Pavement Materials
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Lecture: 07
Strength Properties Of Soil (Part-2)

Hello friends, in the last class, we were talking about the compaction characteristics of soil if you remember.

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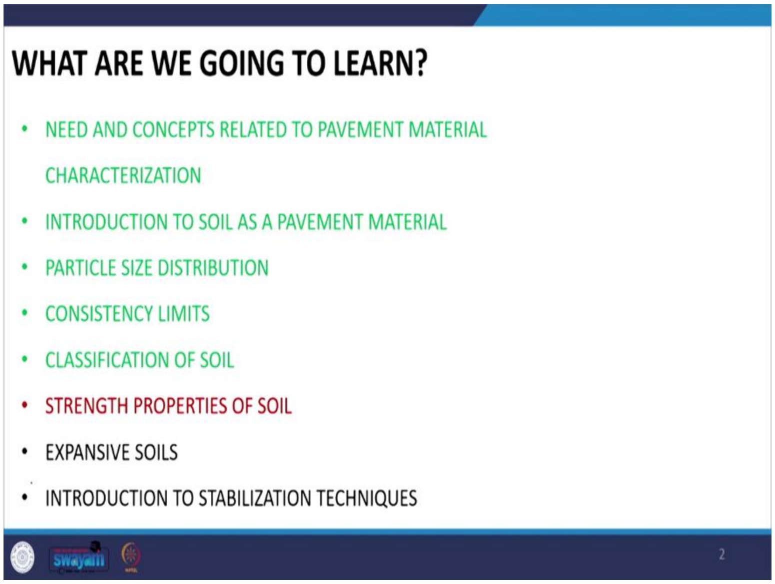

Which falls under the strength properties of soil, which we are discussing, and today we will start talking about the shear strength of soil.

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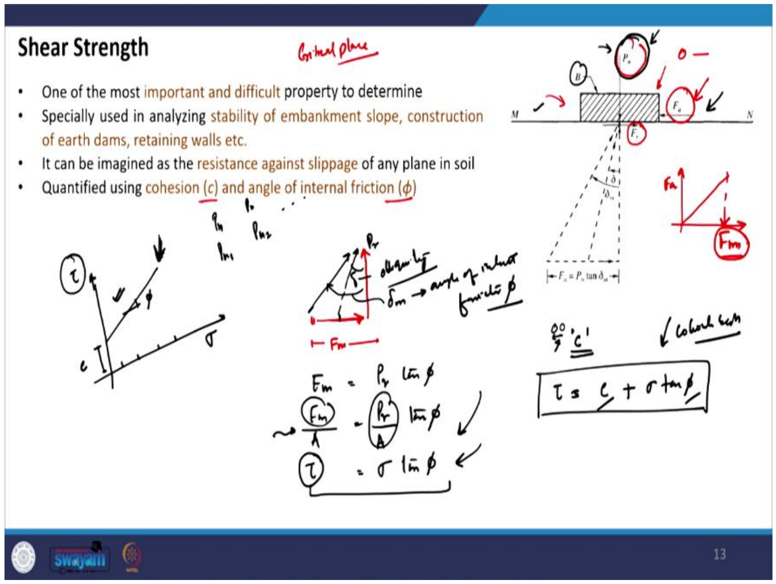

So, shear strength of soil is one of the most important and very difficult property to assess. And most of the times especially when we talk about pavement engineering, we seldom use the concepts of shear strength to analyze the soil. However, being a very important parameter, it is important for us to look into the basic concepts of shear strength and talk about it.

For example, shear strength of soil should be known for analyzing the stability of embankments, we use it during the construction of earth dams, we use it to design the retaining walls and various other structures that involve the use of soil. Now, how to define shear strength? So, shear strength can be defined as the resistance offered by a given soil to sliding and the sliding will occur along a critical plane and how do we quantify the shear strength once we know the definition.

Now, but how do we quantified shear strength can be quantified by two parameters, which are you can see empirical in nature and these parameters are the cohesion of the soil and the angle of internal friction. Now, in order to understand these parameters and the relation with shear strength, let us take an example.

So, on the right side of the screen you are seeing a body B which is resting on the surface MN and is subjected to a normal force PN here and it is also subjected to a tangential force say Fa. Now, in this experiment what we do we keep PN constant and we gradually increase the value of Fa from 0, to a particular value at which this box B will start sliding on the surface MN.

So, you can say that what we are doing here if this is Fa and we are trying to increase the value from 0, to a particular value let us say Fm and this Fm is that value of the sliding force at which the tangential force at which the box B will start sliding. So, if you look at this particular picture, which is shown here.

So, initially the value of Fa is resisted by the frictional force between the surface and this frictional forces denoted as Fr in this particular picture. Now, the resultant of the normal reaction force let us say this is the normal reaction force and this is let us say the value of Fr. So, this the end let us say that, because we are increasing this value.

So, this value can be from here to here and let us say corresponding to this, this is the maximum value at which the sliding occurs. So, if the value of Fr is less than F_N till then we observe no sliding of the block B and see that the resultant force of the normal reaction force and the value of Fr it makes an angle delta (δ) with the normal and this angle is called as the angle of obliquity.

Now, and the corresponding value of delta (δ) at which the sliding occurs, so, this value of delta this is nothing but the angle of internal friction so, this is the maximum angle and beyond this the box will start sliding. So, this is the angle of internal friction phi we denoted using phi. So, if you see this particular force diagram.

So, using this force diagram we can write that $F_M = P_r tan\varphi$,. So, if I divide Fm by the cross sectional area A and similarly on the right hand side I do the same. $\frac{F_M}{A} = \frac{P_r}{A} tan\varphi$ So, what do I get that $\frac{F_M}{A}$ so, this is the tangential force.

So, this is the shear stress here is $\tau = \sigma tan\varphi$. So, this is the relationship between shear strength and the normal stress and includes the value of angle of internal friction. Now, the same concept it can be applied to soil considering that B and MN they are the same materials let us say they are soils.

So, the same equation applies to a soil mass also if we are looking at the critical plane at which this sliding is occurring. So, if you take cohesion less soil and you do not apply any vertical force, the soil will not be stable, let us say that you are taking a, cohesion less soil and you are not applying the vertical force P_N.

So, in that case this soil mass will not be stable because they cannot stay intact in a compacted mass without any confining pressure. But if you take cohesive soil, now, cohesive soil as we have discussed previously, they also have some attraction between the individual particles and this attraction is denoted as the cohesion between the individual particles.

So, owing to the cohesion between the particles, even without the application of P_N, the soil will be able to resist to some extent the tangential force Fa if I want to transform this equation into a general state, so, the shear strength will comprise of two components. So, one will be the cohesion and the other will be corresponding to the angle of internal friction.

