Earthquake Geotechnical Engineering

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Lecture 45

Slope Stability Analysis (Conti.)

I welcome you for this NPTEL online lecture on earthquake geotechnical engineering. And today we are at lecture number 45 of this course and basically, we are under module fifth of this course which is on slope stability and retaining walls. And as mentioned earlier that this module 5 is covered in two part one is slope stability another is retaining wall. So, this is the fifth lecture on slope stability and the slope stability component will be done today with this lecture and then we will continue for the retaining walls. So, what we are going to talk like we already have three chapters in slope stability introduction to earthquake induced landslides which is completed static slope stability analysis.

Third chapter of this module which is seismic slope stability analysis. Today theory part mostly we have covered. I will go in detail of pseudo-static analysis which is widely used by the practicing engineers, and we will discuss in today's lecture one numerical example also. First of all, before we discuss numerical example today, we are going to talk about limit equilibrium analysis and in fact this will be extended to what we call the pseudo-static analysis. You know the limit equilibrium analysis is basically for static case and the counterpart is pseudo-static analysis.

So, we will not only deal with the limit equilibrium but today we are going to talk about pseudo-static analysis also which we discuss already about limit equilibrium analysis and pseudo-static analysis also. But when we discuss limit equilibrium analysis then we talk about what we call method of slices and the Bishops's method. So, let us discuss these two methods in first in detail and then after discussing it we will extend it for earthquake case. Coming to the limit equilibrium analysis this is kind of a review this slide you already that what we need to consider force or moment in the limit equilibrium analysis you consider the equilibrium of forces or equivalent of moment on a mass of soil which is above a potential failure surface. This potential failure surface in geotechnical engineering many times are called slip circle or slip surface.

The soil above the potential failure surface is assumed to be rigid and this is one of the assumption because it is not rigid but for the simplicity. The available shear strength is assumed to be mobilized at the same rate all the points on the potential failure surface as a result the factor of safety is constant over the entire failure surface. This we already discussed in the second lecture of this slope stability. As the soil on the potential failure surface is assumed to be rigid perfectly plastic limit equilibrium analysis though it will give you factor of safety, but it will not provide any information on slope deformation. As for the when we say rigid perfectly plastic the stress strain relationship is given here which we already discussed that is that our soil remains elastic only for a brief strain and then is to start from this point and reaches here and then after that maximum shear stress reach and the strain increases without increase in shear stress.

So, and the soil stability is usually expressed in terms of the index and most commonly this index is called the factor of safety which is usually defined as a factor available shear strength divided by the shear stress required to maintain equilibrium. So, this is like basically ratio of capacity to demand so which we already discussed. Now how to find out this factor of safety for the static case as well as for the seismic case we will discuss one numerical example. Continuing with the method of slices and the method of slices so far what we have done we have considered a triangular section and then we did the analysis for example, if this is the case and then you assume that your failure surface is triangle then it is easy to deal. But most of the time as we discussed that this failure surface may not be the triangle rather it could be a circular surface.

So, how to deal with that in that case because let us say in the method of slices what is considered first of all this method is best it assume the failure surface is a circular surface that is the one of the condition and in this method because it is not a linear it is not a straight line so you cannot have a triangle. So, what we do for the analysis we need to divide a number of components otherwise what will happen suppose if I take a single slice then what will happen the error will be large. As a result, in this method of slices we divide like this is our region wedge ABC. So, we divide here you could see there are 7 slice but it the number of slices in this methods is used is 6 to 12. So, section of a slope with AB trial surface is considered and for a soil which have a C phi strength parameter C is cohesion and phi is angle of internal friction.

The shear strength at the different points of the slip surface varies according to the value of the effective normal stress at those points. So, another issue if we consider single surface the error will be large because the shear strength is not constant rather it will be varying at different points. So, if we divide into a sectors that will be better. The analysis of stability in such a situation is carried out by the what is called method of slices which was introduced by Fellenius 1936 that is why it is also called Fellenius method or because it was from Swedish, or it is also called Swedish circle method. So, method of slices Fellenius method or Swedish circle method all 3 are same thing.

So, basic concept is already there now what we do you pick up one of the slice which is given here let us say slice number 4 it is taken out here. So, let us talk about first static case then we will extend this analogy for the seismic case also. So, weight of the slice will always act vertically downward irrespective of your slope angle. Here slope angle if I say the slope angle is measured with respect to static. So, this is the total slope angle.

