Pavement Materials
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Lecture: 26
Introduction to Viscoelasticity

Hello everyone. Welcome back to the lecture series on Pavement Materials and we are discussing about the properties related to bitumen or asphalt binder. Today, we are going to start a new topic which is on discussion related to the viscoelastic properties of bitumen. Till now, we have completed various other aspects related to the study of bitumen.

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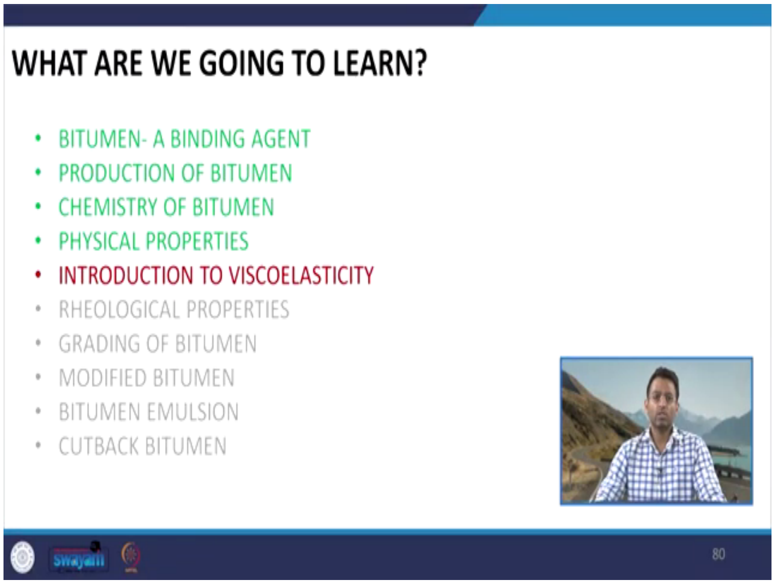

We have discussed about the chemistry of bitumen, we have discussed about the production of bitumen, we have also discussed about the various physical properties and the importance of these physical properties related to the performance of the bitumen in service condition. Bitumen, since it is a viscoelastic material shows time dependence in the response when it is deformed. So, today we are going to learn very basic concepts related to the viscoelasticity of bitumen.

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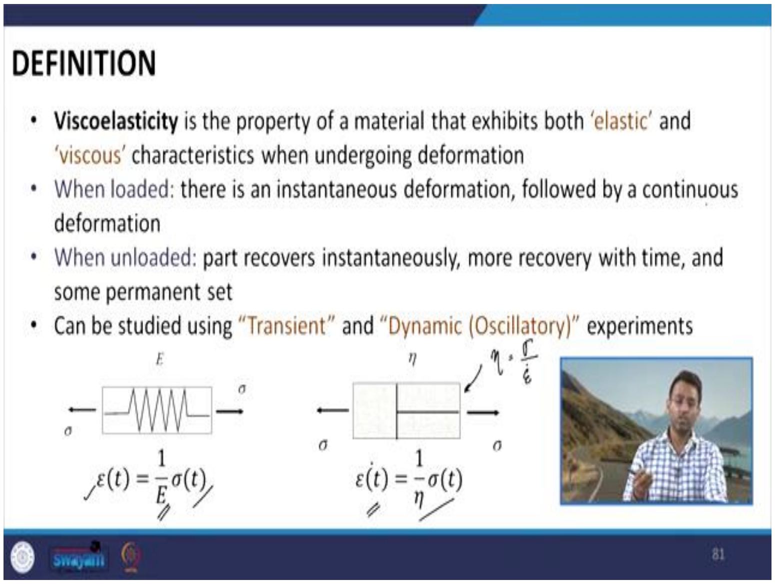

The term viscoelasticity indicates that a material which has such kind of property that is viscoelasticity will exhibit both elastic and viscous response. Now, what do we mean by an elastic and a viscous response? To understand this, let us imagine that we have a spring system which is a representation of an elastic system.

If we consider that we have a Hookean solid, the constitutive equation which can be written for a spring system is $\varepsilon(t)$, which is the strain, is equal to $\frac{1}{E} \times \sigma(t)$. On the other hand, a viscous material such as a Newtonian fluid can be represented using a dashpot or a damper. And the constitutive equation for this dashpot is that viscosity is equal to the ratio of stress to rate of strain.

Therefore, the $\varepsilon(\dot{t}) = \frac{1}{\eta} \times \sigma(t)$.. During the later half of 19th century when researchers started studying various materials, they observed that few materials showed time dependent response while undergoing deformation. They observed when these materials are loaded, there is an instantaneous deformation in the material which is followed by a continuous deformation.

And when these materials are unloaded, the material recovers partially and this partial recovery is instantaneous as soon as the load is removed. When the material is given more time, there is more recovery in the material and also after a considerable time has been given, there is some permanent set in these materials. So, this kind of material is characterized as a viscoelastic material.

Now, such kind of viscoelastic response is typical for polymeric materials and bitumen is also one such material. Now, how do we measure this response? How do we try to understand the viscoelastic response of the bitumen? We can do it using various type of experiments which can be broadly categorized as transient experiments.

Examples of transient experiments can be a creep experiment where we are giving a stress to the material and we are seeing how the strain increases with time. We can also do a relaxation experiment where we are giving a strain to the material, a constant strain and we see that how stress relaxes with time. In addition to the transient experiments, we can also perform dynamic or auxiliary experiments.

The auxiliary experiments being more popular in case of studying the properties of bitumen which we will be discussing later in this presentation. Auxiliary methods or the dynamic methods, they are more popular because using these type of experiments, we can cover a wide range of loading conditions in a relatively shorter period of time and we can study a wide range of properties of the viscoelastic material. Now, in this lecture, we will first try to understand the viscoelastic properties related to the bitumen.

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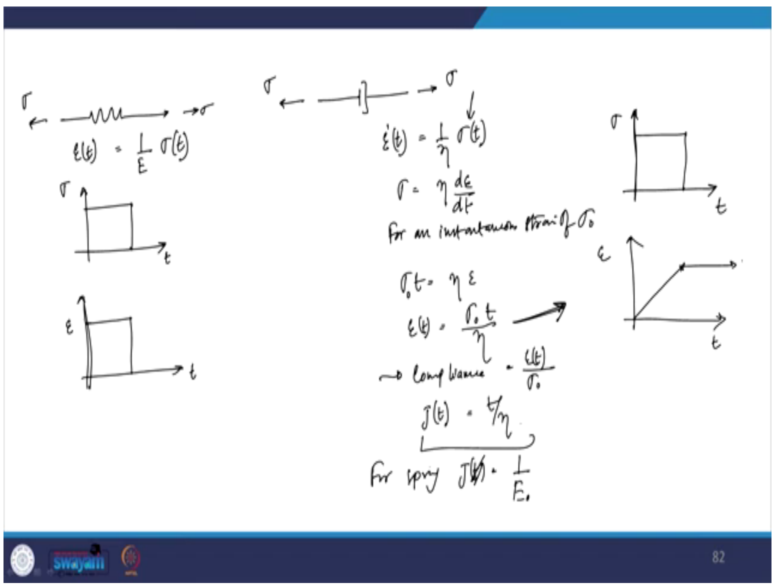

As I mentioned in the first slide that a purely elastic material can be characterized as a spring system. Let us say this is a spring system and we are performing an experiment and we are applying a load or a stress of σ. So, if we try to see the response of the material, this will be equal to $\frac{1}{E} \times \sigma(t)$. If we try to see that how the variation will look like.

So, let us say this is the variation between stress and time and we are applying a load and we are unloading it at this position. So, the response of the material will also be in proportion to the stress applied. So, this is a strain versus time. On the other hand, if we have a dashpot which is a representation for a purely viscous material, let us say we are applying a stress σ and here as I mentioned that the strain rate is equal to $\frac{1}{\eta} \times \sigma(t)$and here σ being constant. So, this is σ_0.

Here $\sigma = \eta \frac{d\varepsilon}{dt}$. Now, since we have an instantaneous strain, σ_0 for an instantaneous strain of σ_0, we can write that $\sigma_0 \times t = \eta\varepsilon$. Therefore, $\varepsilon(t) = \frac{\sigma_0 t}{\eta}$. Commonly, we also can express just like the elastic modulus, we can express a form of stiffness parameter here which is called as the compliance and this is equal to the ratio of strain versus stress.

So, the compliance which is represented as $J(t) = \frac{t}{\eta}$. Now, for spring J(t) is nothing, but the reciprocal of elastic modulus. Now, let us see that how the graphical response will be, how graphically we can understand the response. So, if this is $\sigma(t)$, we are applying a stress and then we are unloading. So, if you see this particular equation, so therefore, if we are trying to see how E is changing with t.

So, this is linear in nature. So, it will linearly increase and after you unload this will not change. So, this is the typical response of a purely viscous material.

