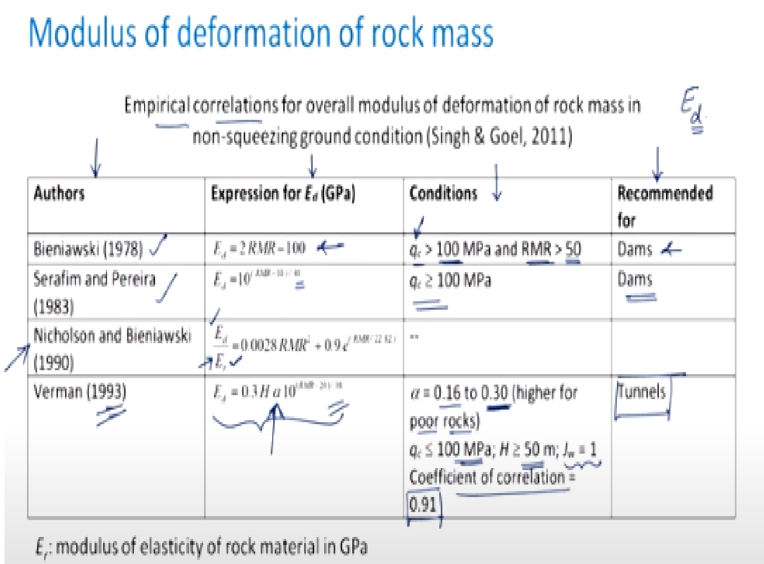
Underground Space Technology
Prof. Priti Maheshwari
Department of Civil Engineering
Indian Institute of Technology, Roorkee

Module No # 08
Lecture No # 38
Modulus of Deformation of Rock Mass Using Q – System, Rock Mass Number, Plate Loading

Hello everyone, in the previous class we discussed about the application of Q system to the underground excavation design, we also learnt about NATM and NMT. So, today we will learn about the modulus of deformation of rock mass and how this can be determined using Q system using the rock mass number N, and one of the field tests for the determination of modulus of deformation of the rock mass, that is plate loading test.

So, first we see that how the modulus of deformation of rock mass can be determined on the basis of various empirical correlations in the non-squeezing ground condition. So, various authors have given different expressions for modulus of deformation.

(Refer Slide Time: 01:24)


Modulus of deformation of rock mass

Empirical correlations for overall modulus of deformation of rock mass in non-squeezing ground condition (Singh & Goel, 2011)

Authors	Expression for E_d (GPa)	Conditions	Recommended for
Bieniawski (1978)	$E_d = 2\,RMR - 100$	$q_c > 100$ MPa and RMR > 50	Dams
Serafim and Pereira (1983)	$E_d = 10^{(RMR-10)/40}$	$q_c \geq 100$ MPa	Dams
Nicholson and Bieniawski (1990)	$\frac{E_d}{E_r} = 0.0028\,RMR^2 + 0.9\,e^{(RMR/22.82)}$	--	
Verman (1993)	$E_d = 0.3\,H^{\alpha}\,10^{(RMR-20)/38}$	α = 0.16 to 0.30 (higher for poor rocks) $q_c \leq 100$ MPa; $H \geq 50$ m; $J_w = 1$ Coefficient of correlation = 0.91	Tunnels

E_r: modulus of elasticity of rock material in GPa

And, this modulus of deformation is represented as E sub d or E with a small d in subscript. So, the first column gives us the idea about the authors and the year in which they propose these expressions all these are empirical correlations, so keep this in mind. Then, the second one is the expression for the modulus of deformation of rock mass, if there are any conditions for which that, these are applicable, that has been given in third column.

And, if there are any particular cases for which these expressions are recommended that has been given in the last column. So, to start with first Bieniawski in 1978 proposed this expression that is, E_d = 2 RMR – 100, where the condition is applicable that Qc should be more than 100 MPa and RMR should be more than 50. And, these were recommended for dams, so here this qc is the UCS of the intact rock.

