Earthquake Geotechnical Engineering

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Lecture 20

Constitutive Relationships of Soils (Conti.)

I welcome you again for this NPTEL course on earthquake geotechnical engineering and this is lecture number 20th that is the last lecture of the module number 2 which is on dynamic soil properties. And in this lecture, we are continuing with the constitutive relationship of soils. And in this chapter what we have already covered is listed on the top with the black and with the violet color cyclic non-linear models and advanced constitutive models we are going to cover these two topics. So, basically we have already covered in the stress system we have cyclically loaded soils and we also talk about in detail equivalent linear models. Today we are going to talk about the advanced constitutive models which consists of two parts one is cyclic non-linear models and the second one is advanced constitutive models.

So, let us start from cyclic non-linear models. Cyclic non-linear models represent the non-linear inelastic behavior of soils using a non-linear backbone curve and a series of rules that governs unloading and reloading behavior. What is backbone curve? We already discussed, in the backbone curve you have kind of S curve which is basically when we talk about equivalent linear model you have to draw the backbone curve you have the stress let us say shear stress and you have the shear strain on x axis and like how the tips of the hysteresis loop will create the backbone curve. So, this is the backbone curve and the curve changes. So, and this the backbone curve is prepared using the coordinates of the tip points of the hysteresis loop. So, this is one part and then in the cyclic this is common the backbone curve was there in the equivalent linear models also, but then there are series of rules which govern unloading and reloading behavior. These backbone curves are usually described by simple functions that reflect the transition from the initial stiffness, initial stiffness means at lower strain level to the ultimate strength of which is at the high strain levels. So, like we will see one of the example and then in this model cyclic non-linear models you have unloading reloading rules control the behavior of the model during stress reversals and ensure that behaves in a manner similar to that exhibited by actual soil subjected to irregular cyclic loading.

So, what is in this case you have first one is the backbone curve then you have a series of rules which will control its loading, unloading and the reloadings. So, when you do unloading then the stress becomes 0 what will you happen to strain when the strain becomes 0. So, these like a different points and when the reversal occurs what will be the path of this curve. So, all these things governs and these rules are similar the rules try to frame the behavior of the soil element like in the actual soil behavior. In contrast to equivalent linear models cyclic non-linear models allows permanent strains to develop.

So, there is no permanent strains are developed in case of equivalent linear models. However, in case of cyclic non-linear models there will be development of permanent strain. Cyclic non-linear models can also be coupled with pore pressure generation models to predict changes in effective stress during cyclic loading. Modeling such behavior requires that the original backbone curve be degraded as pore pressure increase. So, as the pore pressure increase then there will be reduce in the effective stress.

So, the effective stress we discuss is sigma minus u where u is the pore water pressure. So, when there is increase in pore pressure then effective stress will decrease and there will be degradation. So, this generation of pore pressure was not able to deal when you talk about equivalent linear models. So, the pore pressure can be generation of the pore pressure generation models can be dealt in the cyclic non-linear models. So, this is the advancement compared to what we have equivalent linear model.

The further the non-linear stress system in the soils can be represented more accurately by cyclic non-linear models that follow the actual stress system behavior of path during cyclic loading. So, they are more realistic compared to equivalent linear model. Various models are also able to represent the what we call the shear strength of the soil and with an appropriate pore pressure generation model changes in effective stress during undrained cyclic loading. When we say undrained cyclic loading that means if you do not allow the drainage then what will happen there will be development of pore water pressure. So, pore water pressure develops, pore pressure develops number 1 and when pore pressure develops there will be changes in effective stress, the effective stress will decrease. And when the change there are changes in the effective stress then these models will behave differently. A variety of cyclic non-linear models have been developed. All are characterized by two things, one is called backbone curve and a series of rules that govern unloading, reloading behavior, stiffness degradation and other effects. So, these models cyclic non-linear models will be deals in two cases. One is what we call backbone curve and then you have a series of rules.

