Underground Space Technology
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Module No # 02
Lecture No # 08
Basics of Rock Engineering: Empirical Failure Criteria

Hello everyone, in the previous class, we discussed about Mohr's failure theory and Mohr-Coulomb failure criterion. So let us extend that discussion on various failure criteria for rocks and rock masses. So, in today's class, we will learn about some of the empirical failure criterion which are applicable in case of rocks and rock masses.

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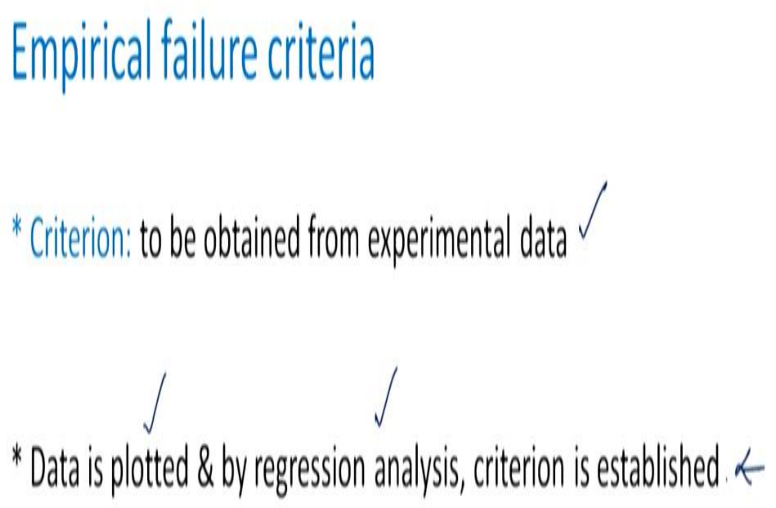

So, when I say that empirical failure criteria so these are obtained from the experimental data. So, in this case, the data is plotted, and by regression analysis, the criterion is established. One thing you need to keep in mind that in case if it is the empirical failure criterion, one needs to be very careful about the units of the input data such as, let us say that one particular failure criteria it uses, say sigma C which is the UCS of the rock.

So, and there can be some empirical factors or the numerical values of those factors, so if the unit of sigma c is to be used as mega Pascal, you should use mega Pascal and not the kilo Pascal or any other unit. So, you need to be extremely careful when we deal with the empirical criterion.

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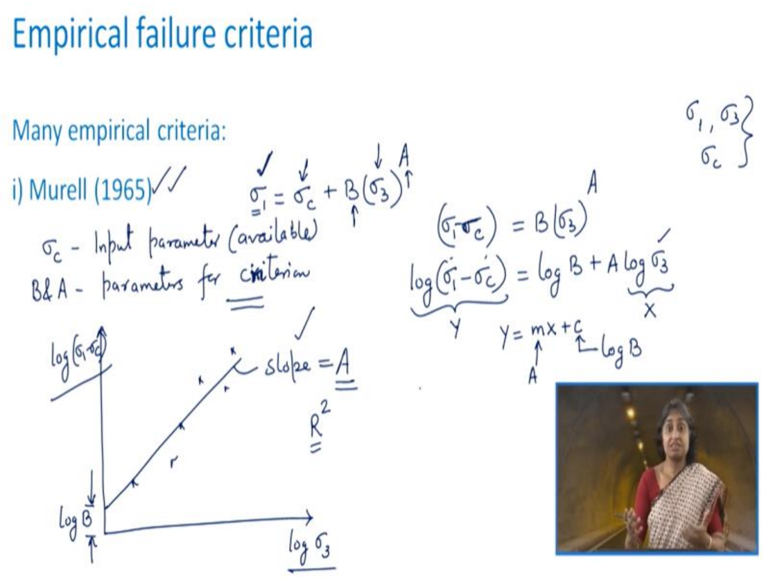

So let us take a look on some of the empirical criteria. So, the first one that I am going to discuss with you is given by Murell, which was given in 1965. So, he proposed that:

$$\sigma_1 = \sigma_c + B(\sigma_3)^A$$

where this sigma C was the input parameter which will be available. So basically, this sigma C was the UCS of the intact rock, and B and A, they were the parameters for the criterion.

Now the questions comes that this sigma 3 is known to me, and if we know B and A, I will be able to get the strength in the form of say sigma 1. So, the question comes how to get this B, and A, so their come's the regression analysis. So, either you can do it mathematically, or you can do it graphically, so how it can be done, let us see. See, we can write here as:

$$\sigma_1 - \sigma_c = B(\sigma_3)^A$$

Now take the log on both sides, so what you will get is:

$$log(\sigma_1 - \sigma_c) = logB + Alog\sigma_3$$

Now, if you just take this quantity as X and this quantity as Y, so see here this will be kind of an equation of a straight line in the form of Y = mx + c where this m is nothing but A, and this c is log of B. So, I have the experimental data here with me so what I do is?

