# Earthquake Geotechnical Engineering

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## Lecture 11

## Stress Conditions

I welcome you again for this NPTEL course on earthquake geotechnical engineering. Today we are going to start a new module of this course that is the second module which consists of 10 lectures and today we will start lecture 11 for this course. Let us discuss what we are going to talk in this module. Before we talk about that one of the important issue, dynamic soil properties. So, this module is completely on dynamic soil properties. Let us have the introduction before we talk about what are the different topics, we are going to talk in this module second module.

Now the nature and distribution of earthquake damage is strongly influenced by the response of soils to cyclic loading. Actually the earthquake loading can be characterized like a cyclic loading and whatever damage which you observe after the earthquake or then it has been observed that they are depending on the how the soil respond to this loading. So, that means it is very important that we know that what is the properties of the soil and particularly for the dynamic loads. And the properties of the soils for the dynamic loads depends on the level of a strain which your soil is subjected.

So, depending on the level of a strain, what we call low level of strain which is dominated by wave propagation and large strains which is normally used for stability of mass of soil. So, in geotechnical earthquake engineering there are the wide range of problems which involve many types of loading and many potential mechanism failure. And different soil properties influence the behavior of the soil for different problems. This response of the soil is controlled in large part by the mechanical properties of the soil. So, which we are going to discuss during the second module.

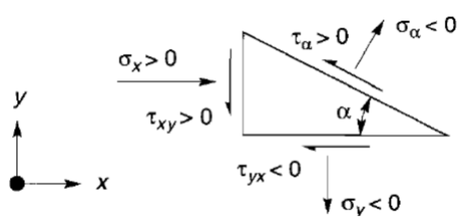
So, the module second the title is dynamic potential properties. We are going to cover this module under these four topics. Concept first is of stresses and stress paths and that today we are going to talk in lecture number 11th for this. Then field test, laboratory test. We will spend approximately three lectures on each on the field test and the lab test.

And then lastly constitutive relationships of soil that is basically stress strain relationship. So, another three lectures will be here. So, all together we have 10 lectures, three lectures on each of the last three topics and in the 11th lecture we are going to talk about concept of stresses and stress paths. So, let us discuss the first one that is the concept of stresses and stress paths. In this chapter 1 of the module 2, here topics to be covered in this lecture is representation of stress conditions, Mohr circle of stresses, principal stresses and stress paths.

And I understand some of these topics you would have already studied during undergraduate in geotechnical engineering course, but we will emphasize what are important particularly for dynamic loads and the last one is stress path may be the new topic for you. So, coming to how we represent the stress conditions for the dynamic loads. The cyclic non-linear and strength characteristics of soil governs the dynamic response during earth peaks. The response will be depending on because there is a non-linearity is involved in the soil. So, the cyclic properties of soils will depend on the level of stress prior to the loading as well as what is the stress condition after the loading.

So, we need to know what the stress condition of the soil before you put any load on it. The stress conditions at any point in a mass of soil it can be described by the normal and shear stresses acting on a particular plane passing through that point, and this is normally you may be aware that this is represented by normal and shear stresses by what you call the Mohr circle. So, normally stresses of soils are either compressive because the soils cannot carry cannot take or resist the tensile stresses. So, normally when we deal with the geotechnical engineering or soils mechanics then we consider that compressive stress is positive while tensile stresses are negative. As far shear force is concerned positive shear stresses which normally tend to cause counter clockwise rotation of the body will be considered positive while other way it will be negative.

So, our sign conventions are here. So, let me use this slide here what you see there is this triangle, and you have three planes one horizontal one vertical and third one is inclined plane. Inclined plane is making an angle alpha with the horizontal. So, the stresses here you have two normal stresses sigma x and sigma y sigma x is compressive. So, that is why it is greater than 0 positive while sigma y is tensile.




So, that means it is negative stress because it is acting downward direction. Then you have shear stress which is denoted by tau xy or tau yx. In fact, tau xy and tau yx is the same thing they will be exactly same and we will explain what is the meaning of notation here. Here in this case when this shear stress is acting counter clockwise direction then it is positive for example, it is positive here and it is negative here in the case when it is acting clockwise direction. Then you have this inclined plane on inclined plane you have this normal stress which is sigma alpha, and you have tau alpha which is shear stress.

