Earthquake Geotechnical Engineering

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Lecture 49

Design of Retaining Walls

I welcome you in this NPTEL online lectures on earthquake geotechnical engineering. We are at lecture number 49 of this series, and this is part of the module 5 which is on slope stability and retaining walls. So, we already finished with slope stability and discussing retaining walls. So, last 3 lectures has already been over on retaining walls. This is the fourth lecture on retaining walls. In the last 3 lectures we covered introduction to retaining walls, static pressure on retaining walls and partly seismic pressure on retaining walls.

So, in this lecture today 49 lecture we will continue the seismic pressure on retaining walls which has been started earlier in the last 2 lectures. And moreover like we will continue the seismic pressure on retaining walls that is the third part of this module which is the longest we will continue even after today's this lecture number 50 also. So, let us talk continuous seismic pressure what we are going to talk in this lecture we already started Mononobe Okabe method in the last lecture, lecture number 48. So, we are going to discuss one numerical example from MO method.

Theory has been discussed already in the last lecture. Then we will have some discussion on MO methods and MO method is you know an extension of pseudo-static analysis which is typically used for the seismic slope stability analysis and it is based on the static methods and but it is not the take care completely dynamic effect. So, one of the method which is based on pseudo-dynamic is Steedman-Zeng method which is also going to discuss and then we are going to talk about non-yielding walls. So, coming to MO methods example and before I go ahead let me acknowledge that most of the material in this lecture is from the Kramer's book which is on geotechnical earthquake engineering, but I will be discussing everything. So, that is like you know, the discussion and explanation will be from my side and the figures and many of the text are from the Kramer's book.

Coming to this one Mononobe Okabe method numerical example. Here the example is like a wall is given which is of 5 meter height. The example is to compute the overturning moment and this moment is required to be compute about the base of the wall which is shown in this case with horizontal seismic coefficient k h equal to 0.15 and vertical seismic coefficient which is half of the k h that is 0.075. So, the value of k h and k v is given to you. Then on the it is vertical wall. So, you could see that this being vertical wall theta will be 0 and horizontal backfill it is horizontal backfill that means there is no inclination on the backfill so beta will be 0. So, two conditions theta is 0 and beta equal to 0 is understood from the figure itself rather than given. Then the mass density of the backfill material is uniform and 1.76 milli mega gram per meter cube phi which is angle of internal friction is given 34 degree and delta in 70 degrees. As for you to understand that MO method basically what we are doing in MO method it is a combination of Culmann's method with and then it is Culmann plus your pseudo-aesthetic analysis method. And the Culmans method is, as we discussed in the last lecture, basically for cohesion as well. So, c is equal to 0. So, c is here there is no cohesion.

What we do to find the value of overturning moment, first you need to calculate the static pressure earth pressure then total earth pressure and then dynamic component will be calculated. So, for let us do the calculation for static part. So, the first estimate the static wave thrust on the wall because the wall is not smooth. Why not smooth because delta is greater than 0 which is 17 degree given. Culmann theory should be used and in Culmans theory for the static case is given from this relation where you have phi theta delta these are the notations and beta.

$$\mathbf{K_A} = \frac{\cos^2(\phi - \theta)}{\cos^2\theta\cos(\delta + \theta)\left[1 + \sqrt{\dfrac{\sin(\delta + \phi)\sin(\phi - \beta)}{\cos(\delta + \theta)\cos(\beta - \theta)}}\right]^2}$$

We already said that theta and beta in this equation is 0. So, that has been put. So, finally, you get the value of k if 0.256. By the way whenever you calculate the value of k you need to make sure that the value of k should be always less than 1.

So, if you are getting k more than 1 that means something is definitely wrong in your calculation. So, this is less than 1 so it is ok. Now the total earth pressure once k is calculated for static case is given by p a equal to half k a gamma h square. All the values once you put up all the values here gamma 1.76 and 9.81 g gamma this is basically rho, and this is g. You know the gamma is nothing but rho into g. So, rho into g and this is height. So, finally, you get the end up that total static pressure 55.3 kilo Newton per meter.

