Earthquake Geotechnical Engineering

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Lecture- 6

Strong Ground Motion (Continued)

Continuing from the last lecture on strong ground motion, that is the fifth lecture. Now we are on the sixth lecture and in this lecture, we are going to talk some of other topics on a strong ground motion. So, this is we are in the middle of the first module of the course and the third topic is on strong ground motion and we are at the second lecture of strong ground motion. Let us have a recap from the last lecture. We talked about introduction to SGM that is strong ground motion and then SGM measurements, characteristics of SGM and acceleration content parameters. So, these was discussed.

Now today in this lecture we are going to talk about two more important issues which was not covered in the last lecture. Last lecture was mostly on amplitude. Now in this lecture we are going to talk about frequency content parameters and the duration and beside that we are going to talk some of the other factors which help you quantify the strong ground motion and one of the factors here you have the power spectra, response spectra, then you have this durations and other quantities which will be helpful for deciding these contents and particularly the Fourier spectra we are going to talk in detail. So, the frequency content parameters normally you know the earthquake time is very complicated with components of motion that normally spawn a broad range of frequencies that means low frequency, intermediate frequency and high frequency and we discussed this thing also in the last lecture.

The frequency content describes how the amplitude of a strong ground motion is distributed among different frequencies. At some frequencies you have a low peak value, but some frequency normally at an intermediate frequency you have very high peaks and then at the end again this becomes almost negligible. Since this frequency content of an earthquake motion will strongly influence the effect of that motion, it is important to study this and this earthquake characterization will not be complete without the frequency content. So, earthquake is a non-periodic function, but in general assuming for simplicity a periodic function means any function that repeat itself after exactly after a certain time that is periodic function that means and that is called period. If you get peak value first positive peak, another positive peak of course, after positive peak you get the negative and then again you get the positive peak.

So, the distance between two positive peak will be called a cycle or that is a fixed interval. If this interval remain constant then we can say that this function is periodic function which is repeating itself after a certain time and normally this can be expressed using what you call the Fourier analysis where it is a sum of series of simple harmonic motions in terms of different frequency, amplitude and phase. So, in the simple word what I can tell you that an earthquake is nothing but sum of a number of sinusoidal waves or harmonic components. So, earthquake how we can define in simple terms sum of harmonic excitations and those excitations are of different frequencies excitations of different frequencies. So, for that purpose we use what you call the Fourier series and you see in the second point here you have a periodic function x t where this is saying that this function is varying with the time it is not constant rather it is a dynamic function which varies with the time and in this case on the right hand side you have series and this series you have many components c0 and cn and what is ca0 and cn they are the coefficients c0 and cn they represent the amplitude and then you have phi n which represent phase and what is omega n, omega n is angular frequency.

$$x(t) = c_0 + \sum_{n=1}^{\infty} c_n \sin(\omega_n t + \varphi_n)$$

So, this represent your frequency component. So, here your n is varying 1 to infinity strictly speaking infinity though then you consider the finite value for the analysis most of the time. So, what you have when the n changes you get omega 1 omega 2 omega 3 and they represent different harmonic components and c1 c2 c3 it will be its amplitude of motion with different phase lag phi 1 phi 2 phi 3. So, cn and phi n are the amplitude and phase angle respectively of the nth harmonic component of the Fourier series. So, then you also have what we call the Fourier spectrum.

The Fourier amplitude spectrum of a strong round motion shows how the amplitude of the motion is disturbed distributed with respect to frequency or period. So, you have Fourier amplitude spectrum. Similarly, you will have Fourier phase spectrum also. So, how this is with respect to frequency that means on what we do in this spectrum or this spectra Fourier spectra on x axis you have frequency on y axis either you have amplitude or phase lag. So, in the Fourier amplitude spectrum it could be very narrow.

When you have the narrow that means most of the energy or most of the signal is confined to for a certain frequencies in a band frequency range only. So, the narrow spectrum will imply that the motion has a dominant frequency or period which can produce a smooth almost sinusoidal time stream. When we say broad it will correspond to a motion that contains a variety of frequencies that produce a more zigzag or irregular time stream. So, in case of narrow you will as confined to some frequencies, but when you have broad it will be spread over many frequencies. The Fourier spectra provides a complete description of the ground motion since the motion can be completely recovered by the what you call the inverse Fourier transform.

If you got a Fourier spectrum if I do the inverse normally this Fourier spectrum is created what we called FFT then inverse FFT. FFT stands for fast Fourier transformation. This is FFT and there is algorithm is available on the internet and this algorithm is freely available on the internet. So, using FFT what you do you convert an acceleration time history into Fourier amplitude frequency like which is called Fourier spectra which we will discuss later also.