Coming to the notations here use for this normal stress and shear stress. Normal stress you have sigma xx or sigma x is nothing, but you can say that sigma xx that is a stress acting on a plane are normal to which is in x direction. While tau xy is the shear stress which is acting in the y direction on a plane perpendicular to x. So, here subscript x is representing what you have the perpendicular direction to this is x and y is its direction. While tau yx is acting on a plane perpendicular to which is y in y direction. So, perpendicular to y is the perpendicular while it is acting in x direction. So, because it is cross xy and yx. So, they are same here. So, this is the sign conventions used for normal and shear stresses. Now, as I mentioned that we use the Mohr circle of stresses and let us say considering both sigma and x and sigma y as positive and we assume let us say sigma y is greater than sigma x this is, and both are positive means both are compressive.

So, rather than one tensile positive means here simply we can say compressive. So once you have both are positive and compressive stresses then considering the equilibrium requirements the normal and shear stress acting on a plane inclined at an angle alpha to the x axis will be given by these equations. Here what these equations two equations are determining the first equation give you the normal stress acting on a plane inclined at an angle alpha at an angle alpha to the x axis. So, and the second equation gives the shear stress. So, you have normal stress and the second one is the shear stress.

$$\sigma_\alpha = \frac{\sigma_y + \sigma_x}{2} + \frac{\sigma_y - \sigma_x}{2} cos2\alpha + \tau_{xy} sin2\alpha$$

$$\tau_\alpha = \frac{\sigma_y - \sigma_x}{2} sin2\alpha - \tau_{xy} cos2\alpha$$

Here one thing is important to note this alpha angle is with respect to x axis and this with respect to x is measured from x axis towards the clockwise direction. So, that is we move from x axis to towards the clockwise direction. So, that is the alpha angle here. Here the sign are important you have plus sign here minus sign here. So, it depends on how you measure the alpha.

So, while using this equation sigma alpha and tau alpha it is important that you need to understand what alpha is here. So, these two equations describe a circle and is called well known Mohr circle of stresses and how this circle is represented it is given here Mohr circle of stresses. In Mohr circle of stresses you have tau what is tau? Tau is shear stress this is shear stress and you have this what we call the normal stress. So, in Mohr circle on x axis you have normal stress on y axis you have the shear stress and then you can draw the circle. The center of the circle is the sum of the stress's sigma y plus sigma x by 2 and then tau equal to 0 this is the center coordinates of the center.

$$\sigma = \frac{\sigma_x + \sigma_y}{2} \text{ and } \tau = 0$$

$$\sqrt{\left[\frac{\sigma_y - \sigma_x}{2}\right]^2 + \tau_{xy}^2}$$

While the radius r is given by this relation r is the radius of the circle that is sigma y minus sigma x by 2 whole square plus tau x y this can be graphically also represented. Here in this case particularly in this case tau x y was 0. So, if you have tau x y equal to 0 then still center will be here which will be the half of this distance sigma y plus sigma x sum of these divided by 2 while the radius will be simply the difference between these two divided by 2. So, sigma y minus sigma x by 2 because tau x y is 0. So, the Mohr circle simply illustrates the stress conditions acting on an element graphically and as such a very useful for understanding the state of stress and the stresses induced by external loading.

So, what you have here in this case let us assume you have a soil element on that soil element normal stresses are only acting no shear stress is acting and normal stresses sigma y in y direction but along on the in a horizontal plane while sigma x is in acting in the x direction but along on a it is acting on a vertical plane. So, you have here we assumed sigma y is greater than sigma x that is the value is higher. So, that is why sigma y is coming here. So, the coordinates of the stresses which is acting on the horizontal plane are sigma y 0 because shear stress is 0 here. So, this value is 0 and similarly sigma x 0.

So, what we do if you have these two points then the center will be in between here and then using this you can draw the circle. So, Mohr circle of stress which is for here for elements subjected to shear major principal stress sigma 1 sigma y and minor principal stress sigma x. What is this principal stress we will discuss later but one thing is more another point is what is called pole. So, p is stand here in this p is called pole. Now, the issues is how we can locate how to locate the pole.

Pole once you draw the Mohr circle then pole can be easily located. For example, sigma y 0 is the stress condition at this point. What we do from this point we draw this stress is acting on the horizontal surface horizontal plane this horizontal plane. We draw horizontal plane passing through this point and wherever it cut on again this Mohr circle that is the location of pole p.

