Underground Space Technology
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Lecture – 45
Rock – Support Interaction Analysis: Required Support Line, Analysis of Available Support

Hello everyone, in the previous class, we discussed about Ladanyi's rock mass tunnel support interaction analysis, which is an elasto-plastic analysis, and there we learnt about the analysis of stresses, as well as the deformation. And using this information, we were in a position to draw the ground response curve. What about the support reaction curve and the analysis of the available support system?

So, we will take a look at some of these aspects today. So, just to give you the idea what this analysis was all about?

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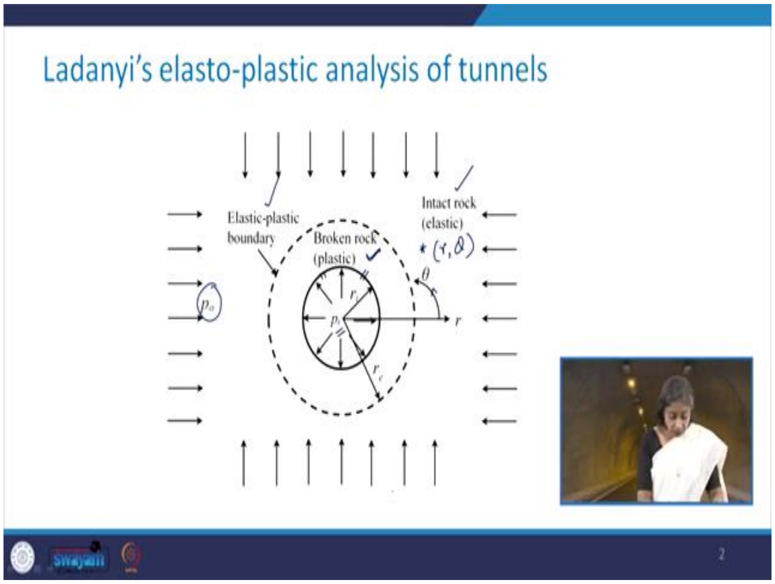

So, we had the circular opening with the radius r_i, it was subjected to the internal support pressure as p_i, any point in the rock mass was designated by r, theta. Where theta was the angle measured in the anticlockwise direction from the horizontal axis. Then you had the elastic-plastic boundary beyond which the rock was the intact rock, and it was behaving as the elastic one. And in between the boundary and the tunnel periphery, we had the broken zone, which we considered to be acting as the plastic zone. It was assumed that this complete system is acted upon by the hydrostatic state of stress represented by these stress p_0.

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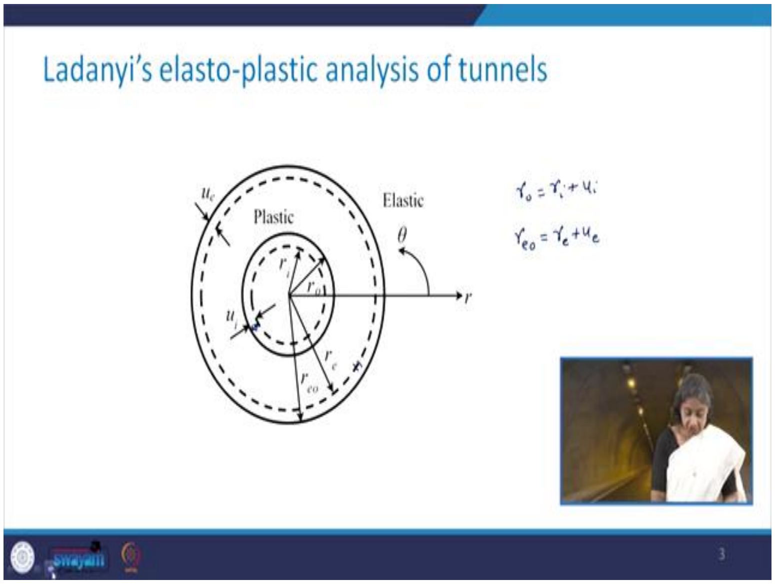

Then, when we were discussing about the analysis of deformation, we saw this particular figure where if you just take a look:

$$r_o = r_i + u_i$$

u_i was the radial displacement of the tunnel periphery. And similarly, as far as the elasto-plastic boundary is concerned, we have:

$$r_{eo} = r_e + u_e$$

So, this is how we carried out the analysis of deformation. Now, today we will focus on the support systems.