In the previous slide, we have discussed the response of a purely elastic material, we have discussed about a response of a purely viscous material, but bitumen as I mentioned is a viscoelastic material. So, we are more interested to see that how the response will be when a viscoelastic material is subjected to different loading conditions. Let us try to understand that with a step strain experiment.

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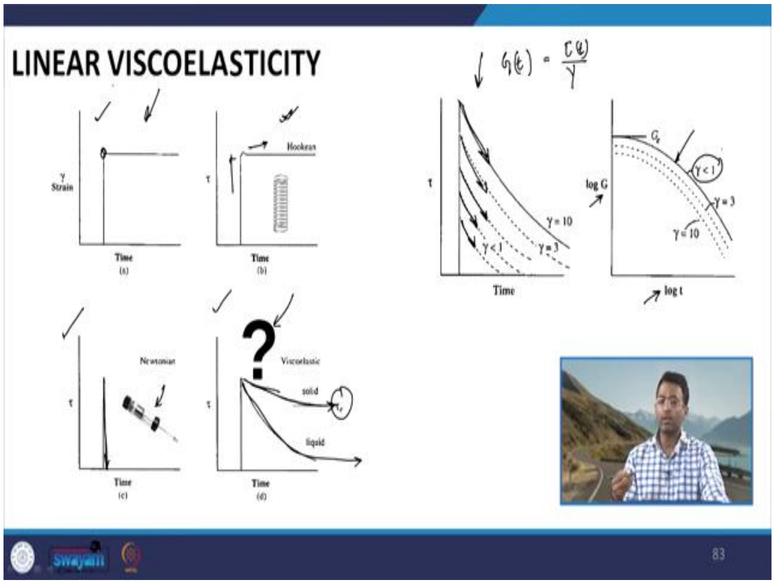

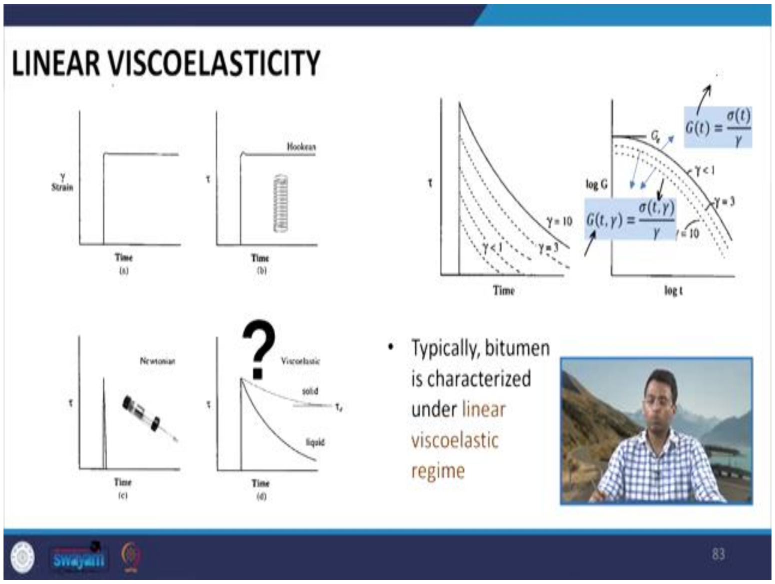

So, this here you can see we are applying a step strain to the material and we are keeping the strain constant after a particular position and we try to see how the response of different materials will be. As I said that a purely Hookean material, it can be represented as a spring element, how it will respond? So, there will be no stress relaxation as such and the stress will be proportional to strain.

So, the stress also increases and then it is constant, which is proportional to the way the strain changes with time. In case of a viscous material which can as I said the element is dashpot to represent it. So, after the load is held constant, the strain immediately relaxes, the material immediately relaxes and the stress drops down to 0 here.

Now, in case of a viscoelastic material, which is somewhere between the response of the elastic material and the viscous material, there will be a relaxation in the stress. So, depending on the type of material, for example, if you have a viscoelastic solid, in that case the stress will relax over a period of time and after a particular time, it will attain equilibrium and we will have a equilibrium stress of $\tau\ (t)$ here.

Whereas, in case of a viscoelastic liquid, if given sufficient amount of time, there will be more stress relaxation and it may happen that after a particular period of time, the stress can completely relax and drop down to 0. So, depending on the type of viscoelastic material, the response or the relaxation spectrum can be different.

Now, the question is if a purely elastic material can be represented using a spring, on the other hand a purely viscous material can be represented using a dashpot, then how using these elements can we represent the response of a viscoelastic material. So, this we are going to touch upon today. We will discuss that how the spring and the dashpot and their combinations can be used to see the response of a viscoelastic material.

And we will also try to see the constitutive equation governing the simple elements which we will be discussing today. But before we can jump into such discussion, few more points should be clarified and this is something which we are going to discuss now. Now, you see in this experiment we saw that this is a stress relaxation experiment, where we saw that if we apply a strain, we keep it constant after a particular point, then how different material responds, how a viscoelastic material responds.

Now, the magnitude of the strain can be varied, is not it. So, let us say that we are carrying out several experiments with several magnitude of the strain which we are applying in the material and we are seeing the response of the viscoelastic material. For example, if you see in this particular graph, this shows that at different strain how the viscoelastic material will respond or the stress will relax.

So, you can see here that if the strain is very low, let us say the strain is less than 1 percent, then these are the graphs, alright. Now, as we increase the strain, the response changes and these are some of the typical responses. Now, since we have the stress response and we have the strain response, this can be transformed into a parameter or a stiffness characteristic, let us say we call it as a relaxation modulus G(t).

So, we are expecting that the value of G will be a function of time. So, this will be equal to the stress which is a function of time divided by strain here. Now, if we try to see the variation of G(t) with respect to time, let us say we are plotting it in a log-log scale which is a very typical way of plotting the variation of a stiffness of a viscoelastic material with change in time. So, we usually use a log-log plot.

So, you will see that the value of G with time, it can be explained through a single curve if the strain in the material is less than 1 percent. So, for lower value of strains, we do not have multiple curves to explain the variation of relaxation modulus with time. This curve can be written in the form that the relaxation modulus with respect to time is $\frac{\sigma(t) or\ \tau(t)}{\gamma}$, the strain which we have applied.

For higher strain levels, when the strain level is let us say more than 1 percent, 2 percent, 3 percent, 5 percent, 10 percent, we see that depending on the strain, we have different curves for the relaxation modulus. And since now the curve is shifted and we have multiple curves at multiple strain levels, such type of response in addition to being a function of time is also a function of the magnitude of the strain which we have applied.

So, for such curve, we can write that g which is the relaxation modulus is a function of time as well as the magnitude of strain and therefore, the stress is also a function of time as well as the magnitude of $\frac{strain}{\gamma}$. And such type of response, so we have to differentiate these two type of response. So, this relaxation modulus for lower value of strain is basically called as the linear viscoelastic response.

So, and this is typical for viscoelastic material only when it is subjected to lower value of strain. For higher value of strain, since the response is a function of time as well as the magnitude of strain, such response is called as non-linear viscoelastic response. Now, for most of the purposes, bitumen is characterized under the linear viscoelastic regime.

So, in today's lecture, we will be discussing only about the basic viscoelastic principles that too under the linear viscoelastic regime. Moving forward, another important principle related to the linear viscoelastic response of a viscoelastic material is the discussion on Boltzmann superposition principle.

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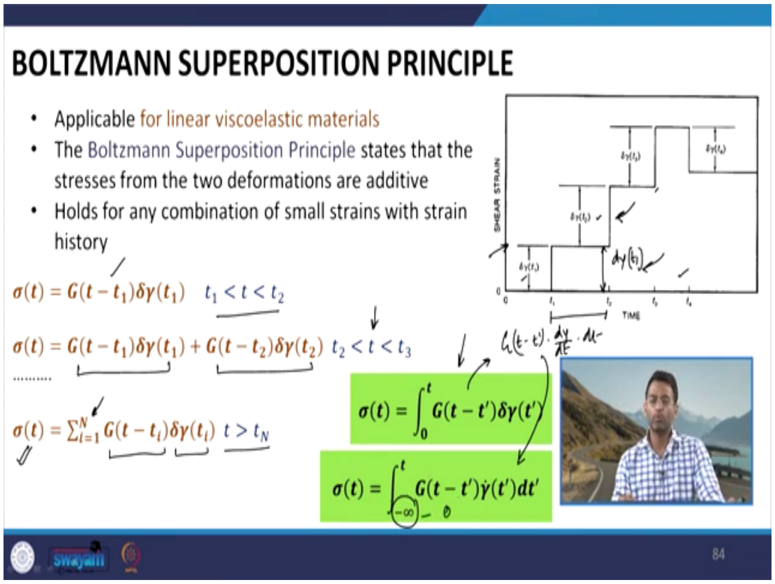

Now, Boltzmann superposition principle is applicable only for linear viscoelastic materials that is viscoelastic materials which are subjected to lower value of strains. So, the Boltzmann superposition principle, it states that the stresses from two deformations or the strain history of the viscoelastic materials are additive in nature. Let us understand this using a step strain experiment.