So, pole p can be located. Other way also if sigma x 0 is the stress condition here but this is acting in vertical plane if I draw a line on vertically here from this and then it will intersect again at the same point. So, this is the p is the location of the pole. Once you locate the pole then it is very easy further for analysis and for further calculation and once pole is located then you can find the stress conditions on any plane which may be inclined. So, that is the case. For continue with this suppose this is a stress conditions acting on a plane alpha.

So, plane you have a plane inclined at an angle alpha. So, similarly using the Mohr circle we can also find using the pole. So, let me explain how you can do. I think this can be done with this very simple example that how the pole have can be used for determining the stresses on inclined plane. Here for the simplicity, we assume that again there is this condition.

In this case this is you can say this stress is your sigma y this is your sigma x sigma x equal to 2 and this is tau x y equal to 1. So, three stresses are there. Both sigma y and sigma x are compressive while tau x y naturally on one side on one faces it will be counter clockwise on another face it will be clockwise. It cannot be counter clockwise on the both faces otherwise the equilibrium will get disturbed. So, in this problem what we do we first plot the Mohr circle locate the position of pole and then we need to find out the stresses acting on a plane which is inclined 45 degree clockwise from the horizontal.

So, let us first create construct Mohr circle. For constructing Mohr circle you need to first identify the coordinates of the stresses acting on horizontal plane and vertical plane. So, horizontal plane is this one. On horizontal plane what are the coordinates which is acting horizontal plane it is acting from sigma y is 4 positive and 1 it is acting in this direction.

So, 1 is also positive. On vertical plane what you have the coordinates the compressive stress is 2 and the shear stress is acting downwards so you will have minus 1. So, you have the this is for horizontal plane and this is for vertical plane. Now, on the Mohr circle what we do to draw the Mohr circle we identify these coordinates 4, 1 and 2 minus 1. So, I locate here this is 4, 1 and this is 2, 2 minus 1 and once you have this one. So, if you identify these two points then what you can do you can draw a circle you draw this line wherever it will pass through this will pass through the center.

So, center is located and once center is located then this is the radius this is another radius you can draw the circle. So, the construction of circle can be simply done if you know the stress conditions acting on a horizontal plane and on vertical plane. So, this is a stress condition on the this is this these coordinates say you the stress condition on horizontal plane stress condition on the horizontal plane and this is on this point shows you the same thing is on vertical plane and once you finalize the stress condition then you can draw the Mohr circle. Now, because this stress condition is on the horizontal plane we draw a horizontal line from here and this cut Mohr circle at a point which is called P. So, this way pole P is located once P is located then you can find the stress condition on any plane.

For example, if you want to find the stress condition on a plane 45 degree inclined. So, if 45 degree it is inclined then we make a line 45 degree with respect to this. So, this angle let us say this angle is 45 degree and wherever it cut again to the Mohr circle that will be your answer stress condition and it cut here. So, the stress condition 4 minus 1 that means you have a normal stress as a compressive while you have the shear stress which will be acting negative that means in clockwise direction. So, this way we can find out like this stress acting on any plane once in the Mohr circle is constructed.

So, I think I already discussed this slides contents of this. Now, one of the important issue for not only for dynamic load even for static loads is called the principal stresses and principal stresses are tied up or they are corresponding to principal plane. What are the principal planes? First of all two points on the Mohr circle are of particular interest. What two particular these two points on any let us this circle or here you can have let me go back. So, this on the Mohr circle the two extreme point this point and this point they are very important and the third one is on the top of it or bottom of it same thing.

So, these two extreme points at these two points you could see in the coordinates also because here there was no shear stress otherwise also because this is lying on x axis. So, there is no shear stresses. So, the planes on which no shear stresses act the points where the circle intersect the normal stress axis describe the normal stress plane where no shear stress acts. So, the normal stresses need to be found on the planes where no shear stresses acts and which are these two points these points are corresponding to the end and the beginning point of this Mohr circle. So, at these points you do not have any shear stress.

So, any plane those planes are called principal stress planes where no shear stress is acting. So, this is called principal stress planes and the normal stresses which act on these planes is called principal stress. So, you need to first identify the principal stress planes and then normal stress acting on those planes will be simply called principal stress. So, the principal stress axis another issue is principal stress axis. We have plane principal we have the principal stresses, and the principal stress axis are those axis which are aligned in the direction of the principal stresses.

