Earthquake Geotechnical Engineering

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Lecture 08

Seismic Hazard Analysis (Cont.)

So, I welcome you again for the second part of the series, chapter 4 that is 8th lectures, for this course. And as I mentioned during the lecture 7 that lecture, this chapter 4 is divided in two component which is on seismic hazard analysis. So, we already discussed partly now we continue with that. So, we have covered four topics and we are under fourth topic seismic hazard analysis the second part of that.

So, this is chapter 4. Now before I go ahead let us recap from the lecture 7. In the lecture 7 we have covered these components seismic hazard analysis which is DSHA and PSHA. And before I talk lecture 8th today so let me like you know that is what are the topics for this lecture.

For earthquake source characterization, spatial uncertainties, size uncertainty, Gutenberg greater recurrence law, temporal uncertainty. So, we are going to talk this under these five topics. And let me acknowledge that most of the information which is taken in this lecture are from the textbook of Kramer, but I will explain each and everything. So, that is going to be very different than what you have only in the textbook. So, first thing is like we continue with PSHA.

In PSHA this lecture is on probability and uncertainty and other things. So, this is basically based on PSHA. We already discussed DSHA and PSHA in the last lecture. In case of PSHA uncertainties are there. So, this lecture is adding to PSHA.

So, probability seismic hazard analysis that is PSHA allows uncertainties in the size, location, rate of occurrence and effects of earthquake to be explicitly considered in the evaluation of seismic hazard. So, that is here that we can consider this uncertainty eternity while dealing with the seismic hazard. A PSHA requires that uncertainties in earthquake location, size, recurrence and ground shaking effects be quantified. So, we need to quantify those effects. Now, when we want to quantify this, so in the earthquake source characterization this require consideration of spatial characteristics of the source, distribution of earthquakes within that source, distribution of earthquake size for each source, distribution of earthquake with time.

So, this uncertainty may be related to these issues and each of these characteristics involves some degree of above characteristics. So, we will be dealing with the uncertainties related to these issues one by one. Now, let us first consider three types of uncertainties. First is what we call spatial uncertainty. Spatial word is here it is very different than what we have spatial.

So, here you have spatial and this is not the special. There is another word spatial and this is kind of a particular. So, this is not the special. This spatial is here related to space. That is the uncertainty which is related to space or distance.

Then you have size uncertainty, then you have temporal. Temporal here is not temporary again this is related to time. Temporal word in use in earthquake engineering related to time. So, and size is normally related to magnitude. So, let us talk about the uncertainty which is related to space.

The geometries of earthquake sources depend on the tectonic processes involved in their formulation. So, how they are created, the sources are created. So, that will depend on. Earthquake associated with volcanic activity normally they are originating zones near the volcanoes which are small enough to allow to be characterized as point sources. So, if you have the small, then you can consider as a point sources.

If you have well-defined fault planes on which earthquake can occur at many different locations, they can be considered as a two-dimensional source. Now for example, this slide is giving you the examples and this is again these figures are taken from the Kramer. So, examples of different source geometries. First one is short fault that can be modeled as a point source. So, you have a like line, it is fault, may be fault line and this is your site. So, you are considering that this is like a point source. So, it is one-dimensional case. Then you have a plane case, shallow fault model linear source. So, you have a 2D case and your site is here. So, source to site linear variation. But the third one is three-dimensional source zone where you have from the source to site like for example, this is a kind of source. So, all the four corners are a kind of a link with the site. So, this is in three-dimensional case. So, the special uncertainty can be deals. Naturally, the first one is the simplest, third one is the most complicated which is in 3D case which require more.

So, this special uncertainty can be deals like this. Earthquakes are usually assumed to be uniformly distributed within a particular source zone that is earthquakes are considered equally likely to occur at any location. So, within the same source zone, you let us say this is same source zone, you have a number of earthquakes which may be like here in the PSHA, we do not only consider one earthquake due to a source or due to a fault line there may be a number of earthquakes. Now, within that what are the chances of occurring whether this small earthquake will occur, large earthquake will occur? No, we consider normally the uniformly like we consider uniformly distributed. However, this assumption of uniformly is required in the sense even if you consider non-uniform distribution in that case, it need to be justified with some information.