So, in this experiment, you see that step strains are applied to the viscoelastic material at different time periods. So, at t_1, $\delta\gamma t_1$ is applied and then it is kept constant up to t_2 and then at t_2, $\delta\gamma t_2$ is applied which is kept constant up to t_3, then $\delta\gamma t_3$ is applied and so on. At t_4, the strain is reduced to $\delta\gamma t_4$and it is kept constant for a particular period of time.

If we are interested to see the stress response for this multiple step strain experiment, then according to the Boltzmann superposition principle, at any given time we can calculate the stress response if we know the strain history, which means that let us say from t_1 to t_2, the stress can be written as multiplication of the relaxation modulus and the strain which we have applied.

So, $\sigma(t) = G(t - t_1)\delta\gamma t_1$ and this is applicable for between t_1 to t_2. Now, at t_2, we are increasing the strain to $\delta\gamma t_2$ which means the actual strain is equal to $\delta\gamma t_1 + \delta\gamma t_2$ plus d gamma t2 and therefore, the stress is also proportionally added and stress becomes equal to the stress for $\delta\gamma t_1 +$ the stress at $\delta\gamma t_2$.

So, this becomes equal to the first stress during the first period that is t_1 to t_2 plus for the second period that is $G(t - t_2)\delta\gamma t_2$ and this is applicable for t_2 to t_3. And subsequently at different time periods, we can do the addition and calculate the stress response. So, in the generalized form can be written in this way that the stress at any time t can be written as $\sum_{i=1}^{n}(t-t_i)\delta\gamma t_i$, where n is the time period up to where we are trying to see the calculate the stress response.

So, this is equal to summation of $\sum_{i=1}^{n}(t-t_i)\delta\gamma t_i$ and this is applicable for $t>t_N$. Instead of a step strain, if it is a smooth experiment, a continuous strain is being applied, then we can write it in the integration form that of $\sigma(t)=\int_0^t G(t-t')\delta\gamma t'$.

Now, the same equation can be transformed as $\sigma(t)=\int_{-\infty}^{t} G(t-t')\delta\frac{d\gamma}{dt}dt$., which is written in the second equation. Now, here you will see that one change is that in the second equation a minus infinity is kept. So, this is just to remind that all the strain history has to be added to get the actual stress response at any given time.

But usually during experiments, when we start with a fresh material that is not loaded or does not have any strain history. Therefore, instead of minus infinity we just put 0 and we are seeing the response up to a time period of t.

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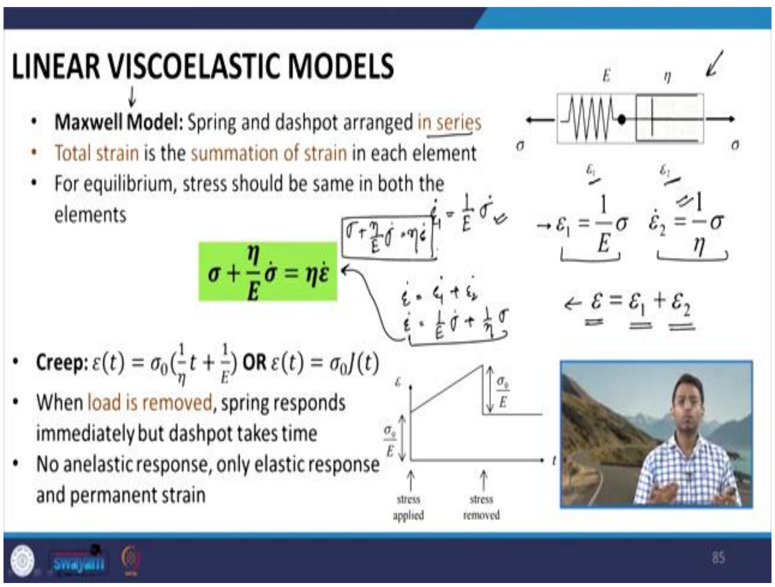

Having said this, now let us try to understand the concept of linear viscoelasticity using some of the viscoelastic, common viscoelastic models. In order to understand the actual model which will represent the viscoelastic behaviour, we have to first go through the constitutive equations of some of the models which are used to form the viscoelastic models.

The first model which we will be discussing is the Maxwell model and remember in all these models which we are going to discuss today, spring and dashpot are used as the basic elements. What is a Maxwell model? It is a combination of spring and dashpot which are arranged in series which you can see here that we have a spring and a dashpot which is arranged in series.

Here we are applying a stress σ, we have ε_1, ε_2 here which is the strain in the spring element and strain in the dashpot element. So, in the Maxwell model the total strain is the summation of the strain in each element, which means the total strain at any particular time period is equal to the strain in the spring element plus the strain in the dashpot element and for the equilibrium of this system the stress should be same in both the elements.

Let us see that how the constitutive equation can be derived. So, we have here first we have written the constitutive equation of the spring which is $\varepsilon_1 = \frac{\sigma}{E}$; we have written the constitutive equation of a dashpot which is the $\dot{\varepsilon_2} = \frac{\sigma}{\eta}$. So, if I take the derivative of the first equation here. So, we have $\varepsilon_1 = \frac{\sigma_0}{E}$.

And if we also take the derivative in both the side of the equation we have total strain is equal to the derivative of the first strain plus the derivative of the second strain. So, $\varepsilon_0 = \frac{\sigma_0}{E}$ which comes from this equation plus if you see this equation this is $\frac{1}{\eta} \times \sigma$ and this is what is written here.

So, here if you take all the stresses on the right hand side and strain on the left hand side. So, you will get that $\sigma + \frac{\eta}{E} \times \dot{\sigma} = \eta\dot{\varepsilon}$ where this dot represents the derivative with respect to time. So, this is the constitutive equation of the Maxwell model and using this constitutive equation we can study the response of the material under various conditions.

For example, in a creep experiment how it will respond; when we are doing a relaxation experiment how it will respond. So, let us see the response of this material under creep and relaxation experiments. So, in creep experiment what are we actually doing? We are applying a stress of σ_0 let us say we are applying a stress of σ_0 t and we are seeing the change in the strain of the material with respect to time.

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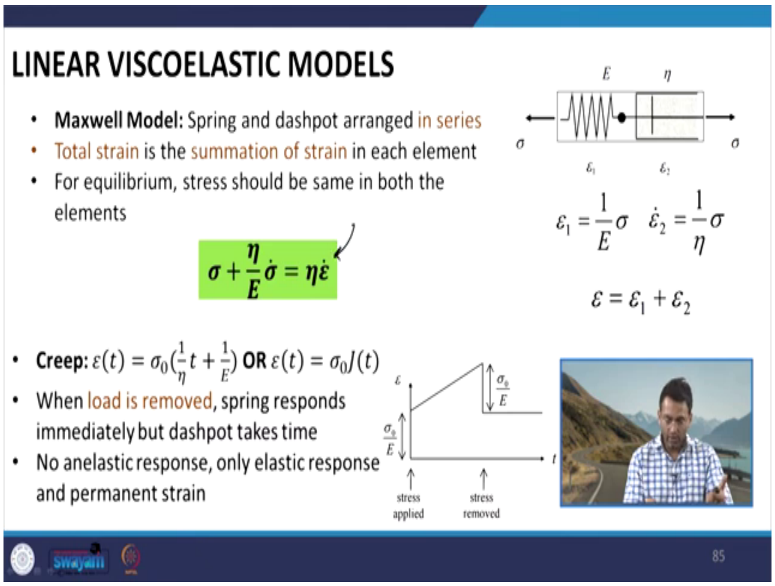

So, we can try to understand the response or derive the response using the constitutive equation. So, which we can do here.