And what this series of rules does that will governs unloading, reloading behavior and stiffness degradation. The simplest of these models have relatively simple backbone curves and only a few basic rules. So, for example, one of the simple model and like we will discuss the one of the simple model which is hyperbolic model, but more complex model will incorporate many additional rules rather than only few rules that allows model to better represent the effects of what we call irregular loading, densification, pore pressure and other effects. So, if you want to more accuracy then this then rules will be more. The applicability of cyclic non-linear models is generally restricted to fairly narrow albeit important range of initial condition and stress path. So, the range for which these models are applicable is narrow, but that range is very important that is the issue here. That range is important for initial conditions and for development of stress paths. So, the performance of cyclic non-linear models can be illustrated by very simple example in which the shape of the backbone curve is described by a simple relation where which is tau equal to FBB gamma, where tau is nothing but shear stress. Tau is shear stress and FBB gamma is a function of strain, shear strain, gamma is nothing but shear strain. So, it says the shear stress will be a function of strain which is logical also.

$$\tau = F_{bb}(\gamma)$$

The shape of any backbone curve is tied to two parameters which is the one is the initial low strain stiffness and the second one is high strain, shear strength of the soil. So, you see how this is represented. This function FBB which is a function of gamma is given by the relation g max into gamma 1 plus g max tau max divided by gamma. So, on the right hand side the value of g max and tau max they are constant parameters while gamma is shear strain which is varying.

$$\tau = F_{bb}(\gamma) = \frac{G_{max}\gamma}{1 + (G_{max}/\tau_{max})|\gamma|}$$

So, this varies. So, when the gamma varies then how this model varies this is given from this relation. We can see the hysteresis loop for the like backbone curve for this model. In the backbone curve you have tau versus gamma this is the curve. The value of g max, g max is nothing but the slope of this backbone curve at very low strain that is when almost gamma is 0 initial strain the slope is g max. And if I draw asymptote to this curve near then at the peak values then you get the tau max.

So, that means, using the value of g max and tau max from these curves g max and tau max is known to you and when the gamma varies then how this the function varies is given from this relation. And this relation is not in the dimensional form of course, it can be made by dividing but g max if I divide like both the sides using g max then tau max divided by g max will be the dimensionless will be simply gamma which is also sometimes like gamma divided by 1 plus g max divided by tau max into gamma. This is further simplified here this gamma the tau max or this is gamma plus gamma over gamma r this is also simplify this form. What is gamma r? Gamma r is said to be the reference strain which is tau max divided by g max.

So, this is here. So, this models becomes very simplified in this form. So, here on the left hand side as well as in the right hand side quantities are in the form of ratio only and gamma r will be the strain which is corresponding to tau max about this point. So, this will be the here gamma r this is not gamma tl reference strain. So, this was what is hyperbolic model, but in this case this equation will give you only the backbone curve or a skeleton curve it is not the complete hysteresis loop. Now, what happens how the model will load and load that will governed by the series of rules.

The quantities g max and tau max may be measured directly computed or obtained by empirical correlations. So, g max and tau max can be either find out in the laboratory or in the field or using empirical relation. For example, model the represents cyclic loading is governed by the following. So, now we have like as we said these models are governed by two factors one is a backbone curve and another is series of rules. Now, we are coming to the on series of rules which are used by these models.

For the series of rules there are four rules for this model for the simple model and we are going to discuss in detail all these four rules for cyclic non-linear models and after discussion of these four rules we will discuss one example also how this like four rules are applied. First rule for initial loading when you start the loading the stress strain curve will follow the backbone curve. So, backbone curve is already known, and the stress strain curve will follow the backbone curve that says the first rule. Now naturally you will ask the question up to what point it will follow. So, if a stress reversal occurs at a point which is defined by gamma r or tau r gamma r, we already defined right gamma r is a reference strain. The stress strain curve follows a path which will be given by this relation this relation. Here the this will follow up to stress reversal. So, up to if there is no stress reversal you will continue to follow the backbone curve. What do you mean by the stress reversal? Like if the stress condition is applied in this direction then all of sudden then it is start in another direction that you will call stress reversal. So, then change in the direction if this is this side this side if this is this side this side if it is up then down.