So, the direction in which principal stress act is nothing but principal stress axis and these are perpendicular to principal stress planes. So, you have the one side plane let us say this is my plane on this plane normal stress is acting is called principal stress and the direction this direction will also called principal stress axis. So, this way we defined the now the largest out of we have three principal stresses and out of these the largest principal stress is called the major principal stress and denoted by sigma 1 sigma subscript 1. Then the smallest is the minor principal stress and that is denoted by sigma 3. So, you may be wondering if you have sigma 1 and sigma 3 where is sigma 2.

There is third stress also in which is also an intermediate principal stress which is denoted by sigma 2 that can be between sigma 1 and sigma 3 and this will be between sigma 1 and sigma 3. So, that means in general I can write that sigma 1 is the largest which will be greater than sigma 2 and then it is greater than sigma 3. So, sigma 3 is the lowest is all out of these three principal stresses. So, continue with this with the Mohr circle the mechanical behavior of so is much sensitive to the what is the relation between sigma 1 and sigma 3. That is the relationship between major principal stress and minor principal stress.

What is the relation between sigma 1 and sigma 3 that governs much about mechanical behavior. However, the sigma 2 is does not influence much. Sigma 2 is intermediate principal stress and normally sigma 2 and sigma 3 are almost equal. So, therefore the value of sigma 2 is normally we do not show not consider separately. Here you see sigma 1 and sigma 2, sigma 3 those there is a lot difference appears in this figure sigma 2 and sigma 3 but they are closer and sigma 1 and sigma 3 one is minimum and sigma 1 is maximum.

So, we consider and most of the time you may not require to consider this value of sigma 2. Continue with this with the principal stress for in the pole can be used to determine the orientation of the principal stress planes. What is the principal stress planes? Principal stress planes are those planes where no shear stress act and the major and minor principal stress act perpendicular to each other. So, like if major principal stress is acting in this direction minor will be acting perpendicular to this one. Intermediate principal stress will also act perpendicular. So, in fact you have three principal planes and three principal stresses. So, one let us say this is major principal stress is acting. Now minor is like this and intermediate will be like this one. So, all three will be orthogonal to each other.

So, all three will be perpendicular to each other. Now in this case how we can use the pole to locate the principal planes? Here the same Mohr circle is there where stress condition is 4 1 and 2 minus 1 here which is plotted and with this stress this Mohr circle what you can find the value of pole by putting this because this is acting on the horizontal plane. So, if I draw this then I can know the pole. Once pole is located you join P with this point of extreme stress and with this points with minimum stress condition. So, the line this sigma 1 which is acting this will be basically principal major principal plane major principal stress plane while this point will be minor principal stress plane. This point this plane is this one in this case also plane is this one. So, you once you join pole with maximum value and minimum value then you get the planes. Continue the last topic of this lecture eleventh lecture is on stress paths and this is little new concept which you need to understand. Here what happens why it is required? When you consider the Mohr circle for variation in stresses. So, for a number of loadings then it becomes little difficult to analyze for many loading sequence. So, what has been done rather than we work with Mohr circle if we work only with a single point on the Mohr circle then it will be easy to show the variation of position on the Mohr circle.

So, what is here the single point is called the stress point the very top point of the Mohr circle is called the stress point. So, stress point is the point which is highest top point of the Mohr circle and the path taken by this stress point during loading is called the stress path. So, what is stress point and stress path? Stress point is the point of maximum shear stress which is on the top point and the path taken by this point during loading is called the stress path. Since many properties of soil are dependent on the stress path induced by the applied loading the stress path is very useful tool in earthquake geotechnical engineering. What you have here? So, here you have the Mohr circle and again you could see sigma 1 is your major principle stress, sigma 3 is minor principle stress, p which will be the average value of sigma 1 plus sigma 3 by 2 which is given here.

$$\sigma = p = \frac{\sigma_1 + \sigma_3}{2}$$

$$\tau = q = \frac{\sigma_1 - \sigma_3}{2}$$

You could see sigma 1 plus sigma 3 by 2 which denotes the basically the center of this Mohr circle while q, q is the here, q is the center this ordinate, this ordinate is q, and this q is sigma 1 minus sigma 3 by 2. So, this is location and definition of stress point on which stress path is, but what happens? These stress parameters p and q need to be represented into effective stress parameters. That means, if they are dry that fine, if these are not dry then you need to take care of pore water pressure. So, total stress minus effective pore water pressure will give you the effective stress.

