Earthquake Geotechnical Engineering

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Lecture 15

Laboratory Tests

I welcome you again for this NPTEL course on Earthquake Geotechnical Engineering. And today, we are going to talk lecture 15, which is the part of the module second, which is on dynamic soil properties. And in the dynamic soil properties, we will talk about laboratory test. So, let us first in the dynamic soil properties, we have the four chapters. Two chapters are already over. First was on concept of stresses in stress path, second on the field test and today we are on the third chapter, which is in the laboratory test.

In this laboratory test, we will have three lectures in this chapter. So, one will be on low strain, then we will have high strain tests and then what is the difference between modal test and the element test. So, we will discuss about that. So, let us start today and before we talk about what we are going to cover in this lecture, it is on the laboratory test in general, what are the like, how they are different and what is the advantage of the laboratory test compared to the field test.

And then we are also going to talk about sampling. And finally, today we are going to talk low strain element test. In the low strain element test, there are resonant column, ultrasonic and bender element test. But today we are going to cover only one part that is resonant column test and we will continue in the next lecture for this low strain element test. Coming to the laboratory test, normally they are performed on relatively small specimens that are assumed to be representative for larger body of soil.

So, that is the difference. In the field when you conduct the test, then you use large soil mass which is representative. Here you are taking a small sample and these specimens are tested either as element if it is subjected to uniform initial stresses and stress conditions or you could have a modal test in the laboratory. In that case, this known uniform boundary conditions can also be considered. So, now here most important point is that sampling is also important.

Sample or the specimen which is tested inside the laboratory that should represent or the representative of what you are having the larger body of soil. So, this is like that sometime it is there, but sometime not. And particularly I think as we discussed, you may have undisturbed and disturbed samples. So, if you have undisturbed samples, then the laboratory tests are okay. And like I can put it here, you have undisturbed sample normally you have in the, it is easy to get the undisturbed samples in case of when you have the clay or the cohesion less soil.

So, undisturbed sample, so you can get in the clay or cohesion less soils, but it is difficult to get undisturbed samples in the sand. So, as a result the laboratory tests are more like that recommended for the clay soil, but in general as we discussed one should conduct both field tests as well as the laboratory test. So, like field test is also important and lab test is also important. Now, the ability of laboratory test to provide accurate measurement of soil properties will depend on their ability to replicate the initial conditions and loading conditions of the problem of interest. No laboratory test can represent all possible stress and strain can pass with general rotation normally worth of the principal access.

As a result rather than in the lab sometimes conducting only one test, we need to conduct for different properties we need to conduct different tests. To conduct the test in the laboratory first you need to carry out what we call the sampling and element tests are performed on soil specimens and these specimens are representative of the larger body. The problems which involve the response of soil are to be placed as a fills specimens can be constructed from bulk or disturbed samples by simulating the compaction process as closely as possible in the laboratory. When the properties of an existing soil are needed the problems becomes difficult. So, how to deal with that. Test on existing soil can be performed on undisturbed or what we call the or reconstituted specimens.

So, reconstituted will be sometimes it is also called remolded or disturbed. So, this is actually or that they are not synonymous rather this is comes under the category this is opposite of the undisturbed sample. So, this is the it is also called disturbed or remolded samples remolded. In many cases the results will be different between these tests because of difference in soil fabric where you have may have natural and reconstituted soil specimens. So, what is issue is here in the soil when you have in the field it is in the natural state, but when you test in the laboratory you have most of the time what we say the reconstituted samples.

Continuing with this dynamic soil properties will be influenced by many factors which include density and stress conditions and it also include other factors. For example, soil fabric, structure, age, stress and strain history and cementation. So, there are so many like you know that they will influence dynamic soil properties and sometime it is difficult to consider all of these factors. Normally the void ratio or stress conditions can be recreated in a reconstituted specimen in the laboratory that we can void ratio like you know we can test in a particular relative density and maintaining the void ratio and the stress conditions also. However, the other effects where it is difficult to consider in the lab.

Continue with that as a result since the effect of these factors are basically particular at low strain level and they like you know that these factors when you consider at the low strain level no issue, but as you go for the higher strain then some of the properties may not be the same as you collect the sample from the field and conduct in the laboratory. Therefore, for the results of the laboratory test to reflect the actual behavior of the soil in the field as closely as possible high quality undisturbed sample must be obtained. But I mentioned this is the requirement that you may get high quality undisturbed samples, but this may not be difficult to get or even sometime impossible to get for the sand, difficult to get in granular soils or what we say in short sand. It may be in the clay. Cohesive soils, procedure for the preparation of test specimen by carefully trimming thin walled tube or block samples are fairly well established.