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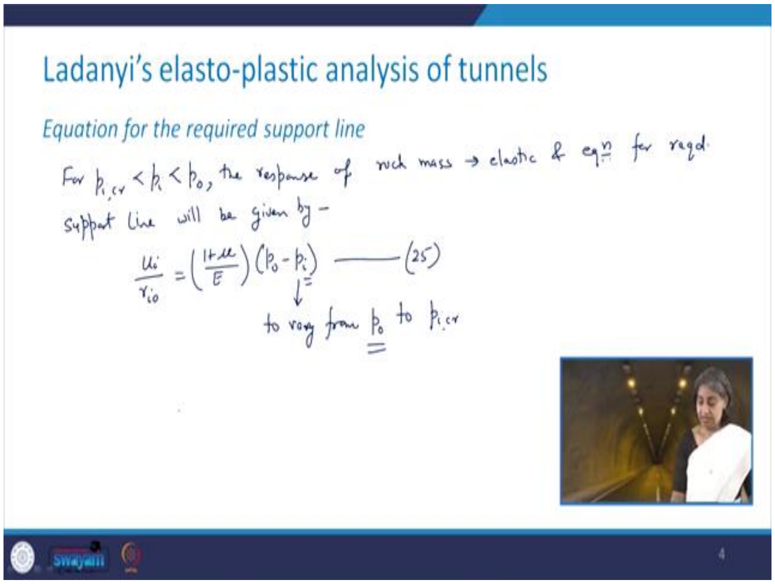

So, what is going to be the equation for the required support line? So, here for p_i critical less than p_i less than p_0, the response of rock mass is elastic. We have seen this in the previous class, and the equation for required support line will be given by the equation,

$$\frac{u_i}{r_{io}} = \frac{(1 + \mu)}{E}(p_0 - p_i)$$

You take a look at equation number 14 of our earlier class, and then you will be able to get that from where this equation is coming. So, we will write this equation as equation number 25.

So, since we are doing this complete derivation of Ladanyi's elasto-plastic analysis of tunnels, for the last couple of lectures so, this is all in continuation. Now, here this p_i to vary from p_o to p_i critical so, this what we will do is after I explain you this analysis, we will take up an example with the help of excel, we will try to solve it, and there you will realize that, how do we vary this p_i? And how do we carry out various steps in the analysis?

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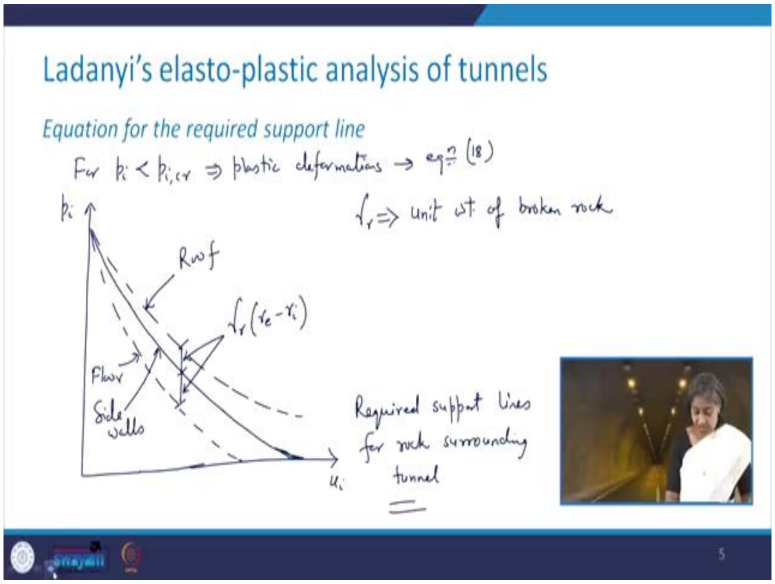

Then we have the equation for p_i less than p_i critical, the plastic deformations are going to be there in this case, and these are represented by equation number 18. So, let us see that how the required support lines for the rock surrounding the tunnel looks like? So, on the x-axis, we plot this u_i and on y-axis, we have this radial support pressure which is p_i. So, we need to draw this so, the top one is for roof, and the bottom one is for the floor.

