Underground Space Technology
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Module No # 02
Lecture No # 07
Basics of Rock Engineering: Failure criteria for rocks-1

Hello everyone, in the previous class, we had the discussion on classification system on rock masses. So today, we will learn few aspects related to failure criteria for rocks. When I say rock means first, we will discuss about the intact rocks, and then we will see about the rock masses. So, before we go ahead and learn about various criteria for rocks, first let us understand about the in-situ stresses and a condition like plane stress condition, plane strain condition, and axisymmetric condition.

And then, we will see that how the failure criterion, such as Mohr failure criterion, Mohr-Coulomb failure criterion. How they work for rocks? So as far as in-situ stresses and strength they are concerned it is extremely important for us to have the idea. So, in this course, we will also learn about how to get or how to determine these stresses by conducting test in the field. However, for the time being, please try to understand what exactly do we mean by in-situ stresses and strength here. So, it is evaluated exactly in the similar manner as it is done in case of the soils.

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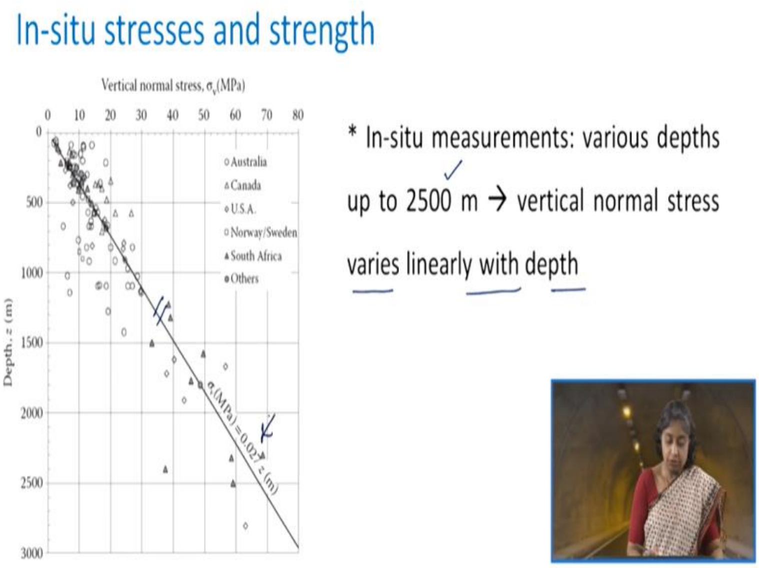

So basically, these are the overburdened stresses within the rock mass. Say, for example, that if you consider the unit weight of rocks as 27 kilo Newton per meter cube for the computation of overburden stresses. So here you can see that there are various points on this plot. These points correspond to some of the test data with reference to in-situ stresses. These belong to the countries such as Australia, Canada, US, Norway, or Sweden, then South Africa and some other countries.

So, all the data when it was plotted on the same space which is vertical normal stresses on X-axis and on Y-axis, it is the depth. Then we could see that one can form a straight line such as sigma v which is in mega Pascal that is equal to 0.027 times Z. So that means that the variation of the in-situ stress is linear. So, these tests were conducted all along various steps and that was varying up to 2500 meter and it was seen that vertical normal stress varied linearly with depth which is represented by this straight line.

So basically, this overburden pressure is equal to gamma times Z in case if we took this as 27 kilo Newton as the unit weight of the rock.

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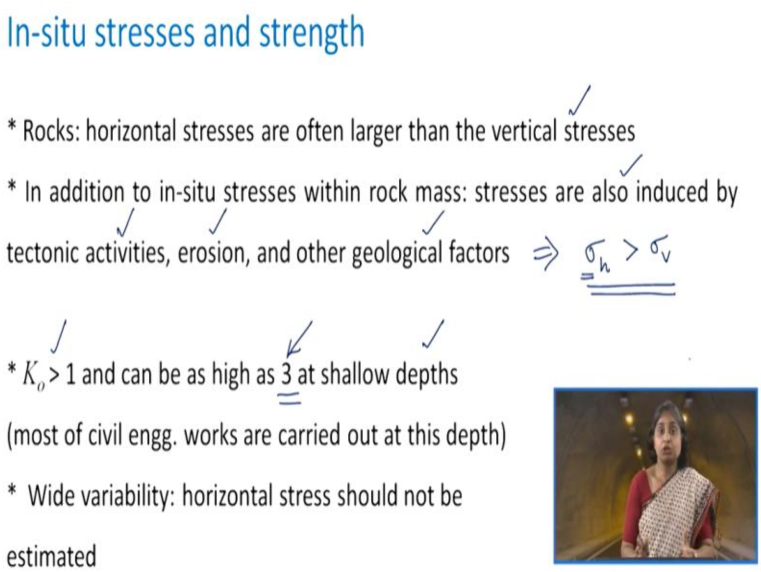

So, as well as rocks are concerned sometimes horizontal stresses are can be larger than the vertical stress, which is a very rare in case of a soils. So, in addition to in-situ stresses with in rock mass stresses are also induced by tectonic activities, erosion, and other geological factors. So, because of these, your horizontal stress can become more than vertical stress in case of rocks.

And therefore, K naught which is the coefficient of lateral earth pressure is greater than 1, and it can be as high as 3 at shallow depths. So, when I say shallow depths means most of the civil engineering works are being carried out at this depth only. So, you see that the reason that sigma v is much less than sigma h in many cases, you can have the value of K naught to be quite high. Then there is wide variability, and hence the horizontal stresses should not be estimated.

