Earthquake Geotechnical Engineering

Prof. B. K. Maheshwari

Department of Earthquake Engineering

Indian Institute of Technology Roorkee

Lecture 48

Static Pressure on Retaining Walls (Conti.)

I welcome you again for this NPTEL online course on earthquake geotechnical engineering. We are at lecture number 48 that is module 5th of this course which is on slope stability and retaining walls. We already covered two chapters from this module introduction to retaining walls and the second chapter static pressure that is basically out of this module like slope stability have the five lectures and then we have the six lectures. So, this is basically chapter number 6 which is static pressure on retaining walls we continue and once we finish static pressure on retaining wall then we will come seismic pressure on retaining walls. So, static pressure we have already started from the last lecture, and we continue from the last lecture, lecture number 47 static pressure on retaining walls where we already discussed Rankine theory and Coulomb's theory.

Now, we are going to talk about two more the approaches for static pressure on retaining walls. One is what is called logarithmic spiral method and the next is a stress deformation analysis. And once this is over static pressure will be over then we will talk about dynamic pressure, then response of retaining walls, then seismic pressure on retaining walls, then yielding walls and then we are going to talk about Mononobe-Okabo method which is active earth pressure and passive earth pressure. Coming to this because these all the topics let me analyze that most of the information is from the Kramer's book.

Coming to logarithmic spiral method, in the case of logarithmic spiral method, in case of Rankine theory or you have the passive the Coulomb's theory, we assume that your this failure surface is linear. That means in this case we have assumed that this is a straight line, this was a straight line if you recall that. However, here it is assumed a curve, and, in this curve, what is done you have logarithmic spiral curve initially and there then you have a straight line. So, this is the basic difference between the logarithmic spiral method and other method. So, why it is required? Although the major principal stress axis may be nearly perpendicular to the backfill surface at some distance behind a rough that is delta when the delta is greater than 0 wall.

The presence of shear stresses on the wall-soil interface can shift its position near the back of the wall. If the inclination of the principal stress axis varies within the backfill, the inclination of the failure surface must also vary. In other words, the failure surface must be curved. So, in fact, this is for accuracy, the failure surface will not be linear which is assumed in case of Rankine theory or Coulomb theory rather it will be curved. A logarithmic spiral function has been used to describe such curved failure surfaces for active as well as passive earth pressure condition.

The active earth pressure distribution is triangular which we just seen in the figure for walls retaining planar cohesion-less backfill. So, this is again for cohesion-less backfill that means assuming c equal to 0, when the c equal to 0 then you get this kind of variation. The active earth pressure coefficients given by the log spiral approach are generally considered to be slightly more accurate than those given by Rankine or Coulomb theory, but the difference is many times so small that the more convenient Coulomb approach is usually used. Rankine theory greatly under predicts actual passive earth pressure and rarely used for the due to this reason because it is underestimating. While Coulomb theory overestimates and this overestimation is being unconservative, an unconservative error is there over predicts passive pressures by about 11% for delta if phi by 2 and about 100% if your delta friction angle is equal to phi.

So, Coulomb theory error compared to logarithmic spiral will depends on delta value. For the low value of delta error is small, but for high value of delta the error is large. So, this was about logarithmic spiral method. Now, for static pressure there is stress deformation analysis also. Since the actual pressure that act on retaining walls depends on interaction between the wall and the surrounding soil which we have discussed.

So, that means this is nothing, but this is basically this we can say delta that depends on the value of delta. It seems logical to expect that they could be estimated by stress deformation techniques such as the finite element method. Finite element analysis are in fact very useful for estimating retaining wall pressures and moments. So, the finite element analysis is used for stress deformation analysis and here we can calculate wall pressure as well as movement that means basically deformation. The accuracy of stress deformation analysis however depends on how well they are able to model the actual field conditions.

So, depending on the actual field condition this can be model. A useful method of analysis should be able to describe the stress distribution behavior of the soil which is non-linear, and wall usually assumed to remain linear. The stress displacement behavior of the soil wall interface and the sequence of wall construction and backfill how it is placed. Without careful attention to each of these factors the results of finite may have limited applicability. So, when we carried out in stress deformation analysis finite element analysis then we need to give the attention to these factors.

Now, this with this chapter number 6 is over and we are going to talk about seismic pressure on retaining walls. But before we before we talk actual seismic pressure on retaining walls, one of the issues dynamic response of retaining walls that when the retaining wall is subjected to dynamic loading then how it may respond. So, we are going to discuss it in brief. The dynamic response of even the simplest type of retaining wall is quite complex. Wall movements and pressures will depends on the response of the soil underlying the wall.