So, when we do one slice then this slope angle total slope angle is important, but right now this will not be used in our calculation this whatever let us say if I say this is beta this total angles of slope is beta. So, this will not be used and then you have the height of the slope also. So, here rather we are carrying out the analysis slice for each slice w is a weight. Now weight will have two component one is normal to the slice which is like this is the center point of the slice. Basically, what happens on the curve code you have the center point. So, a is the center point. Now what alpha how you decide this angle alpha this angle. So, let me explain each and everything about this. So, because we are going to discuss one numerical for this. So, it will be better. So, what you have a slip circle and radius of this slip circle is R and O is the center. So, the total angle which is made at the end is this one this angle, but this angle or this angle of beta that will not come in the calculation rather what we do for each slice we pick up the center point join the center point with the O the center of the circle and you because this is the being that the then you straight line if you draw this will be perpendicular to the asymptote at this point. Now for each slice you need to find the angle with this slice max with the horizontal that is alpha how you do this is alpha line and this straight line is basically an asymptote at this point on this curve that is on the circle. So, this is an asymptotic line.

This way you determine the angle alpha and naturally this angle alpha and this angle alpha here will be the same this will be the angle alpha again. So, this will be also alpha. So, which is shown here. Now n is acting w is acting always vertically down what is n? n is perpendicular to this. So, it is 90 degrees from this one and tau is in tangential direction. So, naturally you know that given this condition what will be the n? n will be simply w cos alpha and what will you tau? Tau in this case will be w sin alpha. So, two components of w and alpha. So, this components we do for each slice wise and then you need to remember that alpha will be varying for each slice alpha is not constant it will be varying for different slice alpha will be different another point w is the weight of the slice only. So, w will also be varying. So, this is a in this method of slices and in this method of slices mass above the assumed slip circle is divided into number of vertical slices of normally equal width it may not be necessary that equal particularly the being first or the last may not be the same because it may be the smaller you could see in this figure also but in between two like for example first and seven may be smaller two three four five six we try to keep equal width.

The number of slices are typically for the good result should vary from six to twelve if you have many slices, it is better but if you increase more number more than tall then your computation will increase that is a disadvantage considering the whole slip circle AB of length L what is L? L is the total length of this code on the curve part. So, AB is not the straight line triangular part like if I go along this curve along the circle then the length along this periphery of the circle from A to B will be capital L and total driving and resisting force the factor of safety against sliding can be determined from this relation where C prime and phi prime is nothing but effective cohesion and effective angle of internal friction and T we already defined. In fact, T is nothing but this T is this is tau is itself is a T tau equal to T equal to w sine alpha. So, what you have we already said that summation of n n is summation of w cos alpha and summation of w sine, and this summation is done with the number of slices if you have five number in this case seven number of slices it will be done for seven times if you have more it will be done. So, summation w so it will be w 1 w 2 w 3 like this cos alpha 1 alpha 2 like this one and then you sum up and C and L already defined L will increase what how if this angle at O is increasing then the code is increasing.

So, if it is angle is small then L will be small. So, depending on periphery so L can be determined simply normally because it is a circle so you can determine for the small angle it can be said as R if suppose I say this angle is theta so it can be L equal to R into theta for a small angle of theta where theta should be in radian. So, this way you can find out the code. Now coming to this like this is the factor of safety which is given from this equation in the condition when you have no pore water pressure that means in dry condition but if there is a pore water pressure then the factor of safety will decrease, and the normal component sigma n should be taken as a sigma n minus u that is where u is a pore water pressure which is given from this equation. So, this equation is same except that here the effect of pore pressure has been considered and this is pore water pressure.

So, the pore water pressure in this slice is denoted from the u capital U. So, naturally when pore water pressure is present the factor of safety will decrease. Coming to this let us discuss one numerical example what has been considered a slope of height 8 meter is considered which is inclined horizontally by an angle beta equal to 30 degree the slope material is cohesion less with angle of internal friction phi 40 degree and unit weight 18 kilo Newton per meter cube. So, this is cohesion less that means once you set cohesion less that means you c equal to 0. So, first of all draw the sketch and so here what you have this is the sketch of the problem.

