Earthquake Geotechnical Engineering

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Lecture 19

Constitutive Relationships of Soils (Conti.)

I welcome you again for this 19th lecture on earthquake geotechnical engineering, under the module second, which is on dynamic soil properties, and we are discussing constitutive relationship of soils in this module that is the last chapter of this module, topics which we have covered in this chapter that is the listed here. So, up to the maximum shear modulus we have covered in lecture 18 and in lecture 19, we will discuss modulus reduction, damping ratio curve which is a big topic and very important and the next lecture the last two topics will be discussed. So, in equivalent linear model in this lecture, we are going to talk about two things, one is modulus reduction, another is damping ratio. In the early years of earthquake geotechnical engineering, the modulus reduction behavior of coarse and fine grained soils were treated separately. Later research has revealed a gradual transition between the modulus reduction behavior of non-plastic coarse grained soil and plastic fine grained soils. So, earlier initially what has been done, this modulus reduction curve has been studied separately for coarse grained soil and fine grained soils. But now later research have pointed out that like this, the PI could be an important factor, plasticity index rather than we divide into coarse grained and fine grained soils. So, depending on the PI value, we can have the modulus reduction curves differently.

So, for example, after reviewing experimental results from a broad range of materials, Dobry and Vucetic (1987) and Sun et al. (1988) concluded that the shape of the modulus reduction curve is influenced more by the plasticity index that is PI than by the old ratio and presented curves of the types which is shown in this figure. So, these are the curves, modulus reduction curve, typical modulus reduction curve which is presented by after Vucetic and Dobry (1991). And what you could see in this typical, on y axis you have g by g max value which is varying from 0 to 1, maximum value is 1 and on x axis you have cyclic shear strain which is represented in terms of percentage. So, when the cyclic strain, shear strain in terms of percentage is 10 to power minus 4, then you see irrespective of the PI value all the curves are 1 up to this point. Then after this strain, the pI equal to 0, the curve for the sand start degrading and it becomes g by g max start decreasing.

While still when PI equal to 15, 30 they continue, for example, pI equal to 200 curves, the g by g max value remains 1 up to quite high strain which is 0.01. So, what we get it here? The shape of these curves for different PI values looks same. However, later on we will discuss this value where these curves start getting less than one value g by g max is called the threshold strain which will be represented by what we will say gamma Tl. For this, for example, gamma Tl this will be for PI equal to 0, but for PI equal to 200 the same will be here.

So, that means as the plasticity index or pI increases, then the value of threshold shear strain increases significantly that we are going to talk again. These curves show the linear cyclic threshold shear strain which is called gamma Tl is greater for high plastic soils than for soils of low plasticity. So, the like it is greater for the clay compared to the sand. This characteristic is extremely important, it can strongly influence the manner in which a soil deposit will amplify or attenuate earthquake motions. So, like because here for the same strain level as this value if I draw on this strain level g by g max value is very much dependent on the value of PI.

So, this becomes like almost 0.7 g by g max for 0, but still, it is almost 1 for PI equal to 200 where plastic soil. For PI equal to 0, the modulus reduction curve is very similar to the average modulus reduction curve which was specifically done for the sands by (Seed and Idriss, 1970) where coarse and fine grained soils were treated separately. So, here (Seed and Idriss, 1970) they treated separately the coarse grained soils and then fine grained soils and then behaviour because the coarse grained soils and fine grained soils their behaviour is quite different. But Vucetic and Dobry through this their investigation found that it is a PI value rather than we say it is coarse grained or fine grained soils that governs this phenomena.

Modulus reduction curve behaviour is also influenced by fine and coarse grained soils. Now, continue with this the difficulty of testing very large specimen gravels, we can test the soils in the laboratory normal laboratory, but it is difficult to test the gravels where they are particular for the large specimens. As a result, the testing of gravel soils in the laboratory is difficult and the data available is very scarce. So, as a result average reduction curve for gravel is similar to that has been found. But the research have pointed out that the modulus reduction curve which is for gravel is similar to what is for the sand, but slightly flatter. Slightly flatter means they will go more up to this like if I say that being slightly flatter that means they will go G by G max for more range before they start decreasing. Modulus reduction also influenced by effective confining pressure particular for soils of low plasticity. If PI is less, than the effective confining pressure is an important parameter for modulus reduction curves. The linear cyclic threshold is greater at high effective confining pressure than at low effective confining pressure. So, gamma tl will be also a function of at what effective confining pressure you are testing, and it is greater if you increase the value of confining pressure.