Then, the next quite popular expression was given by Serafim and Pereira that is

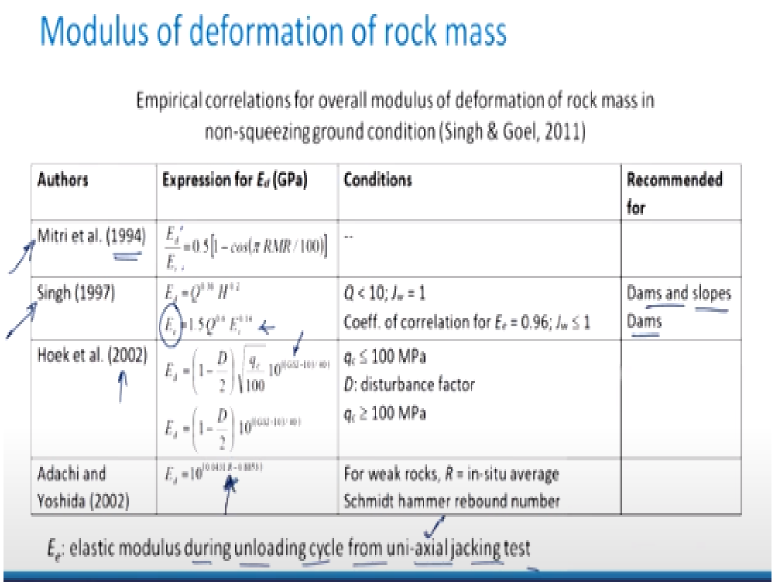
$$E_d = 10^{(RMR-10)/40}$$

Where this qc is more than or equal to 100 MPa and this was also recommended for dams. Similarly, Nicholson and Bieniawski in 1990 proposed this expression, where they gave the ratio of modulus of deformation to the modulus of elasticity of the rock material. So, when I say modulus of elasticity of rock material, essentially, we talk in terms of the modulus of elasticity of the intact rock.

Then, Verman in 1993 came out with this expression for the modulus of deformation where there is a term called alpha(α), so what all are the values for α, so α varies between 0.16 to 0.3. And higher the value of alpha that means, if it approaches towards 0.3 that means it is the poor rock. Then q_c is less than or equal to 100 MPa, H which is the depth of overburden is greater than or equal to 50 meters J_w's ratings to be taken to be equal to 1.

And, since these are all empirical correlations, so, when they had lot of data they tried plotting in one space, in that case, the coefficient of correlation worked out to be 0.91, and this expression was recommended to be used in case of tunnels.

(Refer Slide Time: 04:27)



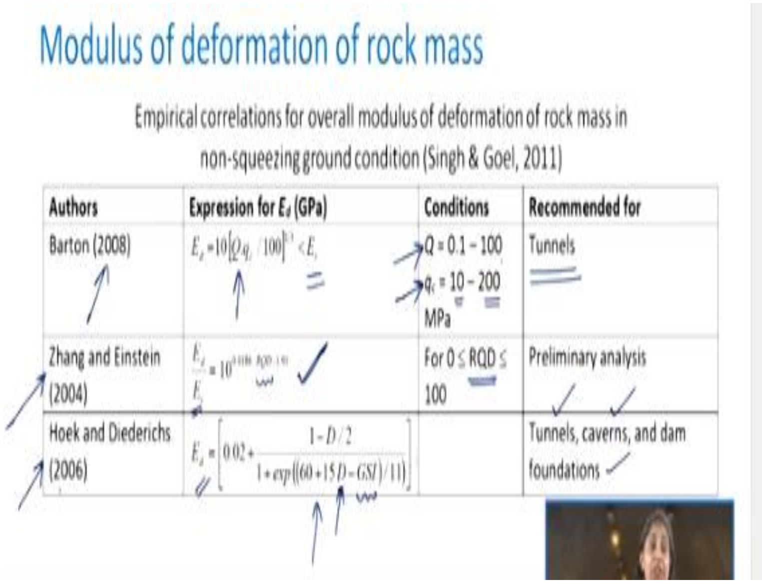
Similarly, there are other correlations for non-squeezing ground condition as it is given by Mitri et al in 1994 it is the expression

$$\frac{E_d}{E_r} = 0.5\left[1 - \cos\left(\frac{\pi RMR}{100}\right)\right]$$

Then, Singh in 1997, this was recommended for dams and slopes, and the second one is for the dams, here this E_e is the elastic modulus during unloading cycle from the uniaxial jacking test. Maybe, in one or the 2 subsequent classes, we will discuss about this test uniaxial jacking test.

Then, Hoek et al in 2002 came up with the expression which made use of geological strength index GSI. Adachi and Yoshida in 2002, gave the expression which used the in-situ average Schmidt hammer rebound number R which is there in this expression at this location.

(Refer Slide Time: 05:42)



Authors	Expression for E_d (GPa)	Conditions	Recommended for
Barton (2008)	$E_d = 10[Q q_c / 100]^{1/3} < E_r$	$Q = 0.1 - 100$; $q_c = 10 - 200$ MPa	Tunnels
Zhang and Einstein (2004)	$\frac{E_d}{E_r} = 10^{0.0186\, RQD - 1.91}$	For $0 \le RQD \le 100$	Preliminary analysis
Hoek and Diederichs (2006)	$E_d = E_r\left[0.02 + \frac{1 - D/2}{1 + exp((60 + 15D - GSI)/11)}\right]$		Tunnels, caverns, and dam foundations

Another, important empirical correlations, they include the one given by Barton in 2008, where he proposed that the modulus of deformation is equal to