So, the stress reversal takes place then you need to note down that point at let us say coordinate of that point is gamma r and tau r basically these curves are tau versus gamma. So, the coordinates of will be tau and gamma at any points. So, once you have gamma r into tau r which will be the point of reversal then the stress strain curve will follow a path which is given by this relation tau minus tau f by 2 into f of gamma into minus gamma r by 2. So, here you are doing nothing, but you are shifting the origin and multiplying by a factor 2. In other words, the unloading and reloading curves have the same shape as the original backbone curve with the origin shifted to the loading reversal point. So, the origins is shifted to a point to instead of tau now you will start tau minus tau f and instead of gamma, gamma minus gamma r and this loop is enlarged by a factor of 2. If this factor 2 would not be there this factor is coming both sides and this factor cannot be cancelled out because this 2 is inside this bracket f v v if it would be outside, then it would have been cancelled out. So, this will be the second points. So, the first rule says you the follow the stress strain backbone curve up to the point where stress reversal occurs. Now, when the stress reversal occurs at that point you need to change the path and the path will be then another backbone curve will be followed which is given by this relation.

$$\frac{\tau - \tau_f}{2} = F_{bb}(\frac{\gamma - \gamma_r}{2})$$

Now we are following the second curve. When you are following the second curve up to what point? Now the third rule says. So, these first two rules which describe matching behaviour are not sufficient to describe soil response under cyclic loading general cycle. As a result two additional rules are needed. If the unloading or reloading curve exceed the maximum pass strain and intersect the backbone curve, then it follows the backbone curve until the next stress reversal.

So, the third point says you have loading then you will have unloading or reloading curve. So, you are on now either on unloading or reloading curve unloading or reloading is the almost like you know the unloading and there is a difference. You load and unload after unloading if you reload again that will be called reloading. So, unloading or reloading curve exceed the value of maximum past strain that was intersected by then it will follow up to the next stress reversal. The last rule says if an unloading or reloading curve crosses and unloading on reloading curve from the previous cycle the stress strain curve follow that of the previous cycle.

So, if it intersect a curve which is coming from the previous cycle then again it need to change its path. So, we will explain these all these four rules using an example. Here is an example. So, cyclic non-linear models how the path is followed? This is like extended meshing rules. The A part says variation of shear stress with time. So, here you have time on x axis and shear stress. So, shear stress is varying. So, how the points A, B, C, D, E, F, G and H are the points. And corresponding points how on the stress strain curve. So, on the A you have stress with time that is called stress time history. On B part you have resulting stress strain behavior. So, you have a stress on y axis and strain on x axis. Point A, point A is the initial point time is 0, stress is 0 naturally strain will be also 0. So, on this curve point A will be corresponding to the point or related to the origin. Now, the backbone curve is already defined because for the given condition you know the tau max and G max.

So, the backbone curve is given by this dotted line, this dotted line where it is dotted line is merged with the solid line here. So, the backbone curve is here D, C, A, B, E, F is the backbone curve that is the initial backbone curve, starting backbone curve. So, the first rule says that when you go from A point you will follow the backbone curve and this should be followed, this backbone curve should be followed up to what point until stress reversal is not coming. So, A the first stress reversal will come at point B. At point B you reach the peak value and then you come down.

So, that means you are changing your direction or like when you are moving this direction now you start coming in another direction. So, the B is a point of stress reversal. So, after B point you need to change your path and what the path will be changed? Path will be changed by the second rule. And in the second rule then you will follow this another backbone curve. The shape of that backbone another backbone curve will be given by this relation second relation here.

That means you will use the coordinates of this point. The coordinates of point B will require gamma r and tau r are the points of coordinates of these points and another backbone curve is from B to C. So, now up to the C point it will be like C is the point where this second backbone curve intersect the first backbone curve. So, C is the point and C is here in this case it is not the peak points or like this one. So, at this C point again the path will be changed and how it will change? At C point this will follow the same backbone curve, the original backbone curve that is A to B and it will continue up to the point D. Point D corresponding to again stress reversal you see in the stress time history after point D it is going up. So, the point D is the, so point B is so I can mark the stress reversal with r. So, this is stress reversal point r, f is stress reversal, this is also stress reversal. So, at D point it will again need to change its path and it will be governed by the rule number 2 and this you have another backbone curve which is E f. Now at point E it intersects again the original in the previous cycle backbone curve.

So, it need to follow the same backbone curve up to point F. Again at point F there is a stress reversal. So, there will be change in the stress reversal another backbone curve will need to follow and what happened G point you get another stress reversal. So, you need to change again the backbone curve and H point you will get the stress reversal, so third back point. So, wherever you get the stress reversal point then you need to keep changing your path.

Of this stress strain loop that the path will be changed due to two reason. One reason there is a stress reversal number 1 even without the stress reversal there will be change if you intersect the backbone curve of the previous cycle any of the previous cycles. So, for example, B, D, F, G and S they are stress reversal point, but why you are changing your path at point C, point E and then point these two points also because at this point when you are moving then you intersect the backbone curve of the previous cycles. So, I think all these are discussed here. Cyclic loading begins at point A and the stress and curve during initial loading from A to B follows the backbone curve as required by rule number 1 which we have discussed that means the original backbone curve.