There should be sufficient information that we are not considering uniform distribution rather than we are considering non-uniform distribution. So, if you do not have any particular information data, then we consider the uniform distribution. Similarly with spatial uncertainty, a uniform distribution within the source zone does not often translate into uniform distribution of source to site distance. Even we consider that our sources are uniform, the effect is similar, but still, it will not lead us to uniform distribution of source to site distance because the distances may not be the same. So, it will not be uniform distribution of source to site distance.

And as we mentioned in the last lecture, we use a predictive relationship which is a recurring, recurrence relationship which is expressed ground motion parameters in terms of some measure of source to site distance, the spatial uncertainty must be described with respect to the appropriate distance parameters. So, you have this spatial uncertainty and naturally this will involve a distance parameter because it is related to space, so a distance parameter will be involved in this case. The uncertainty in source to site distance can be described by a probability density function and this probability density function sometime it is called in short PDF, probability density function. So, that is there.

Continue with this. Now the second uncertainty, first one was related to the spatial uncertainty related to space. Now the second one is related to size of earthquake, size uncertainty. Once an earthquake source is identified and its corresponding source zone characterized, the attention is required towards evaluation of the sizes of earthquake that the source zone can be expected to produce. So, like now we will talk about size of the earthquake and with the size of earthquakes that is expected to be reproduced at that site. All source zones have a maximum earthquake magnitude that cannot be exceeded. This is very important here. It can be large for some and small for some. So, what we do, you have a different, let us say you have a site and let us say three faults are nearby which will influence the site. So, each fault or it is creating a number of earthquakes. Now you decide that this is the maximum value of earthquake magnitude which is expected from this fault and naturally when the next coming from the same source that will not be exceeded.

So, this maximum earthquake magnitude is decided, otherwise if we keep it increasing it will be too much conservative approach and then it will be very difficult to design the structures. So, there we need to decide, we need to put a cap that this is the maximum earthquake magnitude considered for a particular source zone. Then continue with the size uncertainty. In general, the source zone will produce earthquakes of different sizes as up to the maximum earthquake with the smaller earthquake occurring more frequently than the larger ones. So, you may be perhaps knowing that the chances of occurring small earthquakes is higher than the bigger or large earthquakes and the strain energy may be released or in the form of earthquakes assuming that all strain energy is released by earthquake of magnitude 5.5 to 9 and that average fault displacement is one half of the maximum surface displacement. So, with this data if you consider between magnitude 5.5 to 9 then average fault displacement will be approximately half of the maximum surface displacement. Now continue with this on the size uncertainty, this graph shows for different magnitudes return periods in the year and this return period has been found out for different slip rate. So here you have 10 centimeter per year, 1 centimeter, 0.1 centimeter so on. So, when the rate of slip is higher than return period for the same magnitude is very low. That means for example 8 magnitude earthquake it is less than 100 years. But when the rate of your slip rate decreases then return period required is high. That means return period is a period after which a particular magnitude earthquake is expected to repeat. So naturally one thing to be sure, as we just discussed that the chances return period will be low for low magnitude earthquakes.

For example, 6, 7 magnitude earthquakes have a smaller return period than 8 and 9. So when you increase the magnitude of earthquake the return period is going to increase. And the same time if you decrease the rate of the strength slip rate of the strength the return period will increase. Continue with the size uncertainty, the distribution of earthquake sizes in a given period of time is described by what is called recurrence law. So, this will be with respect to recurrence law.

And basic assumption of PSHA is that the recurrence law obtained from past seismicity is appropriate for the prediction of future seismicity. So, this is one of the important, we have data from the past seismicity and we prepare based on those data some relationship for recurrence law and then we suggest that this is expected to be follow in the future also. So, this is the best way like we analyze the data of the past earthquakes and then on the basis of we decide about recurrence law. The Gutenberg-Richter recurrence law which assumes an exponential distribution of magnitude is commonly used with modification to account for minimum and maximum magnitude. So, this Gutenberg-Richter recurrence law we are going to discuss in detail in the next slides.