So, this becomes $\sigma tan\varphi$. And this is the coulomb equation $\tau = C + \sigma tan\varphi$ which tells us about the relationship of C and φ with the shear stress and the value of normal stress. Now, if you look at the equation this equation can be imagined as a straight line relationship between τ and σ where φ is the slope of the straight line and C is the intercept of the straight line.

So, in this experiment, if we keep on increasing the value of P_N means, let us say experiment one is conducted with $P_N = P_{N1}$ experiment two is conducted with $P_N = P_{N2}$ and so, on at different values if we do the experiment, we can get this entire straight line which means we can get the value of φ and C through this experimentation.

So, on the y-axis, it is τ and on the x-axis, it is σ. So, if we keep on increasing the value of σ, so, when I say σ it means the force P_N. So, corresponding to that I every time I get a different value of τ because the value of φ will change and so, we will get the entire graph or the entire straight line equation and using which we can calculate the value of C and φ to define the shear strength parameters.

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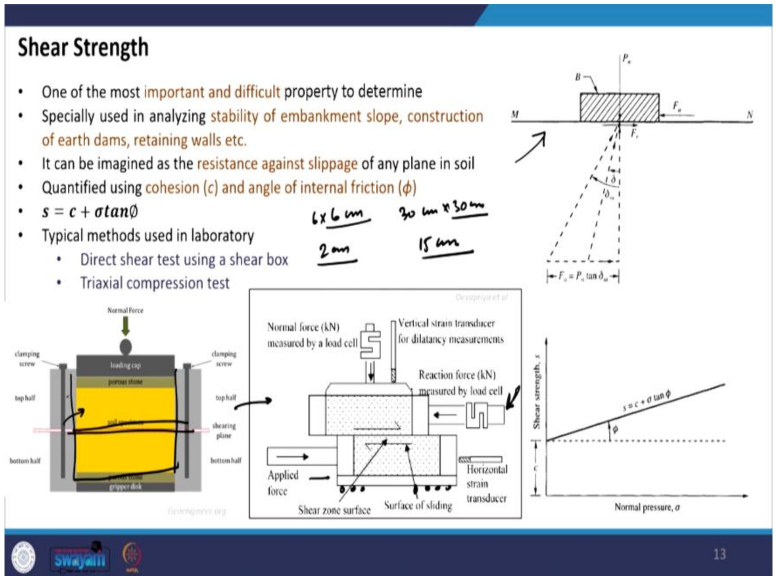

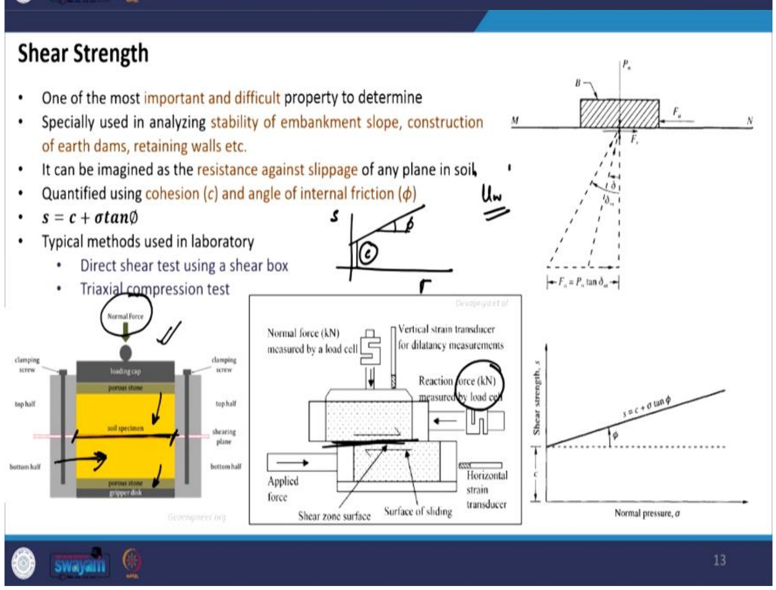

This is what we have discussed. Now, the next question is that, how can we evaluate the value of these shear strength parameters in the laboratory? So, there are two popular methods that are typically used one is the direct shear test using a shear box and the other is a tri-axial test, let us learn about these methods, we will start discussing about the direct shear test using the shear box.

The procedure used in the direct shear test is very similar to what we have discussed while describing the relationship between S and φ which means, this particular experiment, in the direct shear test, we usually have a brass cup. So, you can see that this is the brass cup here.

So, this brass cup has some specific dimension it can be 6 into 6 centimeter or it can be 30 centimeter into 30 centimeter depending on whether the type of soil is cohesive or cohesion less and we can prepare the soil sample or we can take the soil sample which has to be put in this brass cup either in the undisturbed condition directly from the field or we can take the disturb sample remold the sample in the lab at the desired moisture content and press the soil sample in the box to the required density.

Now, the height of the sample is usually 2 centimeter if the smaller size box is used or it is usually 15 centimeter if the larger size box is used. This box is basically split horizontally here. So, you can see that this box is split horizontally at this location and the lower part it rests on rollers.

So, the lower part of the box, which is again shown here, it rests on the rollers and it can be moved horizontally. While the upper half it is clamped, it is usually clamped against a proving ring for recording the reading of shear force. So, you can see where the clamping is done and once this particular box starts moving, this part of the soil will resist this movement and this resistance can be measured using the load cell or to the dial gauge reading in this reaction frame.

In this test, porous stones are also used. So, you can if I remove the marks here, so, you can see that porous stones are also used for allowing the movement of water if it is desired. So, we will talk about this in the next test when we discuss about the tri-axial test in this direct shear test, what happens that a vertical force is applied and it is kept constant.

So, we are applying a vertical force and it is kept constant the horizontal force it is increased gradually. So, we are basically moving this lower part gradually at a constant rate until the sample fails in shear along this particular plane. So, here the failure plane is already decided. Now just a note here, this not necessarily for a given soil this will always be the failure plane this is a forced plane which we are using here to in this particular experiment.

So, the shear force at failure is recorded using the dial gauge placed in the proving ring here and the stress are calculated by dividing these forces that is the normal force and the shear force with the cross sectional area of the failure plane. So, this cross sectional area is already known to us similar to what we have discussed this test is repeated by increasing the value of the vertical load and using which we can plot the relationship between the shear stress and the normal stress. And using this graph, we can calculate the value of φ and we can calculate the value of C.