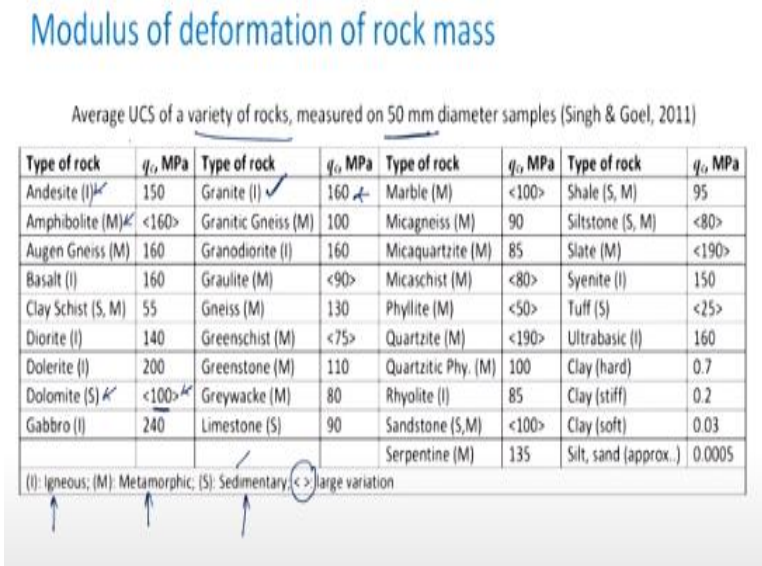
$$E_d = 10[Q.\frac{q_c}{100}]^{\frac{1}{3}}$$

And this should be less than E sub r the conditions which are to be satisfied include Q to be equal to 0.1 to 100, and Q_c varying between 100 to 200 megapascal, and these are recommended for the tunnels, so this expression should be used in case of the tunnels.

Zhang and Einstein, it is given in 2004, they made the use of RQD and E_r to determine the modulus of deformation of course the hopeful range for RQD can be considered. And this was recommended that this expression should be used only for the preliminary analysis and, not for the design stage. Then, Hoek and Diederichs in 2006, they came out with the expression which made use of this disturbance factor D and GSI to determine the modulus of deformation of the rock mass.

If you recall in one of our earlier lectures, we, I have already told you that, how we can determine the value of this D. This expression can be used for tunnels, caverns, and also for the dam foundation. So, this is how you using the empirical correlations, you, can determine the modulus of deformation, and so you have seen that, the use of RMR is there use of Q is there use of RQD is there use of GSI is there. So, various empirical correlations proposed by different research workers, and based upon their study, their fields study their lab study they came out with these expressions.

(Refer Slide Time: 07:54)



Modulus of deformation of rock mass

Average UCS of a variety of rocks, measured on 50 mm diameter samples (Singh & Goel, 2011)

Type of rock	q_c, MPa	Type of rock	q_c, MPa	Type of rock	q_c, MPa	Type of rock	q_c, MPa
Andesite (I)	150	Granite (I)	160	Marble (M)	<100>	Shale (S, M)	95
Amphibolite (M)	<160>	Granitic Gneiss (M)	100	Micagneiss (M)	90	Siltstone (S, M)	<80>
Augen Gneiss (M)	160	Granodiorite (I)	160	Micaquartzite (M)	85	Slate (M)	<190>
Basalt (I)	160	Graulite (M)	<90>	Micaschist (M)	<80>	Syenite (I)	150
Clay Schist (S, M)	55	Gneiss (M)	130	Phyllite (M)	<50>	Tuff (S)	<25>
Diorite (I)	140	Greenschist (M)	<75>	Quartzite (M)	<190>	Ultrabasic (I)	160
Dolerite (I)	200	Greenstone (M)	110	Quartzitic Phy. (M)	100	Clay (hard)	0.7
Dolomite (S)	<100>	Greywacke (M)	80	Rhyolite (I)	85	Clay (stiff)	0.2
Gabbro (I)	240	Limestone (S)	90	Sandstone (S,M)	<100>	Clay (soft)	0.03
				Serpentine (M)	135	Silt, sand (approx..)	0.0005

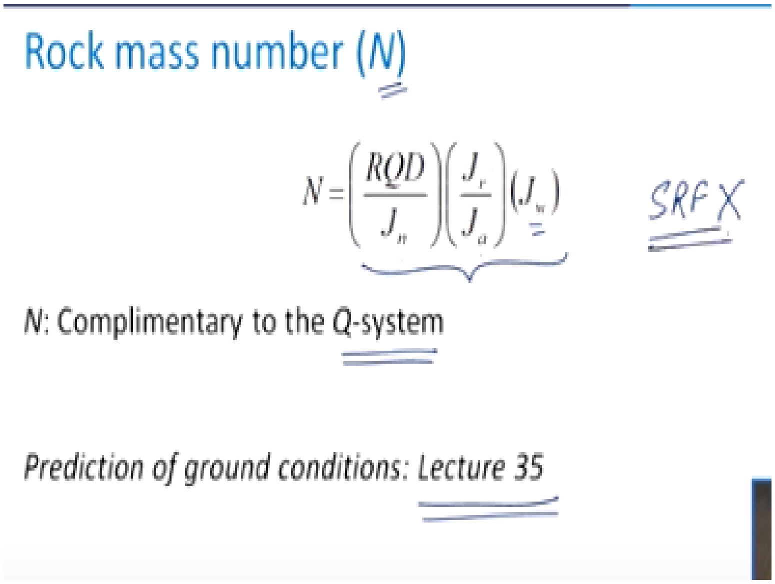
(I): Igneous; (M): Metamorphic; (S): Sedimentary; < > large variation

