Earthquake Geotechnical Engineering

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Lecture 50

Design of Retaining Walls (Conti.)

I welcome you again in this NPTEL online course on earthquake geotechnical engineering and this is lecture number 50. So, this is half century, and we are under module 5 of this course which is slope stability and retaining walls. This is in fact, the fifth lecture on retaining walls where we already covered seismic pressure on retaining walls and today, we are going to talk partly on seismic pressure on retaining walls and that will be over. Once that is over then we are going to talk about seismic displacement of retaining wall also. So, let us start from seismic pressure on retaining walls and which is in chapter number 7 which is continued from the last lectures rather last two lectures this is third lecture on seismic pressure on retaining wall.

And what we are going to talk today in this lecture number 50, 50th effect of water on wall pressure when you calculate the seismic pressure. And then we are going to talk about finite element analysis for the analysis for the seismic basically for seismic pressure. Then once the seismic pressure is over then we are going to talk seismic displacement of retaining walls which will be another chapter and that will be discussed in under three categories. One is Richard's element method, second Whitman-Leo method and the third one is finite element analysis.

Let us start from effect of water on wall pressure, but before going ahead let me acknowledge that most of the material taken is from the Kramer's book. Coming through effects of water on wall pressures the process for estimation of seismic load on retaining walls which is described have been limited to cases of dry backfills. So, whatever we have discussed so far, we assume that backfill was dry, but that is not the real scenario. In real case most of the retaining walls are designed with drains to prevent water from building up within the backfill for example, weep holes are provided. So, weep holes are provided, and water drained out of the backfill then it is okay, but it is not always possible and even it is drained out still there may be some water inside the backfill and this has been not considered.

So, this is not possible for retaining walls in waterfront particularly in waterfront areas where most earthquake induced wall failures have been observed. During the past earthquake it has been observed if your retaining wall is in the waterfront areas for example, near the rivers if you have the bridge of an abutment passing through the river or you have some pond or like this reservoir conditions are there or for example, if you have bridges or maybe like well foundations there if some retaining walls are provided near for those structures, they may be in the near the water bodies. So, in that case it is not possible to that your the backfill is dry. The presence of water plays a strong role in determining the loads on waterfront retaining walls both during and after earthquakes. So, and this will not only during, but after the earthquakes also.

Water outbound of board of a retaining wall can exert dynamic pressures on the face of the wall. Water within a backfill can also affect the dynamic pressures that act on the back of the wall. Proper consideration of the effects of water is essential for the seismic design of retaining structures particularly in waterfront areas. So, that means, when we design these retaining walls it is necessary that we should consider that the amount of water which is the what is the effect of water which apply on the wall pressure including the seismic. The total water pressure that act on retaining walls in the absence of seepage within the backfill can be divided into two components. What are these two components? One first component let us say hydrostatic pressure and this component increases linearly with depth and it act on the wall before, during and after earthquake shaking. So, this component will act on the wall before the earthquake, during the earthquake and after earthquake shaking. That means, it is kind of continuous. The hydrostatic pressure which is applied is nothing to do with earthquake loading because its name is hydrostatic pressure. So, it will continuously be applied without earthquake and with earthquake also.

So, that first component continuous. It does not vary with the time. The second component is hydrodynamic pressure and as dynamic is coming which result from the dynamic response of the water itself and this dynamic response of the water is due to earthquake loading. So, we need to consider both the components hydrostatic pressure as hydrodynamic pressure. Hydrodynamic water pressure result from the dynamic response of body of water that how the waters respond to the earthquake loading, which is the, for retaining walls hydrodynamic pressures are usually estimated from what is we call Westergaard solution.

For the case of a vertical rigid dam, retaining a semi-infinite reservoir of water that is excited by harmonic horizontal motion of the rigid base. So, you have a rigid base, let us say a tank and if I apply the horizontal motion which is harmonic excitation, let us say sinusoidal wave and other things. So, Westergaard solution give you that how you can calculate hydrodynamic pressure. So, for calculation of hydrodynamic pressure can be done using Westergaard solution. Now, in Westergaard what has been showed that the hydrodynamic pressure amplitude increased with the square root of water depth Zw and Zw measured from the top.

When motion is applied at a frequency lower than the fundamental frequency of the reservoir. Fundamental frequency of the reservoir can be calculated f naught v p by 4 h, where v p is the p wave velocity not the shear wave velocity. If let us say v p if I assume as a 2000 meter per second and h thickness of this is 10 meter, in that case v p by 4 h will come like 40, 2000 by 40, so you will get 50 hertz. So, this fundamental frequency of the reservoir is quite higher compared to the earthquake wave which we consider like for the typically or for earthquake you may not have more than 5 hertz. So, this is quite greater than 5 hertz.

