Earthquake Geotechnical Engineering

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Lecture 21

One-Dimensional

I welcome you again for this NPTEL course on earthquake geotechnical engineering. And today we are going to start third module of this course that is lecture number 21. We have total 6 modules each one will be of 10 lectures. So, this is a third module, and, in this module, we are going to talk ground response analysis and local site effect. So, the GRA ground response analysis which will be done by 1D one dimensional ground response analysis and two dimensional ground response analysis.

So, we will have four lectures of GRA that means like four lectures and then because the application of GRA is one of the important application of ground response analysis soil-structure interaction that is called in short SSI and that is also an important topic. So, we will have about two lectures for SSI. Then the last part of this module that is local site effects which will include effect of local site conditions on ground motion, design parameters, ground motion time histories and so on that will be another four lectures. So, we have three chapters in this module first chapter on GRA, second on SSI, third on local site effects and on each chapter, we have four lectures in the first one, two lectures in the second one and another four lectures in the last one.

So, with this introduction on this module third we start from the ground response analysis that is 1D GRA. Today we are going to talk about for the ground response analysis that is 1D. In this case for 1D like ground response analysis in all four chapters what we are going to talk is listed here. First we will talk what is ground response analysis that is the introduction to this. Then the second 1D ground response analysis linear and it will be in three parts one is uniform undamped soil on rigid rock, second uniform damped soil on rigid rock, third when you consider damping on elastic rock.

So, these three will be covered in 1D GRA. Then we are going to talk non-linear approach, then comparison of equivalent linear and non-linear ground response analysis and finally, 2D ground response analysis. So, out of four chapters, four lectures of this chapter one of this module three will be covered today. Then uniform damped soil and uniform undamped on rigid rock and elastic rock will be covered in lecture number the second lecture. Non-linear approach and comparison of equivalent linear and non-linear GRA will be in third lecture and the finally, 2D ground response analysis will be in the third lecture in the fourth lecture of this module this chapter.

Let us start from GRA introduction. Now the issue is this why GRA ground response analysis is important where it is used? It is used to predict ground surface motions for development of design response spectra. Design response spectra is something which is used in the design of the foundation, design of structures and for that we need to find out first what is the ground surface motion prediction of that. You will add dynamic stresses and the strains for evolution of liquefaction hazard. So that when we discuss liquefaction chapter this GRA will be used there also.

To determine the earthquake induced forces that can lead to instability of earth and earth retaining structures. So, when earth dam or earth retaining walls then for those also to what are the forces which are induced by earthquake that also will be covered by this GRA. Continue with this in general an ideal ground response analysis would model the rupture mechanism at the source of an earthquake. So, a rupture mechanism should be modeled including the source. The propagation of waves through the earth's surface motion is influenced by the soils which is lies above the bedrock. So, when the waves start from the source and when they move towards the ground then they are influenced by what we call the properties of the soil. So, that is the thing which we are going to talk in the GRA. And in general we should consider starting from the source itself but considering the mechanism of fault rupture is very complicated and the nature of energy which is transmitted between the source and the site is so uncertain that this approach is not practical that we consider in GRA including its source. So, what we do here? So, the problem of in ground response analysis where we determine the response of the soil to the motion of the bedrock immediately beneath it. So, you have a bedrock where we are going to discuss in detail and motion is given at the bedrock.

Now on the top of bedrock you have a soil layer. So, we are going to see what is the effect on the response of the soil layer and that will be treated as a GRA. Despite the fact that seismic waves may travel through tens of kilometers of rock and often less than 100 meter of soil plays a very important role in determining the characteristics of the ground surface motion. So, normally what happens like before the earthquake wave or what we call the elastic wave reaches to the ground surface they may travel tens of kilometers particularly inside the rock and they travel only may be few meters of soil but still soil play a very important role in determining the characteristics of the ground surface motion and how it plays we are going to discuss that. It is necessary to quantify the influence of local site conditions on strong ground motion. See we already discussed in the strong ground motion because those motions are important for us rather than the weak motion. As an engineer the motion which effect our structures or that is more important for us rather than only the weak motions which may be important for seismologist. Over the years a number of techniques have been developed for ground response analysis and these techniques are normally grouped according to the dimensionality of the problem that means one dimensional, two dimensional, three dimensional. So, types as a result types of ground response analysis can be in three categories 1D ground response analysis, 2D, 3D ground response analysis. So, for the simplicity we start from 1D ground response analysis.