So, this is the case for static active thrust. Now total including dynamic component can be calculated from this relation p a equal to k e gamma h square 1 minus k v. In this relation first you need to find the value of k e that is seismic that is the active earth pressure coefficient for the case of when you consider earthquake. During the earthquake first you need to calculate the value of psi which is calculated from this tan inverse k h 1 minus k v and once you put the number then you find out that this value is 9.2 degree. And you put all the numbers in the equation and the 0 again theta equal to 0 and beta equal to 0 in this equation. And this k a you find out that one thing you can make sure that the value of k a which you obtained should be greater than k a which you calculated here. The k a value was 0.256. So, this value which you obtained is greater than 0.256. So, that is why this is okay. That means k a cannot be less than k a this if you are finding the k a is less than k a this means if you are getting reverse way that k e is less than k a then again there is something wrong in your calculation. So, these things you need to make sure, but still the value of k e will be less than 1. So, it will be less than 1 however more than k a.

So, this is the case. So, it should be less than 1. Now we put the again the numbers here and the same way as we did in that then you find the p a e 72.3 kilo Newton per meter and naturally this cannot be less than it will be greater than p a which you calculated in the last slide. So, p a is known, p a is known as a result you can find the dynamic component of total thrust which is difference of p a e minus p a and with this difference you find out that dynamic component of the total thrust is 17 kilo Newton per meter. Now this total thrust act at a point above the base of the wall and this h will be given by this relation p a into h by 3 plus delta v a.

So, you put all the numbers here and you ultimately get 1.98 meter. Again, you can apply this check that the value of h should be between h by 3 it cannot be less than h by 3 and it should be less than 0.6 h. So, what is h by 3? 5 by 3.

And what is h you find out 1.98 meter and 0.6 h will be 3 meter. So, you this is okay. So, this satisfied. So, you need to apply this check that whatever number you got are getting that falls between h by 3 to 0.6 h. So, roughly and normally this is final answer which you get. In fact, h you get is normally less than or equal to you can say h by 2 simply half of the so half is 2.5. So, it will be you will be ending up near 2.5 or less than 2.5. So, that is answer.

So, this is a simple check. Coming to this now this the moment because the only the horizontal component of the total active thrust contribute to the overturning moment about the base the overturning moment will be given by p a e h horizontal component multiplied by h where this will be into cos delta p a e this is the value p a e and this is cos delta multiple. So, ultimately you get 137 kilo Newton per meter per meter. So, this is the final answer for your overturning moment which you calculate for, and this can be used for the design. So, now the we complete the one numerical example here on MO method. Now, we are going to discuss some discussion on MO method.

Although conceptually MO method is quite simple the MO analysis provide a useful means of estimating earthquake induced loads on retaining walls. A positive horizontal acceleration coefficient causes the total active thrust to exceed the static active thrust. So, you will have the like know that once you apply some positive horizontal acceleration as a result active thrust static active thrust will increase while the total passive thrust to be less than the static passive thrust. So, the compared to static values when you apply the you know earthquake loading or seismic loading active thrust will be increased while the passive thrust will be decreased. Since the stability of a particular soil is generally reduced by increase in active thrust and or a decrease in passive thrust the MO method produce seismic loads that are more critical than the static loads then that act prior to an earthquake.

So, certainly so there are many limitations of MO method, but still within those limitations it take care in the simplified way the effect of earthquake forces. Its simplicity is very appealing. As a pseudo-static extension of the Coulomb analysis which we already discussed the MO method is nothing, but it is a combination of pseudo-static analysis plus Coulomb's analysis. However, the MO analysis is subjected to all of the limitations of pseudo-static analysis as well as the limitation of the Coulomb theory because MO method is a combination of these two. So, whatever the limitations of these two methods it will come here also.

As in the case of pseudo-static slope stability analysis first of all determination of the appropriate pseudo-static coefficient is difficult and the analysis is not appropriate for soil as well. So, and this may not be appropriate for those kind of soils which lose their strength during earthquake for example, liquefiable soils. So, this is okay if you are you do not have the liquefaction, but if your soil is liquefied then this kind of analysis may not be accurate. So, as a result this we discussed what are the limitations of MO method. So, the later on Steedman and Zeng they propose a method which is based on what we call the dynamic, pseudo-dynamic analysis.

MO method is based on pseudo-static, but the Steedman and Zeng consider the dynamic nature of earthquake loading though in very approximate way. It is not a very accurate way, but it is better than what we have discussed in MO method. It is possible however to account for certain dynamic response characteristics in a relatively simple manner. So, and what is done here the phase difference and amplification effect within the backfill that means because along the backfill your amplification and the phase difference is not constant or the complete height of the backfill behind a retaining wall can be considered using a simple pseudo-dynamic analysis of seismic earth pressure and that is from Steedman and Zeng in 1990. So, what is here? What you need to do? Consider the fixed base cantilever wall which is shown in the next figure here and if the base is subjected to harmonic horizontal acceleration of amplitude A h the acceleration at any depth z below the top of the wall can be expressed.