Here in this slide, you could see there are two equations. Using these equations the stress paths can be expressed in terms of total or effective stresses. Normal practice is to plot it is p versus q, what is p? p is effective another q is total stress. So, both effective and total stress are involved, but q or q prime is same as can be proved from the equations here. So, in for example, in the horizontal distance between total and effective stress paths if you plot together is equal to the pore pressure and how it have come? p equal to sigma 1 plus sigma 3 by 2. So, once you have sigma 1 is nothing but sigma 1 minus u and similarly sigma 3 minus u. If I simplify it, then I got sigma 1 plus sigma 3 by 2 minus 2 u by 2 that means p minus u. So, basically p dash equal to nothing but p minus u and p dash is effective parameter. While q is sigma difference of these two sigma 1 minus sigma 3 by 2 and once you differentiate then you end up in the same value as a q if you started from here sigma 1 minus sigma 3 by 2. So, basically here what we do? When we represent p so instead of p in stress path you have p instead of p you use p prime while instead of q prime you use q or let me put other way like here like p dash will be represented by p minus u and q dash will be represented by q because it is same q dash if I write q dash or q it is same. So, p dash equal to p minus u so what we when we plot the stress path and then stress path you have on x axis you have p and on y axis you have q.

$$p' = \frac{\sigma_1' + \sigma_3'}{2} = \frac{(\sigma_1 - u) + (\sigma_3 - u)}{2} = \frac{\sigma_1 + \sigma_3}{2} - \frac{2u}{2} = p - u$$

$$q' = \frac{\sigma_1' - \sigma_3'}{2} = \frac{(\sigma_1 - u) - (\sigma_3 - u)}{2} = \frac{\sigma_1 - \sigma_3}{2} = q$$

So, accordingly stress path is plotted. So, the typical stress path is like here so here this is stress path. In this stress path this is p dash which was nothing but sigma 1 minus sigma 3 by 2 and minus u pore water pressure. So, this is basically p minus u so that means pore water pressure is subjected while q is same as q prime. So, whether we write prime or we do not write for the q it will remain same the value is not changing. Now, the stress path which is shown in this figure have a slope m naught which is denoted here, and this m naught slope can be found out using this equation dq by dp because dq is here on y axis and p is here.

$$m_0 = \frac{dq}{dp'} = \frac{d\left(\frac{\sigma_1' - \sigma_3'}{2}\right)}{d\left(\frac{\sigma_1' + \sigma_3'}{2}\right)} = \frac{\sigma_1' - \sigma_3'}{\sigma_1' + \sigma_3'} = \frac{\sigma_1'(1 - K_0)}{\sigma_1'(1 + K_0)} = \frac{1 - K_0}{1 + K_0}$$

$$K_0 = \frac{\sigma_3'}{\sigma_1'}$$

So, this ratio dq by dp will give you this slope and if I put the value of q and p in this case then I get back this 2 is cancelled out d is cancelled out you end up in this equation 1 minus k naught divided by 1 plus k naught. So, this is the ratio only and this ratio and where the k naught is nothing but the coefficient of lateral earth pressure at rest and that coefficient can be determined nothing but k naught is sigma 3 bar divided by sigma 1 bar. So, this is the definition of both are effective but the ratio of minor principal stress to major principal stress. Here in this stress path, there are 4 paths one is going like towards AC compressive AE extension then LE and LC.

So, 4 paths are there and how they are described 4 paths are here. AC will be applicable if vertical stress increases with constant horizontal stress keeping in view the horizontal stress constant if vertical stress has increased AC will increase opposite will happen for AE if vertical stress decrease with constant horizontal stress. So, AC and AE will be opposite while LE says if horizontal stress decreases with constant vertical stress while if horizontal stress increases with constant vertical stress. So, if it is decreases then we say LE if it is increases then we say LC. So, these are the different notations for stress path here. So, with this I complete the lecture number 11 for this course and we will continue with the second lecture that is lecture number 12 soon. Thank you.