

Underground Space Technology
Prof. Priti Maheshwari
Department of Civil Engineering
Indian Institute of Technology, Roorkee

Module No # 10

Lecture No # 49

Calculation Sequence for Rock-Support Interaction Analysis-02

Hello everyone, in the previous class, we discussed the calculation sequence for rock mass tunnel support interaction analysis using the method proposed by Ladani. So, we will continue that discussion today as well. We saw that there are 6 different steps are involved out of which 2 we already discussed

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Calculation sequence for rock-support interaction analysis

1. Required support line for rock mass ✓
2. Support stiffness and maximum support pressure for concrete or shotcrete lining ✓
3. Support stiffness and maximum support pressure for blocked steel sets ✓
4. Support stiffness and maximum support pressure for ungrouted mechanically or chemically anchored rock bolts or cables



That is required support line for the rock mass and also the support stiffness and maximum support pressure for concrete or shotcrete lining. So today, we will cover the remaining four aspects, which are the support stiffness and maximum support pressure for blocked steel sets. Then support stiffness and maximum support pressure for un-grouted mechanically or chemically anchored rock bolts or cables.

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Calculation sequence for rock-support interaction analysis

5. Available support curve for a single support system

6. Available support curve for a combined support system

Followed by the available support curve for a single support system and then in case if the requirement is such that more than one type of support system is needed for a single application. Then we should know how to determine the available support curve for such a case. So here we will be taking up this that is the available support curve for a combined support system.

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3. Support stiffness and maximum support pressure for blocked steel sets

Input data required –

- W : flange width of steel set
- X : depth of section of steel set
- A_s : cross-sectional area of steel set
- I_s : moment of inertia of steel section
- σ_{ys} : yield strength of steel
- r_t : tunnel radius

So, to start with let us have our discussion on the determination of support stiffness and maximum support pressure for blocked steel sets. Like we did in the case of the shortcut here, also we need to provide the input data. So let us see what all are the data that is required for us to complete this particular step of the calculation sequence. The first input data that is needed is W which is here

W we are representing this for the flange width of steel set. Then we have X Which is the depth of the section of steel set, next is A which is the cross-sectional area of steel set.

Then we have I_s which is the moment of inertia of steel section, next is σ_{ys} this is yield strength of steel. And we have r_i which is tunnel radius.

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3. Support stiffness and maximum support pressure for blocked steel sets

Input data required -

- S: steel set spacing along axis of the tunnel
- θ : half angle between blocking points (radians)
- t_B : thickness of block
- E_B : modulus of elasticity of block material

So these are some of the data. Along with these further, we would need S which is the steel set spacing along the axis of the tunnel. Then we have θ which is the half angle between blocking points. Keep this in mind that this should be in radians. Then we have t_B which is the thickness of block and finally, we would need E_B which is the modulus of elasticity of the block material.

Now having these data with us, we are ready to calculate now the support stiffness and maximum support pressure for blocked steel sets. We have already discussed that what exactly these expressions are, but here we are putting together all the things at one place.

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3. Support stiffness and maximum support pressure for blocked steel sets

Support stiffness and maximum support pressure -

$$a) \frac{1}{k_s} = \frac{S r_i}{E_s A_s} + \frac{S r_i^3}{E_s I_s} \left[\frac{\theta (\theta + \sin \theta \cos \theta)}{2 \sin^2 \theta} - 1 \right] + \frac{2 S \theta t_B}{E_b \omega^2} \quad \checkmark$$

$$b) p_{s, \max} = \frac{3 A_s I_s \sigma_{ys}}{2 S r_i \theta [3 I_s + x A_s \{ r_i - (t_B + \frac{1}{2} x) \} (1 - \cos \theta)]} \quad \checkmark$$

So, we have here support stiffness and the maximum support pressure as the first step will be with respect to the support stiffness, which is:

$$\frac{1}{k_s} = \frac{S r_i}{E_s A_s} + \frac{S r_i^3}{E_s I_s} \left[\frac{\theta (\theta + \sin \theta \cos \theta)}{2 \sin^2 \theta} - 1 \right] + \frac{2 S \theta t_B}{E_b \omega^2}$$

So, this is how the support stiffness can be determined for the blocked steel sets. How about the maximum support pressure? That is going to be:

$$p_{s, \max} = \frac{3 \cdot A_s \cdot I_s \cdot \sigma_{ys}}{2 \cdot S \cdot r_i \cdot \theta [3 I_s + x A_s \{ r_i - (t_B + \frac{1}{2} x) \} (1 - \cos \theta)]}$$

So, this is how we can determine the maximum support pressure and the support stiffness for blocked steel sets. Let us go to the fourth step of this calculation sequence.