At point B the loading is reverse and then unloading portion of the stress restriction curve moves away from B along the path which is required by rule 2. So, from B to C you goes by a path which is governed by the rule number 2, but at the C point you intersect the original backbone curve. So, you have initial unloading modulus equal to G max. The unloading path intersect the backbone curve at point C, at point C you intersect the original backbone curve and according to rule number 3 you need to continue along the backbone curve until the next loading reversals occur at point D. At point D you need to again change your path and then reloading curve move away from D as required by rule number 2 and the process is repeated for the remainder of the applied loading.

So, you keep repeating these rules and accordingly you follow the path. So, this was although this model the cyclic non-linear models is very simple and particularly this one of the type hyperbolic model which we are discussing is here this is hyperbolic model. So, this is described by hyperbola or in general this model is called hyperbolic model. We say this is hyperbolic model and this is one of the most popular model among the cyclic non-linear models.

So, in the hyperbolic models it is simple very simple and is expressed only in terms of effective stresses. It inherently incorporates the stresses nature of damping and the strain dependence of the shear modulus and damping ratio. So, the good point is that in this model there is a shear modulus and damping ratio both are dependent on the strain. To avoid spurious response at very low strain levels some cyclic non-linear models require the addition of a small amount of low strain damping, low strain damping is also required. Note that the cyclic non-linear modulus does not require the shear strains to be 0 when the shear stress is 0. So, that is the point here. Like in case of cyclic non-linear models when shear stress and they need to be 0. The ability to represent the development of permanent strain is one of the most important advantage of cyclic non-linear models over equilinear models. Permanent strain means you remove all the load that means, stress is 0, but still your strain is not 0 that is permanent strain and you get the permanent strains in case of cyclic non-linear models, but it was not the case for equilinear models. This simple example model does not have a, for the determination of shear induced volumetric strain that can lead to what we call hardening under drained conditions or when drained condition that means, or pore pressure development with stiffness, degradation under undrained condition. In drained condition there will be development of volumetric strain, but if you do not allow drainage in that case there will be development of pore pressure.

So, in the model which we have considered volumetric strain or development of pore pressure was not considered, but it is possible in the cyclic non-linear model. Such factors are accounted for the majority of the cyclic non-linear models commonly used in earthquake geotechnical engineering practice. So, the ability to compute changes in pore pressure, hence also changes in effective stress represent another significant advantage of cyclic non-linear models over equilinear models. So, one advantage of was that these models are able to develop the permanent strain. The second thing these models are able to capture the effect of pore water pressure, and which is very important for particularly for the liquefaction analysis. As pore pressure increase effective stress decreases and consequently the values of g max and tau max decrease. So, when the pore pressure increase there will be change in the effective stress and the value of g max and tau max will decrease. So, the backbone curve will be different. So, that is the case there. Since the shape and position of the backbone curve depend on g max and tau max the backbone curve degrades with increasing pore pressure.

So, it will be the difference that it will be down. As with actual soils the stiffness in a stress strain curve model depends not only on the cyclic strain amplitude as implied by the equivalent linear model, but also on the stress strain of the soil this is also be there. When incorporate into a computation models for ground response analysis which was the case for equivalent linear models cyclic linear models allow prediction of the generation redistribution and eventual dissipation of pore pressure during and after earthquake shaking. So, because these models are able to develop the pore pressure and these models are also used for ground response analysis. They will allow generation of pore pressure, dissipation of pore pressure during the shaking and after the shaking. These capabilities are very useful for evaluation of liquefaction hazards as we said because pore pressure generation is the hallmark for liquefaction which we will discuss when we discuss the liquefaction of soils. So, these cyclic non-linear models have many advantage or equivalent linear models. However, it is more complicated compared to equivalent linear models that was used. But as we discussed still they are not enough and advanced constitutive models are developed beyond the cyclic non-linear models also. And the most accurate and general method for representation of soil behavior these are based on advanced constitutive model that use basic principle of mechanics to describe observed soil behavior.