Here on x axis, I plot log of sigma 3, and y-axis, I plot log of sigma 1 – sigma c, so say I get few points like this. So how to get this sigma 3 and sigma 1 – sigma c? See, if you conduct the triaxial test so what you have with you is sigma 1, sigma 3 and if you conduct UCS test then you have sigma c. So, we know all these things, so the question in front of is whether this data follows Murell's failure criterion or not.

So, we plot it on this space that is log of sigma 1 – sigma c and log of sigma 3, and then we try to fit a straight line. Let us say like this, so it is slope would be given as A, and the intercept on the Y-axis will be log of B. So, after plotting this, we can just take the slope and assign that to A and intercept as log of B. So, this way from the experimental data, I can determine the parameters for the criterion.

Now, in this case, R square value that is the whether it is fitting the data, whether it is straight line is fitting the data properly or not that is indicated by R square value. So, if this is R square value is very low, then we will say that this data does not honour the Murell failure. So maybe then we will look for some other failure criteria which this data may follow.

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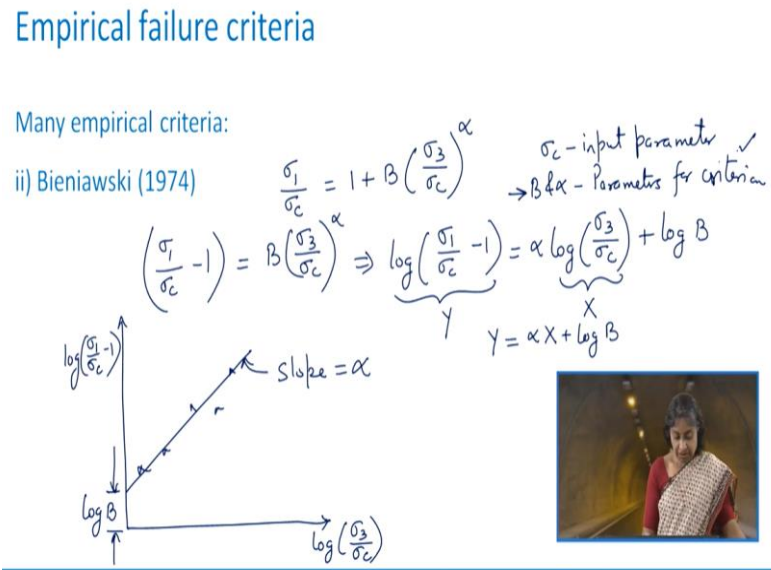

So, the next failure criteria that was proposed by Bieniawski in 1974, so what he proposed was that:

$$\frac{\sigma_1}{\sigma_c} = 1 + B(\frac{\sigma_3}{\sigma_c})^{\alpha}$$

Now again, here sigma C is the input parameter, and B and alpha they are the parameters for criterion. So again, the job with us is to find out these parameters B and alpha, so we follow the same approach and try to play with the terms here in this particular manner.

Let us see I can write this equation in this form, or that is:

$$\frac{\sigma_1}{\sigma_c} - 1 = B(\frac{\sigma_3}{\sigma_c})^{\alpha}$$

Now I take the log, so I will have here as:

$$log\left(\frac{\sigma_1}{\sigma_c} - 1\right) = \alpha log\left(\frac{\sigma_3}{\sigma_c}\right) + logB$$

Now again, if I take this as Y and this as X, so all I am going to have is a equation for the straight line that is alpha X + log of B. So, I will just try to plot it so on X-axis, I will plot log of sigma 3 upon sigma C and on Y-axis, I will plot log of sigma 1 - sigma C – 1.

Say I get say maybe some of these points and then I would like to fit us straight line like this. So, the slope of this line will be alpha, and here and intercept on the Y-axis is going to be give me log of B. So, this is how we can determine the parameters for the criterion that was given by Bieniawski.

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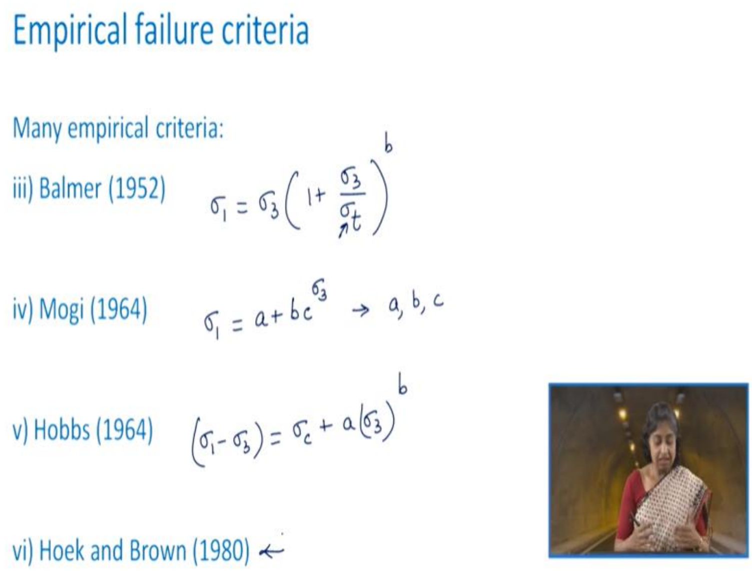