Now, though, this is a very simple test, there are a few shortcomings of this test method due to which tri-axial test is preferred, which will be discussing next in detail, when we will discuss the tri-axial test we will see that the stress distribution in the soil is more uniform in the tri-axial test in comparison to the direct shear test.

So, this is one of the disadvantages. Now direct shear test is usually more suitable when we are analyzing cohesion less soil, but that to accept fine sand and silt, whereas tri-axial test is suitable for almost all types of soil. Also in the direct shear test, the complete state of stress is not known, we only know the stress at failure.

But in the tri-axial test, we get the entire envelope of the state of stress during the test process. As I mentioned that in the direct shear test, we are already fixing the failure plane here, but which may not be the actual critical plane in which we will get the maximum angle of obliquity but when we do tri-axial tests, we will see that we apply normal stresses but the sample will always fail along the critical plane.

So this gives an added advantage. Moreover, in the tri-axial test, we will also be able to measure the pore water pressure which is an again important parameter to be studied though will be not discussing this in detail during this particular presentation. But the measurement of pore water pressure is usually not possible in direct shear test. However, direct shear test has its own advantages also for example, it is quick it is simple, it is less expensive, and these features it adds to the advantages.

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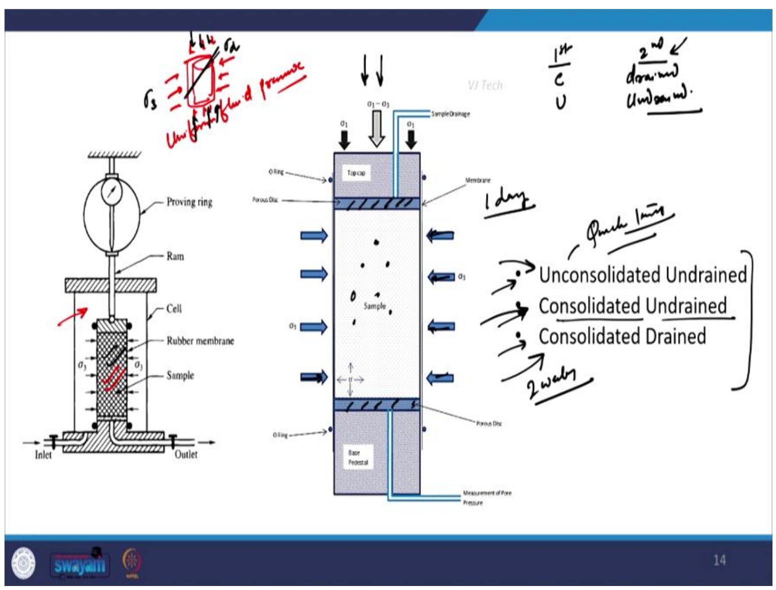

Let us now discuss about the tri-axial axial test. So, these pictures which you see on the screen, it shows the layout of the test apparatus. So, here we have a cylindrical soil sample, which you see here, maybe I will use a different color. So, here you use a cylindrical sample and the cylindrical sample, it is wrapped using a rubber membrane.

So, it is wrapped using a rubber membrane and it is kept inside this tri-axial cell and it is subjected to a uniform fluid pressure all around. So, it is subjected to uniform fluid pressure through all the sides when I say all the sides which means that I am applying a uniform cell pressure here.

Then what we do so, this is the first step that we will keep the sample and we will apply the uniform cell pressure and then we will apply axial load. So, in addition to the cell pressure we also apply axial load here and we will apply this load at a particular rate until the sample will fail in shear.

And the failure will take place basically on internal phases depending on the critical plane and this depends on the type of soil we are testing what we do here this fluid pressure it is denoted as σ_3 whereas, the vertical axial force which we are applying is denoted as deviatoric stress σ_d , we use different combination of σ_3 and σ_d and the failure load are used that means, we see that at what load it fails, so, the failure load are used to assess the value of C and φ for the soil.

So, we will discuss about this that how the tri-axial test is used to estimate the value of C and φ, but, before that, let me also tell you that the test can be usually done in three different modes depending on the soil type and field loading condition. So, this is Unconsolidated Undrained we have Consolidated the Undrained test and we have Consolidated Drained.

So, when we are looking at the tri-axial test, let us divide it into two parts first part and second part. So, the first part is that we are applying the self pressure or the fluid pressure. So, during this process, if we are allowing the pour water because if there is a presence of moisture between the soil particles and when you apply stresses, this pore water pressure will develop and it will try to move.

So, if you allow this movement using probably porous stones, if you allow the movement of water, which means, you are allowing volumetric changes in the material. So, these volumetric changes if you are allowing it is called as the consolidated condition and if you are not allowing the movement of moisture after applying the cell pressure, you are basically talking about the unconsolidated state, we are not allowed any volume change.

So, depending on the first part if it is if you are allowing moisture to go out it is called as consolidated otherwise it is called unconsolidated if you try to relate this with the actual field condition, we are basically talking about the state when the construction has just taken place we are not allowing the actual load to move over it, but we have just constructed the structure or we have just compacted the particular soil on which the construction will come.

So, depending on the type of soil for example, if it is a, sand then typically it has high permeability and the water will tend to move out. So, for testing such type of material, probably using a consolidated condition will give you a more realistic simulation of what will actually happen in the field.

And if you talk about cohesive soil like clays which have less permeability, water does not consolidate very fast. In that case, unconsolidated condition can be used in the tri-axial test, while in the second part, which means you have applied the cell pressure now you are applying the deviatoric stress here. So, if you are applying the deviatoric stress, which means you are increasing the axial load and during this part, this I am talking about the second part and during this part of the test, if you allow the water to go out, then it is called as drain condition.

And if you are not allowing the water to go out, it is called as undrained condition. So, here we will define the volumetric changes in terms of drained or undrained. Unconsolidated undrained test is where you do not allow the movement of water in the first stage as well as in the second stage.

So, this is also called as a quick test in the second test, which is consolidated undrained. So, in the first part, when you apply self pressure, you allow the movement of the water which means you are allowing the volumetric change, but when you are applying the axial force, you do not allow the movement of water.

So, that is called as consolidated undrained, and this test takes longer time, probably 1 day, more than 24 hours to complete. And when you talk about consolidated drain tests so, this is a more time taking test because in the first part of the test also you allow the movement of water and also in the second part you allow the movement of water and this takes about 2 weeks probably to complete for a given type of soil the entire test.

Anyways, we are not going to talk about in detail, because here we are trying to understand the concepts or the strength of soil from the perspective of pavement materials, but, these topics are usually covered as a part of soil mechanics in detail.