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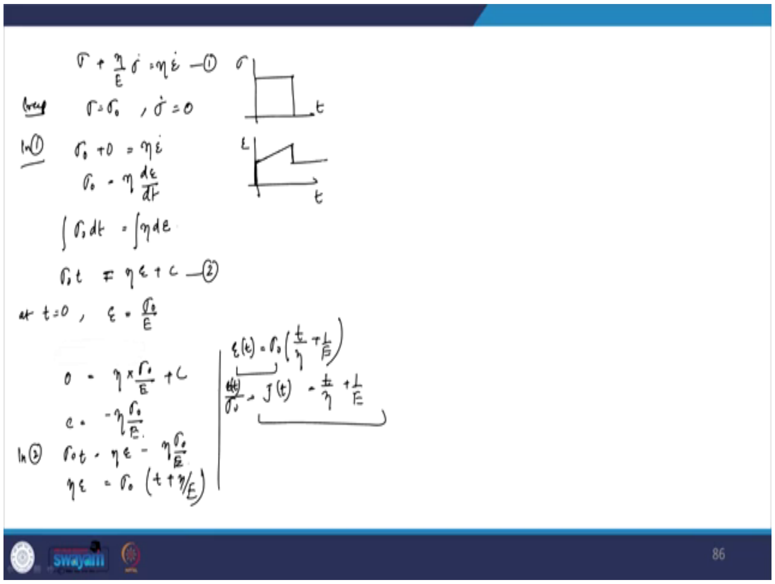

So, our constitutive equation was $\sigma + \frac{\eta}{E} \times \dot{\sigma} = \eta \dot{\varepsilon}$. So, when it is a creep experiment sigma is equal to σ_0 and $\dot{\sigma} = 0$ because this is a constant. So, derivative will be equal to 0. Let us say this is equation 1. So, therefore, in equation 1 we can write $\sigma_0 + 0 = \eta \dot{\varepsilon}$, $\sigma_0 = \eta \frac{d\varepsilon}{dt}$.

So, , $\sigma_0 dt = \eta d\varepsilon$. So, if we integrate both the sides $\int \sigma_0 dt = \int \eta d\varepsilon$ we get $\sigma_0 t = \eta \varepsilon + C$ and here at t=0 what will happen? At t=0 the strain will be the one in this spring element because dashpot takes time to respond. So, $\varepsilon = \frac{\sigma_0}{E}$. So, therefore, at t=0 this is equal to $\eta \frac{\sigma_0}{E} + c$.

So, $C = -\eta \frac{\sigma_0}{E}$. So, if I put this in equation 2, then we get $\sigma_0 t = \eta \varepsilon - \eta \frac{\sigma_0}{E}$. Here, $\eta \varepsilon = \sigma_0 t + \eta \frac{\sigma_0}{E}$ and therefore, the strain with respect to $\varepsilon = \sigma_0 \left(\frac{t}{\eta} + \frac{1}{E}\right)$. Here also we can derive the stiffness parameter.

So, if we take the ratio of $\frac{\varepsilon}{\sigma_0} = J(t) = \left(\frac{t}{\eta} + \frac{1}{E}\right)$. Now, in this experiment if we remove the load after a particular time. So, try to imagine what will happen that once the load is removed the spring will respond immediately. So, there is a spring it will try to come back to its initial position once the load is removed, but the dashpot takes time to, because it will retard the response.

So, it will take time, is not it. So, if we try to see that what happens when there is a load and a recovery, so you will see that initially there will be a response by this spring, this will increase with time, there will be a drop and it will be constant?

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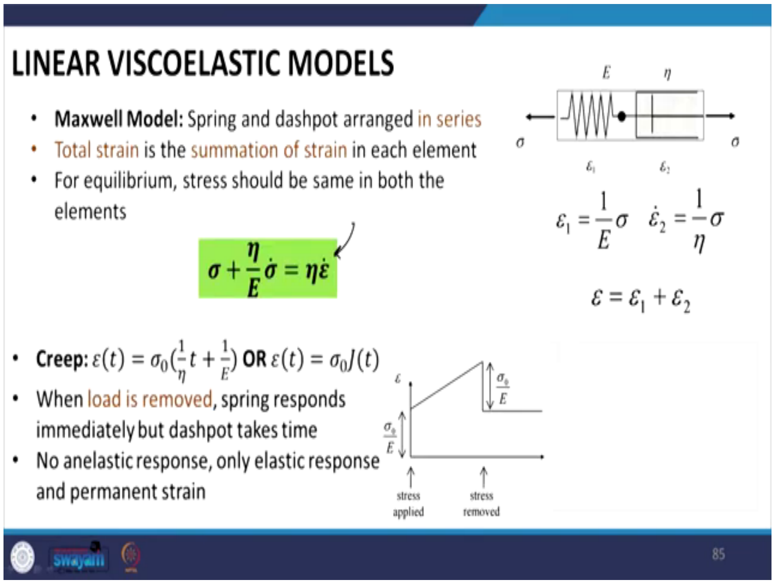

And this is what is shown in this particular figure and this is the response of a Maxwell element when it is subjected to a creep loading. And if you can see in this particular graph that this particular model cannot explain the n elastic response which is typical for a viscoelastic material and only elastic and permanent strain can be explained using this particular model. So, definitely only the Maxwell model cannot be used to describe the response of a viscoelastic material.

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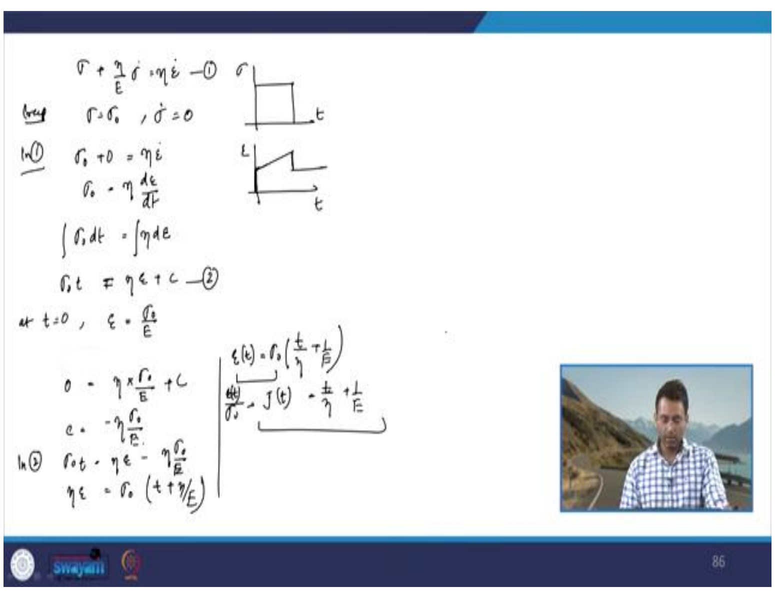

So, before moving to the next element let us see that how the same Maxwell model will respond when it is subjected to a relaxation experiment.

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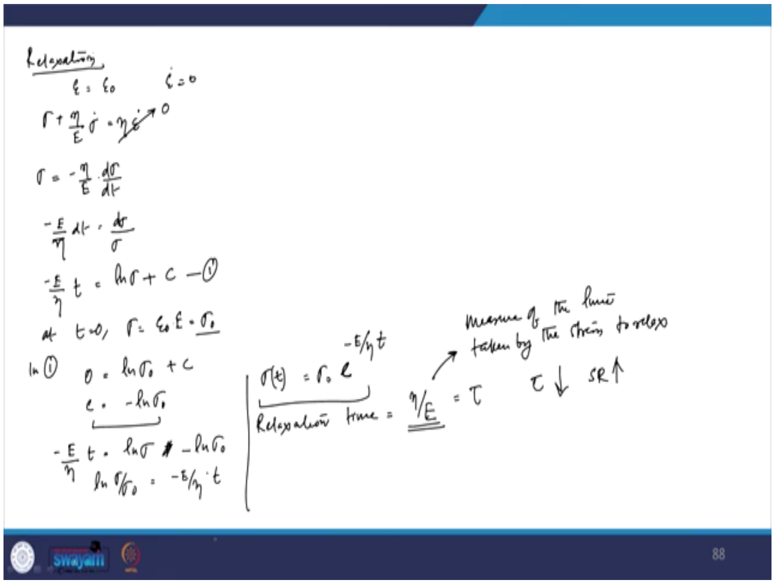

So, in a relaxation experiment, so we are discussing about the Maxwell model here. What we are doing, we are applying a constant strain and we are seeing the stress relaxation. So, if you can recall the constitutive equation was $\sigma + \frac{\eta}{E} \times \dot{\sigma} = \eta \dot{\varepsilon}$. So, here $\dot{\varepsilon} = 0$. So, this becomes equal to 0. So, we have

$\sigma = -\frac{\eta}{E} \times \frac{d\sigma}{dt}$.

Here we can write that $\frac{-E}{\eta} dt = \frac{d\sigma}{\sigma}$. So, minus E by eta into t, if we integrate both sides this becomes $\frac{-E}{\eta} t = ln\sigma + c$ and at t=0, the sigma will be the one from the spring element. So, this is equal to $\varepsilon_0 \times E$. Let us say that this is σ_0, the initial stress. So, this becomes equal to, so at t=0. So, in equation 1, $0 = ln\sigma_0 + c$.

So, $C = -ln\sigma_0$. Putting this in the equation here, so therefore, we can write $\frac{-E}{\eta} t = ln\sigma - ln\sigma_0$ Here we have $\frac{ln\sigma}{ln\sigma_0} = \frac{-E}{\eta} t$, which can be written finally as that sigma which is relaxing with time is $\sigma(t) = \sigma_0 e^{\frac{-E}{\eta} t}$.