So, this Gutenberg-Richter recurrence law we will call it in short GRR law. So wherever rather than repeating this word Gutenberg-Richter recurrence GRR will stand for Gutenberg-Richter recurrence. So, these researcher Gutenberg and Richter in 1944 gathered data for the Southern California earthquakes over a period of many years and organized the data according to the number of earthquakes that exceeded different magnitudes during that time period. So, what they have done they have collected data for past earthquake for many years for decades. Those data are available and they then they said let us say magnitude 5, 6, 7 or like this one how many numbers of earthquake exceed more than 7, how many numbers exceeds more than 6. Of course, when you say more than 6 already which are exceedingly more than 7 will be also included in 6 and how many exceeded more than 5 so that will be there. So that means you categorize in different like what are the chances of occurrence of an earthquake higher than a given magnitude. They divided the number of accidents of each magnitude by the length of the time period define a mean annual rate of accidents lambda m of an earthquake of magnitude. So, this is important. What is the magnitude is m that is the threshold value and mean annual rate of accidents lambda m.

So, lambda m shows the rate of accidents and in a year that means how many times in a year you expect that this earthquake will come more than that given magnitude. So naturally this annual rate of accidents is expected to be higher for the lower magnitude earthquakes. So, if your magnitude is small then chances are more that in a year you will get more accidents but if your magnitude is very high so it may be even 0. Suppose if I say 8 magnitude earthquake it is like it may occur only very if you are like you know many numbers of years rather than every year. So, continue with this GRR law the mean annual rate of accidents of is greater than the large earthquakes which we already discussed.

The reciprocal of this annual rate of accidents for a particular magnitude is commonly referred as the return period of that earthquake. So, return period is important. Return period we already discussed it is a simple term that after how many years an earthquake will be repeated. So that is basically return period. When the logarithmic of the annual rate of accidents of Southern California earthquake are plotted against earthquake magnitude a linear relationship was observed.

$$\log \lambda_m = a - bm$$

So, we will discuss that relationship later in the form of graph but in the form of an equation so this relationship is expressed here lambda m is the manual rate of accidents of magnitude m. So, in this relation m is a magnitude of earthquake and lambda m is the rate of accidents annually. There are two parameters one is a and another is b. Now let us discuss about these parameters. If you put magnitude m equal to 0 then what will happen? Lambda m will be a for m equal to 0 if I put m equal to 0 in this equation then lambda m equal to you get it 10 to the power a. So, 10 is a mean and yearly number of earthquakes of magnitude greater than or equal to 0 as written here and what is b describe the relative likelihood of large and small earthquakes what are the chances of likelihood? b will be applicable when m is not 0 because if m equal to 0 b does not have any meaning here. So if you have some large or small magnitude earthquakes so b describe the what are the chances of occurring of that earthquake. So, this relationship is further plotted here but before that as the b value increases the number of large magnitude earthquake decreases compared to those of smaller if you increase the b value what will happen? Lambda m will be increases lambda m will decrease and lambda m is your annual rate of accidents so that will decrease. So, when the b value increases the number of large magnitude decreases compared to those of a small magnitude. Now this Gutenberg-Richter law is not restricted to the use of magnitude as a description of earthquake size.

Epicentral intensities also been used and these a and b parameters are generally obtained by what you call the regression on a database of seismicity from the source zone of interest. So, we do the regression analysis the database of seismicity regression analysis is done on the database of seismicity and this seismicity should come from the source zone of interest that is source zone of interest means a zone near your site. So, this relationship which we have discussed is plotted in this graph like this which is given by Gutenberg-Richter. So, what you have on x axis you have magnitude this is on the normal scale normal scale on x axis while on y axis you have the log scale. In fact, if I say on the log scale you can remove the log also so you have lambda m value in terms of log scale.

So ultimately and you have tan a that means this intercept will be a here. When I put m equal to 0 magnitude equal to 0 then the value of lambda m is 10 to power a as given here or value of log lambda m will be a. So, and b is the slope of this line this line. So, what you could see as the magnitude increases then your lambda m decreases. So that means higher magnitude earthquake like rate of recurrence will decrease.