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4. Support stiffness and maximum support pressure for ungrouted mechanically or chemically anchored rock bolts or cables

Input data required -

- l : free bolt or cable length
- d_b : bolt diameter or equivalent cable diameter
- E_b : elastic modulus of bolt or cable material
- Q : load-deformation constant for anchor & head

For the analysis so here we deal with the support stiffness and maximum support pressure for ungrounded mechanically or chemically anchored rock bolts or cables. So as usual, as we did in the case of shotcrete and the blocked steel sets. We will need the input data for the determination of support stiffness and the maximum support pressure for un-grouted rock bolts so let us take a look that what all data that we need.

The first one is l which is the free bolt or cable length then we have d_b which is the bolt diameter or equivalent cable diameter. We have E_b which is the elastic modulus of bolt or cable material then we have Q which is the load deformation constant for anchor and head.

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4. Support stiffness and maximum support pressure for ungrouted mechanically or chemically anchored rock bolts or cables

Input data required -

- T_{bf} : ultimate failure load from pull-out test
- r_t : tunnel radius
- S_c : circumferential bolt spacing
- S_l : longitudinal bolt spacing



Further, we would need T_{bf} which is the ultimate failure load from pull-out test. And in the previous class, you have seen that how the typical result from the pull-out test with reference to ungrounded mechanically or chemically anchored rock bolts look like and how we can determine this T_{bf} from that test result r_i as usual is the tunnel radius. We have S_c as the circumferential bolt spacing and S_l that is longitudinal bolt spacing.

If you recall our discussion so if this is the circular tunnel so this is how if these are the directions in which the bolts are installed. Then this circumferential distance center to center this is S_c and S_l is going to be the spacing of the bolts in the direction perpendicular to the plane of screen, which is the spacing along the length of the tunnel or we call it as longitudinal bolt spacing so the next step after knowing all this input data.

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4. Support stiffness and maximum support pressure for ungrouted mechanically or chemically anchored rock bolts or cables

Support stiffness and maximum support pressure -

$$a) \frac{1}{k_s} = \frac{S_c S_l}{r_i} \left[\frac{4l}{\pi d_b^2 E_b} + Q \right] \checkmark$$

$$b) p_{s,max} = \frac{T_{bf}}{S_c S_l} \checkmark$$

The next step is going to be the determination of support stiffness and the maximum support pressure so let us write those expressions here. So, for the support stiffness we have:

$$\frac{1}{k_s} = \frac{S_c \cdot S_l}{r_i} \left[\frac{4l}{\pi d_b^2 \cdot E_b} + Q \right]$$

$$p_{s,max} = \frac{T_{bf}}{S_c \cdot S_l}$$

So, using these 2 expressions, we can respectively find out the support stiffness and maximum support pressure in case of an un-grouted mechanically or chemically anchored rock bolts or the cable.

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5. Available support curve for a single support system

Input data required -

k : Stiffness of support system under consideration
 $p_{s, \max}$: maximum support pressure which can be accommodated
 u_{i0} : initial tunnel deformation before installation of support

Available support curve
for $p_i < p_{s, \max}$:

$$\frac{u_i}{r_i} = \frac{u_{i0}}{r_i} + \frac{p_i}{k}$$

So now we take a look that how should we determine the available support curve for a single support system? So, for that what is the input data that is needed? Let us first take a look at that. So here, we will need the k which is the stiffness of the support system under consideration then we will need $p_{s, \max}$, which is the maximum support pressure which can be accommodated for that support system. Which is we are considering here, then we will need u_{i0} which is the initial tunnel deformation, before the installation of the support system.

So, what is going to be the available support curve so we will have here as available support curve will be that in case if you have for p_i less than $p_{s, \max}$:

$$\frac{u_i}{r_i} = \frac{u_{i0}}{r_i} + \frac{p_i}{k}$$

So, this is how we will be able to obtain the available support curve in case of the single support system. Now this single support system it can be may be the shotcrete or blocked steel sets or the rock bolts.

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6. Available support curve for a combined support system

Input data required -

- k_1 : support stiffness of system - 1
 - $p_{s, \max 1}$: maximum support pressure for system - 1
 - k_2 : support stiffness of system - 2
 - $p_{s, \max 2}$: maximum support pressure for system - 2
 - u_{i0} : initial tunnel deformation before installation of support
- Note: The two support systems are assumed to be installed at the same time & to start responding to tunnel deformation simultaneously.

In case the requirement is such that you need more than one support system for a single application. Then we need to go for the combined support system and therefore we need to obtain the available support curve for such a system so in that case what exactly is the data which is needed? So, we will need the stiffness and the support pressure maximum support pressure for all the support system which are needed to be installed.