Here though as we just said that number of slices should be between 6 to 12 but for the sake of the simplicity right now only 4 slices has been considered in this case to make this numerical simple. In this case what you happen a and b are there. So, here at a and b this is angle theta theta equal to 80 degree which is given and 30 degrees this angle which makes with the horizontal actually this is this angle with the horizontal. 8 meter is the height of the slope these data are given radius is given 10 meter but you will see that radius of the slope and this angle theta is used only one of the calculation of L only L equal to R theta as we just discussed otherwise you will not require and we require the weight of each slice 1 2 3 4 as well as these angles. What angles are listed here and this angle should be considered like you know that from with respect to this like if I draw the vertical line.

So, with respect to vertical these angles is measured in anti-clockwise direction. So, let us see that in the last case also. So, this angle alpha in this equation which is coming in this equation is measured into anti-clockwise direction from the vertical. If it is going on the clockwise direction, then it will be considered negative. So, here in this case in this example all angles 3 angles are 0 and this is vertical.

So, it is here alpha equal to 0 for this slice number 1 alpha equal to 0 20 degree 40 degree 60 degree all are positive because they are going vertically to anti-clockwise direction. Weights are listed here what other data are given or the weight of the slices are 40 kilo Newton 50 kilo Newton 50 kilo Newton 30 kilo Newton. So, the weight of the slice first slice is 40 50 50 and 30. So, these are the data given to you. Now we do the calculation in tabular form one important issue because it is said that cohesion less soil.

So, in this equation C will become 0 if C is 0 then this term will be 0 C into L. So, as such you do not require to calculate R into theta because L is not required to be calculated rather your equation gets simplified. So, in this case your equation limit equilibrium method of slices. So, this method of slices what you have and this is equation in fact this equation is not for limit equilibrium analysis. So, this is for pseudo-static analysis because this equation we are using for earthquake motion also pseudo-static analysis. So, these equations are for pseudo-static analysis. Pseudo-static analysis we already discussed the factor of safety in this pseudo-static analysis given from this relation Fs Cl plus tan phi summation of Wi cos alpha this is Wi cos alpha is the weight and these two factors are Fh and Fv. What is Fh and Fv we already discussed and let me again remind you Fh was horizontal force which is nothing but Kh multiplied by W. What is Fv? Fv is a vertical seismic force this is Kv into W and Kh and Kv are the seismic coefficient in horizontal and vertical direction respectively and alpha are the angles here. So, what you have in this equation this is the effect of horizontal which is going negative here while Fh is positive denominator.

$$F_s = \frac{cL + \tan\phi \sum(w_i \cos\alpha - F_h \sin\alpha + F_v \cos\alpha)}{\sum(w_i \sin\alpha + F_h \cos\alpha + F_v \sin\alpha)}$$

For cohesionless material c = 0, and in compact form

$$F_s = \frac{\tan\phi(\sum N - \sum N' + \sum N'')}{\sum T + \sum T' + \sum T''}$$

So, Fh will always decrease the factor of safety but as for Fv, Fv will be whether it will be positive or negative it depends on the direction in which direction Fv is acting. If Fv is acting in downward direction then it will be considered positive because it will club with the weight Wi plus Fv into cos alpha. So, W and Fv will act in the same direction because both are acting downwards so it will be positive and here also to be positive. But if your Fv is acting upward then here it will be negative both sides it will be negative. But as for Fh is considered we consider only one direction we have negative in the numerator and positive. These things we already discussed when we discussed theory part. So, now this equation that top equation is simplified when c equal to 0 and in the compact form it can be written in this form where what is n, n is this term what is n prime, n prime is this term and n double dot prime is this term. Similarly, t is this term, t prime is this term and t double dot is this term. So, in the compact form and this equation include effect of earthquake analysis also. So, this is pseudo-static analysis this is not simply pseudo-static analysis an extension of limit equilibrium analysis.

$$F_s = \frac{tan\phi \sum N}{\sum T}$$

So, this equation is pseudo-static analysis. Now, we use this equation for different conditions in this given numerical example. So, one of the condition here for static case this equation can be simplified because for static case these two terms will be 0 n prime, n double dot, t and so your equation comes in this compact form. And in this compact form phi is constant phi is angle of internal friction of the soil and this phi angle of internal friction is not going to vary with respect to with respect to this your number of slices that is going to be constant. So, what we do we do the summation for n i, n i is nothing but w. So, in this tabular form w i is given alpha i for each slice is given. So, n i is nothing but w i into cos alpha i which is listed here and t i is w i into sin alpha i because alpha is 0 for the first slice. So, it will be 40 here and 0 here. So, summation of n gives you 140.287 and t gives. So, ultimately you find a factor of safety 1.5649. So, this was the factor of safety which you find out using the what we say the method of slices. But suppose for the simple method if instead of you know that instead of curve if I assume that my the slope is like this slope is a like this only and if I consider a triangle or let us say one single slice with a w weight if I use and then I use this angle which was given to beta beta was given in this problem 30 degree. So, in this case your simple factor of safety will be simply w sin.