So, when you increase the confining pressure as a result the G by G max will decrease after at a high strain rather than at low strain. The effect of effective confining pressure and plasticity index on modulus reduction here were combined by Ishibashi and Zang (1993) and as shown in this here. So, this slide is for G by G max curve. So, in this an equation how to find the value of G by G max if you know the level of strain, if you know the PI and the mean effective principal stress. So, here you have sigma m and this k gamma PI.

$$\frac{G}{G_{max}} = K(\gamma, PI)\left(\sigma_m^{/}\right)^{m(\gamma,PI)-m_o}$$

$$K(\gamma, PI) = 0.5\left\{1 + tanh\left[ln\left(\frac{0.000102 + n(PI)}{\gamma}\right)^{0.492}\right]\right\}$$

$$m(\gamma, PI) - m_o = 0.272\left\{1 - tanh\left[ln\left(\frac{0.000556}{\gamma}\right)^{0.4}\right]\right\} exp(-0.0145PI^{1.3})$$

$$n(PI) = \begin{cases} 0.0 & \text{for PI} = 0 \\ 3.37 \times 10^{-6}\ PI^{1.404} & \text{for } 0 < PI \le 15 \\ 7.0 \times 10^{-7}\ PI^{1.976} & \text{for } 15 < PI \le 70 \\ 2.7 \times 10^{-5}\ PI^{1.115} & \text{for PI} > 70 \end{cases}$$

So, as a result this G by G max and then so, this is the function of three parameters one is shear strain, second plasticity index and third is mean effective principal stress and the exponent which is here given from this relation. So, k gamma PI is a function of strain gamma and plasticity index. The equation long here it is 10 hyperbolic it is not normal 10 this is also 10 hyperbolic in this equation and which is a function of gamma and p i. So, when the p i equal to 0 that is strain for the sand your n p i will be 0. So, n p i can be rightly put in these equations as 0 and in that case this equation is simplified very much.

That means for example, in this equation this term will be over and, in this equation, when p i equal to 0 then this exponent will be become 1. So, this equation simplified very much. So, right now this equation is for c phi soil it could be p i as well as it could c as well as phi that means, p i plasticity index is could be also there. Then how what is the effect we said that confining pressure is also make effect on this modulus reduction curve. For example, when p i equal to 0 that means, for the sand for the first case there are four curves here which are separate out completely for different mean effective confining pressure this is for 1 kPa this is for 50 this is 200 and 400.

As a result value of g by g max which was 0 1 here if you increase the confining pressure then g by g max will continue to be 1 up to higher level of strain for example, this was the strain level 10 to power minus 6 here, but for this curve it will go 10 to power minus 4. So, that means, the value of g by g max will increase for the same level of strain when you increase confining pressures. So, this was the case when p i equal to 0 that is for the sand, but when p i equal to 50 all these four curves come very close and the effect of confining pressure is almost negligible particularly at higher value of strain. So, this is the effect of confining pressure. Now, what has been done that the field test has been conducted and like a lab test as well as the field test and then g by g max what are the factors when you increase those factors what is the effect on ratio of g by g max.

For example, when you increase confining pressure then g by g max will increase with sigma m effect decreases when you increase the p i which we have already just now seen. Void ratio when you increase the void ratio the relative density will decrease as a result this g by g max will increase with e. So, when you increase the void ratio g by for this when we say the g by g max that should be considered for the same level of strain that is the cyclic shear strain should be the same. So, for the same cyclic strain if I increase the void ratio then this g by g max will increase. Geological edge will increase, cementation will increase g by g max, over consolidation ratio because OCR g by g max will not affected.

P I we have seen that when you increase the value of p i so g by g max will increase with p i. Cyclic strain decreases with gamma c cyclic strain. Strain rate then again so these are the number of parameters how they influence on g by g max. Then continue with this now normally we have in the cyclic triaxial as we discussed in the last 17th lecture there are two types of stress version is called stress controlled another is strain controlled. Another stress controlled harmonic loading conditions where you put the maximum value of stress is limited.

Pore pressure generation and structural changes can cause the shear strain amplitude of a soil specimen to increase with increasing number of cycles. So, you have when the number of cycles are increases with the increasing number of cycles this increases. Shear strain amplitude so when you increase the number of cycles then shear strain increases. So, shear strain amplitude will increase. But on another side when you have a strain control test under strain control test where you are putting the limit on the maximum value of strain then in under undrained condition there will be development of pore water pressure and the shear stress amplitude would decrease with increasing number of cycles.

So, in one stress controlled it increases while in strain controlled it decreases. But here in strain controlled it is shear stress amplitude while in stress controlled it was shear strain amplitude. So, both are opposite. Here we are considering the how shear strain amplitude varies in the first while in the second how shear stress amplitude varies. Now, one of the important issue that how this value of g shear modulus varies with the number of cycles when you apply the number of cycles loading.

$$G_N = \delta G_1$$

For cohesive soils the value of shear modulus after n cycles g n can be related to its value in the first cycle g 1. That means first cycle g 1 is the value in the first cycle multiplied by the factor delta where delta is called degradation index and this degradation is nothing but delta equal to n to the power minus t. What is n? n is the number of cycles and t is the degradation parameter. So, as the degradation parameters like your t increases delta value will decrease as a result your g n will be less than g 1. The degradation parameter has been shown to decrease with increasing p i and increasing over consolidation ratio.