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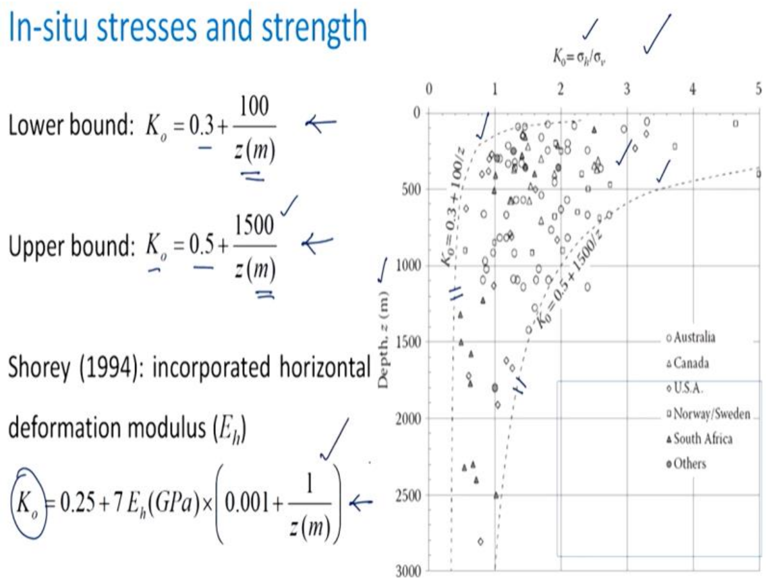

So, see on this plot various data points are there again these are from the countries such as Australia, Canada, US, Norway or Sweden, South Africa, and others. So, when the plotted all this data on K_o versus depth space. You can see that the complete date can be confined between 2 bounds. So, this is what is the lower bound and this is the upper bound so these 2 dotted lines they show the 2 bounds in which all the data points they were found to lag.

So here, the equation for the lower bound can be given by:

$$K_o = 0.3 + \frac{100}{z\ (m)}$$

We need to be careful about the units when we deal with empirical correlations. Similarly, for the upper bound, the equation which was derived which was:

$$K_o = 0.5 + \frac{1500}{z\ (m)}$$

Then, later on few research workers, they pursued this study further, and some of these they incorporated the horizontal deformation modulus.

In this particular manner which is represented by this expression and they tried to obtain the coefficient of lateral earth pressure using this expression which is also a function of horizontal deformation modulus.

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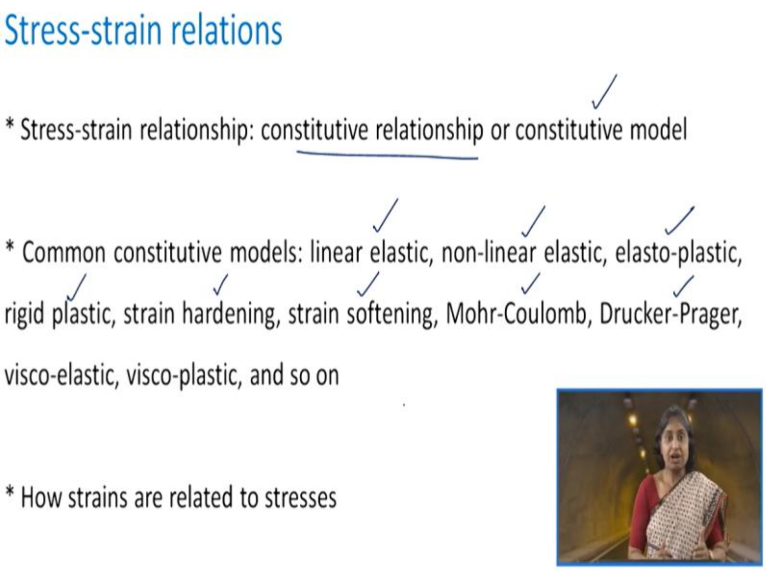

Coming to the stress-strain relationship so when we say that stress-strain relationship we typically mean by the constitutive relationship or the constitutive model. Common constitutive model which are used include linear elastic, non-linear elastic, elasto-plastic, rigid plastic, strain hardening as well as strain softening, then Mohr-Coulomb, you have Drucker-Prager, then you have Visco-elastic, Visco-plastic, and the list is long.

So basically, these constitutive relationships they tell us that how these strains are related to stresses.

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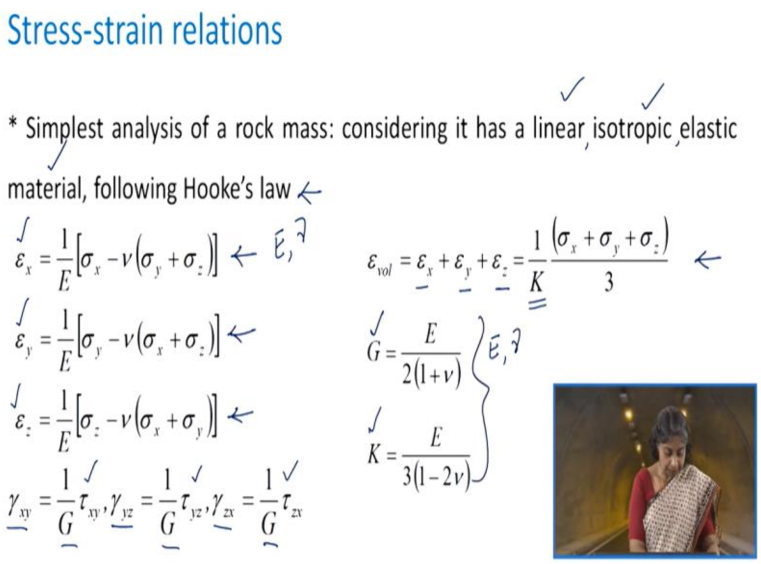

Let us try to take a look corresponding to various ways that stresses are related to strains. So, the simplest analysis of a rock mass would be by considering it as a linear isotropic and elastic material which is following Hook's Law. So, we will have the modulus of elasticity, and you see that in general 3-dimensional situation how the strains ε_x, ε_y, and ε_z, they are related to the stresses σ_x, σ_y, and σ_z along with the elastic property of the rock that is E which is the modulus of elasticity and ν that is the Poisson's ratio.