Coming to the average UC is because this is used in many of the expressions earlier, so this average UCS value for a variety of rocks which is measured on 50 mm diameter samples they are mentioned here. You are seeing some alphabets in the brackets for example you see here I is written this is M, so this corresponds to I as igneous, M as metamorphic, and, S as sedimentary. And, wherever you have such type of symbol, for example here this means that the large variation takes place.

So, let us say for example granite is an igneous type of rock and typically its average UCS value is going to be about 160 MPa. Say, we talk about dolomite, dolomite is a sedimentary type of rock here and the average value is although 100 MPa but the large variation is observed in the value of the average UCS. So, similarly for different types of rocks, you can use this table and you can pick the typical values, although we should conduct the test UCS test in the lab to determine the value of UCS.

But, let us say, if we are not able to do, or if we are going for the preliminary analysis, and we just need to get some idea then maybe this table can be referred. Coming to the concept of rock mass number, although this we have discussed earlier as well, if you refer to lecture 35 you will see that we discussed the prediction of ground condition based upon this rock mass number.

(Refer Slide Time: 09:45)

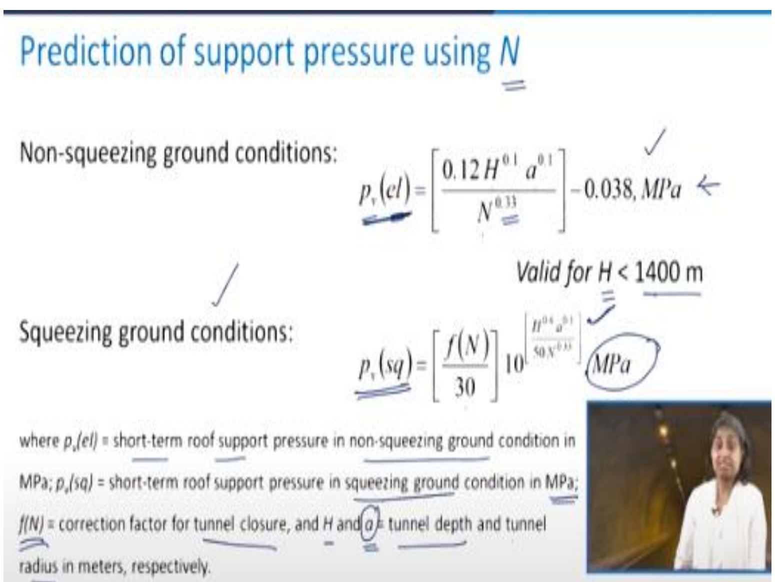


But to have the continuity, let us come back to this concept once again so this rock mass number N is complementary to the Q system. And this is defined by this expression where,

$$N = \left(\frac{RQD}{J_n}\right)\left(\frac{J_r}{J_a}\right)(J_w)$$

here this factor, SRF is not there as it was there in Q system. So, basically, this SRF value is assigned as 1.

(Refer Slide Time: 10:15)



So, in case of the non-squeezing ground condition using this rock mass number N, we can predict the support pressure by using this expression that is

$$p_v(el) = \left[\frac{0.12H^{0.1}a^{0.1}}{N^{0.33}}\right] - 0.038, \text{MPa}$$

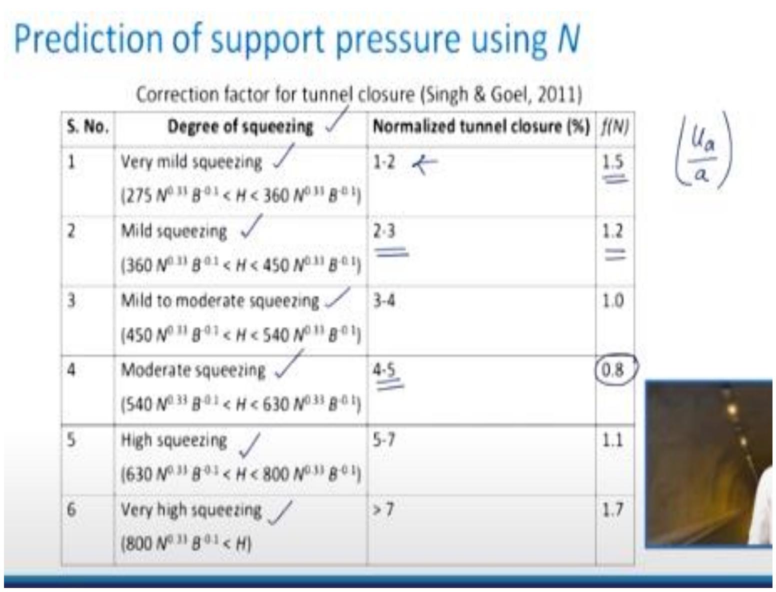
This is valid for h less than 1400 meter, h means is the overburden depth. In case of the squeezing ground condition, the short-term roof support pressure is given as a