So, acceleration time history is converted into Fourier spectrum and a plot of Fourier amplitude versus frequency that is cn versus omega n is nothing, but it is called Fourier amplitude spectrum. Now the similar plot if you plot with respect to what we call the phase lag or phase angle phi n versus omega n then its name will be Fourier phase spectrum. So, you could have Fourier amplitude spectrum and Fourier phase spectrum. So, Fourier amplitude spectrum is much used because it will give you the information about amplitude. When we talk about phase so how much is the lag with respect to the applied force that will give you.

So, this is the typical example for Fourier spectra, and they are again for the Loma Prieta earthquake and two stations Gilroy number 1 which is on the rocky side and Gilroy number 2 on the soil side. So, what do you see in both the spectrum earthquake was same but because these are recorded at two different sites. So, the peak in the first case is occurring at low period and in the second case it is occurring at high period. So, basically when with this period you may be aware this period which is normally t p is opposite of the natural frequency. So, opposite of the natural frequency or the frequency.

So, when you have this frequency f is nothing but k by m. So, you have one root k by m. So, you can write m by k. So, as a result when you go for a stiffer site k will increase or let us say when I go from a Gilroy number 1 to Gilroy number 2 k will decrease if my k decreases, period will increase. So, you can see the peak value is coming here but, in this case, peak value is coming at a higher period value.

So, which is expected from this equation that means because k decreased, mass not much change but there is a difference in the value of k as k decreases. So, it is expected that period will be higher for this case corresponding period. This is Fourier spectrum and in this Fourier spectrum you have Fourier amplitude on y axis and on x axis you may have period, or you could have on some time instead of period you could also have frequency they are interchangeable. But when the period increases frequency will decrease. So, on x axis it will be the other way that means for frequency your values will start from the right hand side and will go towards left.

On y axis whenever we draw a Fourier is like a spectra then it will be Fourier amplitude its unit will be g multiplied by time, g multiplied second where g is acceleration and when we multiply g versus time then you get on the y axis. Continuing with then we have what we call the power spectra. The frequency content of a ground motion can be described by a power spectrum or power spectral density function. So, we will discuss how we define the power spectrum or power spectral density function in next slide. So, this power spectral density function it can be used to estimate the statistical properties of ground motion and to compute the response using what we call the random vibration techniques.

And this is useful in characterizing the earthquake as a random process. So, here is the power spectrum. So, the power spectral density can describe a random process. Here the total intensity of a ground motion and if duration is td let us say td so this is the duration and for this duration if I integrate the square of acceleration, you can see acceleration at any time t and square and if I integrate with respect to t time dt and over the period td then I got total intensity. So, this is total intensity of ground motion.

$$I_0 = \int_0^{T_d} [a(t)]^2 dt$$

So, this is total intensity for duration td. So, you can get, and this total intensity will give you that how much is the strength or how much is the power in that signal. Then you have continued with the power spectra actual strong ground motion accelerograms frequently show that the intensity builds up a maximum value in the early part of the motion. So, this and then remains approximately constant for some time and then finally decreases. So, normally it has been observed in the past earthquakes that initially it is was the 0 value and then it is start building up in the some initial few seconds and after building up some peak values then it starts reducing and then some constant phase will come where the peaks are more or less same and then again it will start decreasing and becomes almost 0 that is the common.

And as a result such random process is often modelled by multiplying a stationary time history for a deterministic intensity function. Continue with the response spectra we have already discussed two types of response spectra one was what we call the amplitude and the second was the phase Fourier phase spectrum. So, a third type of spectrum is called response spectrum, and this is usually extensively used in earthquake engineering and this response spectrum is nothing, but it is very important because many times what is the it is asked particular for the structures for the buildings what is the response of the building. So, that is like you know during a given earthquake how the buildings perform. So, then we say let us see the response spectrum that is the response of the building at a particular point it may be normally at the top of the building or some top story.

So, in this case and this response spectrum is nothing but it defines the maximum response of a single degree of freedom system where this response is that means you model as a single degree of freedom system of your system to a particular motion as a function of the natural frequency. So, on x axis you have the natural frequency on y axis you have some measurement of spectral amplitude at particular damping ratio of a single degree freedom system. So, whenever we talk about response spectrum remember that it is related to response of a single degree of freedom system and this response is for a given earthquake for example if I have Bhuj earthquake. So, what I will do I will input this Bhuj earthquake for a single degree of freedom system and that is I know the natural like frequency of the system.