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right]$$
$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right]$$
$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right]$$

$$\gamma_{xy} = \frac{1}{G}\tau_{xy}, \gamma_{yz} = \frac{1}{G}\tau_{yz}, \gamma_{zx} = \frac{1}{G}\tau_{zx}$$

$$G = \frac{E}{2(1+\nu)}$$

$$K = \frac{E}{3(1-2\nu)}$$

Similarly, shear strains they are related to shear stresses and the shear modulus which is G. Then the volumetric strain can be given by $\varepsilon_x + \varepsilon_y + \varepsilon_z$, and you can write it in this particular manner in terms of the bulk modulus. So, where shear modulus and bulk modulus they can also be represented with reference to or in terms of the modulus of elasticity and the Poisson's ratio in this manner.

$$\varepsilon_{vol} = \varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{1}{K}\frac{\left(\sigma_x + \sigma_y + \sigma_z\right)}{3}$$

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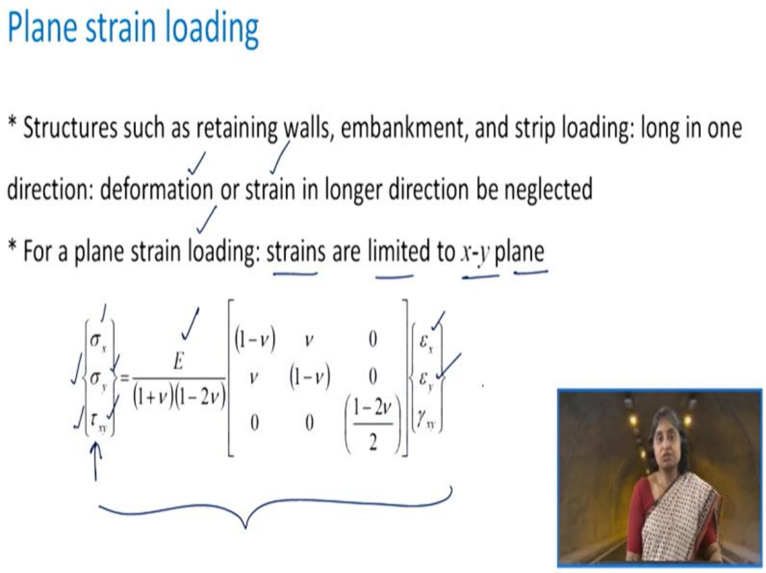

Coming to the other condition like plane strain loading case so the structures such as retaining walls, embankment, and strip footing where the loading is the strip type they are long in one direction. So, the deformation or the strain in the longer direction can be neglected, so such type of situation they fall under the category of plane strain loading. So, for a plane strain loading, the strains they are limited to xy plane so the stresses that are σ_x, σ_y, and τ_{xy}.

They can be represented in terms of strain in this particular manner, so here I have written these 3 equations in the form of the matrix of matrix representation. So here you can see that:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} (1-\nu) & \nu & 0 \\ \nu & (1-\nu) & 0 \\ 0 & 0 & \left(\frac{1-2\nu}{2}\right) \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

So likewise, you can represent σ_y as well as τ_{xy} in terms of the corresponding stresses.

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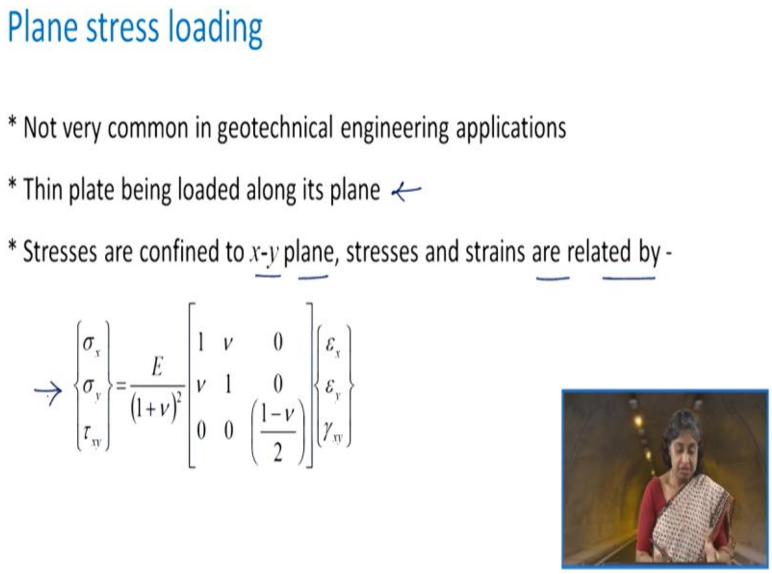

Coming to the other type of loading that is plane stress loading this is not very common in most of the geotechnical engineering applications, and the example of such type of loading include thin plate being loaded along its plane. And in this case, stresses are confined to x-y plane and stresses and strains. They are related by this equation, so you see that this is a bit different than the plane strain loading.

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{(1+\nu)^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \left(\frac{1-\nu}{2}\right) \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

So as far as we as geotechnical engineers we are concerned, we mostly have a plane strain kind of situation than plane stress situation.