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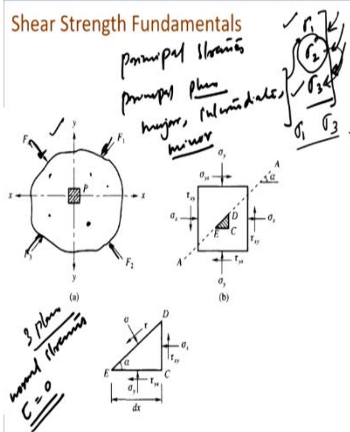
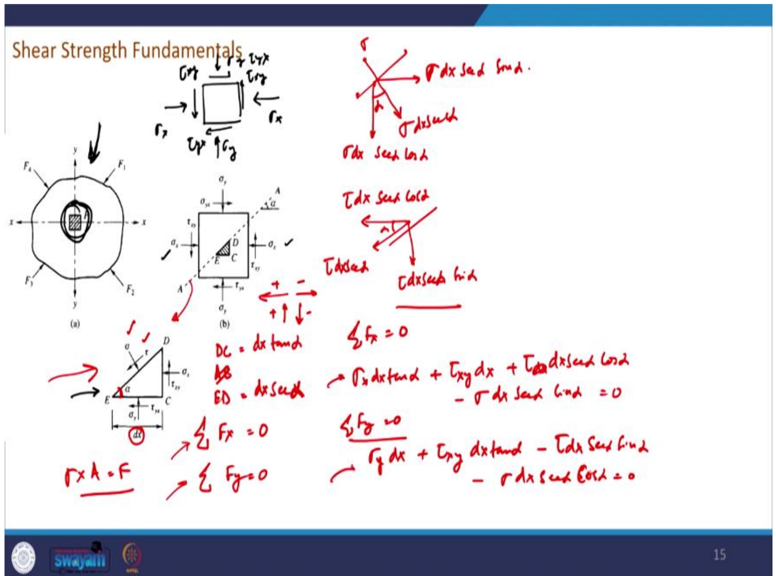
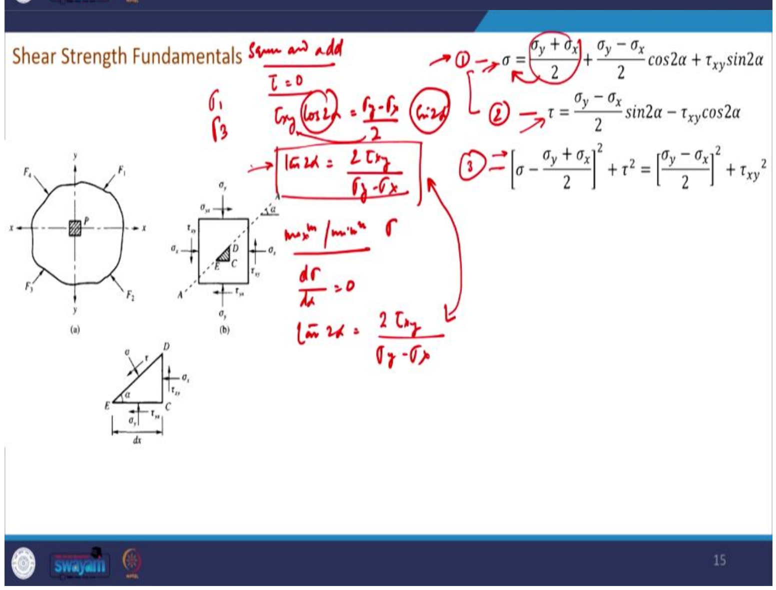
Shear Strength Fundamentals







Before understanding the steps of, assessing the value of C and φ from the tri-axial test, let us understand some basic concepts related to the state of stress in a soil mass, if you see the picture, it shows that a body here if you see this is the first picture where you see a body is stressed due to external forces.

So, there are various forces acting on the body because of which this body is in a stressed state. Now, through every point usually, so, this is a general statement that through every point on a stressed body, there are 3-planes that are unique in nature and that are right angled to each other. So, these planes are unique in some form, and they are usually right angle to each other, and why they are unique?

Because these planes are subjected to only normal stresses and do not have any shear stress component. So, this has normal stresses and they do not have shear stress. So, shear stress = 0 in these planes. So, these stresses are the normal stresses in these planes, these unique planes, they are called as principal stresses.

And these planes they are denoted as principal planes depending on the magnitude of the stress in each of these 3-planes, they are denoted as major principal stress, intermediate principal stress and minor principal stress. So, let us try to denote them using some letters like σ_1 , σ_2 and σ_3 .

So, these are the notations which we can use to describe these 3 stresses in the principle planes. Now, why are these stresses, so, important and why do we specifically mentioned about these unique stresses, because, if these stresses are known, then you can calculate the stress on any other plane through that particular point.

So, that is also one of the major advantage of knowing the principal stresses because once the principal stress through that particular point where the principal stresses, you can also calculate the stress at any other plane for any orientation passing through that particular point. But for most of the design problems, we deal with the knowledge of only.

The two principal stresses σ_1 and σ_3 , they are usually sufficient and many times the effect of the intermediate stress is either ignored or they are taken equal to σ_3 . So, that is why we are more interested to know about the value of σ_1 and σ_3 , which also means that we are talking about a 2-dimensional system rather than a 3- dimensional system here.

If we talk about a state that the body is being stressed as shown in this particular figure, then we are also interested to know that what are the principal stresses and also what is the orientation of the principal plane. Additionally, we are also interested to know for any given orientation or any arbitrary plane, what will be the state of stress at that particular location.

Let us talk about the evaluation process because this will also help us to understand the tri-axial test, let us say that we have an element here. So, this element is P which is taken from some location in this body in which the stresses acting parallel to x and y directions are known, let us say in this particular element.

We know that what are the stresses acting on the x and y direction. So, I am just redrawing this here. So, let us say we know that what is the value of σ_x ,what is the value of σ_y we also know the value of the shear stresses here τ_{xy} . So, this is τ_{yx} , this is again τ_{yx} and this is τ_{xy} . So, now, let us consider a small element of width dx.

So, this is shown here we are considering a small element of width dx oriented at an angle of α with the horizontal so, it is making an angle α with the horizontal. So, you can try to relate just see these two particular pictures. So, in this particular picture which you see, you will agree that the value of $DC = dx\tan(\alpha)$ and the value of ED=dx sec $ED = dx\sec(\alpha)$.

So, here we are interested to find the value of σ and τ on this plane. So, we are interested to find the value of σ and τ. Now, for equilibrium condition from our concepts on mechanics we know that for equilibrium conditions summation of forces in the x-direction should be 0, and summation of forces in the y direction should also be 0.