So, few other empirical criterion are also available. For example, Balmer he gave it in this particular manner that is:

$$\sigma_1 = \sigma_3\left(1 + \frac{\sigma_3}{\sigma_t}\right)^b$$

So, you see that in this case, he use the tensile strength and proposed the criterion the Mogi gave:

$$\sigma_1 = a + bc^{\sigma_3}$$

So, in this case, there were 3 parameters of the criterion a, b, and c. Hobbs, in 1964, he gave:

$$\sigma_1 - \sigma_3 = \sigma_c + a(\sigma_3)^b$$

Then another criterion which was given by Hoek and Brown in 1980, so this is one of the most commonly adopted empirical criteria to be applied in case of rocks, and this is also applicable in case of rock masses. So let us study this particular criterion in more detail.

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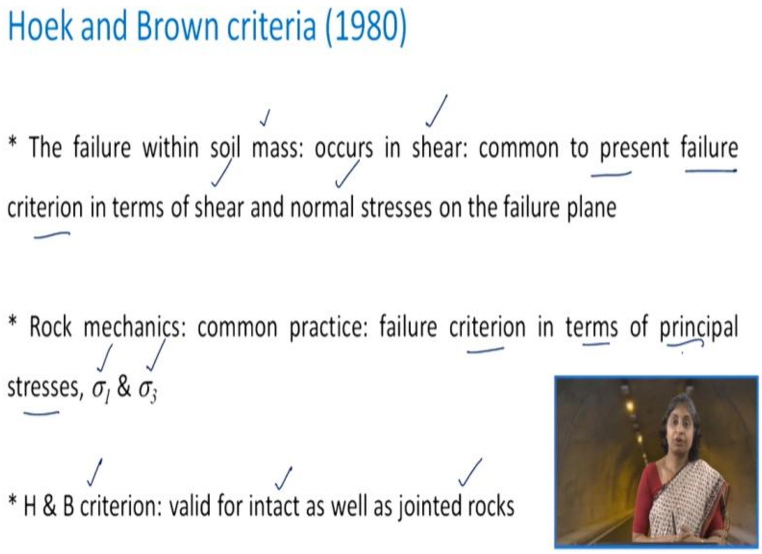

So, the failure within the soil mass that occurs in shear, which is common the present the failure criterion in terms of shear and the normal stresses on the failure plane. In case of the rock mechanics, it is the common practice to represent the failure criterion in terms of principle stresses which are denoted as sigma 1 and sigma 3. As I mentioned, Hoek and Brown criterion is valid for intact rocks as well as for the jointed rocks.

And maybe that is one of the reasons that it is the most commonly adopted criterion in the area of rock mechanics and rock engineering.

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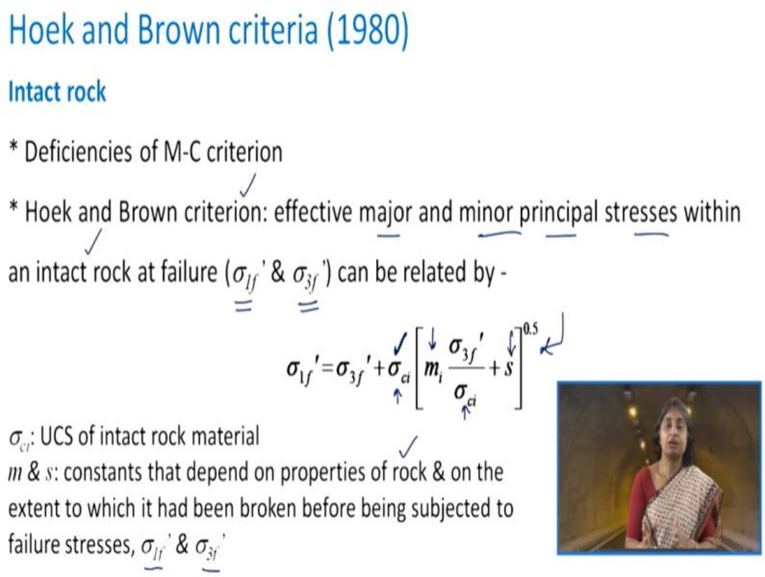

So first, we will discuss about the intact rock, so there are deficiencies with reference to Mohr-Coulomb criterion there we saw that when the minor principal stress. It goes into the tensile region then if we use the simplified version of the Mohr or the extrapolation of the Mohr-Coulomb criterion. Then it overestimates the values, so Hoek and Brown criterion it is quite effective.