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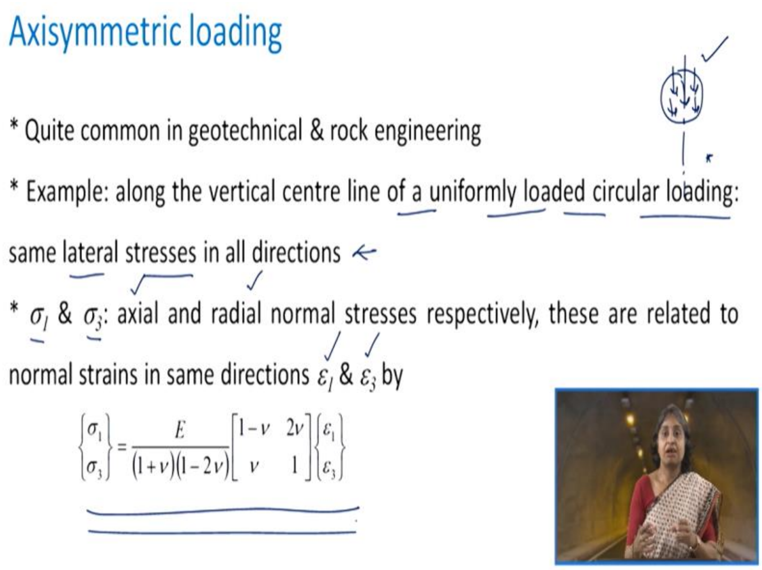

Coming to the next type of loading which is the Axisymmetric loading, this is quite common in case of the geotechnical engineering, including soil mechanics, and also rock mechanics and rock engineering. So, an example of such type of loading may be that along the vertical centre line of a uniformly loaded circular loading. So, it will have the same lateral stresses in all the directions so let us say if you take a look in plants, say this is the circular loading.

Say some uniformly distributed load is there, and this is the centre line, so if we take any point in the soil mass or the rock mass, then such type of problem can be considered as axisymmetric problems. So, in these cases, you will have same lateral stresses in all the directions, and σ_1 and σ_3, these called as axial and the radial normal stresses, respectively. And these are related to normal strains in the same directions that are ε_1 and ε_3, respectively, using this equation.

$$\begin{Bmatrix}\sigma_1\\ \sigma_3\end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix}1-\nu & 2\nu\\ \nu & 1\end{bmatrix}\begin{Bmatrix}\varepsilon_1\\ \varepsilon_3\end{Bmatrix}$$

So basically, here we have the r, theta as the coordinate system rather than x, y.

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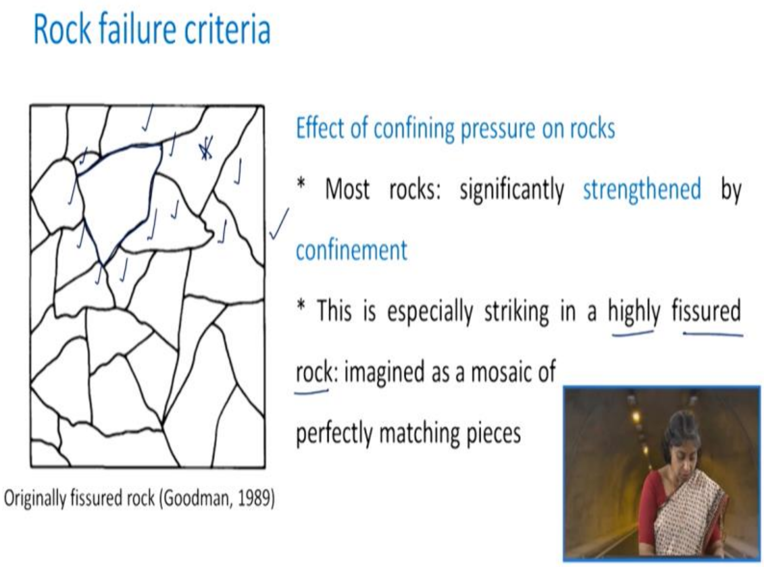

Now, coming to the some aspect related to rock failure criteria, before I go any particular failure criterion, let us try to understand that what exactly is the mechanics behind the failure of the rock? So, the confining pressure has significant influence on the failure pattern of the rocks. So, most of the rocks they are significantly strengthened by the confinement. As we have already discussed earlier that when we increased the confining pressure, what happens is?

The behavior of the same rock gets transition from brittle phase to the ductile phase. So, most of the rocks it has been seen that the strength increases when the specimen is tested at high confining pressure. And this is very significant, especially in case of highly fissured rock. Now just imagine a highly fissured rock as a mosaic of perfectly matching pieces as it has been shown in this particular figure.

So, you see that each and every piece is matching perfectly with the neighbouring pieces you see. Similarly, if you just take this piece, see how nicely it is matching with neighbouring pieces. So, imagine the originally fissured rock as if that it is a mosaic or perfectly matching pieces. Now, how the deformation will take place? how the failure is going to be there?

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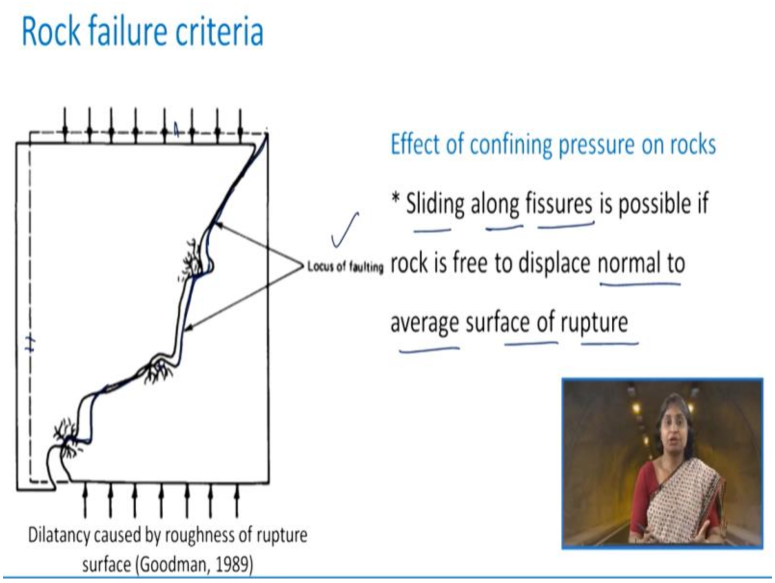

Let us take a look that for this rock to fail, there has to be some dilation there has to be some displacement deformation along the rupture surface. So, for that to happen or sliding along the fresher to happen, the rock has to be free to displace normal to the average surface of rupture. So let us say that this is the average surface of rupture. Maybe I can go to the previous figure and let us say that here this is the average rupture.