So, naturally this condition satisfied that the motion is applied at a frequency lower than the fundamental frequency of the reservoir. So, most of the time motion is applied at a frequency which is quite low compared to the fundamental frequency of the reservoir. As a result, we can use the Westergaard solution. Now, in the Westergaard solution what is there? Computed the amplitude of the hydrodynamic pressure is given from this relation. Ah by g is nothing but this is k h basically if I can say that this ratio will be coefficient of earth pressure, coefficient of like pseudo-static coefficient of in the horizontal direction.

$$p_w = \frac{7}{8}\frac{a_h}{g}\gamma_w\sqrt{z_w H}$$

The resultant hydrodynamic thrust is given by

$$P_w = \frac{7}{12}\frac{a_h}{g}\gamma_w H^2$$

Gamma w is the unit area of water, z w is the depth of the water level from the top and h is the total thickness of the wall. If you integrate this was the distribution of pressure, the resultant hydrodynamic thrust is given from this relation p 7 by 12 ah by g gamma w into h square. And here naturally this is total, so it is coming after integration over the height of the wall. So, z w is does not come in picture other than you get this. The total water pressure on the face of the wall is the sum of the hydrostatic and hydrodynamic water pressures.

So, hydrostatic water pressure which does not vary with the time, and you calculate and the second component hydrodynamic pressure which varies with the time and you calculate the maximum value and then total will be the sum of both the cases. Continue with the water outboard of wall. Another important consideration of the design of waterfront retaining walls is the potential for rapid drawdown of the water outward of the wall. Earthquakes which are occurring near large bodies of water often induce long period motion of the water. For example, such as tsunami of seiches.

Long period you know that long period wave, in the long period wave what you will have? You will have like kind of a flat that means, and then you have another peak which may go. So, your wavelength will be like this kind of, so your wavelength will be peak to peak, or I can say this will be my long wavelength, so this is lambda. So, if your lambda is higher what happens in case of tsunamis? When the tsunamis are traveling inside the sea it is very difficult to visualize from the neckline. Why? Because compared to their amplitude, which may be in meters their wavelength is in kilometers or maybe 100 kilometer, so that is not visible. This wave look like a flat compared to their amplitude wavelength which is quite large.

As a result, large water would induce long period motion of the water such as tsunamis that cause water surface to move up and down. While the upward movement of water outward of retaining wall will generally tend to stabilize the wall. If it is upward movement then it will be going to help, it is expected that it will stabilize the wall. Assuming that it does not rise above the level of the top of the wall, of course it should not go beyond the top of the wall otherwise water will be spilled out. However, downward movement can create a destabilizing rapid drawdown condition that and wall should be designed for that.

When liquefiable soil exists under relatively high levels of initial shear stress, follows can be triggered by very small changes in water level and such flow can originate in the soil which is adjacent to or beneath the retaining structures rather than in the backfill. And this can start which is like backfill which is just below the retaining wall rather than in the backfill. So, this was about water outward of wall, but there could be water which is like this was the water which is you can say that applying some force directly on the wall, but there could be water inside the backfill. And the presence of water in the backfill behind a retaining wall can influence the seismic load which act on the wall and this can influence in three ways. How three ways? First by altering the inertial forces within the backfill, so within the backfill it can alter the inertial forces by developing hydrodynamic pressures within the backfill, by allowing EPP generation that is excess pore pressure generation due to cyclic straining of the backfill soils. The third component is related to the cyclic loading due to earthquake and other thing. While the first two components could be even without earthquake like the first one particularly could be without earthquake, the second one because being hydrodynamic pressure, so it will require the earthquake force. Coming to water the inertial forces, the first one, the inertial forces in saturated soils depends on the relative movement between the backfill and particles of between the backfill soil particles and the pore water pressure that surround them. So, you have one side the soil particle on another side you have this pore water pressure. So, if there is a relative movement between these two, then this inertial forces will depend on the relative movement between these two.

If permeability of the soil is small enough the pore water moves with the soil during earthquake shaking. No relative movement of soil and water or rest and pore water condition, the inertial forces will be proportional to the total unit weight of the soil. So, if unit weight of the soil is more it is you can expect the initial force will be large and this is because it is due to the weight of the soil, so it is expected. If the permeability of the backfill soil is very high, if permeability is high in that case the pore water may remain essentially stationary because it may not escape for the high permeability. While the soil skeleton moves back and forth in such cases inertial forces will be proportional to the buoyant force submerged unit weight of the soil.