So, what you get tan phi or tan beta in the without method of slices and tan phi tan phi means tan 40 degree and this is Tan 30 degree. So, if you compute then it comes out to be around 1.46. So, using that method you finding and here is this is also for static case you are finding out 1.56 why the factor of safety have increased and the reason being simply here because in this method you assume 30 degree throughout for all slices. However, this is not the case here rather for these two slices it is 0 and 20 degree and then it is increasing. So, in that case because rather than is you know this four ways what you are considering this is curve rather than it are trying if I match that is a factor of safety if I go from I draw a line from A to B and then w is acting on the single slice. So, this is not the case. So, this is most flatter which is expected. So, the factor of safety is little bit increased, and this was all about the static case. Now, we consider the second case which is called a seismic case and in the seismic case with the coefficient k h equal to 0.10 that is in the horizontal direction k v equal to 0. So, these two cases has been when k v equal to 0. So, your formula will be simplified to this one 10 phi sigma n minus sigma n dash. So, n sigma n and sigma t is already available to you from the last calculation.

$$F_s = \frac{tan\phi(\sum N - \sum N')}{\sum T + \sum T'}$$

So, you need to calculate n prime and t prime which is done here what is n prime n prime is f h into sine alpha t prime f h into cos alpha i. So, the tabulation has been done. So, ultimately in case of seismic case with k h equal to 0 and k v equal to 0 then you put up these numbers and once you put up these numbers then you get factor of safety equal to 1.24823. So, earlier factor of safety which you find was 1.5649. So, this is naturally the one this is quite less than the 1.5649. So, that means, this was for static case. For dynamic case when you consider the effect of the seismic forces the factor of safety decreases. And in fact, it will always decreased. Never the factor of safety considering the dynamic case will increase than the static case. So, if you find it is more than the static case then there is something wrong in your calculation. Coming to the third case when you have seismic case with k h equal to 0.1 and k v equal to 0.05 and where 0.05 the k v is acting in downward direction. So, in that case we need to use this equation this equation third component summation n and this is what is f v cos alpha f v sine alpha. So, this is calculated here f v cos alpha f v sine alpha and tabulated form. Other things so, in this case we put up these numbers in this equation here n n prime is already known n double dot this is also put up and one you need to one need to understand that this component will be negative on the numerator and positive in the denominator. While the second component n prime and t prime it will be positive in both numerator and denominator or negative it is positive because it is acting in downward. If f v act upward in that case both will be negative. So, if you do the calculation then you find the factor of safety 1.26. So, if you consider both vertical force also then it becomes 1.26 which is little higher than the 1.248 1.248 means 1.25 1.248 so, more or less same. So, you see then when you consider vertical force then there is not much effect of the vertical force on the factor of safety that is why this many times in many pseudo-static analysis vertical force k v is considered to be 0. But you should not consider any case where you in a seismic case you assume that k h equal to 0 and k v is not 0. So, it is must that you should consider k h first then only you should consider k v.

So, this was about the numerical example. Now, the second method which is we discussed in detail about the application of or what we call the Swedish circle method for both static case as well as dynamic case. Then Bishops method have little modified what are the suggested by the Swedish circle method and this in the Fellenius method or the Bishops method the Swedish circle method the effects of the horizontal and shearing forces which act on the sides of the slices were neglected. As a result the factor of safety values were conservative and may lead to the uneconomical design especially in the case of deep slip circles. So, it is okay that Swedish circle method is not like no dangerous because it is on the conservative side but naturally then if you consider some conservation then it may be uneconomical. Because in this case the shearing forces and horizontal forces which act on the slide's slices were neglected.