Here an important parameter to be defined is the relaxation time. So, the relaxation time is written as $\frac{E}{\eta}$. And this is a measure of the time taken for the stress to relax, measure of the time taken by the stress to relax. Now, shorter is the relaxation time more rapid will be the stress relaxation. So, if I say that this is equal to something called as τ. So, when tau is less stress relaxation will be more.

So, this is again an important parameter to describe the response or the behaviour of the material when it is loaded under relaxation.

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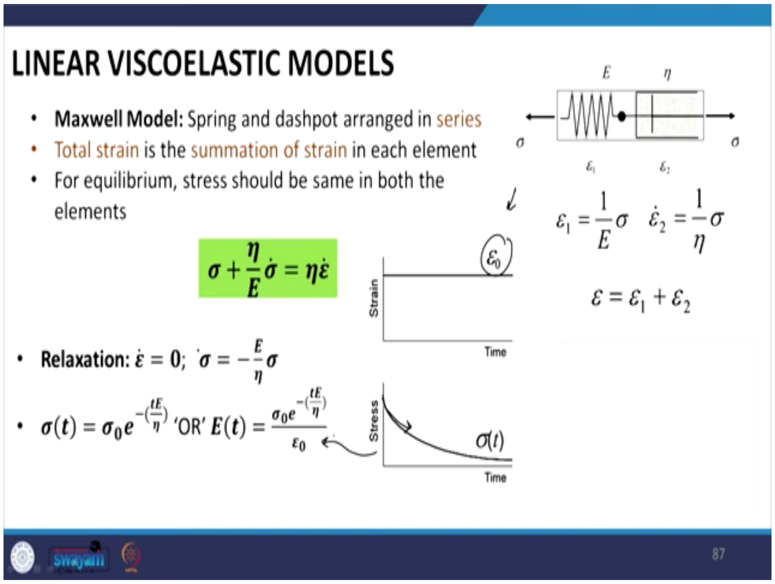

Having said that if we try to see the response you will see it in this form that we are applying a strain and there is a relaxation, which we have tried to understand with respect to this particular equation.

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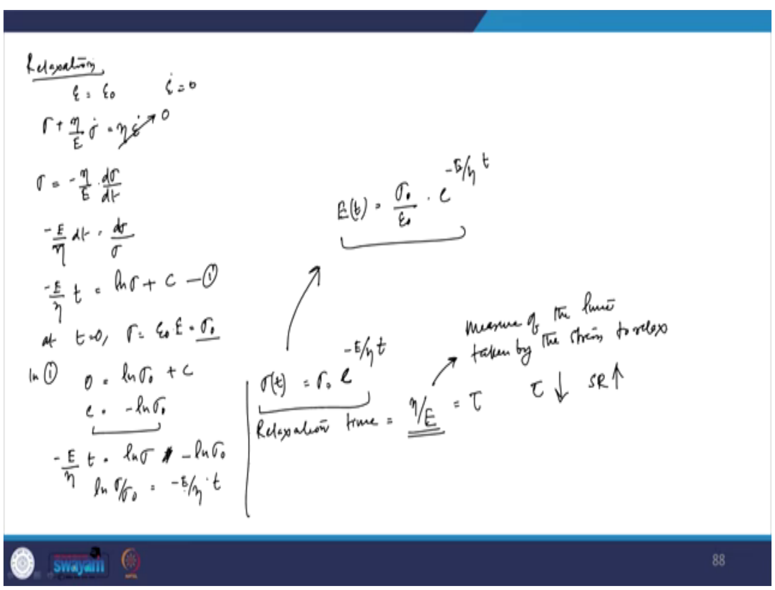

So, in this equation also we can describe it using the stiffness parameter which is the relaxation modulus which can be written as $E\ (t) = \frac{\sigma_0}{\varepsilon_0} e^{\frac{-E}{\eta}t}$. So, we have just taken the ratio of the stress and the input strain. So, now let us see another element or another form of combination of the spring and dashpot element which is the Kelvin Voigt model.

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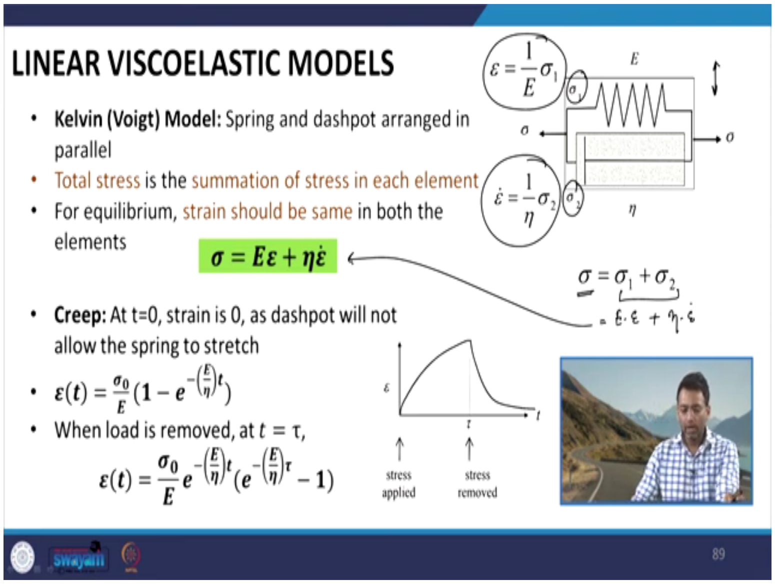

In this model the spring and the dashpot they are arranged in parallel to each other. So, here the total stress is, will be the summation of the stress in each element. So, if we know the stress σ_1, if we know the stress σ_2. So, the total stress $\sigma = \sigma_1 + \sigma_2$ and the strain in both the elements for equilibrium should be same. So, these are the two conditions in the Kelvin model.

Let us try to see the constitutive equation which is simple to understand here. So, we have $\sigma = \sigma_1 + \sigma_2$ and this is the constitutive equation for the spring element, this is the constitutive equation for the dashpot element. So, here $\sigma_1 = E\varepsilon$ and $\sigma_2 = \eta\dot{\varepsilon}$ and this is what is written here.

And this is basically the constitutive equation for the Kelvin element. Now, let us try to see, similarly what we have done in case of Maxwell element that how this model will respond to creep loading and to relaxation loading. So, let us start with creep experiment.

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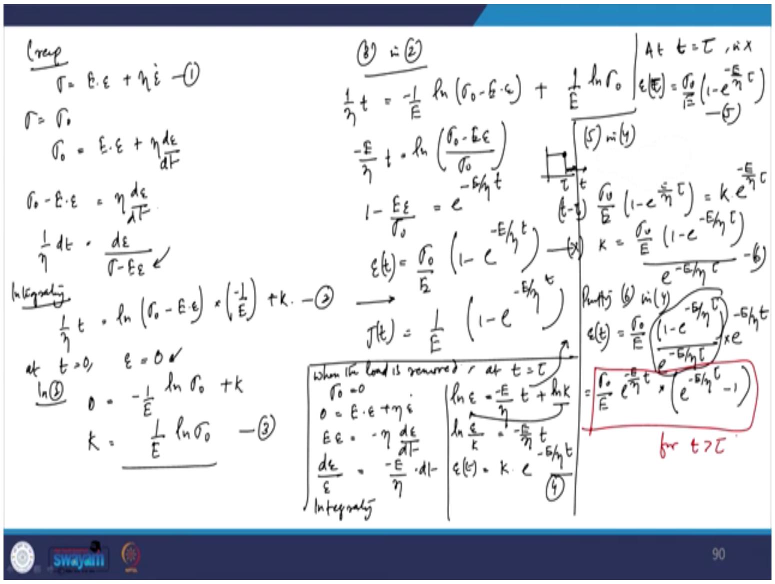

Let us just first write down that the constitutive equation is equal to $\sigma = E\varepsilon + \eta\dot{\varepsilon}$. So, in creep experiment $\sigma = \sigma_0$. So, therefore, if we put it in equation 1, we get $\sigma_0 = E\varepsilon + \eta\frac{d\varepsilon}{dt}$. So, here we can write that $\sigma_0 - E\varepsilon = \eta\frac{d\varepsilon}{dt}$.

Therefore, $\frac{dt}{\eta} = \frac{d\varepsilon}{\sigma - E\varepsilon}$. If we integrate both side, what we will get? That $\frac{t}{\eta} = \ln(\sigma_0 - E\varepsilon) \times \left(\frac{-1}{E}\right) + K$

I am writing σ sigma is σ_0 because that is the stress which we have applied $\varepsilon \times \left(\frac{-1}{E}\right)$. So, this will be the integration of this particular term plus a constant k, the integration constant.