In case of the second condition which is developing hydrodynamic pressure, hydrodynamic water pressures can also develop under free pore water condition and must be added to the computed soil and hydrostatic pressure to obtain the total loading on the wall. The third one for rest and pore water conditions, the MO method can be modified to account for the presence of pore water within the backfill that pore water EPP. EPP stands for excess pore pressure. The active soil thrust acting on a in-link wall can be computed from this relation gamma equal to gamma b 1 minus Ru. What is Ru? Ru is nothing but the ratio, the pore water pressure ratio which is normally you calculate like Ru is calculated by u into some sigma effective overburden pressure.

$$\gamma = \gamma_b(1 - r_u)$$

$$\psi = \tan^{-1}\left[\frac{\gamma_{sat} k_h}{\gamma_b(1 - r_u)(1 - k_v)}\right]$$

So, the pore water pressure ratio and once Ru is known then you can calculate the value of psi tan inverse gamma sat saturated unit weight into Kh into gamma b submerged unit weight and Ru we already discussed Kh and Kv is already known which is horizontal sesmic coefficient and Kv vertical sesmic of it. An equivalent hydrostatic thrust based on a fluid of unit weight gamma equivalent which is gamma w plus Ru into gamma b. So, gamma equivalent will be gamma w, gamma w is the unit weight of the water and if your Ru equal to 0 then this will be gamma w only, but Ru equal to 1 then you will have gamma w plus gamma b where Ru 1 means full liquefaction is there. That should be added. Similarly, the soil thrust from partially submerged backfills may be computed using an average unit weight based on the relative volume of the soil.

So, in that case you calculate the volume of the average unit weight gamma bar, this gamma bar is calculated using this relation lambda is equal to gamma sat 1 minus gamma d. Where what is lambda here? Lambda can be in this case water in backfill, you have the total height of the retaining wall is capital H and lambda H is the height up to which water level is coming. So, this basically lambda H is denoting the location of water table in this case. And in this case again the hydrostatic thrust and the hydrodynamic thrust if present must be added to the soil thrust. So, this was about all together in water in backfill.

$$\bar{\gamma} = \lambda^2 \gamma_{sat} + (1 - \lambda^2)\gamma_d$$

Now, there is third type of analysis like also called finite element analysis. In case of finite element analysis, earthquake induced pressures on retaining walls can also be evaluated using dynamic response analysis. So, we carried out what we call the dynamic response analysis and using the finite element analysis you can do many things that is possible. A number of computer programs are available for such analysis. Linear or equivalent linear because of the soil behavior is not linear or even may not be equivalent linear.

Non-linear analysis are capable of predicting permanent deformations as well as wall pressures. So, the finite element analysis are versatile and particularly if they can deal with the non-linear analysis. So, this is the non-linear analysis. So, this is a non-linear analysis. And particularly if they can deal with the non-linear analysis.

So, this was all about this. Then let us talk about seismic displacement of retaining walls. So, the seismic displacement of retaining walls that is the chapter number 8, some of this one slide is here. The post earthquake serviceability of retaining walls is more closely related to the permanent deformation that occurred during earthquakes. While large permanent deformations may be acceptable for some walls, others may be considered to have failed to much smaller deformations. Analysis that predict permanent wall deformations may provide a more useful indication of retaining wall performance.

So, one side you calculate the seismic pressure that is fine, but if you are able to calculate the deformations also then that is better. So, it is similar to what we have discussed you know that the Newmark sliding block analysis. Newmark sliding block analysis not only give you the factor of safety against the stability, but it also provide you some information deformation. So, similarly there are three methods which are going to discuss in this case for the seismic displacement and first method is Richard's-Elms method which we are going to discuss in detail. Richard's and Elms in 1979 propose a method for the seismic design of gravity wall based on allowable permanent wall displacement.

The method estimates permanent displacement in a manner which is analogous to the Newmark sliding block analysis which is developed for basically from seismic slope stability which we have already discussed. Application of the Richard's-Elms method requires evaluation of the yield acceleration for the wall backfill system. Like in case of Newmark sliding block method also if you recall when we discussed with the slope like you know the slope stability there was a y yield acceleration which was nothing but k y into g, so yield acceleration. So, in similar way Richard's-Elms method also require the yield acceleration for the wall backfill system. So, the value of a y will be required here also.