When a fault rupture below the earth surface body waves travel away from the source in all directions. So, if there is a faulting then what will happen the body waves will travel from the fault in all the directions as they reach these waves reach boundaries between different geologic materials they are either reflected and refracted. Since the wave propagation velocities of cell or materials are generally lower than the materials beneath them. So, normally what happens you have you know one layer, second layer, third layer. So, as we go down as we go deep then because strength of the material is expected to increase as a result because as we see it for example, you see that velocity in shear velocity is simply given by g by rho.

What is g here shear modulus and rho is mass density. It is expected the value of g shear modulus will increase with the depth. As a result so, what happens because wave propagation of cell or materials are generally lower than the materials beneath them. So, inclined rays that is strike horizontal layer boundaries. So, there is boundary between two layers and when this inclined rays is passing, and it is strike then it is usually reflected to a what we call a more vertical direction, and you can see here. Here is this it can be explained here. You have the source here, the source is normally the fault and the waves initially traveling from the fault or the source they are inclined wave. So, it could be a path and when they are traveling through the rock this could be a path of that one. But when these waves start reaching to the ground surface then they encountered a layered soil and you see what is called the surficial layers are here. And in the surficial layer when this waves strike some boundary then it will be reflected.

So, it is reflected to then what will happen it is strike this boundary then it reflected to a more vertical direction. So, it will start once it hit this boundary then it start becoming vertical. And as a result it can be assumed that when these waves reach to the ground surface then they are like a vertically propagating shear waves. Now, 1D ground response analysis are based on the assumptions that the all boundaries are horizontal number one. The response of a soil deposit is predominantly caused by what you called SH waves which are propagating vertically from the underlying bedrock.

So, first of all they are SH waves, if we say a wave is vertically propagating now the particle motion is in the horizontal direction then we say SH waves. So, and if particle motion is in the vertical direction, then we say SV wave. The soil and bedrock are assumed to extend infinitely in the horizontal direction that is the assumption for the 1D analysis. Now, for any ground response analysis including 1D, 2D or 3D there are some certain assumptions certain nomenclature or like you know that terminology. So, for example, there are two figures here in one case soil is overlying bedrock you have a bedrock here on the top of it you have soil here. But in the second case there is no soil overlying bedrock. So, there are four terminology here in fact there are five points, but two points are common this is rock outcropping in this second figure is also rock outcropping. So, let us explain one by one free surface motion is the motion on the top of the soil that is it is on the ground surface number one, but it is on the soil. If on the ground surface, you have directly rock is exposed then it will be not free surface motion rather it is called rock outcropping motion which is given in this point or in this point because rock is coming to the surface.

In the second case if you have soil column and below the soil column at the base the motion is called bedrock motion which is on the top of the rock and below the at the base of the soil column. But the same suppose you do not have this any soil column and rock is neither exposed to the ground surface, but this is at the like at the some depth where no soil cover is there then this is called bedrock outcropping motion. So, you have this is common in this case here outcropping in these two cases and in these two cases you have bedrock is common. So, these are the nomenclature which is generally used for the ground response analysis. Now coming to the linear approach which is used for the 1D analysis we are going to discuss and a term what is called the transfer function has been defined which can be used to compute the response of single degree of freedom system.