Undisturbed sampling of cohesion less soils such as clean sand and gravel is more difficult. Even thin walled sampling tubes can cause significant disturbance of clean sands and densification of loose sands and you have dilation of dense sands. So, if you have the loose soil then there will be densification, but if you have the dense then dilation which is kind of an expansion. So, dilation will take place. So, there are opposite effects.

So, it depends on the relative density of the soil. Now, we come on the low strain element test as I mentioned that we have here in the low strain element test three categories. One is resonant column test, then second ultrasonic pulse test, third one piezoelectric bender element test in the short people call bender element. So, in this lecture we are going to talk about the resonant column and in the next lecture we will talk about the next two ultrasonic and piezoelectric. So, what is in the resonant column test? Concept of resonant column is based on the resonance.

That is in fact, most of these tests are you find one of the property of the soil in these all three tests. The property which you find is common which is give you the what is called the shear wave velocity, shear wave velocity of the soil wave velocity which is nothing but v s. And you know that v s is nothing but square root of g by rho which we already discussed. So, once you know the v s then shear modulus g can be calculated as rho v s square. These things we already discussed.

So, here these tests give you the shear velocity. In the resonant column test, you find what is done in the resonant column test, you vary with the frequency and then you have a different frequency amplitude. So, natural frequency is obtained. So, we are going to discuss that thing in detail. Now resonant column test is low strain test. In fact, all three are low strain test. So, this test is based on the wave propagation theory and most commonly used for the measurement of properties soil properties at low strain not at high strain. In this test, a solid hollow cylindrical specimens is subjected to harmonic torsional or axial loading by an electromagnetic loading system. We are going to discuss in detail. You have cylindrical specimen which is similar to what is used in the triaxial, and it is hollow and it is subjected torsion.

Let us say this is here. So, this is subjected to torsion loading like this or it may be subjected to axial loading which is longitudinal loading and we are going to discuss both these things. The loading systems which are usually apply either it apply harmonic loading for which you can control the frequency and amplitude can be controlled, but random noise loading and impulse loading can also be considered. So, it is here typical resonant column test apparatus is given. A part is the plan which is top view of the loading system and on the right hand side you have profile view of loading system and soil specimen. So, soil specimen is cylindrical. This is soil specimen. You could see here this is soil specimen and it is cylindrical in the shape. So, you have the circular in cross section and height. While you have top view, in top view you have actually four magnets. So, this is like four magnets are there and these magnets have between two drive coil. So, you have drive coil this one and this one and each magnet is between two drive coil and there is a drive coil holder.

So, this is one system, and this is repeated four times here. Then you have proximeter or probe which is circular like it is blade. Therefore, you can say that these are the four blades and then it will rotate this. Then you have accelerometers for the measurements and this side if we come on the elevation side which is profile, you have cylindrical specimen and you have the porous stones which is on the bottom of the specimen as well as at the top of the specimen. And this specimen is kept on what is called the base pedestal.

Then you have the inner cylinder and here you have between this you have the of course, this specimen will be inside the what we call the membrane. And after the membrane, you have the fluid which is normally water is used as a fluid, fluid most of the time like you use water here. Here on the top of it, there is a loading system and in this loading system you have for measurement what are the displacement, to measure displacement you have LVDT and proximeter probe is same as here given. Here on the side you have drive, so this is in the elevation drive coil and magnet can be seen which was on the top here. Top cap is specimen and then you can control the pore water pressure in this case. So, what you have in this typical resonant column test apparatus, once you put this on the top, so you can apply the torsional loading. In the torsional loading all the like the 4 magnets will move in one direction. But when you apply the axial loading, then two are moving clockwise and two other two will move anticlockwise. So, this way you apply this that axial loading or longitudinal loading. Now coming to this like how we find the loading frequency is initially set at a low value almost 0 and then gradually increase until the response which you get strain amplitude reaches a maximum value that is basically resonant frequency.

We in this case we find the frequency response curve in general, or you draw a curve with respect to amplitude on y axis on x axis frequency. So, when the frequency changes your amplitude, initially amplitude was almost nil and it reaches a peak value, wherever you get the peak that is your basically your resonant frequency that we try to assign. The lowest frequency at which the response is locally maximized is the fundamental frequency of the specimen. So, our objective is to find the fundamental frequency of the specimen. Specimen means basically soil specimen.

And this fundamental frequency is a function of the low strain stiffness of the soil, the geometry of the specimen and certain characteristics of the resonant column apparatus. So, once you determine the fundamental frequency using the dimension of the specimen, we try to find the shear velocity and shear modulus of the specimen. So, it is here. The shear modulus can be related to the fundamental frequency using the following procedure. What we do? Consider a resonant column specimen of height h which is fixed against rotation of its base with polar moment of inertia j.