$$p_v(sq) = \left[\frac{f(N)}{30}\right] 10^{\left[\frac{H^{0.6}a^{0.1}}{50N^{0.33}}\right]}$$

and again, take a note of the unit megapascal.

Now, this P_v elastic and P_v squeezing is the short-term roof support pressure in non-squeezing ground condition and squeezing ground condition respectively. And both are in megapascal, f(N) is a correction factor for the tunnel closure, and H and a, they are tunnel depth and tunnel radius in meters respectively. Remember this is tunnel radius, a is tunnel radius, and not the diameter.

(Refer Slide Time: 11:50)

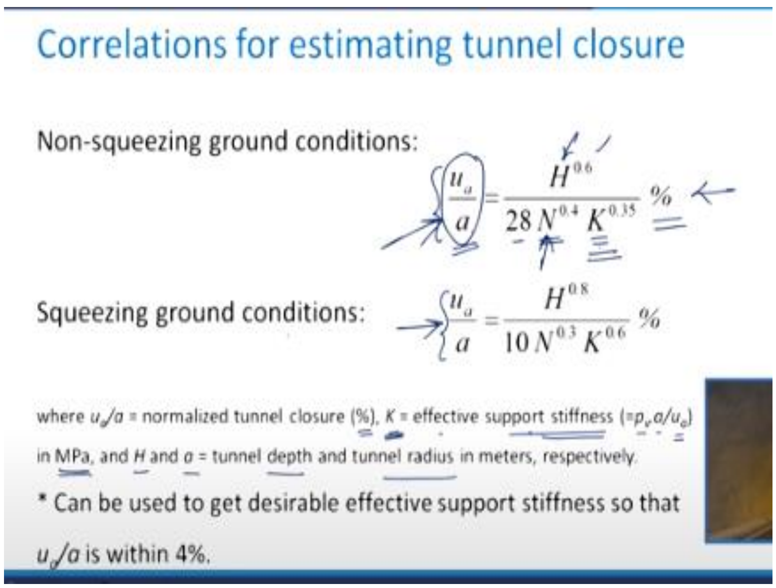


S. No.	Degree of squeezing	Normalized tunnel closure (%)	f(N)
1	Very mild squeezing ($275\,N^{0.33}B^{0.1} < H < 360\,N^{0.33}B^{0.1}$)	1-2	1.5
2	Mild squeezing ($360\,N^{0.33}B^{0.1} < H < 450\,N^{0.33}B^{0.1}$)	2-3	1.2
3	Mild to moderate squeezing ($450\,N^{0.33}B^{0.1} < H < 540\,N^{0.33}B^{0.1}$)	3-4	1.0
4	Moderate squeezing ($540\,N^{0.33}B^{0.1} < H < 630\,N^{0.33}B^{0.1}$)	4-5	0.8
5	High squeezing ($630\,N^{0.33}B^{0.1} < H < 800\,N^{0.33}B^{0.1}$)	5-7	1.1
6	Very high squeezing ($800\,N^{0.33}B^{0.1} < H$)	> 7	1.7

Now, how to determine this correction factor is given in this particular table. So, it depends upon the degree of squeezing and accordingly we can define or we will have the normalized tunnel closure. This also, we discussed in some of the earlier classes that, how on the basis of the normalized tunnel closure and that plot. If you remember, I showed you how on the basis of that some demarcation lines were made which were identifying various zones of very mild squeezing, mild squeezing, mild to moderate, moderate, high, and, very high squeezing.

So, here corresponding to the degree of squeezing you will have the normalized tunnel closure in the percentage. So, how this is defined it is U_a/a in percentage, so, based upon that you have the correction factor given in this table. For example, if the rock mass is experiencing, or going through the moderate kind of squeezing having the tunnel closure between 4 to 5% of the radius of the tunnel, then in that case this correction factor to be used should be 0.8.