For the ground response problem transfer functions can be used to express various response parameters such as displacement, velocity, acceleration, shear stress and shear strength. So, there are so many parameters you have displacement, velocity, acceleration and shear strength to an input motion parameters such as bedrock acceleration. So, input is your bedrock acceleration and output is could be displacement, it could be velocity, it could be acceleration or shear strength. Because it relies on the principle of superposition this approach is limited to the analysis of linear systems. So, in this 1D approach this will be applicable for the linear systems only.

So, because we assume that this based on the principle of superposition. So, what we are going to discuss is for linear analysis not for the non-linear analysis. However, non-linear behavior can be approximated using iterative process and such process are what is it could be equivalent linear soil properties. For example, shake algorithm use equivalent linear model. Similarly, there is another algorithm which is deep soil that is also based on this equivalent linear model.

So, deep soil is software for the ground response analysis. The key to the linear approach is the evaluation of transfer functions. Simplest of these may only rarely be applicable to actual problems. They illustrate some of the important effects of soil deposit on ground motion characteristic without undue mathematical complexity. So, here you can have like simplest in ground response analysis.

They could be based on this like without going in much mathematical complexity those like models can be used and we are going to discuss including derivation for the simple case. Now, I will take up this one later like let me tell you what is the issue here. You have a uniform layer of isotropic linear elastic soil overlying rigid bedrock here. So, thickness of this soil layer is H, capital H is there and this layer is as we discussed assumption of 1D analysis that this layer is going to infinity in both the directions. So, that means this is going to infinity in this direction as well as in this direction and thickness is H. U is the displacement or particle motion in X direction and Z is the depth and Z depth is considered here positive going in that means in downward direction. So, it is linear elastic soil deposit of thickness H underline y and another assumption a uniform layer it is isotropic linear elastic soil which is overlying a rigid bedrock is considered in this case. Now, we have two waves, one is wave going upward another is going downward. The amplitude of the wave which coming in downward is B while it is going upward is A. So, if we consider B positive then A will be negative.

So, naturally if here downward is we said that Z downward is positive. Now, in this case omega what is in this case omega t plus kz, omega is called circular angular frequency while k is called wave number as given here. k is nothing but omega by Vs, k equal to is a wave number which is omega by Vs and Vs is shear velocity which is just we discussed square root of g by rho. Harmonic horizontal motion of the bedrock will produce vertically propagating shear waves in the overlying soil. The resulting horizontal displacement can be expressed using this equation number 1 where A and B are the amplitudes of waves traveling in upward and downward directions respectively.

$$\boldsymbol{u(z,t) = Ae^{i(\omega t+kz)} + Be^{i(\omega t-kz)}}$$

Omega and k is defined, t is time and Z is the depth. So, that means this displacement u is function of two parameter one is it is varying with the depth Z it is not constant rather it will change with the depth Z and at the same time it also varies with the time. So, it is a function of depth Z as well as time and these two parameters time as well as Z is coming in equation 1. Now, our objective is if a waves start from the base and when it reaches to the free surface or ground surface. So, what is the change in that amplitude and the ratio of these two amplitude we need to find which will be defined as a transfer function.

So, here but before that because A and B are such that we need to apply some boundary conditions because there is only one equation and if you have two unknown A and B using single equation you cannot find two unknowns you need require some more information. So, what is the boundary condition here that at the free surface, free surface means Z equal to 0 the shear stress and consequently the shear strain must vanish because what is shear stress shear stress tau is nothing but g multiplied by shear strain. So, if you have shear strain equal to 0 naturally the shear stress will be also 0. So, as a result shear strain is nothing but del u by del z and this shear stress del u by del z should be find out at z equal to 0 that means why it is this is why 0, 0 means this is the value of z. So, this need to be carried out z equal to 0.