You have this specimen. It is fixed at the base and you have its height is h, h is the height of the specimen. So, and polar moment of inertia is j, specimens polar moment of inertia is let us say known and it is subjected to torsional loading, wherein when you apply the torsion loading, this is like this you move like on the. Now, using what wave propagation theory, the elastic resistance of this specimen will produce a torque at its top and this torque is given by this relation, wherein this equation g is shear modulus, j is polar moment of inertia and theta is the angle by which this specimen is subjected. And what we have here, in this specimen z will be downward, z is measured depth downward from here.

$$T = GJ\frac{\partial\theta}{\partial z} = G\frac{I}{\rho}\frac{\partial\theta}{\partial z}$$

So, here z on the top it will be 0, at the base it will be z equal to h. So, in this equation del theta or del z is strain basically and strain multiply by this, this will give you the stress and then torque. So, i in this case is mass polar moment of inertia, basically j is nothing but i divided by rho, where rho is mass density and i is polar moment of inertia, while j was the polar moment, so it will be here mass polar moment of inertia, i is the mass polar moment of inertia, here mass is involved in case of, but j was simply polar moment of inertia. So, that you need to understand, this is with polar moment of inertia only. So, that is why like we multiply by the mass is coming here, mass density multiply by j will give the value phi.

And this torque must be equal to the inertial torque of the loading system. This inertial torque can be find out by the inertial acceleration multiplied by this this polar moment of inertia, so here what you have, i naught is again mass polar moment of inertia at the base multiply with the height into this, in fact this h into this will give you the acceleration. The del square theta, del t square will give you the angular acceleration and if you multiply with the height, then you get the acceleration and then acceleration multiply by this i naught, you get the torque. So, now the torque is given from two equations, one from the stress strength this side, another side from inertial forces. So, you have, if we equate equation number 1 and 2, then we can solve it.

$$T = -I_0 h \frac{\partial^2 \theta}{\partial t^2}$$

But before that, we represent z which is a function of theta and z, like you have, we represent theta in terms of z and t, theta is rotation. In fact, as we discussed that this is your specimen and in this specimen, you have z downward, one parameter that is the depth, but then this theta is also away with the time, so t is time. So, theta is function of z as well as t. To decouple, we have another function phi z which is a function of z only multiplied by c1 cos omega t plus c2 sin omega t, where phi z is given by the relation given c3 cos kz c4. Here, this c1, c2, c3, c4 are the constant which we need to find out using the boundary condition.

$$\theta(z,t) = \varphi(z)(C_1 cos\ \omega t + C_2 sin\ \omega t)$$

$$\varphi(z) = C_3 cos\ kz + C_4 sin\ kz$$

Omega is frequency of excitation; omega is frequency of excitation while k is wave number which is omega by c which is in this case vs. So, k is like, once you know the omega, then this k can be linked with shear velocity, though shear velocity vs is unknown here. So, what we do here, like phi, we separate the variables, this psi z is the variable with respect to z only and the second part is with respect to t only. So, now using equation number 1, 2, 3 and 4 and this condition we apply, 0 rotation boundary condition at the base, at the base z equal to 0, so that means, then here the z is measured with from the base. So, z is measured from the bottom, so upward side positive.

$$\omega = \omega_n = k_n v_s$$

$$G \frac{I}{\rho} C_4 k_n \cos\ k_n h\,(C_1 cos\ \omega t + C_2 sin\ \omega t)$$
$$= -I_0 h\left(-\omega_n^2 C_4 sin\ k_n h\right)(C_1 cos\ \omega t + C_2 sin\ \omega t)$$

So, at this base z will be 0 here. So, as a result when z equal to 0, it will require that once I put in this equation, for z equal to 0, you see that c3 because sin kz this will be 0 and then cos kz will be 1, so c3 will be 1, so c3 will be 0. And then with this condition, we equal, equate equation number 1 and 2 and then we and at particularly, first we find out that c3 equal to 0 by applying this condition. The second condition we apply that at the top where z equal to 0, we say omega equal to omega n, when we consider for the natural frequency omega equal to omega n, which can be represented k n into v s, where k n is the wave number corresponding to this n nth frequency and v s is shear velocity. So, in this case, we when we put these numbers in this equation number 1 and 2 and equate equation number 1 and 2, then you end up equal in equation number 5. Now, in the equation number 5, this term and this term will be cancelled out from both sides and in fact, c4 will be also cancelled out.

So, c1, c2, c4 is, so as a result, you will not be left out with any c1, c2, c3, c4 when we manipulate this equation. So, equation 5 is simplified like this, here, this is coming from the equation 5 only. So, i over i naught equal to omega n square h over k n and rho divided by g, rho divided by g is nothing but 1 over v s, then k n into v s is omega n here. So, k n into this, so omega n is cancelled out, because omega n is nothing but k n over v s. So, ultimately you end up omega n h over v s tan omega n h over v s.