So, you have to remember that these notations which we are seeing on this particular small element, they basically indicate the stresses but here we are talking about forces. So, we need to multiply the stress with the cross sectional area to get the force every time. For example, let us say that if you talk about this plane that is ED.

So, here the value this is here sigma is acting, but the component of the force will be how much $\sigma \times \mathrm{A}_{ED}$.. So, this is $\sigma dx\sec(\alpha)$. So, if I want to, see the horizontal and vertical component of the stress, so, this angle is α, so, this becomes equal to $\sigma dx\sec(\alpha)\cos(\alpha)$ and this becomes equal to $\sigma dx\, sec\,(\alpha)\sin(\alpha)$.

Similarly, if you see the on this particular plane again if you see the orientation of the shear stress, so, this is τ here, but the force component will be $\tau \times A\; dx\sec(\alpha)$ and if I want to see the horizontal and vertical components, so, this is equal to α here, so, this is $\tau dx\sec(\alpha)\cos(\alpha)$ and this is $\tau dx\sec(\alpha)\sin(\alpha)$.

So, I hope this is very clear to you now. So, now, we talk about summation of $F_X = 0$. So, if you look at this particular picture. So, on the x let us say this is positive for us and this is negative for us similarly, this is positive this is negative.

So, if you see we have here $\sigma x dx\tan(\alpha) + \tau_{xy}dx + \tau dx\sec(\alpha)\cos(\alpha) - \sigma dxse\,c(\alpha)\sin(\alpha) = 0$, and if we try to again talk about the summation of forces in the y-direction it will be equal to $\sum F_y = \sigma y dx\tan(\alpha) + \tau_{xy}dxtan(\alpha) - \tau dx\sec(\alpha)\sin(\alpha) - \sigma dxse\,c(\alpha)\cos(\alpha) = 0$.

So, this becomes equal to 0. So, if you solve this equation I am not spending much time in solving so you have these equations now, just try to solve it to find the value of sigma and tau. So, what you will get finally is that $\sigma = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_y-\sigma_x}{2} cos2\alpha + \tau_{xy} sin2\alpha$ and $\tau = \frac{\sigma_y-\sigma_x}{2} sin2\alpha - \tau_{xy} cos2\alpha$.

So, this can be easily derived from the expressions we have discussed. Now, before I proceed forward to explain this equation or to use this equation one more step which we can do here. So, let us say this is equation 1 and this is equation 2 what you do you square this equation square and add these equations and before squaring what you do you just bring this term on the left hand side. So, after you bring then you just square and add both these two equations.

So, again that part I leave to you to do it and once you do it very easily you can find that it will lead to this particular expression, $[\sigma - \frac{\sigma_x+\sigma_y}{2}]^2 + \tau^2 = [\frac{\sigma_y-\sigma_x}{2}]^2 + {\tau_{xy}}^2$ I will come to this expression I am calling this expression is 3 now.

So, using 1 and 2 if you look at 1 and 2 you can find the normal and shear stress at any plane α. So, this expression will give you the normal stress at any plane α, this expression will give you the value of τ at any plane α. Now, for finding the orientation of the principal plane, so, we have discussed that principle plane are those where $\tau = 0$. So, this just put equation 2 equal to 0.

So, from here what you get $\tau_{xy} cos2\alpha$ I am just trying to make it a bit easier for you to understand $sin2\alpha$. So, you see $\frac{sin2\alpha}{cos2\alpha} = tan2\alpha$. So, $tan2\alpha$ becomes equal to this goes there, $tan2\alpha = \frac{2\tau_{xy}}{\sigma_y-\sigma_x}$. So, this is what you get. Here you can see since this is $tan2\alpha$ which means you have two planes and these planes are perpendicular to each other and by now, we know that obviously, there are two planes.

Because one plane will be corresponding to σ_1 and one plane will be corresponding to σ_3 and there will be perpendicular to each other, if you see equation 1 and if you are also interested to find out the maximum or minimum normal stress rate maximum or minimum value of sigma what you do you just do $\frac{d\sigma}{d\alpha} = 0$.

So, once you do that again you land upon the same expression here that $tan2\alpha = \frac{2\tau_{xy}}{\sigma_y-\sigma_x}$. So, this is very interesting that both these two expressions are same which means that the plane having these zeros shear stress or the principal planes are also the planes that have the maximum and minimum normal stresses. So, this is also an important conclusion from this particular discussion.

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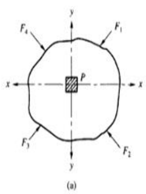
Shear Strength Fundamentals

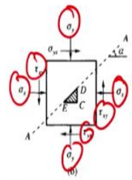
$(x-a)^2 + (y-b)^2 = r^2$

$$\sigma = \frac{\sigma_y + \sigma_x}{2} + \frac{\sigma_y - \sigma_x}{2} cos2\alpha + \tau_{xy} sin2\alpha$$

$$\tau = \frac{\sigma_y - \sigma_x}{2} sin2\alpha - \tau_{xy} cos2\alpha$$

$$③ - \left[\sigma - \frac{\sigma_y + \sigma_x}{2}\right]^2 + \tau^2 = \left[\frac{\sigma_y - \sigma_x}{2}\right]^2 + \tau_{xy}^2$$