(Refer Slide Time: 13:15)



In the non-squeezing ground condition, the tunnel closure can be determined by this expression that is

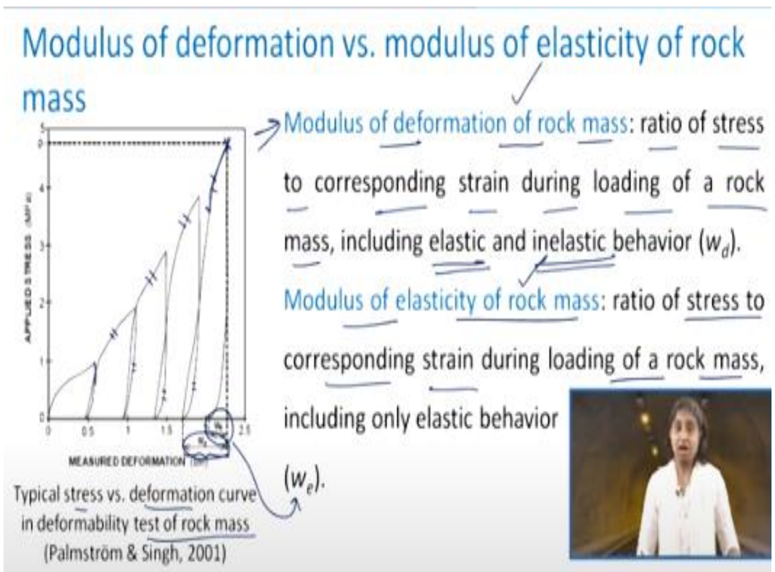
$$\frac{u_a}{a} = \frac{H^{0.6}}{28N^{0.4}K^{0.35}}\%$$

and this is in percentage. Similarly, for the squeezing ground condition one has to use this expression to determine the tunnel closure, which is the normalized tunnel closure here.

So, this is a normalized tunnel closure in percentage K is the effective support stiffness which is written as Pv into a, upon U_a here this is Ka, is also in megapascal and H and a, they are tunnel depth and tunnel radius in meter respectively. Now you see here there is a limit to the allowable U_a/a it should be within 4%. If I have that permissible limit of U_a/a, with me and say if it is the non-squeezing ground condition.

Then, I can focus on this particular expression I know this quantity because it is the permissible one, I know h, I know n. So, using this expression I can find out that what will be the required stiffness of the support system so that, the closure or the normalized closure is limited to that much of the percentage which I am giving as input here. So, this expression although if we know the support system stiffness, we can find out what will be the tunnel closure, but, it is used other way round in a better manner, that if we have the permissible value of u_a upon a, then we can find out that what will be the required effective support stiffness.

(Refer Slide Time: 15:27)



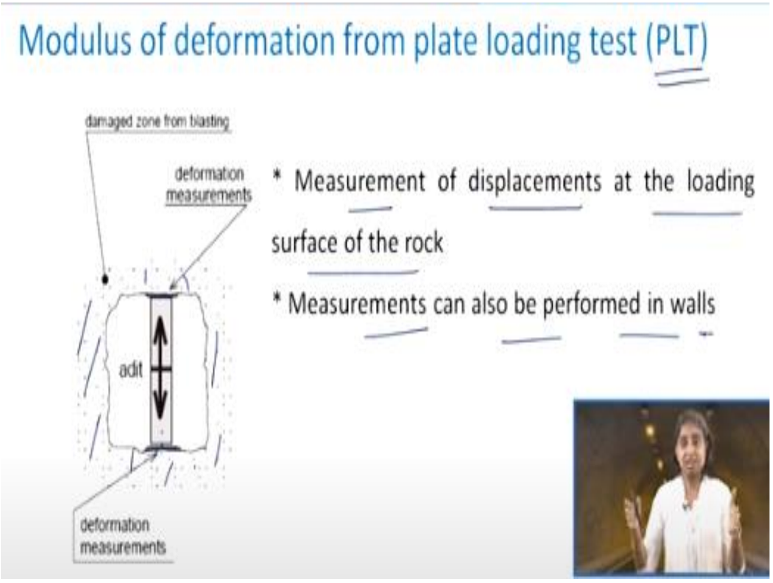
Now, before I go for the field test for the determination of modulus of deformation. Once again let us discuss about the difference between modulus of deformation and modulus of elasticity of rock mass, in some of the earlier lectures we have already discussed this, but let us do this again. The modulus of deformation of the rock mass is defined as the ratio of stress to corresponding strain during loading of a rock mass including elastic and inelastic behavior.

Now, let us say that I conduct a test, where I load it and then unload it and further load it, and likewise, I have few cycles, as it has been shown as a typical stress versus deformation curve in the deformability test of the rock mass. So, the first cycle loading then unloading, then second cycle loading unloading, third cycle, fourth cycle and then fifth cycle, and similarly you have the unloading part as well.