So, I will get the response. So, I will get one point now again if I change the natural frequency of my single degree of freedom system, I will get second response. So, what we do we plot response on y axis and on x axis we have the natural frequency and the natural frequency versus this response is nothing but this response spectra. For example, continue with earlier example of Gilroy which is again for Loma Prieta earthquake of 1989. So, therefore here it is given for rocky site on the left hand side and then you have the soil site on the right hand side. What you see you have one top one is for acceleration then velocity and then displacement.

So, what we could see that for rocky site you have again higher acceleration compared to the soil site. However, when we talk about velocity it is other way your peak values is higher in case of soil site compared to what you have for Gilroy number one site here. Similarly, for displacements you get quite high displacements in case of soft soil compared to what you have in the rock site. Here one thing is important there is not only change in the values that values are higher or lower. So, here you have higher in this case also high but the peak values also shifting you could see here that period is increased here.

And as I mentioned because why it is increased because simply k have decreased in case of stiffness have decreased. When you go from Gilroy number one to Gilroy number two then stiffness decreases and as a result this peak value have shifted towards the on the higher side on this. Continuing with the response spectra the frequency content of the two motions are reflected in the response spectra. So, here the Gilroy number one rock motion produce higher spectral acceleration at low periods then did the Gilroy number two which is at lower spectral at higher periods. The higher longer period content of the Gilroy two which is on soil motion produce spectral velocity and displacement much higher than those of the Gilroy number one which we already discussed.

The shape of the typical response spectra indicate that the peak spectral acceleration or peak spectral velocity and displacement values are associated with the different frequencies. So, that means what we get it here you are talking about the peak values, but peak value occurs at different locations peak value for acceleration is at different frequency, peak value for velocity occurs at different frequency, peak value for displacement is occurring at different frequency. So, frequency or period they are interchangeable as we discussed. So, here in this graph on the x axis you have period, but you can easily frequency is opposite of the period you can work on that and then accordingly. So, one thing is sure that peak of acceleration that coming here it is different and then you have very much different for the displacement.

So, this is normally low frequencies the average spectral displacement is nearly constant. So, displacement is for the at low frequencies spectral displacement is almost constant but contrary to this the acceleration spectral acceleration fairly constant at high frequency. So, the displacement will be is expected to be constant at low frequencies while when we talk about acceleration it happens at high frequencies. Continue with the response spectra now you have low frequencies and then you have high frequency but there is a intermediate range also lies in between there and that intermediate range your velocity or spectral velocity is almost constant and because of this behavior response spectra are mostly divided into three categories. First is acceleration controlled response spectra where you see higher frequency than velocity controlled which is intermediate frequency and then you have displacement control which you have low frequency portions.

Continuing with the spectral parameters, the Fourier amplitude spectrum and the closely related power spectral density which is combined with the phase spectrum can describe at a ground motion completely. So, how many parameters have come here you have spectral density you have phase spectrum and the Fourier amplitude spectrum. So, that is like it is in two category one is phase spectrum another is amplitude spectrum they can define the complete time step. The response spectrum does not describe the actual ground motion, but it does provide valuable additional information on the potential effect of the structure. In fact, response spectrum is giving you the more towards the response of your system which could be a building, or it could be a structure against the given motion. While this itself will not be exactly give what kind of motion you have because let us say if I apply the same motion input motion to different buildings or different structures then I may expect different even the motion input is same, but your output motion will be different.

Now the next part duration of strong ground motion is also very important, and this have very strong influence on earthquake damage. Many physical process such as the degradation of stiffness and strength of certain types of structures and the built up of pore water pressure which occurs in loose saturated sands are very sensitive to the number of load or stress reversal that occur during an earthquake. A motion of short duration may not be able to produce enough load reversals for damaging response to build up in a structure even if the amplitude of the motion is high. So, you know to build up a damage you require some time even you got very high peak values, but it occurs only very fraction of a second then it may not be able to do any damage.

But if you have the low even this moderate amplitude but it is for long duration then it may produce enough load reversal to cause substantial damage. And the duration of this strong round motion is mostly related to the what we call the time which is required for release of accumulated strain energy by rupture along the fault. As the length or area of fault rupture increases the time which is required for rupture increases and as a result the duration of strong motion increases with increasing earthquake magnitude. So, like it is way like when the duration increases it is expected the earthquake magnitude will increase corresponding. If you have earthquake magnitude is increasing it is expected that duration of that earthquake motion will be large great it will be going to be larger.