So, at t = 0, $\varepsilon = 0$. This is because when we have applied the stress, so immediately the spring will want to respond, but since there is a dashpot there, it will stop the movement of the spring. So, therefore, just at t=0, we will not see any response from this particular combination of element.

So, therefore, at t =0, the strain becomes equal to 0. So, if we put this in equation 2, we get $0 = \frac{-1}{E}\ln(\sigma_0) + k$. So, k becomes $k = \frac{1}{E}\ln(\sigma_0)$. Now, if we put 3 into, what we get here? That $\frac{t}{\eta} = \frac{-1}{E}\ln(\sigma_0 - E\varepsilon) + \frac{1}{E}\ln(\sigma_0)$.

So, this can be further solved that $\frac{Et}{\eta} = ln\frac{(\sigma_0 - E\varepsilon)}{\sigma_0}$. Further solving it, $1 - \frac{(E\varepsilon)}{\sigma_0} = e^{\frac{-E}{\eta}t}$. So, here the strain becomes equal to with respect to time of course, becomes $\varepsilon(t) = \frac{\sigma_0}{E}(1 - e^{\frac{-E}{\eta}t})$.

So, this is how the response of the strain will look like in case of the Kelvin-Voigt model, when it is loaded in creep. If we want to see the stiffness parameter which is the compliance, this is can be this is the ratio of strain to stress. So, this becomes $J(t) = \frac{1}{E}(1 - e^{\frac{-E}{\eta}t})$.

Now, here we are also interested to see that what happens when the load is removed. So, I will try to complete this in this particular slide. So, I am just dividing it in parts. Here when the load is removed, so let us say that the load is removed at $t = \tau$. So, at $t = \tau$, we are basically writing that $\sigma_0 = 0$. So, if you see the constitutive equation which is 1, here $0 = E\varepsilon + \eta\dot{\varepsilon}$.

If we try to solve this particular equation, what we get that $E\varepsilon = -\eta \frac{d\varepsilon}{dt}$. So, $\frac{d\varepsilon}{\varepsilon} = \frac{-Edt}{\eta}$. I am just saving some space here. If we integrate what we get, $ln\varepsilon = \frac{-Et}{\eta} + lnK$ Now, here you have to remember this t is actually after the load removal.

So, if let us say that the load is removed at this particular position and we are trying to see what happens here. So, this t is nothing but, if you are trying to define t beyond the time of load removal τ. So, this will be, this equation which we are discussing will be related to $t - \tau$, the location or the position $t - \tau$. Anyway, so this is fine here. So, here if you bring it on the left hand side, so this becomes $ln\frac{\varepsilon}{k} = \frac{-Et}{\eta}$.

So, $\varepsilon(t) = ke^{\frac{-E}{\eta}t}$. Let us say this is 4. Here our initial condition is that at $t = \tau$, let us say that this is the equation x. If you see the in x, $\varepsilon(\tau)$ becomes equal to at $t = \tau$, this becomes equal to $\varepsilon(\tau) = \frac{\sigma_0}{E}(1 - e^{\frac{-E}{\eta}\tau})$.

Now, putting this in 4, what we get? Let us say this is 5. So, 5 in 4. So, putting this in 4, what we get? That $\frac{\sigma_0}{E}\left(1 - e^{\frac{-E}{\eta}\tau}\right) = ke^{\frac{-E}{\eta}\tau}$ So, we are looking at $t = \tau$ here. So, $k = \frac{\frac{\sigma_0}{E}\left(1-e^{\frac{-E}{\eta}\tau}\right)}{e^{\frac{-E}{\eta}\tau}}$.

Now, we have the value of k, let us say that this is 6. Now, putting 6 in 4, what we have?
$\varepsilon(t) = \frac{\frac{\sigma_0}{E}\left(1-e^{\frac{-E}{\eta}\tau}\right)}{e^{\frac{-E}{\eta}\tau}} e^{\frac{-E}{\eta}t}$.

If we take this expression and if we rearrange it, we will get $\varepsilon(t) = \frac{\sigma_0}{E} e^{\frac{-E}{\eta}t}(e^{\frac{-E}{\eta}\tau} - 1)$. And this is the strain for $t > \tau$. So, this is what will happen just after the recovery of the sample in the creep condition.

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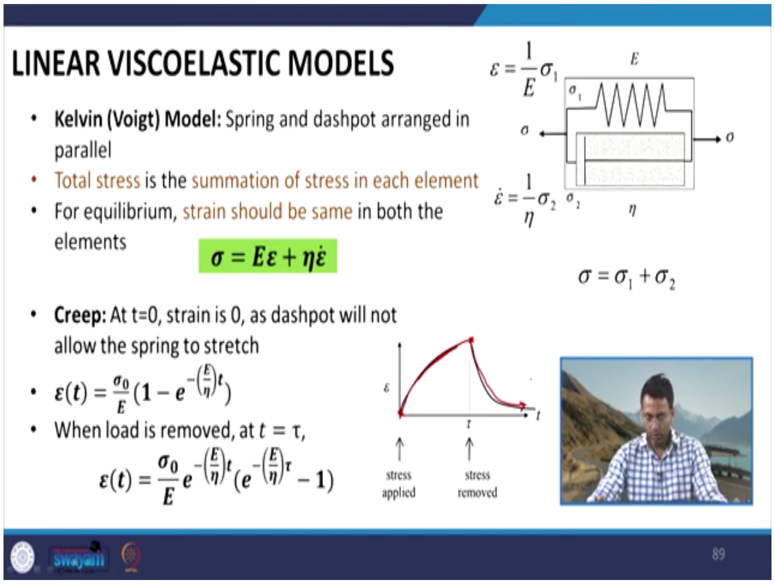

So, therefore, if we go back and try to see what we have just derived, we have the value of epsilon t in the creep loading and when the load is removed at $t = \tau$, we have the expression for the strain after that particular point. Here what we see that at $t = 0$, we have an viscous response.

So, it starts, the strain increases and after it is unloaded, we have the retarded strain because of the viscous component of the dashpot and this is how the strain reduces after the removal of the load. This cannot be a true representation of a viscoelastic material because here we see that mostly the viscous response is captured and we are not able to see any delayed elastic response or an elastic response in the material.

But of course, Maxwell's Kelvin model will be used in some form in integration with the Maxwell model to describe the response of a linear viscoelastic material. Having discussed about this, now let us talk about the relaxation which is more simple to understand in case of a Kelvin Voigt model.

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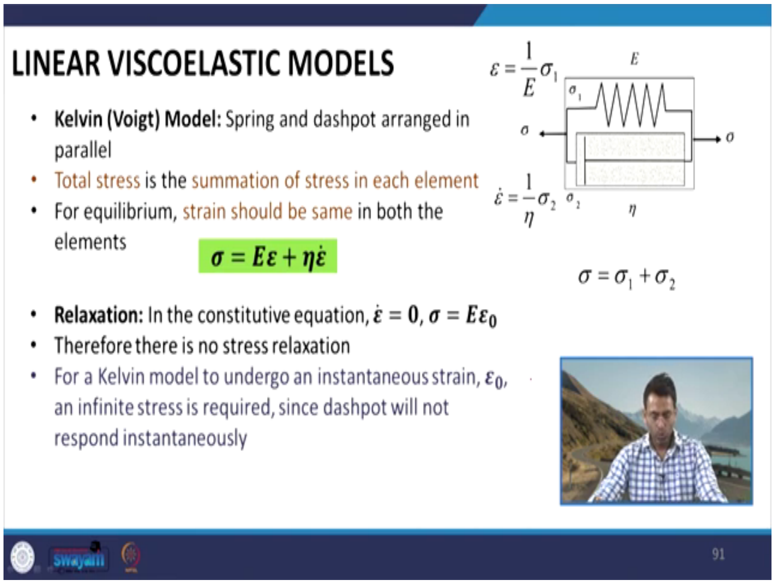

So, what we do in the relaxation? We are basically applying a constant strain. So, if you see the constitutive equation here, so this becomes equal to 0, straightforward is not it. So, we have $\sigma = E\varepsilon_0$. So, therefore, there is basically no stress relaxation and if you try to understand it from the perspective of the Kelvin Voigt model.

So, what happens that if you want the system to undergo an instantaneous strain which means you are applying a strain and leaving it, but for applying a strain, instantaneous strain, you have to actually induce a infinite stress in the material because we have the dashpot which would not allow this instantaneous strain to be induced in the material, because it does not respond instantaneously.