In order to do away with that particular deficiency of Mohr-Coulomb criterion so in this case that is with reference to Hoek and Brown criterion, the major and minor effective principal stresses within the intact rock at failure which are represented sigma 1f prime and sigma 3f prime, respectively. They can be related by this particular equation. So, you can see here that there are 3 parameters of Hoek and Brown criterion that is sigma ci, mi, and s.

$$\sigma_{1f}' = \sigma_{3f}' + \sigma_{ci}\left[m_i \frac{\sigma_{3f}'}{\sigma_{ci}} + s\right]^{0.5}$$

So basically, this represent sigma ci represent UCS of the intact rock material, and mi and s these are the constant which depend upon the property of rock and on the extent to which it has been broken before being subjected to the failure state of stress which is the defined by the major and minor principal stresses as sigma 1f prime and sigma 3f prime. Although we say here that this sigma ci is the UCS of the intact rock material, but as far as its determination is concerned, we kind of try to find this out as one of the fitting parameter only.

And do not rely on the test that we conduct under uni-axial compressive condition and assign this value from the test. No, this is not assigned the value as the UCS of the intact rock material, but this is also treated as the one of the fitting parameter which is to be found out from the tri-axial test data which is conducted on the intact rock specimen.

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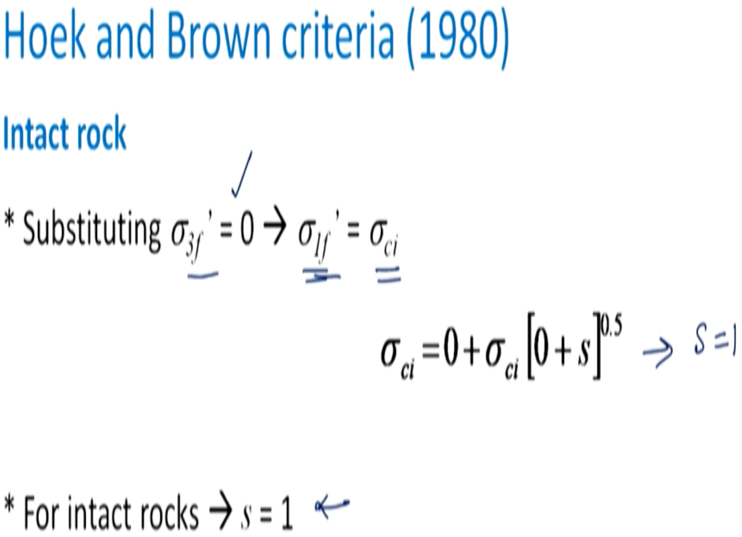

Now with reference to intact rock, you have seen the expression, so if you just substitute confining pressure sigma 3 f to be equal to 0, what will happen to sigma 1f prime? It will become equal to sigma ci.

$$\sigma_{ci} = 0 + \sigma_{ci}[0 + s]^{0.5}$$

So that, substitute these 2 values in the expression and what you will get from here is see this parameter s will work out to be equal to 1. So kindly remember that for intact rocks this parameter s of Hoek and Brown criterion becomes equal to 1.

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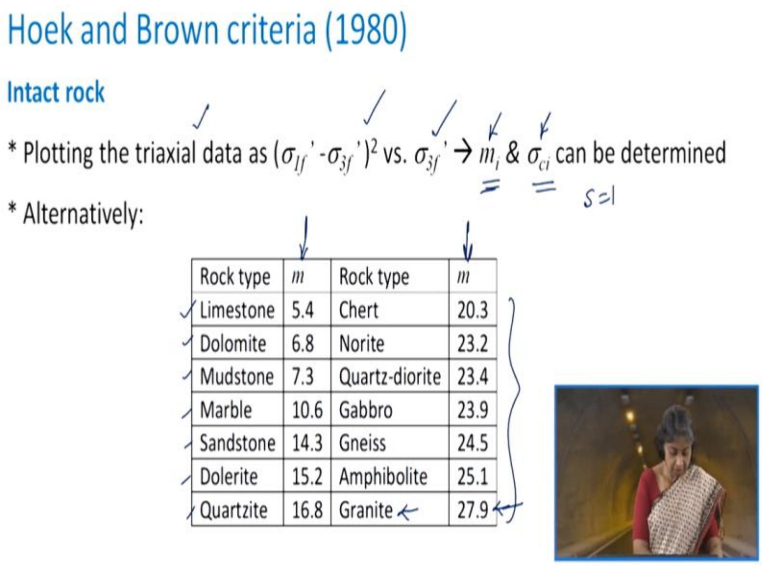

Now, when we plot the tri-axial test data as sigma 1f prime – sigma 3f prime whole square versus 3f prime, you can determine mi and sigma ci. Because for intact rock s = 1 so we end up having 2 parameters mi and sigma ci. So that we can obtain from the tri-axial test data alternatively, let us see if I am not able to conduct the test and find out what is the value of mi. So, for different rock types, the typical values of m for the intact rock has been mentioned here.