So, you have the soil and below the wall there is soil how this soil movement takes place accordingly. The response of the backfill the inertial and flexural response of the wall itself and the nature of the input motion. The test and analysis the majority of which have involved gravity walls what that has indicated the wall can move by dry by translation or rotation. So, there can be movement of the wall either by sliding or it can rotate. The magnitude and distribution of dynamic wall pressures are influenced by the mode of wall movement, that is mode of the wall movement whether wall is moving in the translation mode or rotation mode.

The maximum soil thrust which is acting on a wall generally occurs when the wall has translated or rotated toward the backfill. The minimum soil thrust which occurs when the wall has translated or rotated away from the backfill. So, the soil thrust will be the minimum when the either wall is translated or it rotate away from the backfill. The shape of the earth pressure distribution on the back of the wall change as the wall moves. So, when there is a movement of the wall accordingly the shape will change.

Dynamic wall pressures are influenced by the dynamic response of the wall and backfill and can increase significantly near the natural frequency of the wall backfill system. Increased residual pressure may remain on the wall after an episode of strong shaking has ended. So, this was the dynamic response. Given these complex increase interacting phenomena and the inherent variability and uncertainty of soil properties it is not possible to analyse all aspects of the seismic response of retaining wall accurately. As a result, simplified methods that makes various assumptions about the soil structures and input motions are most commonly used for the seismic design of retaining walls.

What are those assumptions for seismic pressures on retaining walls? So, we will discuss first some assumptions are made what are the assumptions and then we will discuss one. One common approach to the seismic design of retaining walls which involve estimating the loads imposed on the wall during earthquake shaking and then ensuring that the wall can resist those loads. So, this is the way like you know the in the study case also we said the design will be not done based on the deformations rather you what you do the design is done based on the forces and you assume with some factor of safety. So, that the deformations are within the permissible limit. In the similar case here in the case of seismic pressure also you estimate the loads which is imposed on the wall during earthquakes shaking and then we need to make ensure that these forces are the wall is sufficient enough thick to resist these loads.

Because the actual loading on retaining walls during earthquake is extremely complicated, seismic pressures on retaining walls are usually estimated using simplified methods. So, what are the simplified methods? Retaining walls that can move sufficiently to develop minimum active or maximum passive earth pressure are referred to as yielding walls. So, the yielding walls are those walls they can move sufficiently that means they have like flexible they can move. So, as a result there could be development of minimum active earth pressure or maximum earth passive pressure. The dynamic pressure acting on yielding walls are usually estimated by pseudo-static process that share many features of those described by seismic slope stability analysis.

We have already discussed in detail seismic slope stability analysis using what we call as pseudo-static analysis. So, this pseudo-static word is common between slope stability as well as or retaining walls. So, using pseudo-static process, seismic pressure of the yielding walls is calculated. And in these process what is the methodology given there is what is called Mononobo-Okabe method which is called in the short MO method, this is also called MO method. This MO method was proposed by Okabe in 1926 and Mononobo and Matsuo 1929 which develop the basis of pseudo-static analysis.

As we discussed, this is based on the pseudo-static analysis of seismic earth pressure on retaining structures that has become popular known as the Mononobo-Okabe method MO method. The MO method is a direct extension of the static Coulomb theory to pseudo-static condition. So, you can say MO method is nothing but static Coulomb theory plus pseudo-static analysis. For the in a MO analysis pseudo-static accelerations are applied to a Coulomb active or passive ways the pseudo-static soil thrust is then obtained from force equivalent of the ways. So, what is done here the forces which is acting on acting on the ways in dry cohesionless backfill, cohesionless means C equal to 0 and this is shown in this figure.

So, in a what is shown here you have all the forces as a static case, but 2 more forces are added here compared to static case. One is k h w which is the lateral force acting outward for active earth pressure condition, k h w will act away from the backfill and k v w it could be upward and downward. So, it is in this case assumed upward but in the k h w will be always considered away from the backfill for the case of active earth pressure condition. Here what is k h and k v? k h and k v are nothing but they are dimensionless pseudo-static coefficient which is same as we discussed in case of seismic slope stability. When we talk about seismic slope stability, we discuss in detail what k h and k v.

In addition, the forces that exist under this case is also acted upon by horizontal vertical pseudo-static forces and these forces are nothing, but this is mass multiplied by A h. So, basically what you have you can say like this your F h you have F h in equal to nothing but k h into w. What is k h? k h is A h by g into w or this can also be written w by g mass multiplied by A h. So, this is the same thing. Similarly, F v is nothing but k v into w where k v was nothing but A v by g into w again it can be written m into A v.

What is A h and A v are horizontal and vertical acceleration acting. So, ultimately because w is known so if you know k h then you can find F h if you know k v then you can find this one. So, that means once we know the k h and k v pseudo-static coefficient then we can calculate these forces. And how to fix the pseudo-static coefficients we have discussed in detail when we talk about seismic slope stability. This can be linked with the seismicity of the area basically you can link the seismic zone nation, seismic zone of the country.