Here let me first go through this figure. In this figure what you have a wall, a retaining wall, height of the wall is constant H, capital H. And then what you have this is kind of a failure surface let us say we have this failure surface this is linear, and this failure surface make an angle alpha with the horizontal. Now inside the base what are the forces acting W? W is nothing but capital W is the weight of the vase which is always acting vertically in downward direction. While Q h is the horizontal earthquake force which is acting for active earth pressure case it will be considered outside like away from the wall side. Then you have this total active earth pressure P ae considering earthquake loading will make an angle delta because wall is vertical here, the wall considered is vertical in this case.

So, the delta it will be the with the like normal to this wall will be the horizontal and this angle with this max Pae with respect to horizontal is delta. And phi is angle of internal friction where F is the force applied. So let us consider now this P ae at a point which is at a height h from the base. So, what we need to find out using this Steedman-Zeng method what is your P ae? So, our unknowns are we will do analysis and find and also find what is h. So, after doing the analysis this need to be find out.

Here this what has been assumed the it is like in the pseudo-aesthetic analysis, or you can say MO method it has been assumed that acceleration is constant along the height of the wall, but here acceleration is assumed to vary along the height using this equation. In this equation if I put z equal to 0 let us say for at the base at the base of the wall what you have at the base your z will be capital H. If I put z equal to H, then A at z equal to H at any time t will be Ah sin omega t simply this will be the here. So like so here you have this equation is very much simplified A sin omega t and how this simplified what is omega in this equation? Omega is angular frequency of excitation and t is time. So, this varies so here A which shows you acceleration varies with two factor one is z the depth and time.

$$a(z,t) = a_h \sin\left[\omega\left(t - \frac{H-z}{v_s}\right)\right]$$

So, because time variation is considered so that is why it will take care into account for dynamic loading and z will take care of your amplification factor. So, we continue with this here if the seismic wall pressure are assumed to result from the soil within a triangular wedge which we have discussed which is inclined at angle alpha to the horizontal the mass of thin element of the ways at depth z will be given from this relation gamma into H minus z g into tan alpha. Here you can have the bracket here dz and this mass will be for one strip which is at distance z from the top. The total inertial force acting in the wall can therefore be expressed if you integrate this mz over the multiply with this force will be mass multiplied by acceleration. So, this is mz which is coming from this top equation and Azt is already we discussed. So Azt is from this equation and mz is from this equation. You put all the mz multiplied by Azt integrate over the dz or the height of the wall. So finally you end up in this expression lambda gamma Ah, gamma is unit weight Ah we already defined alpha is already known in this figure. So, everything is there but you may be wondering what is lambda and what is zeta.

$$m(z) = \frac{\gamma(H - z)}{g \tan\alpha} dz$$

The total inertial force acting on the wall can therefore be expressed as

$$Q_h(t) = \int_0^H m(z)\, a(z,t)\, dz$$

$$= \frac{\lambda \gamma a_h}{4\pi^2 g \tan\alpha}[2\pi H \cos\omega\zeta + \lambda(\sin\omega\zeta - \sin\omega t)]$$

So that has been shown in the next slide. Lambda is nothing but wavelength which is can be 2 pi Vs or omega. Vs is nothing but shear velocity of the material. So you can say the Vs is the shear velocity of the material inside this v. And zeta is t as a represent time t minus h over Vs.

$$\lim_{v_s \to \infty} (Q_h)_{max} = \frac{\gamma H^2 a_h}{2g \tan\alpha} = \frac{a_h}{g} W = k_h W$$

So naturally zeta which you find will be less than t. So, the value of zeta will be less than t at any time. So, t minus h over Vs will give you zeta. If in this equation, we apply the limit for when Vs tends to infinity then you can simplify this equation and you will end up in this Qh max is nothing but Kh into W and which is gamma h square Ah 2z tan alpha. So, you end up in this Kh into W. Now with this simplification we find out a pseudo-static force equivalent to the this which is equivalent to the pseudo-static force assumed by the MO method.

The total static plus dynamic soil thrust can be obtained by resolving forces and how you find out here Qh t into cos alpha minus phi while another you have two component. Qh t is this component which is you find out one and another is W. So Qh t into cos alpha minus phi, alpha is this angle, minus phi will be this difference of this angle and this angle and cos component of this, cos component if you see that like this will be alpha so this will act in this direction and W will also act in this direction. So they will act along this one.