So, the value of delta will decrease as you increase the value of p i. So, that means this degradation will be more for the higher value of p i. The effects of stiffness degradation on modulus reduction behavior is shown in next figure. So, here is the case here and this figure is for OCR equal to 1 that means there is no effect on this curve of the over consolidation ratio. On y axis you have g by g max on x axis we have cyclic shear resistance amplitude gamma c.

So, effect of cyclic degradation on shear modulus and this is after Vucetic and Dobry (1991). And what we see here for different values of p i you have different values of p i. Here there is when we understand these curves, this curve is for 0 for the value of p i 15 there are 4 curves depending on the n number of cycles 1, 10, 100, 1000. For 30 again one curve for p i equal to 50 again you have the 4 curves 100 and 200. So, the effect of the degradation is done for p i equal to 50 and p i equal to 30.

So, as you could see when this curve n equal to 1 is for this curve. So, when the value of n is increasing then you are getting the higher degradation that is the g by g max decreases. So, this was given by the Vucetic and Dobry (1991). Now the second part of this we have discussed about modulus reduction what happens to damping ratio curves for the level of strain. Theoretically no static dissipation of energy takes place at strains below the linear cyclic threshold shear strain gamma tl.

So, gamma tl is a cut off strain below this strain g by g max remains 1 and when you increase gamma tl your strain level is more than gamma tl then g by g max decreases less than 1. Special evidence shows that some energy is dissipated even at very low strain. So, the damping ratio is never 0. Theoretically damping ratio should be 0 when you have very low strain, but this is not the case when you have this. So, above the threshold strain the breadth of the stress is low exaggerated by cyclically loaded soil increase with increasing cyclic strain amplitude which indicates that the damping ratio increases with increasing strain amplitude.

So, it has been observed that when you increase the strain amplitude then the damping ratio increases because why the breadth of the stresses loop will increase when you have the for the higher strain amplitude. Just as modulus reduction behavior is influenced by plasticity characteristics, similarly the damping ratio is also influenced by the pI of the soil and the effect will be shown. Damping ratio of highly plastic soils are lower than those of no plastic soils at the same cyclic strain amplitude. For example, the effect of pI on the damping ratio is shown in this figure. So, in this figure what you have on y axis you have damping ratio, which is in percentage, while on x axis you have the p like you know the cyclic strain amplitude.

First of all, for given value of pI as the strain amplitude increases the value of damping ratio increases. So, the damping ratio increases with the strain level of strain. But you get the higher value of damping ratio for pI equal to 0 that means for the sand damping will be higher than the clay for the same value of strain. And as the pI increases for the same value of strain the value of damping ratio decreases. So, you could see if I draw a line here at 0.1 percent and you see that how much the difference is, the difference is very large. Here damping is almost more than 15 percent while for this case it is about 3 percent. So, sand will have in general for the same level of strain the sand will exhibit higher damping compared to the clay, while G max it was the reverse it was more for the clay compared to the sand. So, for pI equal to 0 damping curve was similar nearly identical to the average damping curve that was used for coarse grained soils.

So, that is okay. Now continue with this. The similarity suggests that the damping curves can be applied to both fine and coarse grained soils because the pI is the factor rather than we say the fine grained soil or coarse grained soils. The damping behavior of gravel is also similar to that of sand. So, for the we do not have the separate curve right now for the like you know here for the gravels. So, like but they could be similar to the sands. And the damping behavior is also influenced by effective confining pressure particularly for soils of low plasticity.

If they have the soils for low plasticity then the that will be also influenced by effective confining pressure. Similar to modulus reduction curve like G by G max we also have a equation for damping ratio and in this case rather than the level of strain this on this damping ratio is function of simply if I can write that this is a function of G by G max and pI. So, only two parameters are coming. However, you have seen that G by G max you need to be known and then in the last equation G by G max was a function of pI, it was function of gamma and it was these two factors. So, the gamma the influence of shear strain has gone in the G by G max and then once you know this G the influence of strain in the G by G max through that the damping ratio will be different at different level of strain.

Similar to what we discussed for G by G max ratios that they are influenced by many factors. Similarly damping ratio is also like influenced by many factors. On the left hand side on this table the first column says that when these factors are increasing on the second column says what happened to damping ratio. For example, when you increase the confining pressure G by G max was increasing, but damping ratio will decrease with effective confining pressure, effect decreases with increasing pI. For void ratio E this decreases with increase in E while this was opposite for G by G max. T g decreases, concentration may decrease with C, over consolidation did not affect it was the same in G by G max, pI decreases with pI. So, most of the effect here are opposite what we have seen for the case of G by G max. So, this was about damping ratio. So, we have discussed G by G max as well as for damping ratio how this varies and with this, we have completed equivalent linear models for the soils and only one chapter one lecture is left for this topic which is on cyclic non-linear models and advanced constitutive models.

Thank you very much for your kind attention. Thank you. .