$$\tau(0,t) = G\gamma(0,t) = G\frac{\partial u(0,t)}{\partial z} = 0$$

So, applying this condition substituting this equation in equation 1 and number 2 and differentiating then we get this condition g ik A minus B A u. Now to satisfy this equation only there is one solution possible A equal to B or another solution could be that you have both A and B equal to 0 both A is also 0 B is also 0 but that solution will not give if you do not have the any amplitude. So, here rather than only that A should be if A equal to B then this equation will be satisfied and once we put A equal to B then we can you know simplify the equation number 1 and once we simplify because A and making equal then we can take A in common and 2 divided by 2 will be cancelled out. So, we rearrange this equation number 1. So, equation number 3 is directly coming from equation number 1 only but now it is written in different form.

$$Gik\left(Ae^{ik(0)} - Be^{-ik(0)}\right)e^{i\omega t} = Gik(A-B)e^{i\omega t} = 0$$

$$u(z,t) = 2A\frac{e^{ikz}+e^{-ikz}}{2}e^{i\omega t} = 2A\cos kz\, e^{i\omega t}$$

Still, you have two variables in this equation on the right hand side, one is z and another is time t both are coming in this equation. The equation number 3 describes a standing wave of amplitude 2 A cos kz. So, that is 2 A cos kz is the amplitude of the wave. The standing wave is produced by the constructive interference of the upward and downward traveling waves and has a fixed shape with respect to depth. So, here now once you define that this the value of u at any depth and any time is given from this equation number 3 then we apply this equation 3 to find out the transfer function. A transfer function describe the ratio of displacement amplitude at any two points in the soil layer. So, here choosing these two points one is at the top of the layer and another is the bottom of the soil layer which we have considered. So, at the top find the maximum value of u that is u max when z equal to 0 and t. At the bottom the value of z will be simply H capital H. So, we put z equal to 0 then you get 2 A ei omega t and when you put z equal to s then you will get 2 A cos k h e i omega t.

$$F_1(\omega) = \frac{u_{max}(0,t)}{u_{max}(H,t)} = \frac{2A\, e^{i\omega t}}{2A\cos kH\, e^{i\omega t}} = \frac{1}{\cos kH} = \frac{1}{\cos(\omega H/v_s)}$$

In this ratio 2 A and ei omega t is cancelled out as a result you get transfer function equal to 1 over cos k h and k h because k is nothing but omega H over Vs. So, simply I can write that F1 my F1 omega and even if I find the absolute value of this function F1 omega this will be simply 1 over cos omega H over Vs absolute value of this. And here try to understand that in this equation earlier equation which was there you have iota here. In equation third e i omega t was coming so that means iota complex number was coming here. But e i omega t is cancelled out when you find this ratio as a result your denominator is a real number only it is not a complex number.

$$|F_1(\omega)| = \frac{1}{|cos(\omega H/v_s)|}$$

So, this was the case when you do not consider any damping in the soil. So, this is undamped soil whatever we are discussing is neglecting the damping in the soil undamped soil. So, in this case you got it the transfer function and the transfer function is come in very compact form and this is the equation 5 which we already discussed. The modulus of transfer function is the amplifying function which indicates that the surface displacement is always or at least as large as the bedrock displacement. Because why the value of denominator omega H over Vs could be written as theta.

So, if I say 1 over cos theta where theta is nothing but omega H over Vs. So, you know cos theta value could be minimum value is minus 1 and it pass minus 1 to 0 and then it go to 1. So, minus 1 to plus 1. But when you find its absolute value if you find the absolute value of cos theta then it will be 0 to 1.

So, minimum is 0 and 1. So, because when it is 0 if you put 0 cos theta equal to 0 in equation 5 you get infinity f 1 omega. If you put 1 then you get 1. So, that means f 1 omega minimum value of f 1 omega will be 1 in case of undamped soil. Using this equation like f 1 omega is the already discussed the ratio of free surface motion amplitude to the bedrock motion amplitude or the bedrock outcropping motion. As omega H over Vs approaches to pi by 2 plus n pi that is the value of this theta tends towards pi by 2 plus n pi then denominator coming in this equation 5 will tend to 0. As a result, when the denominator becomes 0 then you will get infinite amplification or resonance which can be shown in this figure. So, you have this when pi by 2 n pi so you have in this equation pi by 2 plus n pi. So, the minimum when n is 0 what is the value of n here and the value in this equation could be 0, 1, 2, 3 or like this but it is integer. So, when 0 then you get pi by 2 when you have 1 then you will get 3 pi by 2, 5 pi by 2, 7 pi by 2 and so on it will continue.