So, the component will be basically along this line. So cos component from here and W sin component from the, so these together will make the numerator and then in the denominator you have cos delta plus phi minus alpha. All together once you solve this equation PAT and this PAT actually, so this PAT is total force. Then if I differentiate this equation with respect to z, del by del z, then you find the active earth pressure variation with respect to time. So PAT, delta PAT del by del z of this top equation, then you end up in this equation.

$$P_{AE}(t) = \frac{Q_h(t)\cos(\alpha-\phi) + W\sin(\alpha-\phi)}{\cos(\delta+\phi-\alpha)}$$

and the total earth pressure distribution by differentiating the total soil thrust

$$p_{AE}(t) = \frac{\partial P_{AE}(t)}{\partial z} =$$
$$\frac{\gamma z}{\tan\alpha}\frac{\sin(\alpha-\phi)}{\cos(\delta+\phi-\alpha)} + \frac{k_h \gamma z}{\tan\alpha}\frac{\cos(\alpha-\phi)}{\cos(\delta+\phi-\alpha)}\sin\left[\omega\left(t-\frac{z}{v_s}\right)\right]$$

Now in this equation you have two term. Two term, this is the let us say A term and this is the B term. So I say this last one is B term. So, A is the first term and B is the second term. You could see the A first term give you the variation with depth because z is coming in this case that how it varies, but t is not coming. In the second term t is coming, but of course z is also coming in the second term also, but the first term will give you the equation increases linearly with the depth.

It will linearly increase with the depth and does not vary with time. This represented nothing but static earth pressure. Why static earth pressure? Because this term is not varying with the time, it is constant with respect to time which is acting on the wall. The resultant static thrust act on accordance with static earth pressure theories at point HS equal to H by 3 above the base of the wall. This is the static pressure and this term, the force which is applied by this term will be acting at a height H by 3 above the base of the wall.

The second term represent the dynamic earth pressure. So, this term will represent the dynamic earth pressure. It increases as a non-linear function of depth with a shape that depends on the ratio H by lambda. So here if I combine the result of the first and second term that is a and b, then I get this kind of variation using a Steedman Zeng method. In this case on y axis, you have z by H and z equal to 0 at the top, so it is 0 here and z equal to H at the bottom, so it will be 1 at the bottom.

On x axis you have the ratio p i into gamma H. So basically whatever you find from this equation, then this should be divided by gamma H. So this is the ratio here. And in this case, one another thing you have this for different value H for a given value of H by lambda 0.3. These curves are for two values k H is assumed 0.2 and H by lambda is assumed 0.3. So you see the result, a comparison between Steedman and M-O method. This comparison is very good particularly at the middle heights around the 0.6, around 0.6 like you know that from 0.5 to 0.7 around that one, so on average they are matching. And at the top also when at the, it is also matching, but at the base the variation is, there is a variation and M-O method are basically predicting the higher value of PAE compared to Steedman-Zang method. So, the M-O method is more conservative than compared to Steedman-Zang method because it predicts the more value of earth pressure. So this was about the active earth pressure. Similarly, calculation has been done for the points of action where this dynamic pressure H that is given by Hd.

Since the dynamic pressure increases non-linearly with the depth which you could see here, that means when the depth increases the pressure is increasing from both M-O and both Steedman-Zang method. So the position of the dynamic thrust varies with time and this Hd is not a constant or rather this you can put this Hd is a function of time, this is a function of time. Here we already discussed what is lambda and we also discussed zeta. So lambda was here like 2 pi over vs omega 2 pi and then t minus H over vs. So here lambda is 2 pi vs over omega while zeta your is t minus H over vs.

$$h_d = H - \frac{2\pi H^2 \cos\omega\zeta + 2\pi\lambda H \sin\omega\zeta - \lambda^2(\cos\omega\zeta - \cos\omega t)}{2\pi H \cos\omega\zeta + \pi\lambda(\sin\omega\zeta - \sin\omega t)}$$

So, this is coming. So this is also representing time, this is also representing time. So this was all about. Then the point of application of the dynamic thrust for very low frequency motions, very low frequency motions with H by lambda for small H by lambda because when omega is less, so you will get this when omega is less what will happen? Lambda will be large and when lambda is large H by lambda will be small. So, for the small H by lambda, backfill moves away from phase and then in that case Hd is roughly H by 3. For higher frequency motions that means where the H by lambda will large the point of application moves higher on the wall.