Now, if mass polar moment of inertia at the base, which is i naught here for a given specimen i, i naught and h are generally known at the time of cyclic loading and the fundamental frequency is obtained experimentally and then you can find the value of v s can be calculated and shear modulus. Once v s is known, then shear modulus can be calculated before that. But before that, how to use the equation 6? For equation 6, a condition when i naught equal to 0 in this equation, so what will happen? For i naught equal to 0, the left hand side will be infinity and this is possible only when this omega n h over v s is basically pi by 2, because tan pi by 2 is infinity. So, omega n h over v s is kept as a pi by 2 for this condition and with this condition, when we put this, we find the shear velocity using this relation. So, shear velocity of the specimen can be simply calculated by fundamental frequency f n into h.

$$\frac{I}{I_0} = \frac{\omega_n^2 h}{k_n} \frac{\rho}{G} \tan(k_n h)$$

$$\frac{I}{I_0} = \frac{\omega_n^2 h}{k_n} \frac{1}{v_s^2} \tan(k_n h)$$

$$\frac{I}{I_0} = \frac{\omega_n h}{v_s} \tan \frac{\omega_n h}{v_s} \qquad (6)$$

So, the issue is here, we need to find only this what you do. In this case, you draw amplitude versus frequency curve. So, you have the frequency on x axis, which is let us say f and you have the amplitude on y axis and your curve let us say comes like this. Wherever the peak value is coming, this is called f n. So, you just need to find out the value of f n. Once f n is known, then v s shear velocity can be found simply 4 f n into h and this is for the first for fundamental frequency. Basically, it should be written 4 f naught h, which is the fundamental frequency. So, f naught and this will be basic natural frequency or like the first fundamental frequency. So, this way once that is, so objective is here to obtain the resonant frequency. Once resonant frequency is known, then we can find the shear velocity and if shear velocity is known, then you can find the shear modulus using this equation.

$$G = \rho v_s^2$$

So, this was the case when you apply the loading in a torsional mode of vibration. However, if you apply and you find the value of v s and the shear velocity, but if you apply the loading in longitudinal loading or axial loading, then the equations, equivalent equations which was for the case for the torsional case is now given here, this equation. Again, in this equation, at the resonant condition, you have tan omega n h or v l should be pi, that is pi by 2, that means this should be pi by 2 and v l in this case is nothing but E by rho. So, using this again you find the natural frequency for the longitudinal wave and once this way you find the longitudinal wave velocity v l, so v l is calculated. So, v l is calculated in the similar way as like in the last equation, you have 4 into f naught h, here v l will be also calculated as 4 f naught into h, where f naught is a natural frequency or first fundamental frequency in this case.

$$tan\frac{\omega_n h}{v_s} = \infty \qquad \frac{\omega_n h}{v_s} = \frac{\pi}{2}$$

$$v_s = \frac{2\omega_n h}{\pi} = 4f_n h$$

So, once v l is known, then you have E, Young's modulus can be find out using the equation rho v l square. So, this is the test where you can find, this is the apparatus using resonant column apparatus, you can find the shear modulus as well as Young's modulus independently. And like once like you can find the shear modulus and Young's modulus independently, then you know there is a relation between shear modulus and Young's modulus that is g equal to E divided by 2 1 plus nu. So, as a result you can find the Poisson's ratio of the soil, Poisson's ratio nu can be calculated E divided by 2 g minus 1.

$$v_l = \sqrt{E/\rho}$$

So, using this relation we find the Poisson's ratio. If you know the E and g, but E and g are from the same resonant column apparatus find out from two different mode of vibration. One is E, g is find out using torsional loading, while E is found find out using longitudinal load. Continue with this, the shear strain in a solid cylindrical resonant column specimen which is loaded in torsion varies from 0 at the center line of the specimen to maximum value at the outer edge. The situations in which the shear modulus varies with shear strain amplitude, the effects of non uniform strain can be significant and for that the use of hollow cylindrical specimens minimize the variation of shear strain amplitude across the specimen. The resonant column apparatus test allows stiffness and damping characteristic to be measured under controlled conditions.

The effects of effective confining pressure, strain amplitude and time can readily be investigated. So, we can like you know these parameters can be considered. However, measurement of pore water pressure is difficult and the material properties are usually measured at resonant frequencies which is normally lies above those of the most of earthquake motions that is the frequency of interest. So, one limitation is that pore water pressure is difficult to consider. The second limitation is that we have this, we test the samples at a frequency which is higher than the of interest for earthquake loading, earthquake motions.

So, with this, I thank you very much for your kind attention and as I mentioned that in the low strain testing, we have discussed resonant column. Now, we will discuss ultrasonic pulse test and bender element test in the next lecture. Thank you very much for your time. Thank you.