So, let us for an application Richard's-Elms method consider the gravity wall which is shown in the figure and in this gravity wall what you have like this is a wall. What is w? Capital W is the weight of the wall itself which is always at vertically downward direction. F h is the horizontal force which is applied due to the let us say earthquake which is F h is normally k h into w which is acting outward direction for active pressure condition. The total active pressure p a e can be divided in two components one is horizontal component p a e h and then another is p a e v. Then at the base of the wall you have the reaction in the form of normal reaction n and thrust T.

So, n and T will be balancing other forces and we need to consider the force equilibrium. So, when the active edge is subjected to acceleration acting toward the backfill the resulting horizontal forces will act away from the backfill. So, this horizontal force will when you want to move the wall towards the backfill then reaction will come outside and this the inertial force will act away from the backfill. The level of acceleration that is just large enough to cause the wall to slide on its base is the yield acceleration. So, what you do if I find out equate the total horizontal forces in this direction T is acting.

So, T capital T should be equal to F h plus p a e into h. So, p a e h and F h. Similarly n will be w plus p a e v. So, this is the equilibrium in the horizontal direction this is in vertical direction. Now, subtracting T there is a relation between T and n.

T is n multiplied by tan phi b. What is phi b? Phi b is angle of friction between soil and wall at the base. Why b? b means base at the base. So, tan phi b and F h is nothing but a y by w g which is we said a y b g w g which just I said k h into w. So, F h is also known to you. So, if I and then what is p a e h which p a e cos delta plus theta and well p a e is sine delta plus theta.

$$\mathbf{T = F_h + (P_{AE})_h}$$
$$\mathbf{N = W + (P_{AE})_v}$$

Substituting

$$\mathbf{T = N\tan\phi_b} \qquad \mathbf{F_h = \frac{a_y W}{g}}$$

$$\mathbf{(P_{AE})_h = P_{AE}\cos(\delta + \theta)} \qquad \mathbf{(P_{AE})_v = P_{AE}\sin(\delta + \theta)}$$

What is delta and theta in this equation? As you know that here wall is not vertical. So, theta will be this angle which this wall max with the vertical theta will be this angle. Theta is this angle. What is delta? Delta is basically if I draw the normal to this wall here and delta will be what this p a e max with this is this angle delta. That is the friction between the soil and wall at this backfill near the backfill.

So, delta and theta is known then p a e into cos delta plus theta and p a e into sine delta plus theta. So, that means p a e h and this is known. We substitute and then we solve this equation. So, ultimately the yield acceleration can be computed using this expression a y equal to tan phi b minus p a e and what is in this equation? Phi b is the angle of friction at the base, delta and theta we already discussed and p a e and p a e is the total earth pressure active earth pressure and then whatever you get should be multiplied by g to calculate the acceleration. The Richard-Sanglensky recommends that p a e be calculated using the MO method.

$$a_y = [\tan\phi_b - \frac{P_{AE}\cos(\delta + \theta) - P_{AE}\sin(\delta + \theta)}{W}]g$$

Since the MO methods now here the calculation of p a e requires MO method. However, MO methods also require that a y be known that is the yield acceleration should be known in the MO method and yield acceleration is not known. So, what do you do? You assume some value of yield acceleration then using MO method calculate the value of p a e and once p a e is known then calculate the value of a y. Now this a y will be different than the a y which we have assumed if it is same then you can say it is final value final answer, but if there is a difference then you again revise the p a e calculate a y and this need to be calculated iteratively until you reach the convergence. Using the results of sliding block analysis in the same manner as Newmark 1965, Richard-Sanglensky proposed the following expression for permanent block displacement where permanent displacement is given by 0.087 v max square a max cube divided by f a y 4. So, in this expression what is v max? Maximum wave velocity, what is a max? PGA, maximum acceleration due to the earthquake which is peak ground acceleration, a max can be compared with pga and a y is yield acceleration. And this equation will be applicable when the ratio of a y by a max is greater than or equal to 0.3. The above equation provides displacement estimates that are close to the estimated maximum displacement of Newmark of 1965. So, it has been observed that the displacement which is estimated from the above equations are very close to the displacement calculated from Newmark methods here.

$$d_{perm} = 0.087\frac{v_{max}^2\, a_{max}^3}{a_y^4} \qquad \frac{a_y}{a_{max}} \geq 0.3$$

So, this was all about Richard-elms method. There are another researchers, Whitman and Liao methods which they also did for to find out the seismic displacement retaining walls. The Richard-Elms method offers a rational deterministic approach to estimation of gravity wall displacement. Its simplicity comes in part from assumption that neglect certain aspects of the dynamic operation problem. Whitman and Liao in 1985 identified several modeling errors that result from the simplifying arrangement of Richard-Elms procedure. So, some errors which are in the white like Richard-Elms method has been overcome by Whiteman and Liu.