Now the failure will take only when there is the sliding normal to average surface of rupture, which is shown here that see this dotted line was the original portion and here this is what is the locus of faulting that has been shown here? Now, what will happen if you have large confining pressure see, when the confining pressure is large, the rock has to be exert lot of energy in order to slide along these fissures, and the result is going to be more strength.

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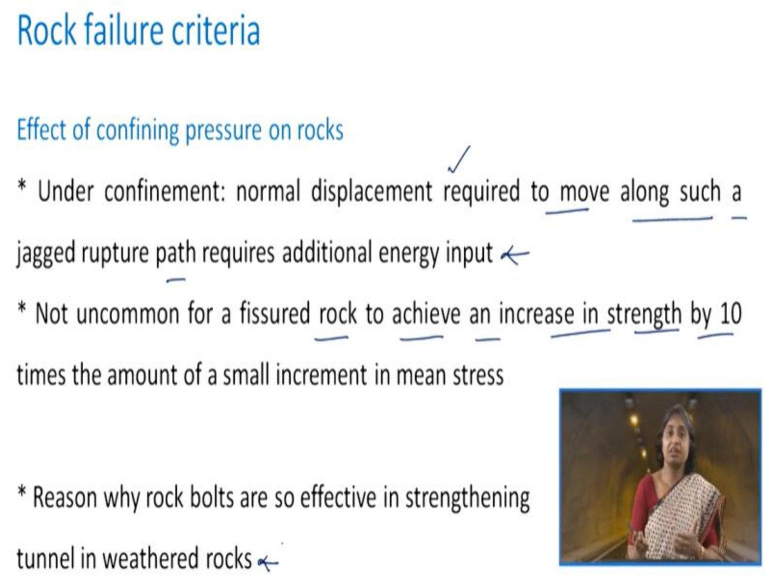

So, under the confinement, the normal displacement which is required to move along such a jagged rupture path it required additional energy input. Which is not uncommon for a fissured rock to achieve and increase in strength by 10 times the amount of a small increment in mean stress. So let us say that I increase the confining pressure by a very small amount, and as a result the strength can go up may be to the tune of 10 times.

And that is the reason, why the rock bolts are extremely effective in strengthening that tunnel in weathered rocks. So, please understand when there is this rupture path, when the confining pressure is more, the rock has to exert additional energy to have the normal displacement, which is required to move along such rupture part, and this result into the increase in its strength.

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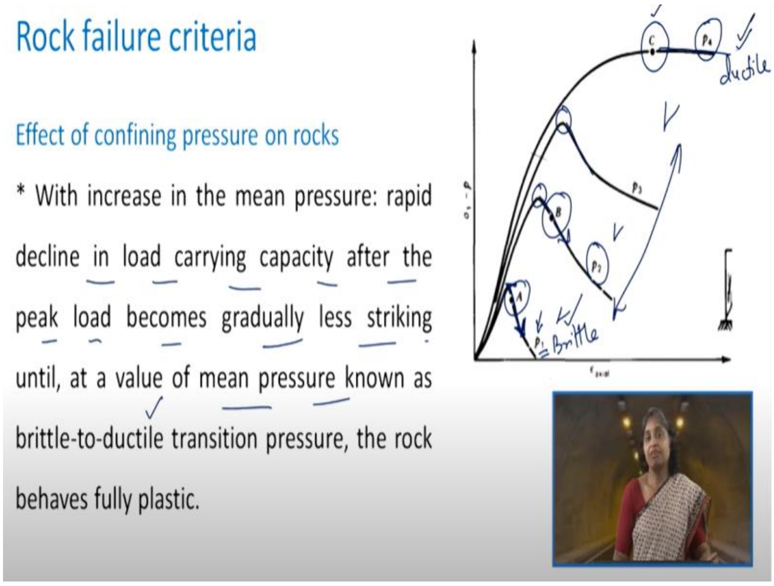

So, you see here that here the various stress-strain relationship has been shown under different values of confining pressure. So, you see here this is P1 which is less than P2, this is less than P3, and P3 is less than P4. And please note down these 3 points A, B, and C, and I will explain that why I ask you to focus on these points. So, what happens that when we increase this mean pressure, there is the rapid decline in load carrying capacity after the peak load.

It becomes gradually less striking until you have a mean pressure which is known as brittle to ductile transition pressure. See, what I mean by this is when you have the low value of mean stress or the confining pressure. See, after the peak load, there is a sudden drop here, but then the confining pressure is increased that is P2. So, you see here, this was the peak and see here the slope was so steep.

But here, it becomes bit gradual further increase the value of P3, and beyond the peak, this is gradually reducing. And then you have the peak situation here, and it is not reducing, but it is more or less becoming kind of a ductile material. So here, this is brittle, and here this is ductile, so slowly, in between this, there is one pressure that is called as brittle to ductile transition pressure, and after this pressure, the rock starts behaving as a fully plastic material.

So, you see, it is the same rock which was behaving as the brittle material at low values of confining pressure. But starts behaving as a ductile material at high values of confining pressure.