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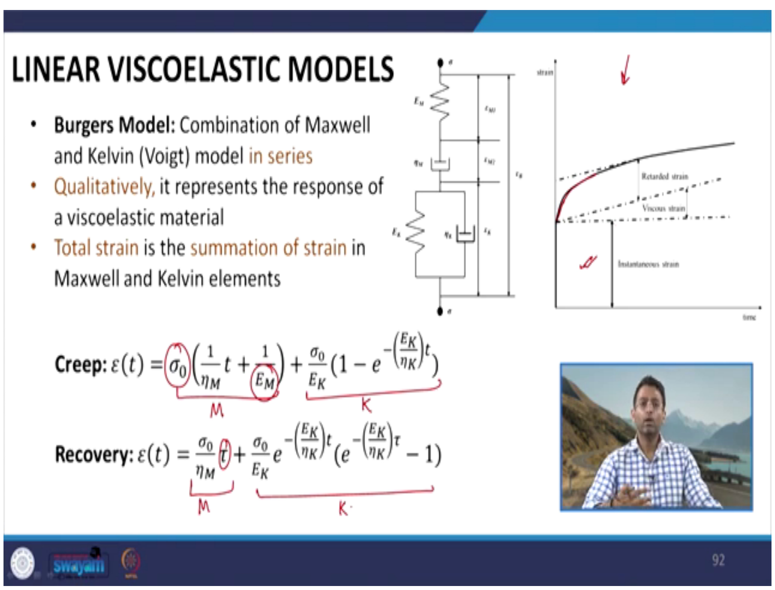

Now, we discuss about the Burgers model which is basically a combination of Maxwell model and Kelvin Voigt model in series and Burgers model have been very commonly used to describe the response of a viscoelastic material. So, the model looks something like this. Now, since you have a Maxwell element, so you can see that this is the Maxwell element and the parameters has been assigned here with a suffix m.

So, the elastic modulus or the spring constant is E_M, this is eta suffix m, m is for Maxwell model, the strain induced is ε_{M1}, ε_{M2}here. And then you have the Kelvin model here. So, the spring constant here is represented as capital E_K, $\eta_K$and the strain which is basically the same in both the elements is represented as ε_K.

And the total strain in the Burgers model is ε_b where b actually represents or denotes the Burgers model. So, we do not have to do any further derivation here because we have already done while discussing about the previous models. Now, since this is a model arranged in series, so we know that when a model is arranged in series, the strain can be decomposed as the summation of all the strains.

So, this will be $\varepsilon_{M1} + \varepsilon_{M2} + \varepsilon_K$. So, this is very simple to understand. So, in case of a creep loading if we apply. So, total strain is the summation of strain in the Maxwell and the Kelvin elements and we know that what isε_{M1}, we know what is ε_{M2}, we also know that what is the strain in the Kelvin element in case of creep loading which we have already discussed.

And if you try to club the previous equations which we have discussed through derivations, you will see that under the creep loading this is equal to sigma naught into this plus this. So, this is the derivation which we did for the Kelvin element and this is the derivation we did for the Maxwell element. Now, this is under the creep loading and this is explained using this particular figure that at t=0, there is an instantaneous strain in the material which is because of the presence of the spring here.

You can see that if you put all the t=0, so you will have the $\frac{\sigma_0}{E_M}$here. So, which is the response from the spring element and then after that the dashpot will try to retard the behavior. So, the strain will increase at a retarded rate and the entire strain after this position can be divided into two parts. So, we will have a viscous strain which is because of the presence of the dashpot and the retarded strain.

So, in the retarded strain spring is trying to respond, but dashpot is stopping it to respond. So, the summation is retarded in nature. So, we have this retarded strain here. Similarly, if you want to see what happens after unloading, once the load is applied and it is unloaded which means we are trying to see the recovery phase, again we have derived all the equations previously. So, we do not have to worry about it.

So, in the recovery phase again it is $\frac{\sigma_0}{\eta_M}\tau$ where τ represents the value of t beyond the recovery point. So, this is $\frac{\sigma_0}{\eta_M}\tau$ which is the point of recovery plus this equation, which we have already derived in case of recovery of a Kelvin model. So, this is from the Maxwell model, this is from the Kelvin model. Let us say we have an experimental data.

So, we can model this behavior using the creep and recovery response. So, as I said Burgers model has been commonly used in the literature to explain the response of different types of polymeric materials, not only bitumen, but in different fields of sciences.

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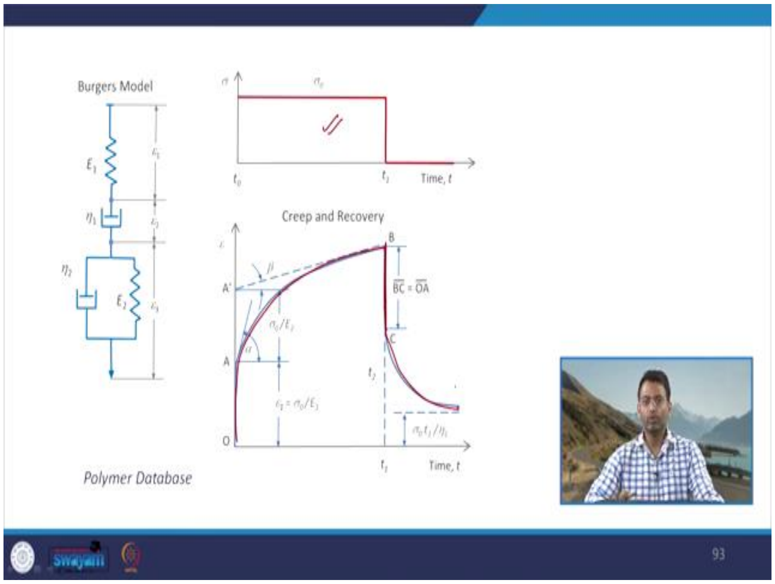

So, again this is just a picture to explain how the entire creep and recovery response will look like which we have already discussed. So, this is the creep phase you can see that the strain increases and after the recovery we will have an instantaneous recovery because of the presence of the spring and then a delayed recovery because of the presence of the dashpot. So, depending on the property of the bitumen the form of the graph will change.

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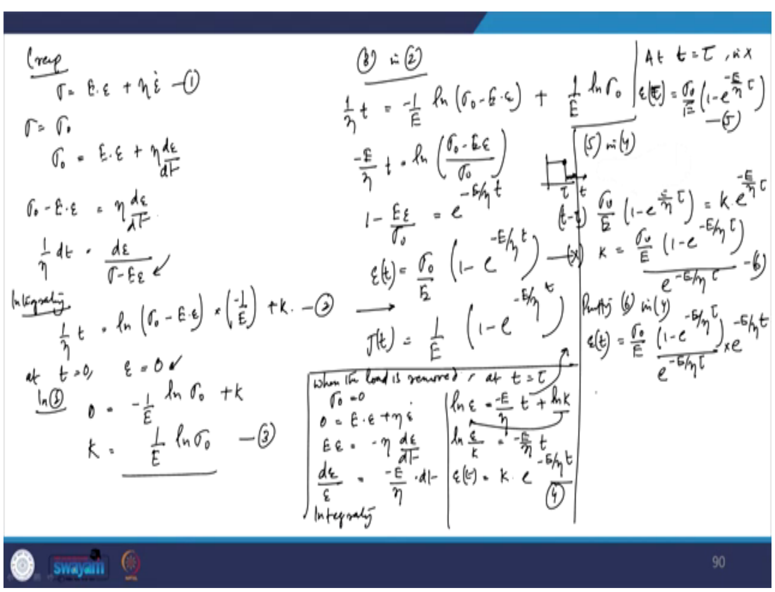

Now, Burgers model qualitatively do it explains the response of a viscoelastic model, but quantitatively if we see a single Kelvin element or Kelvin model within the Burgers 4 element model it is insufficient to describe the retarded strain over longer period of time. So, because we have only one Kelvin element which is responsible to represent the retarded strain behavior of the viscoelastic material.

But the presence of one Kelvin model is insufficient. Therefore, a generalized form of the Burger's model is that we have the spring element which is one Maxwell model which is arranged in series with n number of Kelvin models and this is called as the generalized model and this has been used very frequently in different programs in different modeling techniques to understand the entire response of a viscoelastic system like bitumen.

So, here we have the same set of equation which we saw in the previous slide only that we have to do the summation corresponding to all the Kelvin elements we have. So, the Maxwell component will remain the same and in the Kelvin element we will have the summation to sum the response of all the Kelvin elements which we have. So, therefore, this is i =1×n and we have i number of such elements present in the system.

And how many element should be used? It depends typically on the experiment which we are performing and the response of the material. Similarly, in the recovery phase this is from the Maxwell component and again the generalized form of, so I think here there is a correction required that there should be a summation here from i =1×n and this is from the Kelvin element which is summation of all the Kelvin elements.

So, for shorter loading times which we typically use while performing the experiments the viscous response because of the Maxwell element can be actually neglected and we can use only the elastic response. So, if we are interested to find out the stiffness parameter such as creep compliance we can basically use it in a generalized form.

And here you see this is the ratio of strain to stress which can be derived using the first equation here and here we are omitting the viscous response from the Maxwell component. So, till now I hope that the understanding about the viscoelastic response using the basic elements that is spring and dashpot is clear to all of you.