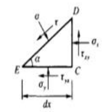


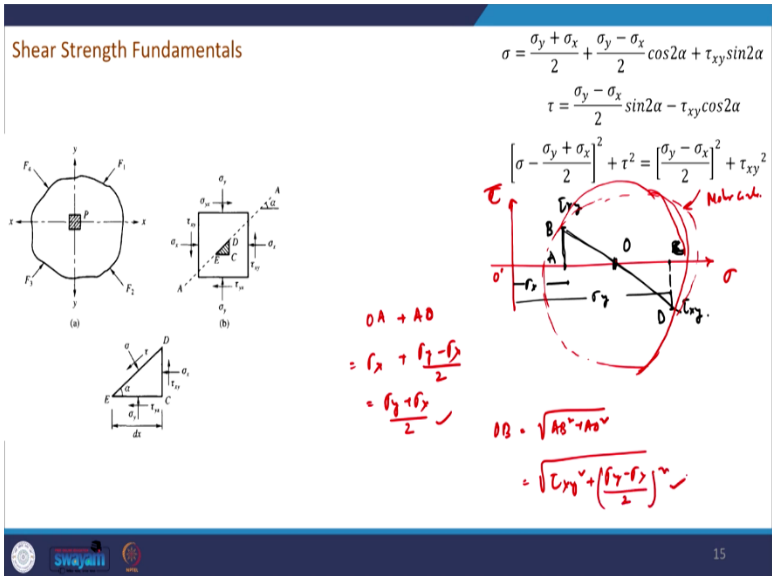


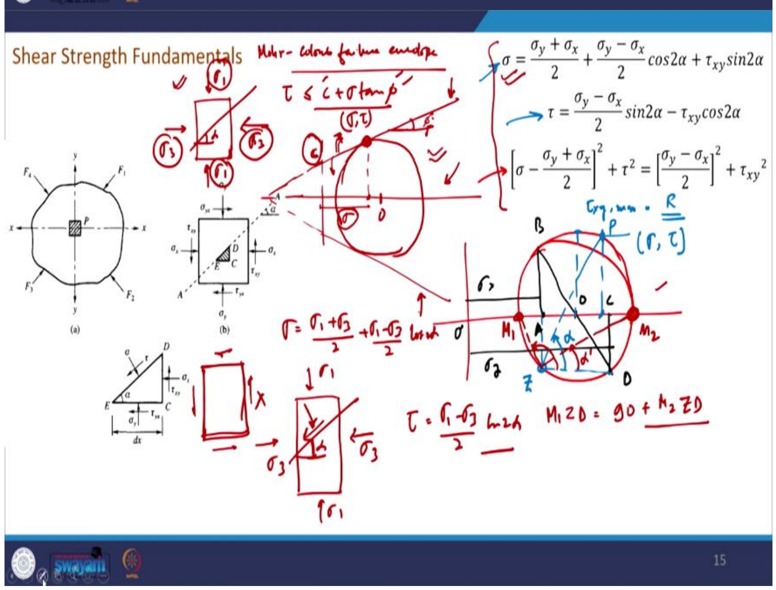
$a = \frac{\sigma_y + \sigma_x}{2}$

$b = 0$

$r = \sqrt{\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau_{xy}^2}$







Now, if you look at equation 3 which I said this is equation 3 it is basically an equation of an circle I hope you understand that $(x-a)^2 + (y-b)^2 = r^2$ this is what we know the equation of the circle. So, this is of the same form where you see a is nothing but $\frac{\sigma_y+\sigma_x}{2}$, b = 0 and r $r = \sqrt{(\frac{\sigma_y-\sigma_x}{2})^2 + \tau_{xy}{}^2}$ which means that the coordinates of the center are this and the radius is this.

But how do we draw the circle if we know the value of σ_x and σ_y and τ_{xy} in a particular plane, then using those values because these are already known if you remember. So, if this then how do you draw the Mohr circle.

So, I will tell you the steps very quickly what you do here just draw an axis system. So, this is τ and this is σ. So, here what the value of σ_x so, just mark the value of σ_x. So, this is σ_x you also marked the value of σ_y. So, this is let us say σ_y then what you do through σ_x you take a you just take a dimension corresponding to τ_{xy}.

Because this is also known and τ_{xy} will also will be constant in the plane so, this will also be τ_{xy}. Now, you draw join these two lines sorry these two points this intersects at a point O, let us say that this is A, and this is B. So, now this is a B, this is C and D. So, taking OB as the radius if you draw a circle.

So, this circle is nothing but the Mohr circle just let us reconfirm if this is the Mohr circle according to our definition, let us say this is O dash the coordinates of O here the coordinates of O here are what $\sigma_x$which means all OA + AO, So, a plus arrow is what $\sigma_x + \frac{\sigma_y - \sigma_x}{2}$. So, this is $\frac{\sigma_y + \sigma_x}{2}$ so, this is correct. And if you see the value of OB, so, OB = $OB = \sqrt{(AB)^2 + (AO)^2}$.

So, this is nothing but $\sqrt{{\tau_{xy}}^2 + (\frac{\sigma_y - \sigma_x}{2})^2}$ so, this is also correct. That is why we say that this is the Mohr circle and this is the Mohr circle and it matches with our equation number 3. So, let me quickly draw the Mohr circle because it got disappeared.

Now, if you see this particular Mohr circle one more thing which you can see here is that the maximum value of the y-coordinate is nothing but the radius I hope you will agree to it that the maximum value will be corresponding to the radius here and this is nothing but the maximum value of the shear stress τ_{xy} maximum.

So, τ_{xy} maximum is nothing but the radius of the circle again a point to remember in the Mohr circle. Now, the question is that how to use the Mohr circle to calculate the stress at any plane α which means how to calculate σ and how to calculate τ using the Mohr circle. So, let me tell you further steps here.

So, what you have to do in the Mohr circle you just extend the line AB and join it to that particular point of the circle. And similarly, if you extend AB you will reach a point D and you mark this value is Z this Z is called as the pole of the circle of the Mohr circle.

So, this is the pole here any plane in the Mohr circle when it is joined with Z it gives the orientation of that plane with the horizontal. So, if I draw any line through the Mohr circle and reach a particular point in the circle, so, this value is nothing but the orientation of the point P with the horizontal α and the coordinates of the point P which means the coordinates of the point P it will give me the value of normal stress and shear stress at point B.

So, this is how you can identify or use the Mohr circle to calculate the normal shear stress and shear stress at any given plane α. Now, the next question is identifying the principal stresses and the principal plane, we know that at the principal plane these stresses normal stresses will be either maximum or minimum.

So, if you see this particular Mohr circle very clearly you can identify that these points are this particular point let us call this as M2 and the next the first point is this M1, O'M1 and O'M2 are the principal stresses σ_3 and σ_1 respectively. As per the definition and we have learned about the pole, so, what will be the orientation.

So, if I want to find the orientation corresponding to the principal stress σ_1 I will just join Z with M2 and this is that angle α'. Similarly, the orientation of the minor principle plane will be corresponding to this so, this is the angle this is the angle and you can very clearly C, that M1 Z.

So, which means that M1ZD is nothing but 90 +M2ZD so which means that these planes are oriented at right angles to each other. Now, one more additional concept which we should discuss before talking about the tri-axial test is the Mohr coulomb failure envelope, according to coulomb.

Because we have discussed about the coulomb theory the condition of failure in the soil is what the $\tau \leq C + \tan\varphi$ is not it if we know the value of σ and α we can plot the Mohr coulomb envelope and how do we do that it is very simple we take the same axis system.

So, if you see that this is the equation of a straight line, so, let us say this is the value of C. So, corresponding to this I can draw a straight line similarly, I will have a straight line on the other side. So, this is phi and this is the value of C.

Here I am talking about a stress condition which is very simple to the tri-axial test. So, in the tri-axial test we are applying a value of σ_1 here and we are applying a value of σ_3 here and we are trying to see the failure is taking at which particular plane if this is the particular envelope.

And I know the value of let us say sigma at the failure plane I know where the sample has failed if I try to draw a circle such that the straight line is tangential to this particular circle. So, this gives me a Mohr circle why it is a Mohr circle because this point represents the value of σ and τ I am drawing it tangential.