So let us list all the input data down here the first one in the list is k_1 : the support stiffness of system 1. Then we have $p_{s, \max 1}$ which is the maximum support pressure for system 1. We have k_2 similarly so this is going to be the support stiffness of system 2 and we have $p_{s, \max 2}$ that is maximum support pressure for the second system. Then we will also need u_{i0} which is the initial tunnel deformation before the installation of the support systems.

Now here if; you recall our discussion earlier also mentioned so again here I am emphasizing that both the support system should be installed at the same time. So here we need to take a note of the fact that the 2 support systems are assumed to be installed at the same time and they also respond simultaneously to the deformations. So, that is also an assumption which is involved and to start responding to tunnel deformations simultaneously so this is what you need to keep in mind as the note.

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6. Available support curve for a combined support system

Calculation response for available support curve -

$$a) u_{max1} = \frac{r_i p_{smax1}}{k_1} \checkmark$$

$$b) u_{max2} = \frac{r_i p_{smax2}}{k_2} \checkmark$$

$$c) u_{12} = \frac{r_i p_i}{(k_1 + k_2)} \checkmark$$

$$d) \text{ For } u_{12} < u_{max1} < u_{max2} \checkmark$$

$$\frac{u_i}{r_i} = \frac{u_{i0}}{r_i} + \frac{p_i}{(k_1 + k_2)}$$

What should be the calculation response for the available support curve? Let us try to see in case of the combined support system here, you need to provide some logic. If you recall, I discussed with you that if you have the shotcrete and the steel sets of course, the deformation for the shotcrete will be more as compared to that of the steel set, and the stiffness of these set is much larger than the stiffness of the shotcrete.

So how to choose that which one will be governing in case, if you have the combined support system where you have shortcut as well as the steel set? So here we are going to write a generalized algorithm to obtain the available support curve in case of the combined support system. So, what all going to be the calculation sequence? Let us see this is going to be first as u_{max1} which is equal to:

$$u_{max1} = \frac{r_i p_{smax1}}{k_1}$$

So, you see that here we are using the input data for the first support system and we are trying to find out that what is going to be the maximum deformation knowing k_1 and p_{smax1} along with the tunnel radius. Then we will find out for the second system, which is u_{max2} this we can write as:

$$u_{max2} = \frac{r_i p_{smax2}}{k_2}$$

If these 2 are combined, what will happen? We will have u_{12} which is equal to:

$$u_{12} = \frac{r_i p_i}{k_1 + k_2}$$

So this is how we can find out the deformation for the first system second system and the combined one. The question is, how should we get the available support curve for the combined support system? So here we need to provide a logic now that is for u_{12} more than $u_{\max 1}$ and this is also less than $u_{\max 2}$. That means out of these 3 deformation u_{12} is the minimum so in that case how are we going to find out the support curve. That will be:

For $u_{12} < u_{\max 1} < u_{\max 2}$

$$\frac{u_i}{r_i} = \frac{u_{i0}}{r_i} + \frac{p_i}{k_1 + k_2}$$

Since here u_{12} is minimum, so the stiffness of the combine support system is going to govern the available support curve.

And therefore, instead of k here we are writing $k_1 + k_2$ but if this is not the condition satisfying, then how should we go head?

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6. Available support curve for a combined support system

Calculation response for available support curve -

e) For $u_{12} > u_{\max 1} < u_{\max 2}$

$u_{\max 1}$ is attained first

\therefore Deformation of system -1 governs $p_{s, \max}$

$\therefore p_{s, \max 12} = \frac{u_{\max 1} (k_1 + k_2)}{r_i}$

f) For $u_{12} > u_{\max 2} < u_{\max 1}$

attained first as it is least

$\therefore p_{s, \max 12} = \frac{u_{\max 2} (k_1 + k_2)}{r_i}$

So for such thing we need to take another logic that is for u_{12} more than $u_{\max 1}$ but this is less than $u_{\max 2}$. So, this means what out of the three deformation values that is $u_{\max 1}$, $u_{\max 2}$ and u_{12} which one is the minimum? It is $u_{\max 1}$ which is the minimum, so in this case which support system will govern it will be the system 1. So, in this case, here you will have $u_{\max 1}$ attained the first so this will be attained first.