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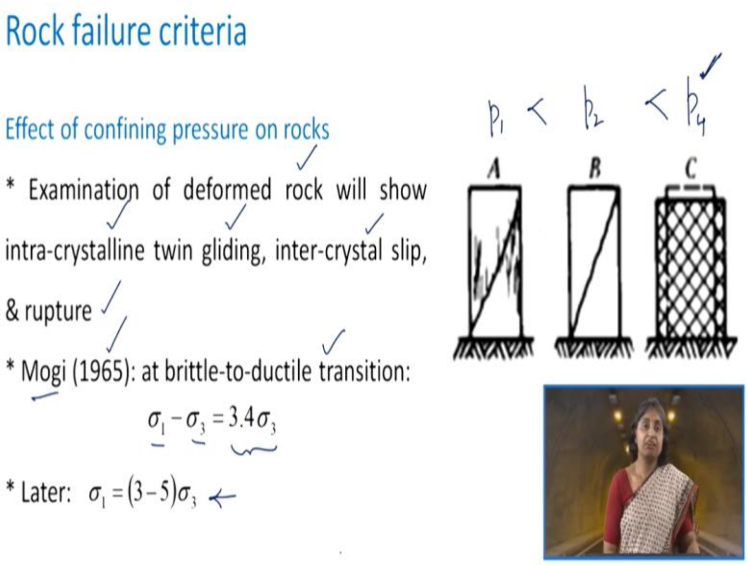

Now, I mention to you that you need to take a note of those 3 points A, B, and C. So, you see, the failure pattern at those points how it is changing with increase in the confining pressure. So here, the confining pressure was P1 which was less than P2, and in this case, it was P4. So, you can notice that how the failure mode is changed when you are increasing the confining pressure. So, if you examine that deformed rock, that will show you that there is intra-crystalline twin gliding, inter-crystal slip, and the rupture.

So, the question is, what should be the brittle to ductile transition pressure? How to get that? So various research workers they conducted so many studies, and in 1965 Mogi came out with the relationship that at brittle to ductile transition, the difference between the major and minor principal stress that becomes approximately equal to 3.4 times the minor principles stress. And later on, when few other research workers they carried out similar type of study. So, they came out with this relationship that sigma 1 to be equal to 3 to 5 times sigma 3.

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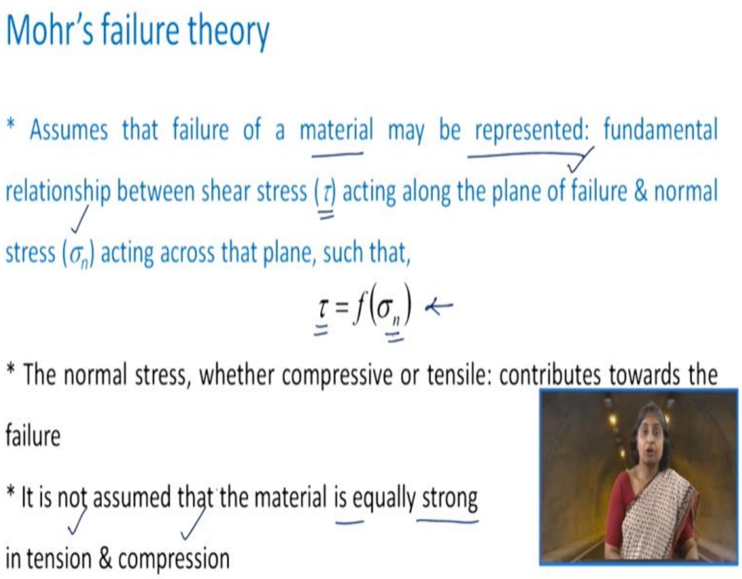

Now, with this background, let us start our discussion on some of the failure criteria. So, one of the first and the most basic one is Mohr's Failure theory. Many a times, we confuse between Mohr's failure theory and Mohr's Coulomb failure criterion. These are 2 different things, so first, let us understand what was Mohr's failure theory? And then, we will learn about Mohr Coulomb failure criterion.

So, in Mohr's failure theory it assumes that failure of the material that can be represented by fundamental relationship between the shear stress that is acting on the plane of failure and the normal stress which is acting across that plane. Such that it can be written in this particular that how as this function of σ_n, τ is the shear stress, and σ_n is the normal stress that is acting across that plane.

Now the normal stress, whether it is compressive or tensile that contributes towards the failure. And another thing which was assumed that the material was not equally strong in tension as well as compression.

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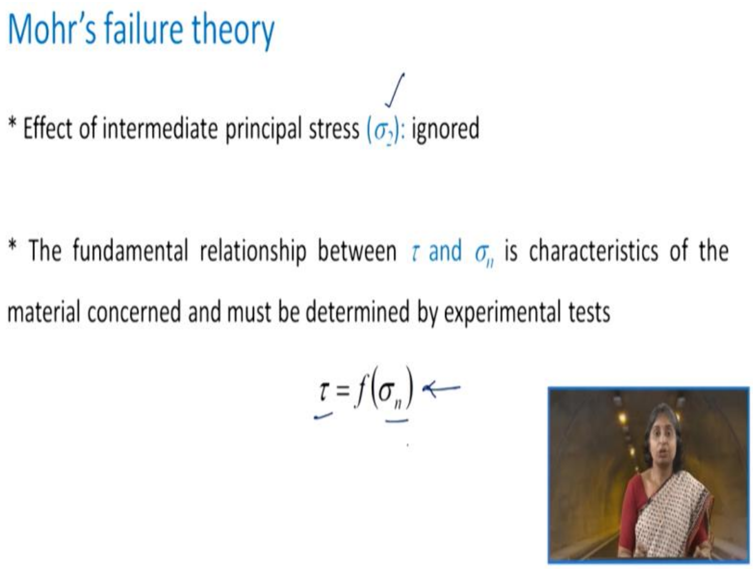

In this case, the effect of intermediate principal stress that is σ_2 was ignored and it was said that the fundamental relationship between the shear and the normal stress is typical characteristic of the material concern. And this can be determined by conducting the test in the lab or maybe in the field. So accordingly, if there are 2 different materials, they may have 2 different functional relationship between τ and σ_n.