And the middle one is for the side walls, and then these two distances they are γ_r into r_e minus r_i each of these. Where you have this γ_r as the unit weight of the broken rock. So, this is what is the required support lines for rock surrounding the tunnel.

So, if you recall our discussion, I mentioned to you that there is going to be the broken zone in the vicinity of the excavation and they will exert an extra weight on the roof of the tunnel. That is why what we did that? We consider the analysis of the tunnel without this additional weight. And then I mentioned to you that we will take care of this weight after the analysis is complete. So, this is how that we are going to do this?

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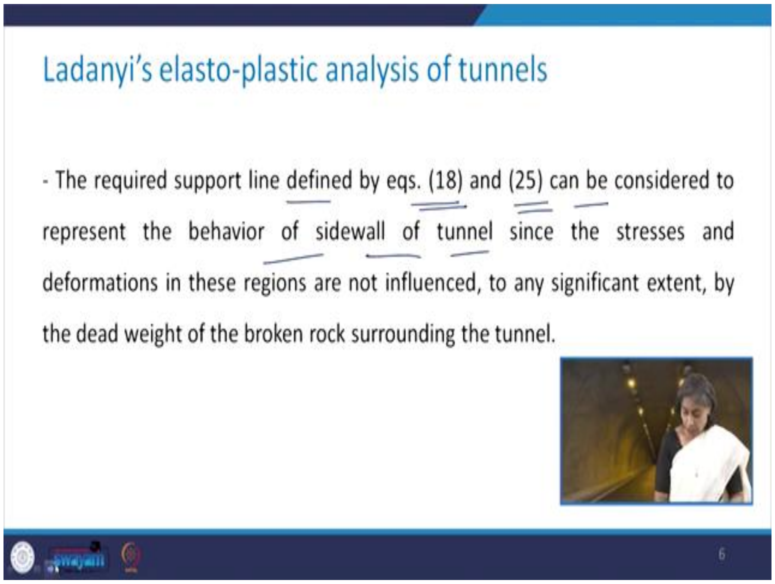

That is the required support line which are defined by equations 18 and 25, these can be considered to represent the behaviour of side wall of the tunnel. Because the stresses and deformation in the side walls, they are not really influenced significantly by the dead weight of the broken zone surrounding the tunnel. But what happens at the roof and the floor of the tunnel? Yes, that weight of the broken zone, broken rock is extremely important.

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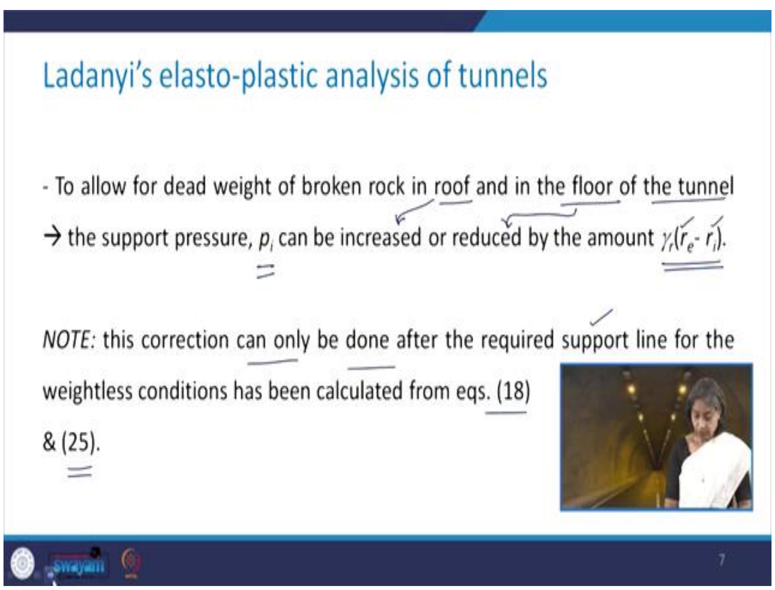

So, in order to allow for the dead weight of broken rock in the roof and in the floor of the tunnel, what we need to do is? We need to increase or reduce the support pressure p_i, so it is increased in case of roof and it is reduced in case of the floor of the tunnel and what will be the amount by which it should be increased or reduced? It will be γ_r into r_e minus r_i, where r_e and r_i, I just showed you with the help of a figure.