This table is not new to you. We have discussed this earlier as well when we were discussing about the classification system of the rock mass. So better the quality of the rock and better will be value of m with reference to intact rock.

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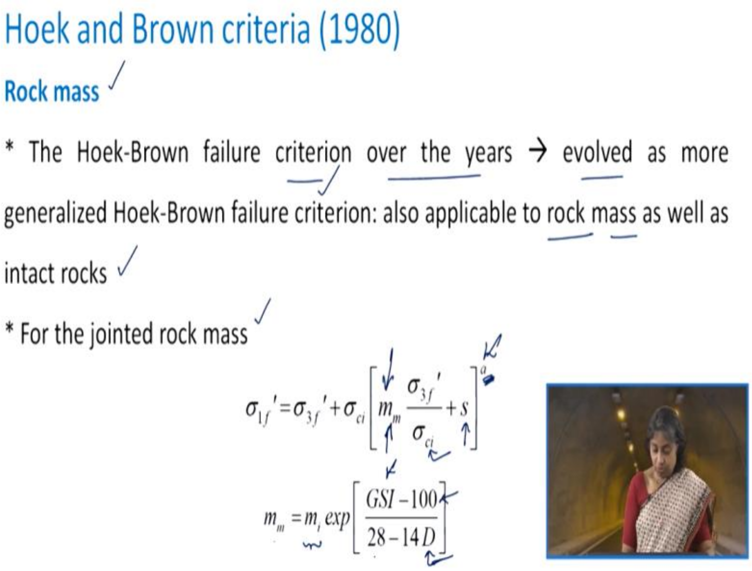

Now, coming to the rock mass, the difference between the intact rock and the rock masses. Once again, it is the intact rock; when you have the discontinuities into it, we call that as rock mass. So, the Hoek and Brown failure criterion is also applicable in case of the rock mass, but of course the parameters are not going to be same as they were there in case of the rocks. So, since this has been used over the years, it has evolved as more generalized Hoek and Brown failure criterion.

And therefore, always applicable to rock masses as well as the intact rocks, so in case of the jointed rock mass, the parameter remains the same but you should take a note here that you have m_m, s, a as the parameter and you have also sigma ci. So, in case of the intact rock, if you recall, this power was 0.5 however, in case of the jointed rock mass, this is another parameter which is to be determined from the test data focus this m_m that is the parameter m for the rock mass.

$$\sigma_{1f}' = \sigma_{3f}' + \sigma_{ci}\left[m_m \frac{\sigma_{3f}'}{\sigma_{ci}} + s\right]^a$$

$$m_m = m_i\, exp\left[\frac{GSI - 100}{28 - 14D}\right]$$

This is a function of the parameter of the Hoek and Brown criterion for intact rock that is m_i, GSI this we have learnt earlier, and a disturbance factor called D. And rest all numbers they have been obtained empirically, and here you have 100, and then in the denominator you have 28 and 14.

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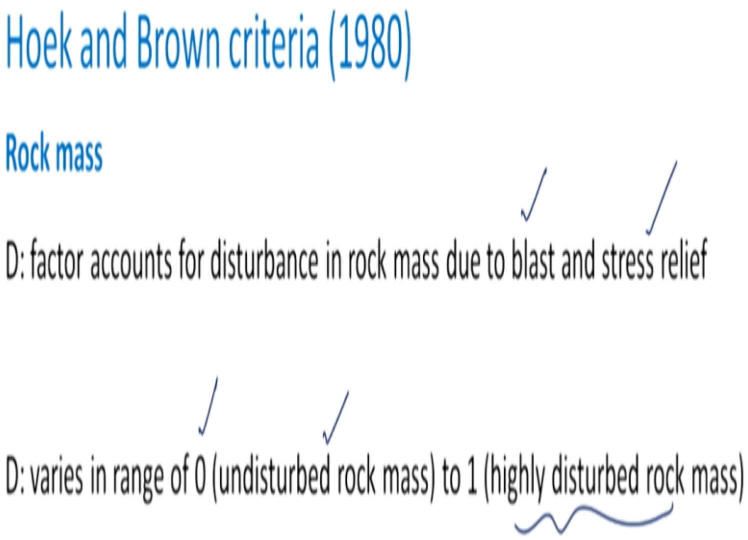

Now this D factor or the disturbance factor it accounts for the disturbance in rock mass due to blast and hence the stress release. So, this various in the range of 0, which corresponds through the undisturbed rock mass to 1 that relates to the highly disturbed rock mass. So let us see, how we can get the value of D corresponding to various situations.