Steedman and Yang found that the sole thrust for backfill of different stiffnesses were close to those obtained when the mean shear wave velocities of the backfill were used in the pseudo-dynamic analysis. So if we use the mean shear velocity because the shear velocity may also vary that may not be constant over the height of the wall like here, it may vary from bottom to the top. In that case what has been suggested that one can use the mean shear wave velocities rather than vs, the mean value of the vs can be used in the calculation. So here this figure shows how this ratio Hd by H varies with lambda H by lambda. Here you need to understand because lambda we already defined; lambda is nothing but 2 pi vs over omega.

For a small value of omega lambda will be large and H by lambda will be small. So, H by lambda is small means omega is small. For large value of omega this ratio will increase. So when the H by lambda, for small value of omega let us say omega equal to a kind of a 0 which is the static case, so you will have this is basically nothing but this ratio is around this is H by 3, 1 by 3rd, 0.33 or like this, this number and H by lambda increases. Initially it is almost flat then after some value of H by lambda it exponentially increases.

So, this is the and may go from 0.3 to 0.55 or 0.5 around that. So, this was all about Steadman and Zeng method. Then let us talk about non-yielding walls. Some retaining structures such as massive gravity walls which are founded on rock or basement walls which has bashed both top and bottom do not move sufficiently to mobilize the shear strength of the backfill soil. Sometimes what happens you may have kind of a walls.

Whatever we discussed so far was yielding walls, then walls when they get yielded. But there could be a case where you can have massive gravity walls like in this case. For example, this is given by Wood in 1973 where wall geometry and notation has been shown and this has been used for analysis for unknown yielding walls. So, you have two rigid walls. The distance between two rigid wall is capital L while H is the height of the wall. What is the property of the soil inside, linear elastic soil? So, H is the height, gamma is the unit weight and v s is the shear velocity.

So, shear velocity and unit weights are there. So, in this case as the limiting condition of minimum active or maximum active pressure cannot be developed for this cannot be many minimum active or maximum pressure cannot be developed for what case for known yielding walls. We have discussed this was for yielding walls. So, and as, we discussed that we would analyze the response of a homogeneous linear elastic soil trap between two rigid walls connected to rigid base. So, two rigid walls are on the sides and both walls are connected through a rigid base. If the two walls are assumed to be spaced far apart, the pressures on one wall will not be strongly influenced by the presence of the other.

If they are very far away, in that case both will act like you know that no influence of this wall and this wall. So if the difference is large, then they will act separately. But if they are coming closer, then the effect is different. We showed that the dynamic amplification was negligible for low frequency input motion. So if your input motion is low in that case that means what you mean by input motion is low that is the motion which is less than half of the fundamental friction of the unrested backfill.

And how you find the fundamental frequency of the backfill can be found out from this relation f naught equal to Vs by 4h. What is Vs in this equation? Vs is nothing but your shear wave velocity and h is the height of the wall which we already discussed. For this range of frequency in which many practical problems that is wall pressures can be obtained from the elastic solution for the case of a uniform constant horizontal acceleration which is applied throughout the soil. Now for smooth rigid walls, Wood in 1973 expressed the dynamic thrust and dynamic overturning moment about the base of the wall can be calculated from these equations. What is gamma in this equation? Unit weight of the soil h is the height, Ah is acceleration in the horizontal direction where normally if you have the Ah can also be linked with k h into g with the whole and Ah by g will be basically k h only and then you have f p and f m.

Here the point of reflection, the dynamic thrust is at a height above the base of the wall which is the ratio of the moment delta m e q divided by delta p. Typically the value which you obtained h e q is around 0.63 h. Now the issue in these two equations what is f p and f m? f p and f m there are dimensionless parameters which is listed here and for example f m which is and these parameter vary with the Poisson's ratio.

There are four curve for Poisson's ratio 0.2, 0.3, 0.4 and 0.5. Naturally for the soil 0.4 and 0.5 curve will be more applicable rather than 0.2 and 0.3. When the L by h increases L by h when increases the value of f p increases and the value varies between 0 to 1.2. Initially there is no effect of Poisson's ratio at least when you have L by h equal to almost 0.2 and then after that there is a difference is coming higher the Poisson's ratio higher the value of f p. Similarly, f m also the variation with L by h and f m you see here it is varying from 0 to 0.6 around that and again initially there is no effect of Poisson's ratio but at higher value of L by h higher the Poisson's ratio higher the value of f m you get it.

So, this was about non yielding walls. So, this like these f e, f m these can be used in these equations and then you can calculate the point of application of the dynamic thrust acting at a height above the base of the wall. So, this with this we complete non-nielding walls also. Thank you very much for your kind attention.