Now, viscoelasticity anyway is a very big branch of study in polymer sciences and there is a lot which can be discussed about it, but in this presentation we are limiting our discussion only to the basic principles which are required to form a generic understanding about the response of the bitumen when it is subjected to different temperature conditions, different frequency conditions, different loading conditions.

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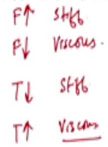
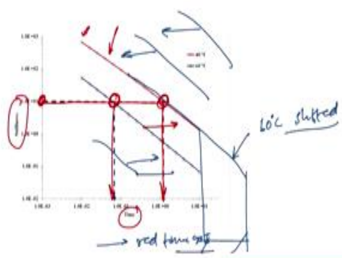
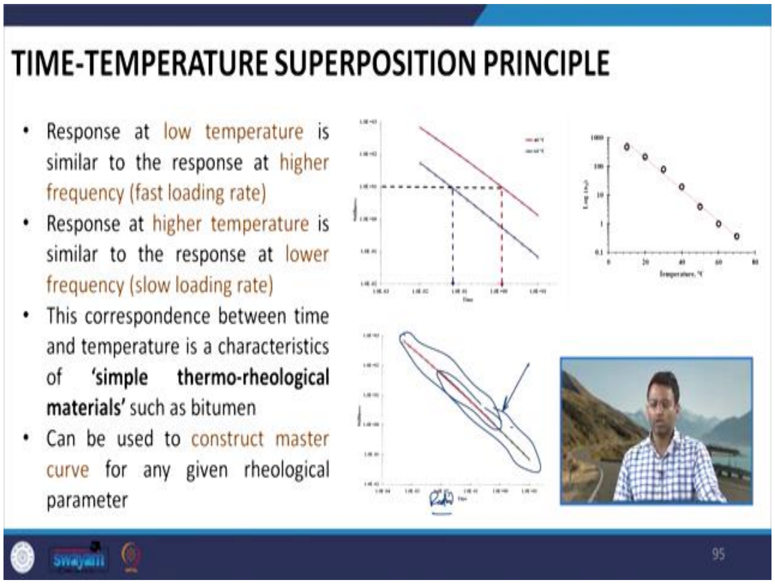
TIME-TEMPERATURE SUPERPOSITION PRINCIPLE



TIME-TEMPERATURE SUPERPOSITION PRINCIPLE

- Response at low temperature is similar to the response at higher frequency (fast loading rate)
- Response at higher temperature is similar to the response at lower frequency (slow loading rate)





So, now let us discuss about another important principle related to linear viscoelastic material which is the time temperature superposition principle. Now, if you remember in the first class I showed you the bitumen sample and we did a very simple experiment. So, what did we do there? We took the bitumen sample and we stretched the bitumen sample. So, first we stretch the sample at a very slow rate.

So, while stretching it at a very slow rate we saw that the bitumen sample could be stretched to a considerable distance and again the same sample we stretched at a very fast rate and we found that the sample broke instantaneously when we actually pulled it from both the side. So, there we discussed that the response of the material is time dependent which means that when the time of the loading is short which means we are applying a high frequency the response is more brittle in nature, it is more stiff in nature.

And when we are increasing the loading time or we are reducing the frequency of load the response is more of viscous in nature. So, what we understand that a higher frequency is equal to, maybe I will write it here that a higher frequency is a representation of a stiff behavior and a lower frequency is basically a representation of the viscous behavior. Now, let us try to understand this concept with respect to temperature.

So, what happens when we basically change the temperature? So, at very lower temperature if we freeze the bitumen sample it will just behave like an elastic solid or it will be a very stiff material. So, when the temperature is low the behavior is stiff in nature and as we start increasing the temperature of the bitumen the bitumen will start becoming soft and it will start flowing.

So, when the temperature is increased the behavior is more viscous in nature. So, this means if we try to understand the relationship between the frequency and the temperature as a loading parameter on bitumen you will see that high frequency or you can say lower loading time is similar to the behavior at lower temperatures. Similarly, lower frequency or higher loading time is similar to the behavior we will get at higher temperature.

So, this means that there is a relationship between time and temperature to describe the response of the linear viscoelastic material. As I said response at low temperature is similar to the response at higher frequency or faster loading rate and response at higher temperature is similar to the response at lower frequency or slow loading rate.

And this we can understand using this particular graph where I am showing you in the y axis we have the stiffness parameter and in the x axis we have the time of loading. Let us say we have done an experiment and we have measured the response of the bitumen at two different temperatures 40 degree Celsius and 60 degree Celsius. We can definitely do at several other temperatures.

So, this is just for explaining the concept of time temperature superposition principle. So, of course, the stiffness at 40 degree Celsius will be higher than the stiffness at 60 degree Celsius. So, you can see that the graph for 40 degree Celsius is above the graph we get for 60 degree Celsius. Now with increase in time let us see how it behaves with increase in time.

When the time of loading is increased, which means we are going towards more of a viscous response. So, the stiffness of the material is reducing. So, if you take any stiffness value and we just draw a straight line, this straight line as we see will intersect both the stiffness curve at 40 degree Celsius and stiffness curve at 60 degree Celsius. However, the value of time of loading at which this stiffness is arrived is different for different temperature.

So, here what you see that the value of stiffness at higher loading time at a lower temperature is equal to the value of stiffness at a higher temperature, but lower loading time. So, this is something which is called as the time temperature superposition. So, usually what happens that the experiments which we do in the laboratory using several different equipments.

For example, we will be discussing that typically for bitumen we will use a dynamic shear rheometer. So, such equipments has their own limitations. For example, they can give you the dynamic data or any stiffness parameter only up to a range of particular frequency, maybe a window of about 4 decades let us say and beyond that we the machine would not be able to tell you about the response or the behavior of the bitumen.

In such a case the use of time temperature superposition principle helps us to analyze or to assess the response of the binder at a range of frequency which is beyond the range of the measured frequency. And how do we do that? As you see in this curve that if I want, let us say I am interested to find the stiffness value at 40 degree Celsius, but at a different frequency range and I know that this response is equal to this response.

What does it mean that if I start shifting this curve on the right hand side, then if I start just shifting the curve on the right hand side, then what I will get, this curve will come just above this curve, something like this and you see this is the curve of 60 degree Celsius which has been shifted and you see that we are able to get the response of the material at other frequency range, which is beyond the measured frequency range.

And therefore, after this shift we call this axis as reduced time axis. So, how do we do the shifting? We use shift factors. So, what is this shift factor? The shift factor is a multiplication factor which you multiply with the time axis or the frequency axis at a particular temperature or a set of temperatures to shift the behavior at a particular reference temperature which means at the reference temperature we are not doing any shifting.

So, the shift factor is equal to 1. Now, depending on whether the curve is on the right hand side or on the left hand side, the shift factor will either be more than 1 or less than 1. So, if you are trying to shift the behavior from lower temperature to higher temperature which means you have to do a shifting on the left hand side and if you are trying to shift the behavior from higher temperature to lower temperature you have to do a shifting on the right hand side.

So, therefore, finally, what you get is a smooth shifted curve where you are able to see the response at a wider range of time or frequency whatever you are interested in and such correspondence between time and temperature is a typical characteristic for materials which we generally call as simple thermos-rheological materials.

Now, for bitumen there are also conditions such as partially simple thermo-rheological material all right, but of course, detailed discussion about viscoelasticity is not within the scope of this particular presentation, but I hope that you know this discussion which we have done today will give you some insight to explore more about the viscoelastic characteristics and understanding the response of the bitumen.

So, this time temperature superposition principle as I said using shift factor is typically used to construct what we call as master curve. This master curve is nothing, but the variation of any viscoelastic parameter with respect to reduce frequency or reduce time scale. In this particular example which you have taken let us say if you are interested to shift the behavior from 60 degrees to 45 degrees, this is how the smoothen curve will look like.

You can see that this blue curve has just overlapped the red curve which is at 40 degree Celsius. And similarly, if you have more temperature, more shifting can be done and the range or the reduced frequency, so this is reduced time, the reduced frequency range can be increased. And as I said that you can also see that at different temperature, for different temperatures you will have to use different shift factors.

So, there will be a variation of shift factor with respect to temperature. So, typically if you plot the variation of log of shift factor with temperature, you obtain a straight line which looks something like this. This is just an example which has been shown here. So, this is all about the concepts related to time temperature superposition principle.

With this, we will end our discussion today and I hope that this discussion will help you in exploring more about the viscoelastic characteristics of bitumen. And in the next lecture, we will continue from here and we will start discussing about the rheological properties which we typically use in the laboratory to characterize the viscoelastic response of the bitumen.

And we will also see the available specifications which requires the measurement of such rheological properties or parameters that governs the performance of the bitumen in service condition. Thank you.