The most important of these are neglect on the dynamic response of the backfill, neglect of kinematic factors, neglect of tilting mechanism and neglect of vertical acceleration. So, so many things have been neglected in case of Richard-Elms method. Higher consideration of vertical acceleration produces slightly larger displacement than when they are neglected. At least for motions with high peak round acceleration that means a max is greater than 0.5 g when and a y a max is greater than 0.4. So, a y means yield acceleration will be 0.4 times of PGA while PGA should be itself is greater than 0.5 g. So, in this case if you have very high PGA value then an errors will be large. Using the results of sliding block analysis of 14 ground motions by Wong, Whiteman and Liu found that the permanent displacement was log normally distributed within mean value which is given from this relation and again we discussed v max and e max, a y and e max and uncertainty due to statistical variability of ground motions was characterized by a log normally distributed random variable which is Q with a mean value of Q bar and standard deviation log sigma log Q.

$$\bar{\mathbf{d}}_{\mathbf{perm}} = \frac{\mathbf{37v_{max}^2}}{\mathbf{a_{max}}}\mathbf{exp}\left(\frac{-\mathbf{9.4a_y}}{\mathbf{a_{max}}}\right)$$

So, the effects of uncertainty in soil properties, specifically the friction angles on permanent displacement were also investigated by Whiteman and Liao. Using standard deviations of sigma phi let us say 2 to 3 degree for soil angles about delta sigma delta about 5 degree, for wall soil intersection angles, so this 2 to 3 degrees for soil friction angles. So, this is one group and this delta phi for soil wall intersection angles. The computed yield acceleration that is a function of phi and delta was defined as a random variable with mean value a y bar and standard deviation sigma a y. The mean value a y bar is the yield acceleration computed using the mean values of phi and delta.

To calculate the value of a y bar you will require phi as well as delta. Combining of all of these sources of uncertainty the permanent displacement can be characterized using a log normally distributed random variable with mean value which is given from this relation which is quite similar as the relation which we have seen last 37 amex exactly. Only there is two these two factors Q and Q bar and M bar has been added to overcome the uncertainties. And with the variation log d, this sigma log d can be calculated from this relation where sigma a y standard deviation in yield acceleration sigma log M in the factor magnitude and this is in the thrust. So, all these combines you can find the variance also, so that with that uncertainty can be accounted.

Weitman and Liao methods mean and standard deviation values for gravity wall displacement analysis is shown here. For modal error this is like M bar 3.5, a y bar a y phi bar delta bar and Q bar 1. So, standard deviation are listed here. For the first case, standard deviation for M is 0.84, for a y it varies from 0.04 to 0.065 and Q from 0.5 to 1.05. So, there is a large variation in standard deviation, it is minimum for a y, but it is maximum for Q. So, the last one for calculation like seismic displacements of retaining wall can also be calculated using what we call finite element analysis. Earthquake induced deformation of retaining walls can be predicted by dynamic stress deformation analysis. Obviously, prediction of permanent deformations require the use of a non-linear analysis.

$$\bar{d} = \frac{37 v_{max}^2}{a_{max}} \exp\left(\frac{-9.4 a_y}{a_{max}}\right) \bar{Q} \bar{M}$$

Variance

$$\sigma_{\ln d}^2 = \left(\frac{9.4g}{a_{max}}\right)^2 \sigma_{a_y}^2 + \sigma_{\ln M}^2 + \sigma_{\ln Q}^2$$

So, non-linear analysis is required to be carried out. A rigorous analysis should be capable of accounting for non-linear inelastic behavior of the soil and of the interfaces between the soil and wall elements. Regress 2D finite element analysis that predict permanent deformations are those reported by some of the researchers. For example, Alampalli and Elgamel in 1990, Finn et al in 1992 and Lai and Kameka in 1993. So, there are so many like researchers which we work to find the seismic displacement of retaining walls using finite element analysis. With this, this lecture of 50th lecture is over and we are done with seismic pressure on retaining walls as well as seismic displacement on retaining walls.

In the next lecture, we will discuss seismic design consideration for retaining walls and that will be the last lecture on this retaining walls. Thank you very much for your kind attention. Thank you.