It can be described duration can be described in absolute or in relative terms then there is a term which is called bracketed duration, and this is nothing but time between two threshold value. So, you have the let us say you have decided this is my threshold value. So,

you pick up time between these two let us say first threshold value. So, then your time window will start and again now after this threshold value you will get some peaks and then again it come down. So, again when it come down then that will be another time window.
So, the time between like when my this time required for that threshold to cross one time and another time. So, the time that is will call bracketed duration on the both sides of the bracket you have the threshold value. For any purposes the bracketed duration is most commonly used and here is some examples here. Typical earthquake durations at epicentral distances less than 10 kilometer and this first table will be applicable for at a epicentral distance which is less than 10 kilometer. So, one thing is sure when the magnitude of earthquake is increasing the duration is going to increase for both rocky sites as well as soil sites.

So, you could see when the magnitude is going 5 to 5.5, 6 and 6 so it is number of duration in seconds is increasing 4, 6, 8, 11, 16 and ultimately at magnitude 8.5 it could duration could go up to 43 seconds or like this. This is the typical this may not be the exactly the same when earthquake comes. But on the soil sites the duration is almost doubled at most of the time it is more than the double which was on the rock sites and sometimes it is more than doubled also like you have here 22, 45.

So, ultimately you get for the soil site the higher durations compared to the rocky site. The second table represent equivalent number of uniform stress cycles and that equivalent number of uniform stress cycles depends on your magnitude of earthquake. For example, if you lower a magnitude 5.25 you could have 2 to 3 cycles, but typically the standard magnitude of earthquake which is considered to be 7.5 and for this magnitude of earthquake you get 15 number of cycles.

And further if I increase the magnitude then I get a greater number of cycles for duration of strong ground motion. Then the last point for this today's lecture is called predominant period or predominant frequency. So, as I mentioned this period and frequency is interchangeable so instead of sometimes you may get that it may be not period rather it could be predominant frequency. So, a single parameter that provides a useful although somewhat crude representation of the frequency content of ground motion is the predominant period which is normally done by Tp. The predominant period is defined as the period of vibration which correspond to the maximum value of the Fourier amplitude spectrum.

So, the period or frequency which corresponds to the maximum peak value in the Fourier amplitude spectrum measure so that will be there. And this predominant period to avoid that it is not influenced by the individual peaks, spikes of the Fourier amplitude spectrum often this is obtained from a smooth spectrum rather than you may have many zigzags so we clean it and make a smooth spectrum and then it is find out. This predominant period provides some information regarding the frequency content. It is easy to see that motions with different frequency contents can have the same predominant period.

So, normally we have the same predominant period. For example, here there are two motions which are completely different but the predominant period could be same. Like for example this dotted line which is at the base it is Tp the period. So, the Tp is nothing but predominant period. This one predominant period and this predominant period remains same for both the cases irrespective of the maximum amplitude is coming here in this case also it is here.

But the variation in their shape is quite different. So, this is possible that you could have same predominant period for two hypothetical Fourier amplitude spectra with the same predominant period but very different frequency content. So, here now the last part which is called magnitude and distance effects. So, much of the little energy is released by rupture along a fault text in the form stress waves or what we call the elastic waves. Since the amount of energy which is released in an earthquake is strongly related to its magnitude the characteristic of the stress waves will also be strongly related to magnitude of earthquake. Each earthquake can came from essentially the same source and each accelerogram was measured at about the same distance from the source.

So, what we do finally the variation amplitude frequency content and duration with magnitude are apparent and that help you in the magnitude and distance effect. For example, on this case there was 6 earthquake, and this is record of accelerogram from 6 earthquakes of the Pacific coast of Mexico. Each accelerogram was measured at nearly the same epicentral distance. So, what you see here you have different magnitude acceleration time history top one is simply for 3.1 magnitude 4.1, 5.1, 5.2, 7 and then finally 8.1. So, when you see the top one when the magnitude was less than the peak values was also less and when the magnitude is increased 7 the peak values jumped. Similarly, when magnitude is increased from 7 to 8.1 again you get so many peaks, and the peak values are increased here which was very small here. So, this is for 6 earthquake which was recorded at the Pacific coast of Mexico and each accelerogram was measured at nearly the same epicentral distance.

So, epicentral distance was same here. So, important point is here that when you increase the magnitude of earthquake it is expected that this acceleration time history will get higher peaks. The peak values want to increase. Then there is what you call the specific energy which is nothing but energy per unit volume and it has been seen that it decreases with increasing distance from the source. Since the characteristics of stress waves are strongly related to specific energy they will also be strongly related to distance.

So, with this I conclude this lecture. So, many things have been covered here. Fourier spectrum, then you have power spectrum, response spectrum, magnitude duration, magnitude distance effect. So, thank you very much for your kind attention. Thank you.