Now, we need to take a note here that this correction can only be done after we have got the required support line for the weightless condition. Which we can get from equations 18 and equations 25 for elastic and the plastic zones, respectively. So, this is how the support pressures for the roof and the floor they can be obtained. Because in case of the side wall, whatever that you get, you need to add γ_r $(r_e\text{-}r_i)$ and then you will be getting it for roof. If you subtract, you will be getting for the floor.

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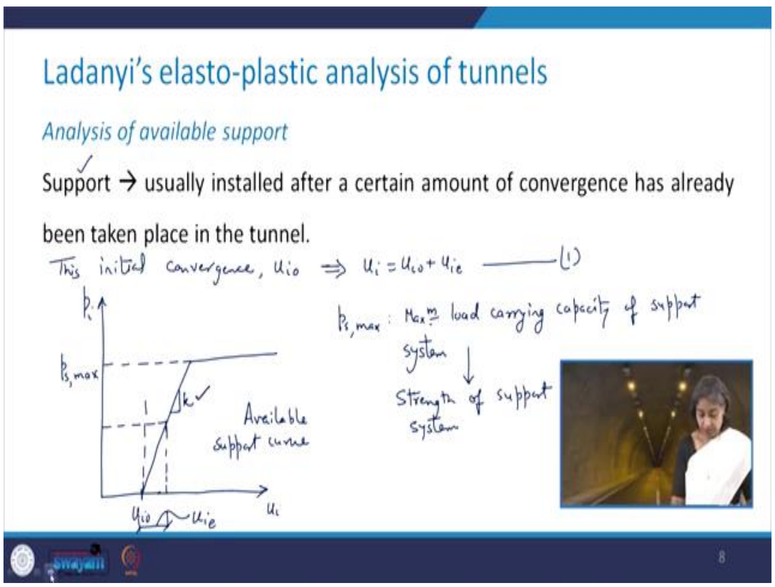

Now, coming to the analysis of the available support. Basically, these support systems are usually installed after a certain amount of convergence has already taken place in the tunnel. Why this is so? We have discussed already that in case, if upon the excavation, you do not want to allow little bit of the deformation. Then you have to go for very, very rigid or stiff support system, which may be quite uneconomical.

Therefore, usually certain amount of convergence is allowed before the installation of the support system. This initial convergence which I am denoting by may be u_{io} this will be

$$u_i = u_{io} + u_{ie}$$

Give me a moment, and I will explain you, how this is going to be done? So, let me draw a figure, so on the x-axis, I have u_i and here I have p_i and see here this initial convergence is allowed, which is say u_{io}.

And then we have some stiffness of the support system, and after that it becomes like this horizontal. So, this we are calling as $p_{s\ max}$ and if you take any point corresponding to which you will get one value of u_i and here you will get the corresponding value of p_i. So basically, the elastic part of the total deformation is u_{ie}. So, here if I want to show it in this figure so, basically it will be this distance that is going to be given by u_{ie}.

And of course, it is slope will be k which is the stiffness of the support system. So basically, what I have drawn here is? The available support curves. So, here this $p_{s\ max}$ is the maximum load-carrying capacity of the support system, or you can say that this is the strength of the support system. Now, the question comes that, what is the stiffness of the support system, which is k?