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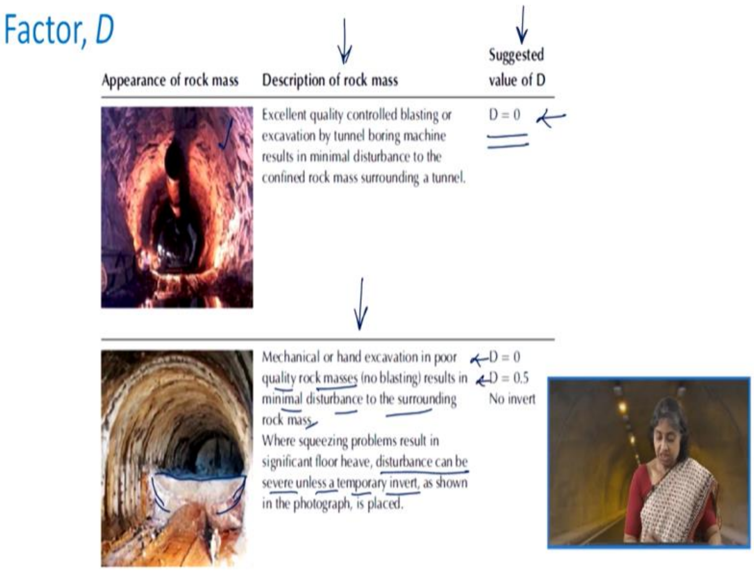

So, here the typical figure have been given showing the appearance of the rock mass and the second column describes the rock mass, and the last column gives you the value of D. So, in case if you have the excellent quality control blasting and the excavation and see the appearance it looks like this in that case you can consider this to be D to be equal to 0. In case, if you have this type of the rock mass that is mechanical or hand excavation in poor quality rock masses which results into the minimum disturbance to the surrounding rock mass.

Then, in that case, you can have D to be equal to 0 in case if you have the squeezing ground condition; then, in that case, the disturbance can be severe unles1s you provide temporary invert, which is shown here. You can see here this portion is the temporary invert which is provided. So, in that case, one needs to go for the larger value of D that is 0.5.

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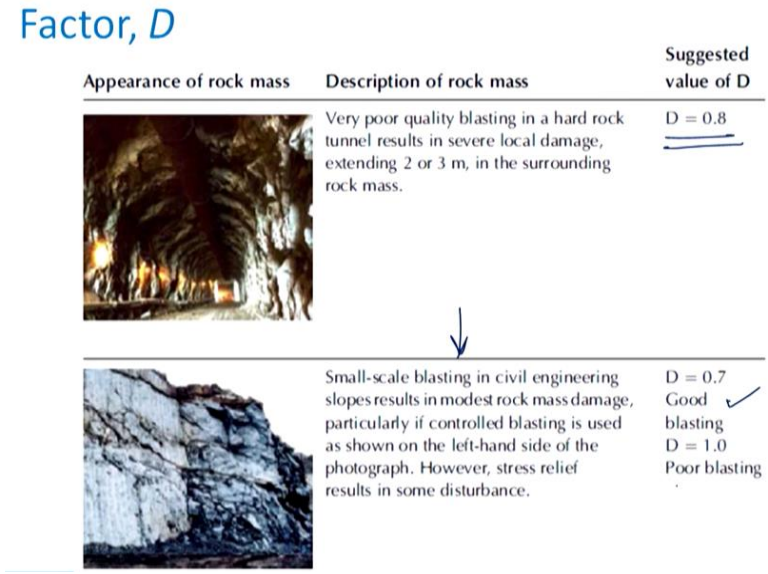

In case you have the very poor-quality blasting in the hard rock, this results in the severe local damage. So, you have to assign the larger value of D that can be given as 0.8 similarly, for this description, you have D to be equal to 0.7 for good blasting and D to be equal to 1 for poor blasting condition.

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Factor, *D*

Very large open pit mine slopes suffer significant disturbance due to heavy production blasting and also due to stress relief from overburden removal.
In some softer rocks, excavation can be carried out by ripping and dozing and the degree of damage to the slopes is less.

D = 1.0
Production blasting
D = 0.7
Mechanical excavation

And finally, if you have very large open pit mine slopes which suffers significant disturbance due to heavy production blasting. So, in that case, one is to go for the value of D as 1, and in case if you have the mechanical excavation, then you can reduce this disturbance factor to 0.7. So, once we know this disturbance factor, we can apply it to the expression that was given in some of the earlier slide, and you can find out the e parameter of Hoek and Brown criterion with respect to rock mass.