And therefore, we can say that the deformation of system 1 will govern $p_{s, \max}$ of the combined support system. So therefore, we can write that $p_{s, \max 12}$ will be equal to $u_{\max 1}$ into $k_1 + k_2$ divided by r_i because $u_{\max 1}$ is minimum. So, the deformation of the system 1 will govern the available

maximum support pressure, and therefore for the combined support system this maximum support pressure which is $p_{s, \max 12}$ will be equal to:

$$\text{For } u_{12} > u_{\max 1} < u_{\max 2}$$

$$p_{s, \max 12} = \frac{u_{\max 1}(k_1 + k_2)}{r_i}$$

In case if you have $u_{\max 2}$ as the minimum deformation out of $u_{\max 1}$, $u_{\max 2}$ and u_{12} then what will happen? So, we have the next step in this situation that is for u_{12} greater than $u_{\max 2}$ and less than $u_{\max 1}$. So, in this case, this will be attained first why because it is least in all the 3 situations this is the least once. And therefore, we will have $p_{s, \max 12}$ as:

$$\text{For } u_{12} > u_{\max 2} < u_{\max 1}$$

$$p_{s, \max 12} = \frac{u_{\max 2}(k_1 + k_2)}{r_i}$$

So, this is how you can find out the maximum support pressure in case if you have the combined support system most of the time it is only 2 type of the support system which is combined.

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Allowance for dead weight of broken rock

- Required support line defined by eqs. -

$$u_i = r_{io} \left[1 - \underbrace{\left(\frac{1 - e_{iv}}{1 + A} \right)^{1/2}} \right] \quad \& \quad \frac{u_i}{r_{io}} = \left(\frac{1 + \nu}{E} \right) (p_o - p_i)$$

can be considered to represent the behavior of sidewalls of tunnel since stresses & deformations in these regions are not influenced,

to any extent, by dead weight of the broken rock surrounding the tunnel.

Now what about the allowance of dead weight of the broken rock? I discussed with you that when you excavate, what happens to the surrounding rock mass is that in the roof portion, there is going to be some zone of the broken rock mass whose dead weight becomes very important as far as GRC for the roof is concerned and also for the floor. But as far as the side walls are concerned,

their GRC will not be affected that significantly with the help or by this dead weight of the broken rock.

And therefore, I mentioned to you that first, we carry out the analysis without considering the deadweight of the broken rock, and then later on we will incorporate this. So how it is done here, we are going to discuss. So, the required support line which were defined by these equations:

$$u_i = r_{io} \left[1 - \left(\frac{1 - e_{av}}{1 + A} \right)^{1/2} \right]$$

$$\frac{u_i}{r_{io}} = \left(\frac{1 + \nu}{E} \right) (p_o - p_i)$$

So, these equations we have learnt already in the earlier classes. So, these 2 equations can be considered to represent the behavior of the side walls of the tunnel because the stresses and the deformation in the side walls they are not influenced to any extent by the dead weight of the broken rock which is surrounding the tunnel. But coming to the roof and the floor portion of the tunnel, we cannot say the similar thing. So, there we need to provide the allowance for the dead weight of broken rock.

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Allowance for dead weight of broken rock

- To allow for dead weight of broken rock in the roof and in the floor of the tunnel, the support pressure p_i can be increased or reduced by the amount $\gamma_r(r_e - r_i)$, where $\gamma_r \rightarrow$ unit weight of broken rock.



Note: this correction \rightarrow be done only after the required support line for weightless conditions has been calculated

So, in order to allow for the deadweight of the broken rock in the roof and the floor of the tunnel. So, you see that if this is what the tunnel is so, this portion will be considered as the side walls, this is roof portion, and this is the floor portion or we call it as invert as well so in this case, what

we do is? We have already obtained the support pressure without the allowance for the dead weight of the broken rock.

So, if it is the roof portion, we will increase that support pressure p_i which we have obtained without the dead weight, and for the floor, we reduce the support pressure that we have obtained. So how much increase or how much reduction should be done is γ_r into $r_e - r_i$ should be added or subtracted to the support pressure that we have obtained without the allowance for the deadweight of the broken rock with respect to roof and the floor, respectively.

Where this gamma r represents the unit weight of the broken rock, you need to keep this in mind that this particular correction should be done only after the required support line for the weightless condition has been calculated. So repeatedly, I have been telling you that first we do not consider the deadweight of the broken rock in the analysis. Once we do it without the weight of the broken rock and then we incorporate indirectly in this particular manner to take care of the GRC for the roof and for the floor.

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Allowance for dead weight of broken rock

- This method: provides reasonable estimate of effect of weight of broken rock ✓
although it is a gross simplification.

So when we do this method more or less provides us with a reasonable estimate of the effect of the weight of broken rock. Although it is a very gross simplification for all practical purposes, it gives us the reasonable estimate of the effect with reference to the allowance of the dead weight of the broken rock for GRC of the roof and the floor. So, this finishes our discussion on Ladani's elastoplastic analysis for the rock mass tunnel support interaction.

What we will do is in the next class, we will take the help of this calculation sequence, and then we will try to solve few examples of different support systems. And also, we will try to see how we can determine the ground response curve for roof side walls and the floor of the tunnel. This example will give you the idea how to attempt to any problem related to the ground response curve, and the support reaction curve. Thank you very much.