Although both may be set to follow the Mohr failure theory but then this relationship is a typical characteristic corresponding to a particular material.

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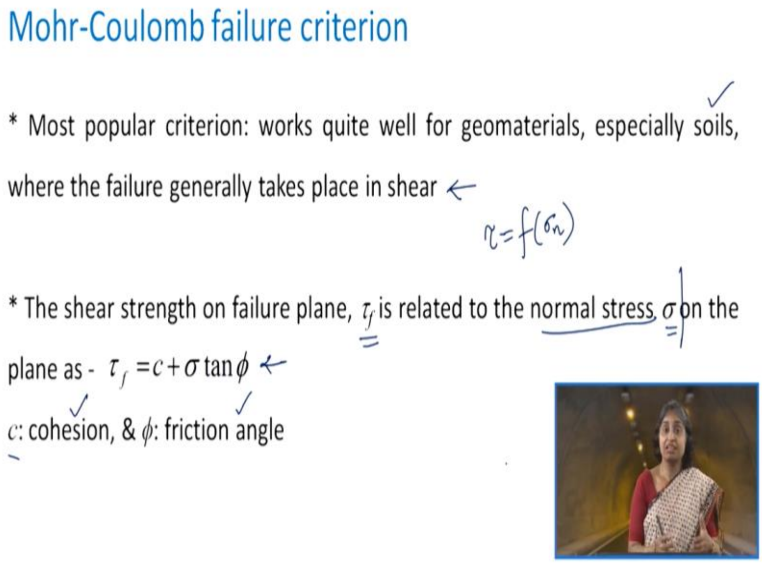

Now, with this background, let us come to the failure criterion that is Mohr's Coulomb failure criterion, most commonly used in the geotechnical engineering. It works quite well for various geo-materials especially, for soils where the failure generally takes place in shear. So, the shear is strength on the failure plane, tau f is related to the normal stress sigma, see up to this much it is the Mohr's failure theory only that is τ is equal to some function of σ_n.

So, what Mohr's Coulomb did that they defined this functional relationship as this expression that is:

$$\tau_f = c + \sigma tan\varphi$$

where c is defined as the cohesion and φ be the friction angle. This is you must have used n number of times where during your studies with respect to soil mechanics or foundation engineering.

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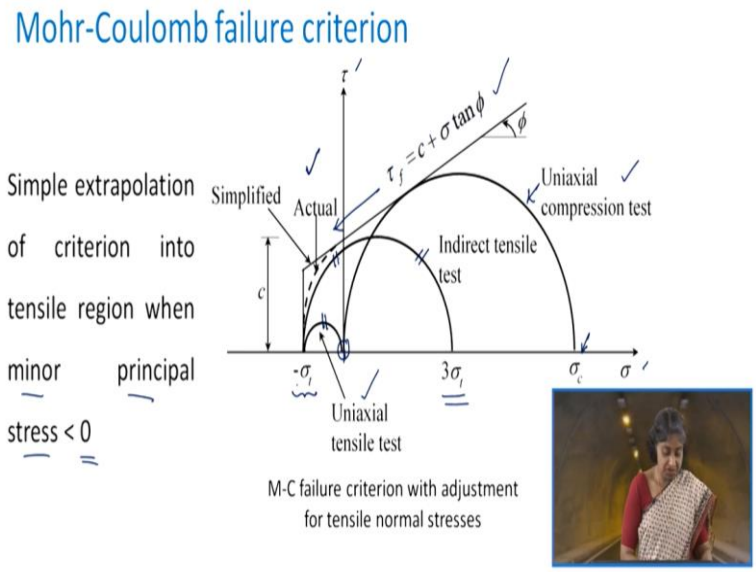

Now, that state of stress is represented in the space τ versus σ and with the help of Mohr's circles. So here that, I have drawn 3 Mohr circle one is corresponding to uniaxial compression test, and therefore you can see that its sigma 1 is equal to the UCS of the rock, and sigma 3 that is the minor principal stress is equal to 0. Because it is uniaxial compression test then, the second circle it corresponds to the indirect tensile test, and we have discussed this if you have conducted the Brazilian test, that is one of the indirect tensile test.

So, in that case, you get the major principal stresses 3 times sigma t which is the tensile strength of the intact rock. And the minor principle, a note here that it is negative that is equal to the tensile strength of the rock. You remember I mentioned to you that tension we will represent by a negative sign. And in case if you have a direct tensile test that is uniaxial tensile test.

So, in that case your major principal stress is going to be equal to 0 because it is uniaxial tensile test. And the minor one is going to be minus sigma t that will be equal to t tensile strength of the material. So, what we do is in case of the rocks, there is the tensile strength which is available. However, in case of soils, there is no tensile strength, so in order to represent that what is done is that the Mohr-Coulomb failure criterion this was extrapolated on to this side. So that means that is extrapolated in the tensile region when you have the minor principal stresses to be less than 0 as you can see that this circle and this circle for that these 2 circles minor principal stresses, they are less than 0.