So, you have uniform undamped soil on rigid rock. So, this is influence of frequency on steady state response of undamped linear elastic layer. So, you have for resonance at n equal to 0 for the first case when I put n equal to 0 then omega h over v s should be pi by 2. Now, for this case omega h over v s equal to pi by 2 you can rearrange corresponding frequency. Frequency will be simply omega is angular frequency f naught will be omega over 2 pi which can be rewritten as v s by 4 h. So, the first fundamental frequency you get for the soil layer is v s by 4 h where v s is shear velocity and h is the thickness of the soil. So, what you see here the peak values first peak will come at pi by 2, the second will be at 3 pi by 2, 5 pi by 2, 7 pi by 2 and so on. And all these minimum values 1, so if I draw a line of one line here then let me put then this will pass through this one. So, all this like this. So, what is here when between 0 to pi by 2 when it is 0 then you go towards pi by 2 your f 1 omega increases. At pi by 2 it reaches to infinity this these are going to infinity.

Again, when you increase further the value of this omega h this is k h that is k h is nothing but omega h over v s. So, when you increase this value further so what will happen that it will start decreasing and then it reaches to the value 1 but never 0 or less than 1. Then again this point is pi this coordinate here it will be 2 pi this will be 3 pi and so on 4 pi. So, at pi 0 pi 2 pi you will get 1 this factor 1 while pi by 2, 3 pi by 2, 5 pi by 2 this will get the resonance that is the maximum value. Now, so before we illustrate for one model let us how to deal this was the case for one single frequency if you have harmonic excitation. But in case of you have a real earthquake motion then how to carried out this using the analysis we are going to discuss here. So, in case of real earthquake what happens you it known time history of bedrock input motion is represented using what is called the Fourier series or what we called FFT. So, first FFT is an algorithm which is FFT full form stand here fast Fourier transform and using FFT you transfer a time history into frequency domain. So, you have time history and this is converted into a frequency response curve. In time history you have time on x axis while in frequency response curve you have frequency omega or f on x axis that is the difference and on y axis in FFT you have Fourier amplitude.

So, you can use the FFT to find the Fourier spectra for the time history. Each term in the Fourier series of the bedrock motion is then multiplied by the transfer function to produce what is called the Fourier series of the ground surface or output motion. Once it is done the ground surface motion can then be expressed in the time domain again using the inverse FFT. First thing is that you do the FFT find the Fourier response spectrum and then multiply this Fourier spectrum with the transfer function and whatever you get inverse apply the inverse FFT to get back the time history and which will not be the same as the original time history because it is multiplied with the transfer function.

So, we will discuss one example here for this case. Now like in this case let us so in the very simple model the response of soil deposit is highly dependent upon the frequency of the base motion and the frequency at which strong amplification occurs depend on the geometry and material properties that is the material properties means as wave velocity of the soil layer. Here just I will like to emphasize that this equation which determine the transfer function for undamped case on rigid rock looks a very simple but this equation carrying three factors. One is H capital H which represent the geometry of your problem then Vs which is the shear velocity which represent the material property and omega is your frequency of excitation. So, and this transfer function equation 5 is suggesting that the value of this transfer function will keep varying as the omega changes.