So, you see to find out the modulus of deformation, we have to consider the total deformation, that means including the linear as well as the non-linear portion. But, in case, of the modulus of elasticity of the rock mass we have to use the ratio of stress to the corresponding strain during loading of a rock mass including only the elastic behavior. So, how we can do that which deformation should we take when we unload it the elastic part of the deformation is recovered and the plastic portion remains as it is.

So, you see that corresponding to the last cycle this has been shown after this loading the system was unloaded and here this much is the deformation which is recovered which is We. So, this is being used for the determination of modulus of elasticity of the rock mass, however this complete Wd will be used for the determination of modulus of deformation of rock mass, so that is what is the basic difference between the 2.

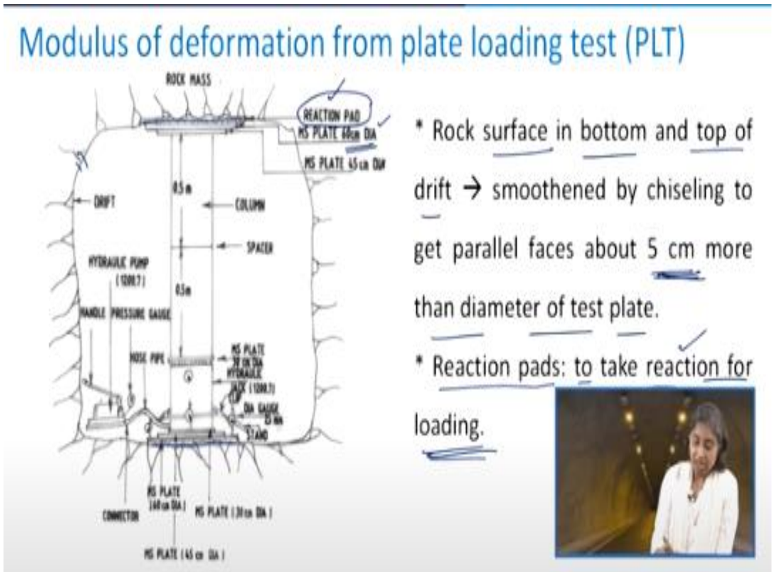
(Refer Slide Time: 17:51)



Now, let us take the first in-situ test for the determination of modulus of deformation this is called as plate load test. So, here what we do is in the rock mass, we prepare an edit and there we apply or we put the plates on both the ends and then we apply the load onto these plates and then we measure the deformations. So, this is how we carry out the plate load test, so, this is the measurement of the displacement at the loading surface of the rock.

So, it is not inside the rock mass but at the surface of the rock mass. Then, the measurements can also be performed involved, so here an example is given for the roof but then if you conduct the test in the horizontal position then you can take the measurements in the walls as well.

(Refer Slide Time: 19:00)

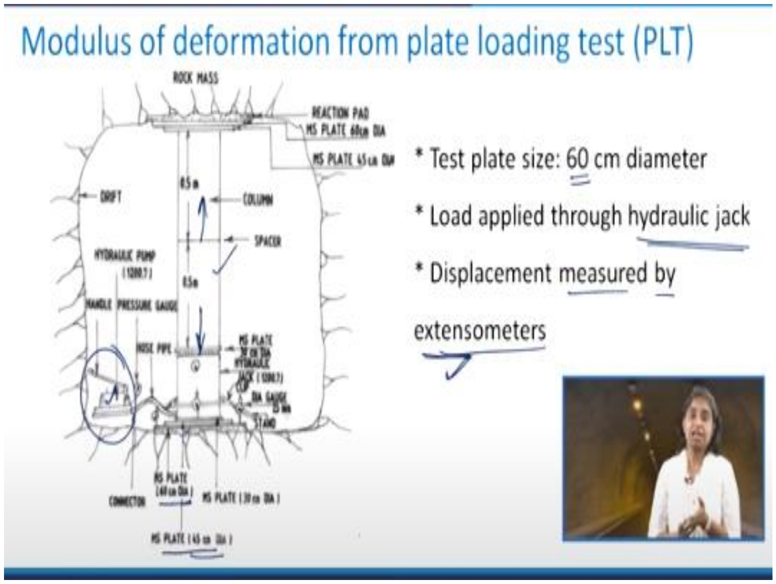


Let us take a look at the test setup, and this will make things clear to you. We use the rock surface in a drift so you see that this is the drift that is prepared. So, the rock surface in the bottom and top of the drift that means here this is what it is the rock surface this is smoothened by chiseling to get the more or less parallel faces about 5 centimeters or more than the diameter of the test plate.

So, if the test plate has let us say the diameter of 60 centimeters accordingly the smoothened surface, or smooth and parallel face will have the dimension about 5 centimeters more than that. Then, we need to provide the reaction pads which take the reaction for the loading, so, you see here how the plates are placed and on bit, and in between the plate and the rock mass there is a reaction pad.

So, it not only ensures that both are in contact with each other but then also takes the reaction for the loading because if it is not in contact with each other how will you get the deformation characteristic of the rock mass surface?