Because I know that failure will not take place below this and it will not be possible to stress the soil beyond this particular point. So, failure has to take place within this particular envelope this circle which is tangential to the coulomb equation. So, this entire system this is called as the Mohr column failure envelope.

And in fact the principal objective of the tri-axial test is actually to establish this Mohr coulomb envelope so, I will just repeat that how we have drawn we have done a tri-axial test let us say we know the value at which the sample has failed. So, we know the value of sigma at which the sample has failed. So, I have just drawn the value of σ here.

And I have just drawn this envelope with a value of C and φ we have still not talked about how we get this value of value of C and φ this is something which we have to talk about. So, please remember that we do not know a till now, how do you get the value of C and phi but let us say the value of C and φ.

So, I will have a circle such that the coulomb equation becomes tangential to that particular point at which the failure has taken place. So, I think that this is clear to you. I will come back to this Mohr coulomb when we talk about it with respect to the tri-axial test in the tri-axial test and I mean what is the relationship between the concepts of the tri-axial test and all those theories which we have discussed.

Now, in the tri-axial test what we are doing we are already defining the principal planes here, because in this particular cylindrical sample, I am not applying any shear stresses here I am not applying any shear stresses here.

So, these planes, since it is free of shear stresses are the principal planes. So, if you see this equation 1 very simply you can say that the value of $\sigma = \frac{\sigma_1+\sigma_3}{2} + \frac{\sigma_1-\sigma_3}{2} cos2\alpha$. So, in this cylindrical sample if I know the value of σ_1 if I know the value of σ_3 at any plane alpha, I can find out the value of normal stress and shear stress.

So, that is the advantage. So, this makes the tri-axial test a Mohr simpler form of the generic state of stress we are talking about. Similarly, the value of a $\tau = \frac{\sigma_1-\sigma_3}{2} sin2\alpha$ the same we can understand using the Mohr circle.

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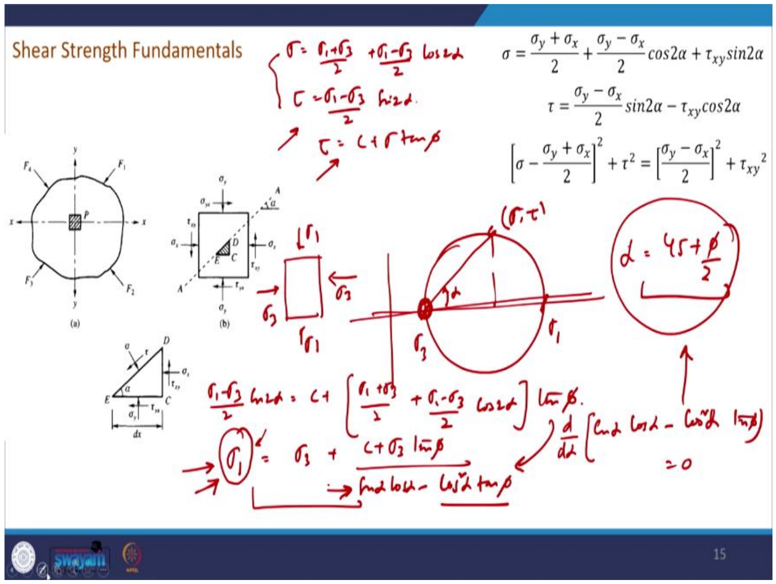

So, if you try to draw the Mohr circle, so, the Mohr circle will be a similar circle, but here this will be equal to $\sigma_3$this will be equal to σ_1 and where is the pole the pole is actually these points the, because we know the value of σ_3 and σ_1 and τ is equal to 0 so, it will coincide with the x-axis.

So, here corresponding to any angle alpha at any plane α I can find out the magnitude of the normal stress and shear stress. So, I hope you understand that how Mohr circle in the tri-axial test will look like. Now, using the simpler form of the equation that is $\sigma = \frac{\sigma_1+\sigma_3}{2} + \frac{\sigma_1-\sigma_3}{2} cos2\alpha$ and $\tau = \frac{\sigma_1-\sigma_3}{2} sin2\alpha$. So, if I want to use coulomb's equation here what it will become we know that $\tau = C + tan\varphi$.

So, if I put τ and σ in this particular equation from here what do we get? We get that $\frac{\sigma_1-\sigma_3}{2} sin2\alpha = C + [\frac{\sigma_1+\sigma_3}{2} + \frac{\sigma_1-\sigma_3}{2} cos2\alpha\ ]\ tan\varphi$. So, if you try to rearrange this equation taking $\sigma_1 = \frac{\sigma_3+C+\sigma_3\ tan\varphi}{sin\alpha cos\alpha - cos^2\alpha\ tan\varphi}$.

So, this you can solve you can get from here to here just by rearranging the expression. Now, in this particular expression, the plane which will have the least resistance to shear will be which one if you again try to imagine the tri-axial test so, which will be that particular plane that plane where you will have the failure at the minimum value of σ_1.

So, the plane having carrying the minimum value of σ_1 at failure will basically be the critical plane when if you look at this expression when will σ_1 be minimum σ_1 can be minimum if the denominator is maximum and to make this denominator maximum I can write $\frac{d}{d\alpha}(sin\alpha cos\alpha - cos^2\alpha\ tan\varphi) = 0$. If you solve this you get that α is nothing but equal to $45 + \frac{\varphi}{2}$, c this is the critical plane of failure in the tri-axial test.

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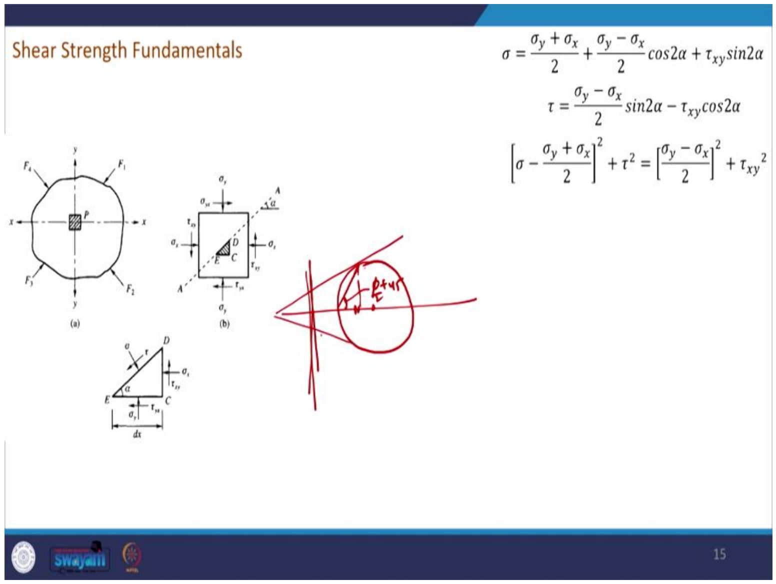