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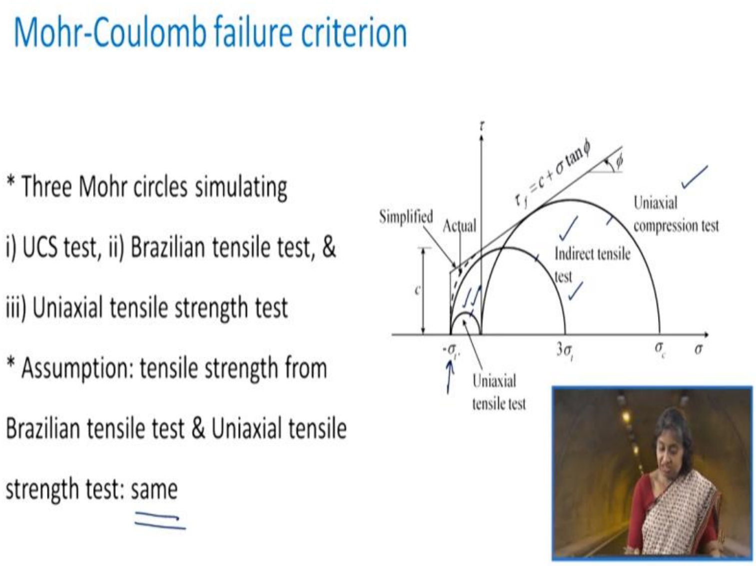

Now this I have already explained you that we have 3 Mohr circles, one for the UCS, one for Brazilian test, and another one for the uniaxial tensile strength test. The assumption which is involved here is that the tensile strength that you obtain from the Brazilian test and from the direct test or the uni-axial tensile test that has been assumed to be same. So you can see that is why both the circles they have minor principle stress as minus sigma t.

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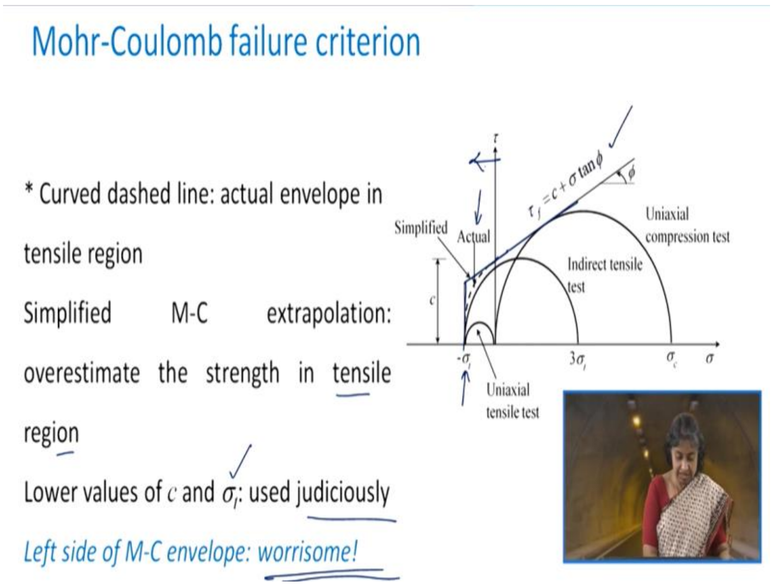

Now, if we see the actual envelope in the tensile region, it is going to be this curved portion which is shown by the dotted line here. But then we go ahead with the simplified, Mohr-Coulomb extrapolation. So, you just extend this tau f which is equal to phi, to the tensile zone. And then you provide tension cut-off here corresponding to the value of sigma 3 as minus sigma t.

So, when the simplify this Mohr-Coulomb criterion, when we apply this what happens is we kind of over-estimate the strength in the tensile region. You see at any point, the actual one is lower than the simplified one. So, we need to use the lower values of C and sigma t, so these has to be used judiciously, and that is the reason that the left-hand side of the Mohr-Coulomb envelope that is in the tensile zone is always worrisome.

So, we need to use our engineering judgment if we are using this extrapolation of the Mohr-Coulomb failure criterion in the tensile zone.

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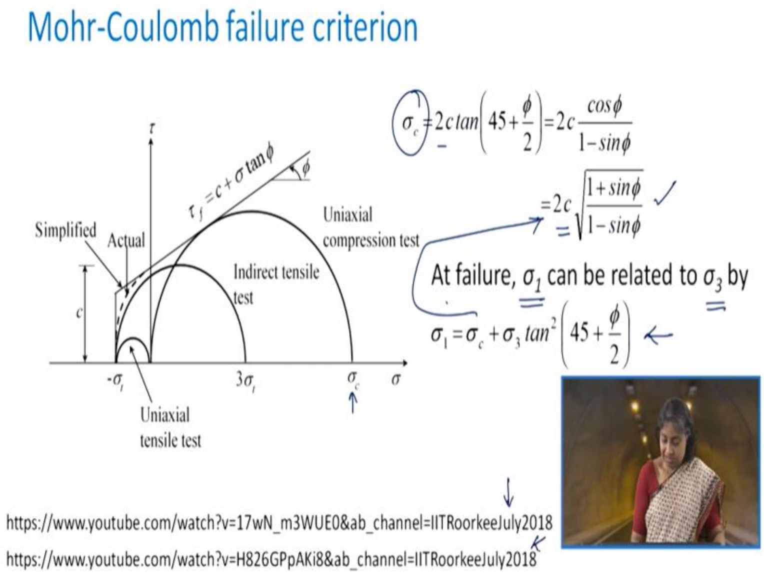

So, how do we represent it with reference to the mathematical expression so you can see that here:

$$\sigma_c = 2c\,tan\left(45+\frac{\varphi}{2}\right) = 2c\,\frac{cos\,\varphi}{1-sin\,\varphi} = 2c\sqrt{\frac{1+sin\,\varphi}{1-sin\,\varphi}}$$

$$\sigma_1 = \sigma_c + \sigma_3\,tan^2 \quad \left(45+\frac{\varphi}{2}\right)$$

Where this σ_c can be represented by this expression in case if you want to know more details about such failure criterion, maybe you can refer these 2 links where have discussed these failure criterion with respect to rocks in more detail manner. So, what we learnt today was about the in-situ stresses, then plane stress, plane strain, and axisymmetric loading condition. What is the effect of confining pressure on the strength characteristic of rock? Then we learnt about Mohr failure and Mohr-Coulomb failure criterion.

So, in the next class, we will learn about some of the empirical failure criteria that we apply in case of rocks as well as rock masses. Thank you very much.