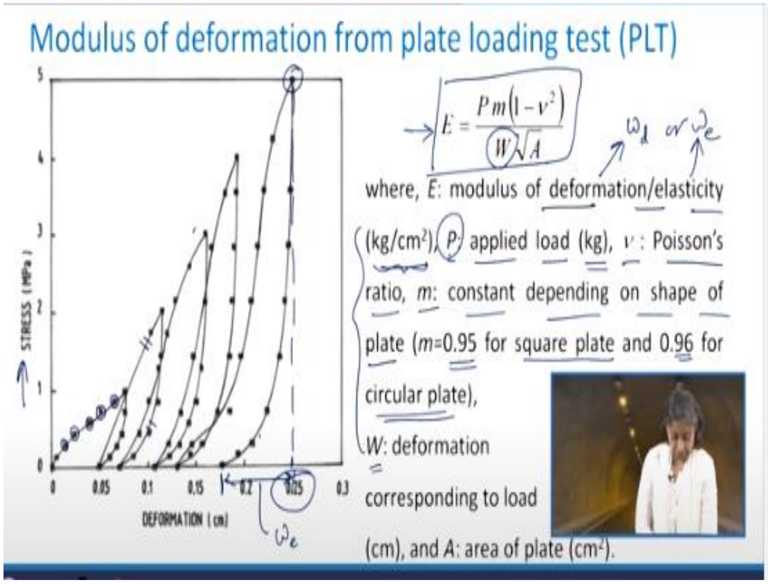
(Refer Slide Time: 20:24)



So, the test plate size of 60-centimeter diameter is used so you can see here that, 60-centimeter plate is there which is in contact with the reaction pad. And on top of that, we put another plate which is 45-centimeter diameter, and then somewhere in between we have these spacers. And, with the help of this hydraulic pump here, that is shown we apply the load that is being mobilized and it tries to push the plates in this direction.

So, once the loading is applied there is going to be the deformation at the surface that displacement is measured by the extensometers. And, the load that is applied is through the hydraulic jack which is shown here. These extensometers, in case, if you have to measure the rock mass deformation then we go for the use of extensometer. When we discuss the chapter on instrumentation and monitoring, we will discuss in detail different types of extensometers they are working etcetera.

(Refer Slide Time: 21:43)



So, what we do is that we apply the load then we wait for some time the deformation to take place, then we remove that load in the stages it is exactly the same as you conduct cyclic plate load test in case of the soils. So, you see various points they tell you that, ok this was the load increment and the corresponding deformation. And, while unloading also it is done in steps not that immediately whole of the load was removed.

So, first cycle unloading then again it is being loaded, unloaded and the similar thing is done. So, from here we can obtain what is the deformation corresponding to any load. For example, let us say corresponding to a particular load if you divide that by the area of the plate you will get the stress in megapascal. So, let us see corresponding to 5 MPa what is going to be the total deformation so we just drop it here and maybe approximately 0.25 centimeter.

But then, what is going to be the elastic deformity displacement here, so upon unloading this much has been recovered, so this is what is going to be We in case of the last cycle. So, how can we determine the modulus of deformation or the elasticity? See both we are doing and I just explained you that, what is the difference between these 2. So, based upon that here W value will be either taken as Wd or We accordingly you can determine either the modulus of deformation or the modulus of elasticity.

So, the units are kg per square centimeter, P be the applied load in kg, nu is the Poisson's ratio, M be the constant depending on the shape of the plate and this is going to be equal to 0.95 for square plate, and 0.96 for the circular plate. And W be the deformation corresponding to the applied load which is P and A be the area of the plate. So, we use this expression that is $E = \frac{Pm(1 - v^2)}{W\sqrt{A}}$

to determine modulus of deformation or modulus of elasticity, where all the other variables are mentioned here like this.

(Refer Slide Time: 24:26)

References

* Palmström, A. and Singh, R. (2001). The deformation modulus of rock masses-comparisons between in situ tests and indirect estimates. Tunnelling and Underground Space Technology, 16 (3), 115-131.

* Singh, R. (2009). Comparison of modulus of deformation of rock mass by different methods. Journal of Rock Mechanics and Tunnelling Technology, 15 (1), 37-54.

So, this was all about the plate load test, the 2 references apart from the one that I gave you in the beginning of this course are mentioned here. So, this material is taken from these research papers, may be if you want to look into more details maybe you can refer back to that. So, what we discussed today is about the modulus of deformation, how it can be determined from the empirical correlations how it can be determined using RQD, using RMR, using Q system, using GSI.

Then, we also discuss some aspects related to rock mass number N, followed by our discussion on the in-situ determination of the modulus of deformation. And in this connection, we discussed the plate load test. So, in the next class we will discuss few other in-situ tests for the determination of modulus of deformation, thank you very much.