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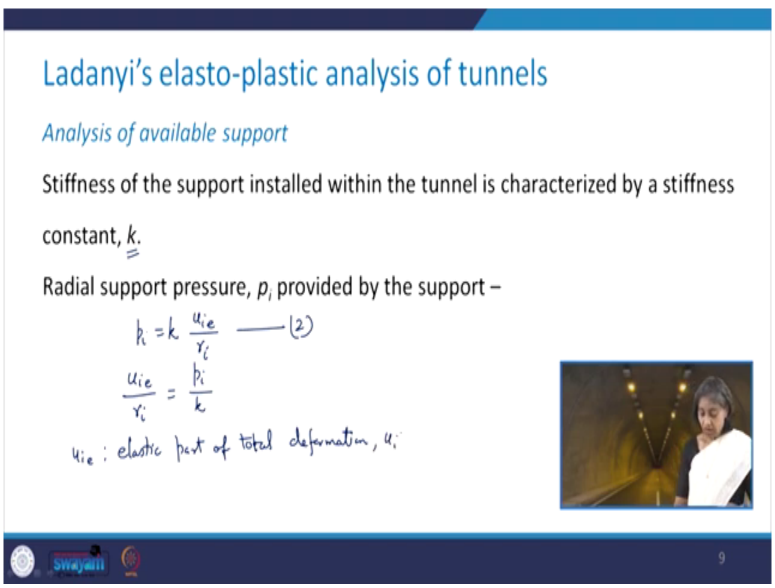

So, the stiffness of the support that is installed within the tunnel is characterized by the stiffness constant, which is represented by k. So, the radial support pressure p_i which is provided by the support can be given by

$$p_i = k\frac{u_{ie}}{r_i}$$

I will mark this as equation number 2 and what we will get from here is:

$$\frac{u_{ie}}{r_i} = \frac{p_i}{k}$$

So, here this u_{ie} as I mentioned, it is the elastic part of the total deformation which is u_i.

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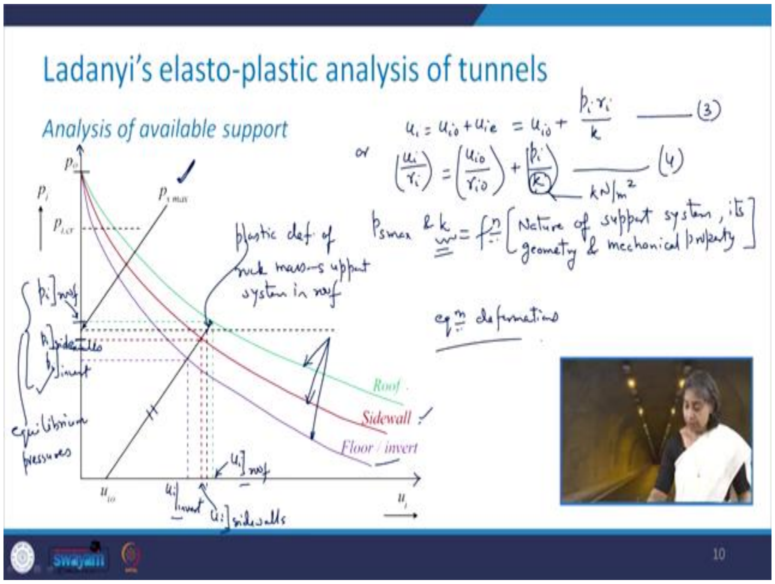

So, from this equation number 1 that I gave you that was

$$u_i = u_{io} + u_{ie} = u_{io} + \frac{p_i r_i}{k}$$

If I just substitute the expression from equation number 2. So, this is what that we are going to get as equation number 3 or we can also write it as

$$\frac{u_i}{r_i} = \frac{u_{io}}{r_{io}} + \frac{p_i}{k}$$

This is equation number 4. So, basically, this k here is in kilo Newton per meter square. And you have this $p_{s\ max}$ and k as the function of nature of the support system it is geometry and the mechanical property. Now, take a look at this particular figure where this u_i is represented on the x-axis and p_i is represented on the y-axis. We have a value of p_i critical so, here what we get is? The first curve that is roof which, is represented by green colour curve here.

Then, we get the side wall, and the last one is for floor or invert. So, you see that here, corresponding to, for example, say floor so what you will get here is? u_i invert ok and for this one roof, you will have this as u_i as roof and this red one will be u_i of the side wall. So basically, these three u_i invert, u_i roof and u_i side walls these are the equilibrium deformations.

And corresponding to each of these, we have here p_i invert, then we have p_i side walls this one red one and the green one is p_i roof. So, these three are the equilibrium pressures, and here this is what is? $p_{s\,max}$. So basically, this portion is going to be the plastic deformation of the rock mass support system in proof. So, this is how first we draw the GRC for roof, sidewall, and floor or inward from the analysis that we learnt in the previous class.