The Bishops method eliminates errors to a large degree and provides more accurate results. The analysis based on effective stress approach the shape of slip surface is assumed circular. So, this is the same thing as it is basically an extension or modification of Swedish circle method. So, what you see in Bishop's method forces on a typical slice and these forces are listed here you have the forces on one side x n minus x n plus 1 and another side here and this will be so this is shown here also. The slope is here those you have the a to d slip circle and forces two types of forces are here one is horizontal force shearing force. So, here you will have the horizontal force which is acting like x n x minus this is like and then you have the shearing force. So, shearing force this is shearing force rather the horizontal force is E n this side and E n plus 1. So, this is a horizontal component so this is slice and x n and x n plus 1 is shearing force on one side it is x n on another side x n plus 1 and then you have E n and E n plus 1 for the as a horizontal force. So, this thing was neglected this was not considered in the Swedish circle method. So, when we consider these forces then what is the change in the factor of safety.

$$F = \frac{1}{\Sigma W \sin\alpha} \Sigma \left[\frac{c'b + W(1 - r_u)\tan\phi'}{m_\alpha} \right]$$

In which

$$m_\alpha = \left(1 + \frac{\tan\alpha \tan\phi'}{F} \right) \cos\alpha$$

And pore pressure

$$r_u = \frac{ub}{w}$$

So, the equation of factor of safety which is F now yeah you can say it is the same as F s earlier is given from this relation. Now this relation is little long c dash what is b, b same as in this equation somehow it is written b it is l then w is also same and here the pore water pressure is considered using a factor pore pressure ratio R u where u is pore pressure and b is as we said length of the code and w is a total weight. So, here w is same so this should be treated as u into l divided by w so the pore water pressure. Now if you do not consider any pore water pressure in that case simply R u will be 0 R u is not required to be considered in dry case and you will have w tan phi only. But one of the factor m alpha is coming in this equation and for m alpha equation use tan alpha tan phi divided by F into cos alpha where F is nothing but factor of safety found out from this equation. Alpha what is alpha in this case alpha is again the angle which is slice are making phi is the angle of internal friction. So, this way it is done now what happens when we try to find the factor of safety from this equation in right hand side also you have F s which is not unknown. So, what is done this need to be done the calculation need to be done the iterative form what we do we assume some factor of safety initially and then find out let us say you assume F s equal to 1 F s equal to 1 and then calculate m alpha once m alpha is known put on the top equation find the value of F s and whatever F s you get you put again in this equation m alpha again find revise the value of m alpha for the next F s and then again find the F s. So, you need to do this iteration until the value of F s you find and the F s you assume which you put in the value of m alpha are matching no not much difference. So, this way the factor of safety is found and in this case for both sides it comes, it is done.

So, which is already discussed the calculation is repeated. Junbu et al in 1968 given them some curves to make the calculation easy plotted values of m alpha for various values of 10, 5 and F which is shown in the next figure. Use of this chart facilitated computation the Bishops simplified method gives more accurate results than the felinist method the error which result from the use of the Bishop's simplified method is 7% or less or usually less than 2%. So, mostly the results are and the charts which is given here in this figure. So, you have an excessive value of alpha in degrees which is varying from 0 to 70 degree and on negative side also because this angle alpha we discussed could be positive or negative if it is anticlockwise direction positive or clockwise direction negative. So, depending on alpha value you select the alpha value and these curves are for different value of phi or F. So, that means, again for using this chart you need to assume value of F initially. So, let us say I start my calculation assuming F equal to 1 and if phi is I say let us say 45 degree then 10 phi will be 1 and 10 phi 1 divided 1. So, then in that case you need to use a top chart. If my phi is 30 degree so, 10, 30 you calculate and F equal to and then you will be somewhere 0.2, 0.4 and then select that chart and depending on the value of alpha you find the M alpha. So, basically this chart is helping to calculate the value of M alpha which is used in the equation of that. So, this is just for helping for calculation. So, this was all about factor of safety how to find the factor of safety using what you call the Swedish circle method or like Fellenius method and where we discuss not only limit equilibrium analysis, we discuss also the pseudo-static analysis that and which is widely used in practice where the earthquake is represented by either k h and k v, k h and k v should be known to you. How to find k h and k v we already discussed in the theory in detail with this I think this we complete the course part, course work on the slope stability and now in the next lecture which is 46th lecture we will talk about retaining walls. So, thank you very much for your kind attention. Thank you.