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Hoek and Brown criteria (1980)

Rock mass

* The constant m_m can take → positive value in the range 0.001-25

* Highly disturbed poor quality rock masses in lower end & hard and almost intact rocks at the upper end.

* Intuitively: $m_m < m_i$ → rock mass weaker than intact rock

Now, the constant m_m that is the constant parameter m for rock mass it can take the positive value, which can vary in the range 0.001 to 25. This lower value represent the highly disturbed poor-quality rock mass. And hard and almost intact rocks, they will be at the upper end that is 25 now in intuitively we can say that the parameter for the rock mass is less than the m parameter for the intact rock this shows that the rock mass of course is weaker than the intact rock because of the presence of various discontinuities.

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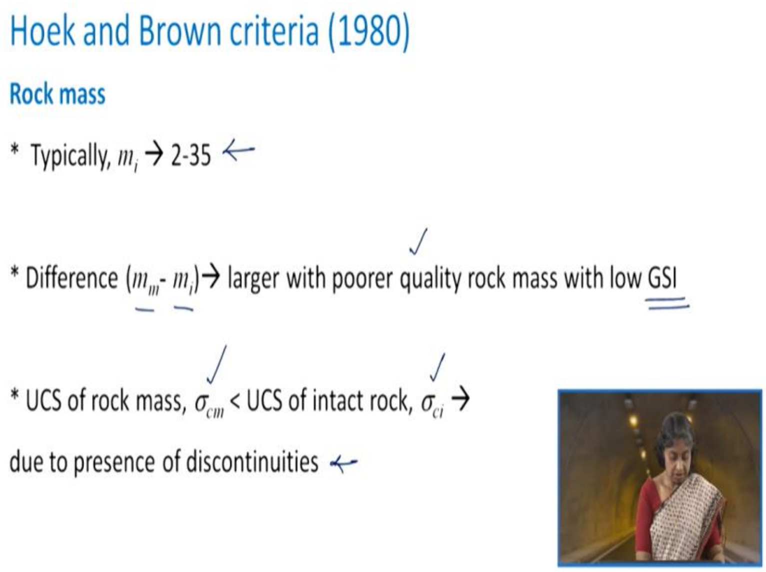

So typically, this m_i it varies between 2 to 35 for various rocks, so the difference between the parameters m for the rock mass and the intact rock that is the difference $m_m - m_i$. It will be larger for poorer quality rock mass having low value of geological strength index or GSI. UCS of the rock mass that is sigma cm is less than the UCS of intact rock, which is very obvious statement it will be of course, because of the presence of discontinuities.

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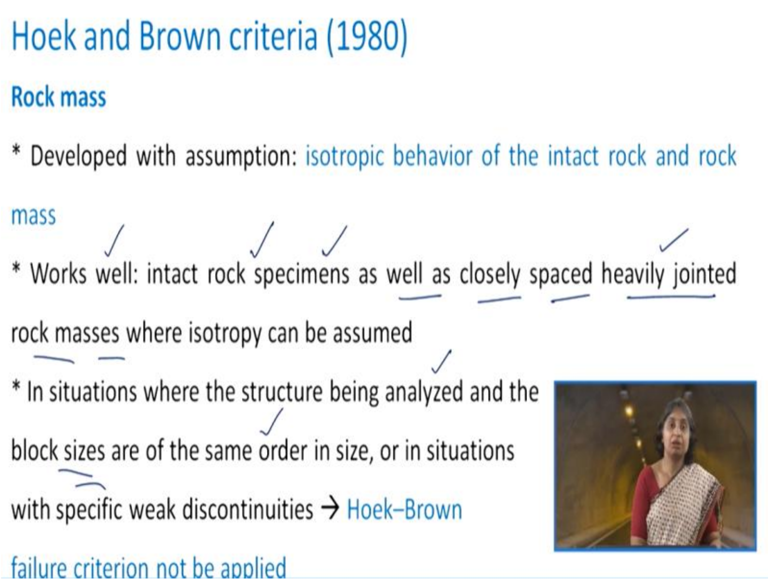

Not this Hoek and Brown criterion when it was applied for the rock masses then the assumption which was involved that the behavior of the intact rock as well as the rock mass it will be isotropic. So, therefore, the criterion works very well for the intact specimens as well as closely spaced heavily jointed rock masses where you can assume the isotropic. If you recall, in one of the previous lectures, I also mention to you that massive rocks or highly jointed rock masses both can be considered as the isotropic material.

So, the Hoek and Brown criterion it works very well in case of the intact rock is specimen and also in case of the closely spaced heavily jointed rock masses. So, in situations where these structure is which is being analyzed and the block sizes that are of the same order in size and situation with the specific weak discontinuities. Hoek and Brown failure criterion, they may not be applicable so in that case you have other failure criterion which is available specifically developed for rock masses.