So, when the omega varies the value of F1 omega will be different. So, the transfer function is highly dependent on frequency but it also depend on thickness and shear velocity. Now coming to one small example it says the compute the time history of acceleration at the surface of the linear elastic soil deposit which is subjected to horizontal component of an earthquake motion with time interval. Interval means the time at what steps are given delta t, delta t equal to 0.02 second that means you have first stop at t equal to 0 then t equal to 0.02 then 0.04, 0.06 or so on. Now what we do to while using this delta t small time interval then this in time domain if you want to convert into frequency domain then you consider what is called the Nyquist frequency and this Nyquist frequency is nothing but 1 over 2 times of delta where delta t is nothing but 0.02 seconds. So, if I put this 0.02 second then I get 25 hertz. So, 25 hertz is the maximum frequency which you need to consider for this analysis and for this was the Nyquist frequency.

On another side you have the natural frequency of the system first fundamental frequency which is given by vs by 4 h and vs is shear velocity which is given in this problem is 1050 meter per second and thickness is 10 meter. So, if I divide 1050 by 10 then I get 26.25 hertz that is a natural frequency of the soil layer which is quite high, but this you assume that this data may be hypothetical rather than because like this is too high frequency. Now in case of real earthquake excitation which consists of a number of omega rather than a single omega how to deal with that.

So, we need to rotate what is called FFT and inverse FFT. So, the first case B figure B it is an acceleration time stream and when this acceleration time stream is processed through what we called FFT fast Fourier transform then you get Fourier response which is in terms of Fourier amplitude on y axis and on x axis you have frequency. So, frequency versus Fourier amplitude give you the this is the this give you the Fourier response. Now we find out the transfer function using this equation number 5 and when you find the transfer function using this equation number 5 then value of f 1 will keep varying with your frequency. At very low frequency you allow value even less than 1, but as the frequency increases then this continuously increases and it goes to high very high value when frequency is 25 hertz. But why it is 25 hertz it is the peak is going near 25 hertz why not at 10 hertz or some other value.

So, the reason being if we find out the natural frequency of the soil layer is 26.25 hertz. So, as a result and in your case you are not going more than 25 hertz because Nyquist frequency 25 hertz. So, you are calculating transfer function up to 25 hertz.

So, as you reach near to the natural frequency of the soil layer which is 26.25 hertz then your peak will start and particularly for undamped case it may go to infinity, but here it is because 25 only it is not here 26.25. So, but this is the transfer function f 1. Now what we do in this transfer function you are getting the peak values only at higher frequencies that is at 15, 20 or so not at the lower frequency less than even less than if you have less at certain frequencies this f 1 is also going to be less than 1 which is not expected, but at certain frequency it happens. Now what we do this is your transfer function which is independent of your input motion and this is the Fourier amplitude of input motion.

We multiply the transfer function with the Fourier amplitude and then we get another Fourier amplitude as shown in the figure E. But here you will see this time history for c and e is almost same they looks appear to be same and the reason being because the value of transfer function is almost near 1 up to when the major peaks are coming. When all the peaks get diminished then only you are realizing the effect of this frequency here effect of Nyquist frequency. So, you see here that if I go from 20 to 25 hertz then it is continuously increasing exponentially increasing. When you multiply this value f 1 with the Fourier amplitude in the b c given, then we get the product of these two.

So, the product of these two curve will come in third. So, this third curve is the simply product of these two curves. That means at each frequency you need to pick up the value of f 1 and this and then you need to multiply by the value given Fourier amplitude in the second figure and then find out. So, you see there is not large difference between c and e they look similar and the reason being here because the value of transfer function was very low in the range of interest where the maximum peaks are coming. It is influencing much more after 20 hertz, but that range has gone out.

So, that range is like at this range you do not have any peak values here. So, the product of c and e give you the figure e, in this case figure e if I mention using e it is in product of c and d. And once this though theoretically this is different than the input but here many things are same here. So, we reverse it and then if we do what we call the inverse f of t then we again find the time history, acceleration time history and which is your final answer for this case. So, thank you very much for your kind attention. Thank you.