And this is how we can get the support reaction curve for any kind of support which is available to that particular system? The question comes here that how to determine this stiffness. So, all the different types of the support systems, such as shotcrete, steel sets, rock bolts. You have this separate expression for this stiffness that we will see in a short while.

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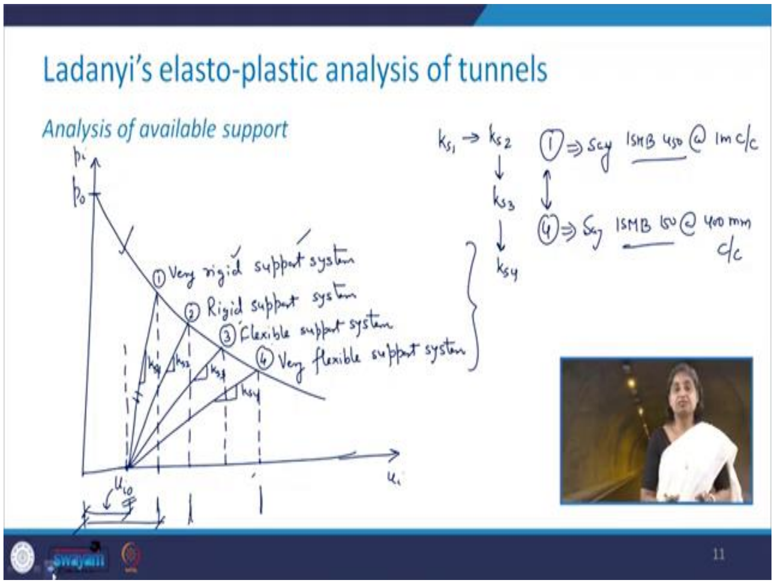

So therefore, if we want to draw now, let us say both together that is GRC, as well as the support reaction curve, this is how it is going to look like? This is what is u_i, and here p_i and beginning I have p_0. And let me draw just one ground response curve, and the initial deformation which is allowed is say u_{io}. And then from here, let me draw and explain you how the different types of the support systems they will have their support reaction curve?

So, this is the first one, second, third and the fourth one. So, corresponding to the first one, let us focus first say that it has the stiffness as k_{s1}. Similarly, the second one has k_{s2}, the third one is k_{s3}, and the fourth one is k_{s4}. So, here this u_{io} is this distance, now corresponding to this the first one here, you will get a value of u_{ie}. Similarly, for the second one, it will be more. The deformations are going to be more as the stiffness of the support system it reduces. So, the first one, we can write as very rigid support system.

You can see that the total deformation is only this, now we change the stiffness of the support system from k_{s1} to k_{s2} and the resulting support reaction curve corresponds to this situation two. So, here, although it is more than the k_{s1} but still, it is not that large as compared to k_{s3} and k_{s4}. So, maybe we can write it as the rigid support system. Coming to the third one, when we go from k_{s2} to k_{s3} that means stiffness is further reduced so, in this case, maybe we can recall this as the flexible support system.

And in case, we further reduce it to k_{s4} what we have is? Very flexible support system. So, you see, you go for more rigid support system, the resulting deformations are low in all the four cases, we have made the tunnel to experience the initial convergence of the magnitude of u_{io}. Now, an example of these different types of support systems may be let us say just for an example corresponding to 1 let us say that maybe it is ISHB 450 at the rate 1-meter centre to centre.

And then maybe 4, we can say that it is ISMB 150 at the rate 400-millimetre centre to centre. So, depending upon what type of the support system it is we can choose the appropriate cross-section corresponding to different types of steel sets. So, it is not only the steel set that we can have, you can get different thickness of the shotcrete or maybe different types of the rock bolts. So, the point I want to make here is that the moment you start reducing this stiffness, the resulting deformations are going to be larger as compared to a support system which is stiffer in nature.

So, this is how we can obtain or we can draw the combined ground response curve and the support reaction curve. The question still remains and answered that is how to determine the stiffness of the support system. So, we will try to explore ah these questions answer in the next class with the help of the support systems, such as shotcrete and the steel sets. Thank you very much.