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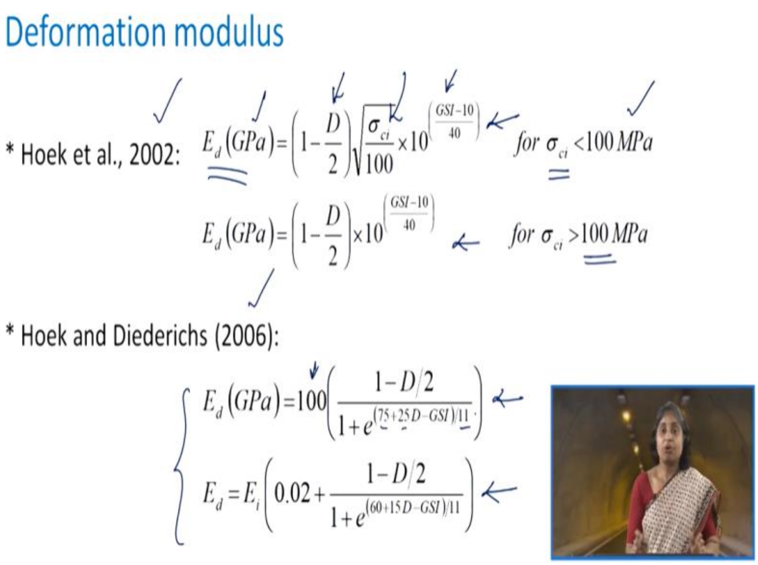

Now as far the deformation as far as the deformation modulus is concerned, some aspects are there. So, this deformation modulus it can be estimated from the index Q, that is:

$$E_d(GPa) = 25\log(Q)\,[for\ Q > 1]$$

This relationship was given by these authors in 1993. And this is applicable for Q index to be greater than 1. Please remember that in one of the previous class, I described you the difference between the modulus of elasticity and the deformation modulus.

Now coming to the next one, that is was given by Bieniawski in 1978, the modulus of deformation which is:

$$E_d(GPa) = 2RMR - 100\ [for\ RMR > 55]$$

Further, this is one of the most commonly used expression for the deformation modulus, which is given by Serafim and Pereira that is deformation modulus is:

$$E_d(GPa) = 10^{[\frac{RMR-10}{40}]}$$

So once again, let us say that you have these stress-strain relationship, and let us say that you have conducted the cyclic test.

So, the first cycle and then you unload and then further load so like this so deformation and here you have the load. So, you see that corresponding to any cycle you have one is the total deformation and when you unload is some deformation is retrieved it is recovered. So that is what we call as the elastic deformation, and some you have as the total deformation that is w_d.

So, when you consider the behavior as a linear as well as the non-linear behavior and the modulus that you obtain, that is what is your deformation modulus?

However, in case of the elastic modulus, you only consider the linear portion, and from there you find out the elastic modulus. So, in case of the rock mechanics and rock engineering, it is the deformation modulus which is the quite often used rather than the elastic modulus. So further, some expression which is given by Hoek et al. in 2002, they make use of GSI this disturbance factor and also sigma ci to obtain the deformation modulus.

$$E_d(GPa) = \left(1 - \frac{D}{2}\right)\sqrt{\frac{\sigma_{ci}}{100}} \times 10^{\left(\frac{GSI-10}{40}\right)} \quad for\ \sigma_{ci} < 100\ MPa$$

$$E_d(GPa) = \left(1 - \frac{D}{2}\right) \times 10^{\left(\frac{GSI-10}{40}\right)} \quad for\ \sigma_{ci} > 100\ MPa$$

Kindly take a note of the unit in view of the fact that these are all empirical correlation. You have to be careful about the units. Now, this is the first expression is applicable in case you have the UCS less than 100 MPa, and the second one is applicable for UCS greater than 100 MPa. Then few expressions which were given by similar set of authors in 2006, they have been given here in this particular manner.

$$E_d(GPa) = 100^{\left(\frac{1-D/2}{1+e^{(75+25D-GSI)/11}}\right)}$$

$$E_d(GPa) = E_i\left(0.02 + \frac{1 - D/2}{1 + e^{(60+15D-GSI)/11}}\right)$$

Again, all are the empirical correlations because you can see that many numbers are there 100, 72, 25, 11. So, all these have been obtained from the experimental data and hence you have got the empirical correlations. So, this was all about some of the basics of rock mechanics and rock engineering, so what we learnt today was the empirical failure criterion and finishes our discussion on these basics of rock engineering.

Now from the next class onwards first, I will introduce you to the various types of underground excavations that what exactly is difference between a tunnel and a cavern. And many things on similar lines and then we will continue our discussion on the particular our course which is the underground space technology. Thank you very much.