If you are in the higher zone, then assume the value of k h higher. In fact, k h k v is typically taken half of the k h or two-third of k h. But k h we discuss that it can be taken as one-third to half of the value of the seismic zone factor. So, value of A max by g. So, if you know the A max by g so one-third to two-half can be taken k h.

We discuss it in detail when we talk about that. So, this was about active earth pressure condition. If we draw the force polygon, the force polygon for active earth pressure will w will act always downward then k f is acting along this direction and k h w horizontally outward k v w upward and then you close the polygon with p a e. So, you close this so from this point to this point you come. Here p a is the active earth pressure which is shown here including the effect of earthquake.

So, therefore the dynamic case and p a is given from this relation half k e gamma h square into 1 minus k e where the total active can be expressed in form similar to the developed for a static condition. So, this equation gives you a combination of total this is total means static plus dynamic. So, this equation gives the complete earth pressure including static also. And what is k e where the dynamic active earth pressure coefficient k e is given from this relation where phi theta psi there are four angles one is delta theta delta theta phi and beta which is same as we discuss in the static case, and which is shown on the wall. But what is psi now the issue is what is psi here delta theta delta theta phi and beta are the same.

But one more angle is involved here which is psi and psi depends on your seismic coefficient. Psi can be found out from this relation tan inverse k h 1 minus k v where k h and k v are the seismic coefficient. Even if I assume k v equal to 0 in that case psi could be simply tan inverse k h this will be the case when k v equal to 0. And as we discussed earlier you can assume k v equal to 0 but k h is not 0. However, you cannot do reverse that you assume k h equal to 0 and k v is not 0.

So, you need to consider value of k h in any case. So, as a result numerator can here cannot be 0. So, tan inverse you can find and one another condition you need to check that the difference of phi minus beta should be greater than psi for the seismic case and gamma for the will be considered as gamma d here. So, everything is in known in this equation. So, you can find the value of k e once k e is known you put in the top equation find the value of p a e.

$$\mathbf{P_{AE} = \frac{K_{AE}\gamma H^2}{2}(1 - k_v)}$$

$$\mathbf{K_{AE} = \frac{\cos^2(\phi - \theta - \psi)}{\cos\psi\cos^2\theta\cos(\delta + \theta + \psi)\left[1 + \sqrt{\frac{\sin(\delta + \phi)\sin(\phi - \beta - \psi)}{\cos(\delta + \theta + \psi)\cos(\beta - \theta)}}\right]^2}}$$

And now you need to find the angle alpha a e with the horizontal where this is acting the failure surface. So, the alpha a e is given from this relation phi minus psi plus tan inverse tan this phi minus plus c 1 e and c 2 e. What is c 1 e and c 2 e? c 1 e and c 2 e is given from these relations where you have again 5 angles now instead of 4 5 angles phi, psi, beta then you have theta and then you have delta also. So, 5 angles are involved and this is based on all the angles. Similarly, c 2 e 1 2 3 4 and theta is phi psi theta then you have beta fourth angle and delta is not coming here in this equation c 2 e.

Where $\phi - \beta \geq \psi$; $\gamma = \gamma_d$; $\psi = \tan^{-1}[k_h/(1 - k_v)]$

The critical failure surface, which is flatter than the critical failure surface for static conditions, is inclined at an angle

$$\alpha_{AE} = \phi - \psi + \tan^{-1}\left[\frac{-\tan(\phi - \psi - \beta) + C_{1E}}{C_{2E}}\right]$$

$$C_{1E} = \sqrt{\tan(\phi - \psi - \beta)\,[\tan(\phi - \psi - \beta) + \cot(\phi - \psi - \theta)][1 + \tan(\delta + \psi + \theta)\cot(\phi - \psi - \theta)]}$$

$$C_{2E} = 1 + (\tan(\phi + \psi + \theta)\,[\tan(\phi - \psi - \beta) + \cot(\phi - \psi - \theta)])$$

In that delta was coming and the top equation. So, although the MO analysis implies that the total active thrust should act on at a point h by 3 above the base of a wall of height h but that will act when you have the static case. Experiments results suggest that it actually acts at a higher point under the dynamic loading condition and that is okay because the dynamic load will carry the load on the top. Now how we determine the location where this load act? So, for that we what we need to do we need to divide the total thrust acting in the static component and the dynamic component. So, we already calculated the value of p ae and p a from static conditions.