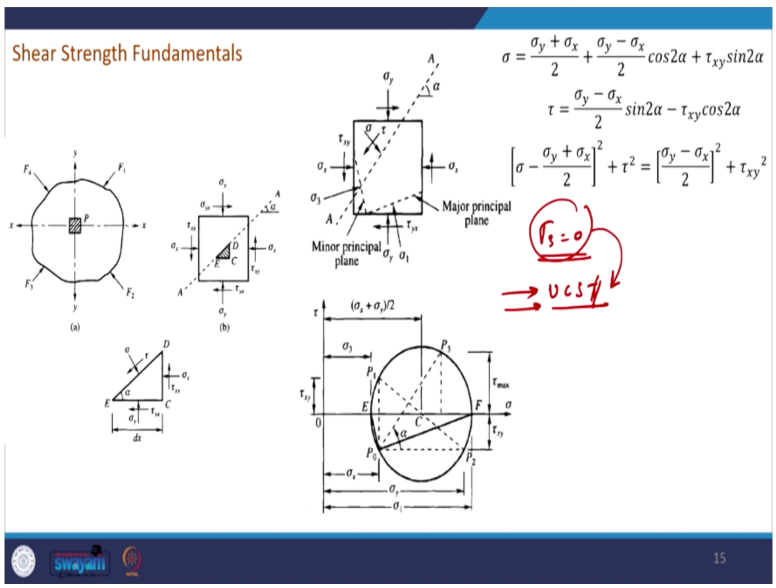

If you talk about the Mohr coulomb failure theory, ifσ_1 and σ_3. Now, see the Mohr coulomb envelope, so, this will be the plane of failure. So, this will be equal to $\frac{\varphi}{2} + 45$and now, using this also we can try to understand the Mohr coulomb failure envelope so, we know the orientation of this plane now.

Now, the important question is that how do you determine the value of C and φ just by using or running the tri-axial test. So, before we again talk about the tri-axial test let me just also give you one additional piece of information that in the tri-axial test if you put $\sigma_3 = 0$ then we call this test is unconfined compression strength test.

So, the UCS so, this is just for your information just if you understand the general tri-axial test, if you put $\sigma_3 = 0$ then you can also understand everything regarding the unconfined compression strength test again this is one of the tests which is popularly used to measure the strength of the soil.

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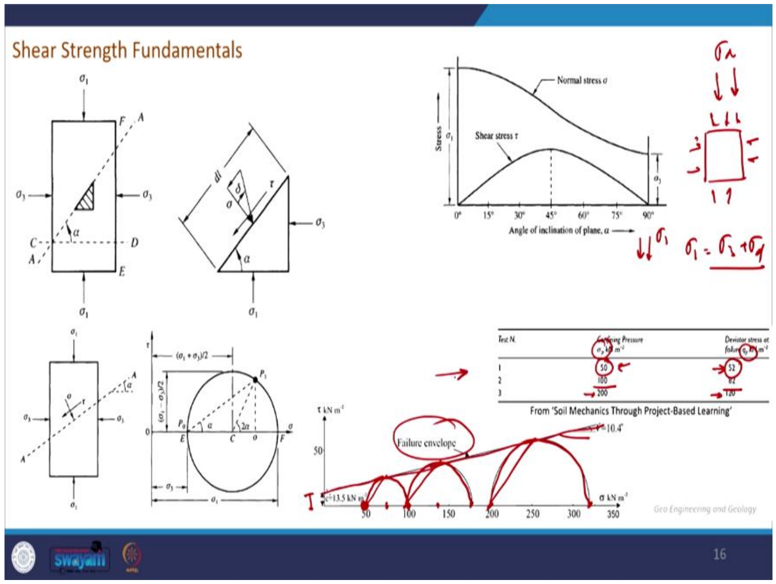

Now, we will just very quickly we will try to complete our discussion on the tri-axial test. So, these concepts we have already discussed. So, the last point is that how do you use the tri- axial test to get the value of c and phi of the soil. In the tri-axial test what you do we will take different combinations of σ_3 and we will take different combination of σ_1. So, a different combination of σ_3 and σ_1 what we will do for each combination we will plot the Mohr circle.

So, here σ_3 here the deviatoric stress if this is let us say σ_1. So, this is basically the deviatoric stress. So, $\sigma_1 = \sigma_3 + \sigma_d$. So, here what is the value of σ_1 then the deviatoric stress the value of σ_1. So, it is 52 +50 =102 so, you mark σ_3 you mark 102 you just draw the circle because the radius. In the next experiment, we are taking 100 as σ_3 and 182 as σ_1.

So, we are taking 100 here 182 here we know the radius we can draw the Mohr circle in the next experiment it is 200 and then 200 +120. So, here it is you draw the Mohr circle and using this Mohr circle, we will try to plot a line such that this line is tangential to all the circles, tangential to all the because we know the angle of failure here, we know that this angle is $\frac{\varphi}{2} + 45$ is not it $\frac{\varphi}{2} + 45$and φ because this is the same soil should be similar at the particular failure point.

So, that is why this line will be tangential here and once we are able to draw this failure envelope then directly we can find out what is the slope of this line and where it is actually intercepting at the y-axis when x = 0, so you get the value of C. So, using this we understand that how using the tri-axial test, we can calculate the value of C and φ.

In the actual case when we talk about visually pavements our loads are more of repetitive in nature and soil properties are usually defined considering those repetitive load. So, in addition to the value of C and φ, the tri-axial test in repeated loading mode is also used to estimate another parameter which is called as the resilient model of the soil.

And this resilient model of the soil is further used in the design of pavement, so, we will stop here today and in the next class, we will talk that how tri-axial test itself can be used to calculate the value of resilient modulus. And today, just to recap, we have discussed about the tri-axial strength test, we have tried to understand the general state of stress in a particular soil mass.

And we have discussed that if we know the value of vertical horizontal and shear stresses, then how we can calculate the value of normal and shear stresses at any particular given plane. And we have also talked about using these general equations to identify the principal plane and the principal stresses.

And these concepts were further used to discuss about the case of tri-axial test which is a more simpler form where we already know the value of principal stresses. And we have also discussed about the construction of Mohr circle and the use of Mohr circle to calculate the value of normal and shear stresses. So, we stop here today and we will meet in the next class. Thank you.