So, these models will be based on the basic principles of mechanics rather than on few rules' backbone curve and few rules. And what are these basic principles on which these models are governed? First general initial stress condition. So, number one a wide variety of stress paths, rotating principal stress axis. For example, when we do the cyclic triaxial test we already assume our principal stress axis are fixed. Major principal stress is applied in the vertical direction and minor is applied in the horizontal direction.

But in this case in these models we assume that this their principal axis are rotating. Then you can have the cyclic loading or monotonic loading both can be applied. High level of strain or low level of strain can be dealt and drained or undrained conditions that means consideration of pore pressure can also be dealt in these models. The last item is related to the liquefaction it is important in the when liquefaction occurs undrained conditions are considered. So, there is a development of pore pressure, but if you do not allow if you allow the drainage then pore pressure may not develop, but there will be change volumetric strain will develop. So, these need to be captured by these models. Advanced constitutive models use basic principles of mechanics to describe soil behavior for general initial stress condition and a wide variety of stress path with rotating principal stresses cyclic or monotonic loading high or low strain and drained or undrained condition. Therefore these models are more general than equivalent linear or cyclic non-linear models. So, they are able to capture many of the behavior of the soil which was not considered in the equivalent linear model or cyclic non-linear model. However, there is a penalty for accuracy and the penalty for this increased generality comes in the form of increased complexity and increased number of model parameters and some of these parameters are difficult to determine the laboratory and increase completion effort when incorporated into ground response or soil strength interaction analysis. So, when these we incorporate or we apply these models in our algorithm, for example, SSI algorithm or ground response analysis, then we face difficulty because they are complicated, they are not easy simple.

Such models generally requires a yield surface. There will be yield surface that means, yielding occurs and that occurs describe the limiting stress condition for which elastic behavior is observed. Yield surface and the hardening law is also governed that describe changes in the size and shape of the yield surface as plastic deformation occurs and a flow rule. So, one is the yield surface in these models, second is hardening law and third is a flow rule. How this soil will flow that relates increment of plastic strain to increment of stress. So, in general rather than representing stress strain relationship in these typical models you have stress invariant on X. For example, for one of the type of his model or let us say like what you have the von Mises model. So, you have the deviatoric stress J 1 D on the x-axis y-axis while you have I 1 they are called stress invariants and in this case for example, if you deal with Drucker-Prager model. So, Drucker-Prager model will look like this you have this intercept and then it will go like this hardening parameters. So, you have the intercept here negative intercept. So, this is like open remain and then when you have the hardening rule then there will be cap and then it may come down or like this one.

So, plasticity so these parameters. So, these are the advanced models. So, it will be difficult to deal, but they are incorporate yield surface, hardening law and flow rule. Most of the time associated flow rule is considered for, but when associated flow rule is considered that is okay for isotropic soil, but not for anisotropic soils. Although advanced constitutive models allow considerable flexibility and generality modeling the response of soil their description usually requires many more parameters than equivalent linear models or cyclic non-linear models. Evaluation of these parameters can be difficult and the parameters obtained from one type of test can be different from those obtained from another.

Same parameter if you find using one test it will find the different answer. If you conduct a different test, then you will find the different test. And sometime for finding the parameters of these tests some specific you know that like equipment is required. For example, if one parameter is need to find out from cyclic reaction then you may not be able to have the another substitute or you need to find direct shear like you know you have this direct simple shear test then need to be done using that only. So, this is the limitation of these models.

They are advanced, they are good, they are accurate, but it comes with higher cost. Although the use of advanced constitutive models will undoubtedly increase these practical problems have to date limited their use in earthquake geotechnical engineering practice, but still their use is continuously increasing and the number of advanced constitutive models are already available and they start range from Von Mises, Drucker Prager and we also implemented many of these models in our research with my research group. And particularly one of the advanced constitutive model which was developed by Prof. C.S. Desai and his team at University of Arizona is his model advanced constitutive HASS. And his model stands for High Archical Single Surface Model. Prof. Desai also developed DSC models which is disturbed state concept that is also DSC model is also one of the advanced constitutive model.

And these models are developed us, I assisted Prof. C.S. Desai and at University of Arizona in US. And like one of our research groups, particularly my PhD students have implemented these models in what we call the SSI, soil structure Interaction Analysis. So, many of the research paper you can find using these models on my website. So, with this, thank you very much for your kind attention and we have done with the module 2 which is on dynamic soil properties.

So, today we finished 20th lecture. So, that means one third of our course have finished today and thank you very much for your kind attention and patience. Thank you.