$$P_{AE} = P_A + \Delta P_{AE}$$

So, that what we do we find delta p ae using this equation which can be find out simply p ae minus p ae. So, using this you find the value of delta p ae and now p ae is acting at h by 3 and delta p ae is normally considered to be acting at approximately the dynamic component by Seed and Whiteman recommended that it approximately at 0.6 h. The total thrust will act at height h above the base where this is the contribution coming from the static component and this is contribution from the dynamic component divided by the total pressure. So, using this you can find the value of h from the base where this total thrust is at and naturally the answer will come somewhere between h by 3 to h by 2.

$$h = \frac{\frac{P_A H}{3} + \Delta P_{AE}(0.6H)}{P_{AE}}$$

So, it will not be past the middle path. The value depends on the relative magnitude p ae and et but often ends up near the mid height of the wall. Seed and Whiteman concluded that vertical accelerations can be ignored when the MO method is used to estimate p ae for typical wall designs. So, this was all about active pressure. Similarly, you can have passive earth pressure on the same line as we discussed for active earth pressure. And in the case of passive earth pressure, total passive thrust which act on a returning on a dry cohesion less backfill, again when we say cohesion less backfill means it is c equal to 0.

$$P_{PE} = \frac{K_{PE}\gamma H^2}{2}(1 - k_v)$$

Where the dynamic passive earth pressure coefficient, K_{PE}, is given by

$$K_{PE} = \frac{\cos^2(\phi + \theta - \psi)}{\cos\psi\cos^2\theta\cos(\delta - \theta + \psi)\left[1 - \sqrt{\frac{\sin(\delta + \phi)\sin(\phi + \beta - \psi)}{\cos(\delta - \theta + \psi)\cos(\beta - \theta)}}\right]^2}$$

For the c equal to 0, it is given by p pe k pe gamma h square by 2 1 minus k v. It was if you recall it was in the earlier equation it was plus it was like yeah same 1 minus k v was there. So, here but the difference here that k p is given from this relation here now. So, it was k e earlier and again when you find the value of k p e, all 5 angles are coming 5 theta psi delta and beta. So, using 5 angles, the definition of psi is same as we discussed in case of psi can be calculated tan inverse you have tan k h divided by tan 1 minus tan k v. So, this definition remains same as we discussed earlier. So, here k h tan inverse k h 1 minus k v k h 1 minus k v. So, this may be like I think I need to erase this one. So, this is psi tan inverse k h divided by simply 1 minus k v.

$$\alpha_{PE} = \psi - \phi + \tan^{-1}\left[\frac{\tan(\phi + \psi + \beta) + C_{3E}}{C_{4E}}\right]$$

$$C_{3E} = \sqrt{\tan(\phi + \beta - \psi)\,[\tan(\phi - \beta - \psi) + \cot(\phi + \theta - \psi)][1 + \tan(\delta + \psi - \theta)\cot(\phi + \theta - \psi)]}$$

$$C_{4E} = 1 + (\tan(\delta + \psi - \theta)\,[\tan(\phi + \beta - \psi) + \cot(\phi + \theta - \psi)])$$

So, using this equation, you find the value of psi. And k p is given from this relation. Here the triangular ways, now the difference between active and passive pressure that the k h w will act inward, it was acting outward in this direction. As for k v w, whether you consider upward or downward that does not make a difference. But in the passive earth pressure condition, we need to apply the value of k h w in upward in the inward direction that is means towards the backfill.

W will act downward, alpha p. Now earlier p p was acting here like this, but now it is on the top of here. So, this was p a, if you recall p a was acting like this and f we will make an angle. So, all are known delta, theta, beta, phi, alpha v is not known which can be calculated. So, first of all alpha p and then if you draw the force polygon, so this will be force polygon, f will be acting here.

So, f will be in this direction here. And from that k h w in inside and then k w in top and then you close the polygon from this. So, this is the direction of p p can be found. What is the value of alpha p? Alpha p can be calculated from this relation, this is psi minus phi plus tan inverse this. So, you have one angle, two angle and then beta third angle and then phi then you have 1, 2, 3 and then last angle is here beta like 1, 2, 3 and then you have another angles involved.

All five angles are involved here. So, five angles are here delta, theta, beta and phi and alpha p. Delta is coming in this case, delta is here, then psi is coming, theta is coming, phi is coming, beta is coming. So, all five are involved here in calculating alpha p e. So, alpha p e is calculated from this. So, with this like and then p p total force will be like in case of active, here also it will be p p plus delta p e, where p p is static passive pressure and delta p is dynamic component which can be find and the dynamic components acts in the opposite direction of the static component thus reducing the passive resistance.

So, this will be the total active thrust. This is similar, it is similar way as we discussed for in case of active earth pressure. So, with this thank you very much for your kind attention. This was about seismic earth pressure and then we will continue for the retaining wall in the next lecture. Thank you.