So, chances of the higher occurring reoccurring of higher magnitude earthquake are less compared to a smaller magnitude earthquake. Now let us have a one example on application of GRR law. Let us compute the return period of 8 earthquakes on the two belts one is Circumpacific and alpide belts. So, and this return period we need to compute for a magnitude of 8 earthquakes. So, this is given in the next slides what you have in this slide you have a GRR relationship and this GRR relationship has been for two sides first is alpide belt down and Circumpacific belt.

$$T_R = \frac{1}{\lambda_m}$$

- **Circumpacific** $T_R = \frac{1}{1.76/yr}$ **= 0.6 year**
- **Alpide** $T_R = \frac{1}{0.31/yr}$ **= 3.2 years**

So, for 8 magnitude earthquakes if I draw so this is my point here which is about the value if I read this value on of course it is on the log scale so it is about 1.76. While for alpide belt I have here this value which is I think somewhere you have 0.31 so this is 0.31. So, this value of lambda m is per year that is the number of earthquake accidents which exceed a certain threshold limit. Here threshold limit we are putting is 8 magnitudes of earthquake. So, what says in the Circumpacific belt the chances are there 1.76 times the 8 magnitudes will be exceeded while in alpide belt it will be chances are 0.31. So that means now if we determine the return period for both these like 8 magnitudes of earthquake in these regions then the return period Tr corresponding return period which is like said in the Tr this will be opposite of lambda m. So, 1 by lambda m so Circumpacific chances are there that within 0.6 year that means 0.6 into 12 so around 7.2 months. In 7.2 months, it is chances that the 8-magnitude earthquake will repeat while in alpide belt it will take 3.2 years to repeat that 8 magnitude earthquakes. So that means, sub-smicity of Circumpacific is more than the alpide which is obvious from this figure also. So, this was about application of GRR law. Now the last part of this lecture that is what we call temporal uncertainty.

So, when we talk about temporal uncertainty as I mentioned this temporal here mean is related to time it is not temporary it is related time here. To calculate the probability of various hazards occurring at a given time period the distribution of earthquake occurrence with respect to time must be considered. So, what is the distribution of chances of occurrence and this should be done with respect to time. Earthquakes have long been assumed to occur randomly with time and in fact if we have examined the available seismicity records that has revealed little evidence when aftershocks are removed of temporal patterns in earthquake occurrence. So, what has been observed that if we remove the data related to aftershocks then we get that there is not much variation with respect to time.

But that is why we need to consider the data of aftershocks also so we to get the temporal uncertainty. Now continue with this. So, what is there? The assumption of random occurrence that earthquake may occur randomly it allows the use of simple probability model, but it is inconsistent with the implication of elastic rebound theory. So, the temporal occurrence of earthquake is most commonly described by a model called Poisson model. Poisson is the name of the person who invented this model and it remains the most commonly used models for determination of PSHA probabilistic seismic hazard analysis.

So in this case temporal uncertainty in the Poisson's model, it provides a simple framework this model provide you a simple framework for evaluating probabilities of events that follow a Poisson's process one that yields values of random variable which describe the number of occurrence of a particular event during it given time interval or in a specified special region. Since PHSA deal with temporal uncertainty the special application of the Poisson’s model may not be considered if you deal with the like, you consider the like special here that means related to space again. So that may not be if you are considering temporal uncertainty and if then again, a special uncertainty that will complicate it for a particular for this model. So, this is done one by. Now temporal uncertainty in case of Poisson's model how it is dealt for a Poisson's process the probability of a random variable that is n representing the number of occurrences of a particular event during a given time interval is probability.

$$P[N = n] = \frac{\mu^n e^{-\mu}}{n!}$$

Here what is p? p is a probability capital P is here probability what are the chances, the number is between you have this number this is probability. This number p is stand between could be between 0 to 1 that is 0 to 100 percent and this is between capital N should end what is mu here mu is the average number of occurrences of the event in that time interval what are the chances average number of occurrences. So, with this we can find out the probability the time between events in the Poisson's process can be shown to be exponentially distributed. So, like here now the last part of this is accuracy of PSHA what are the chances like accuracy of probabilistic seismic hazard analysis. The accuracy of PSHA depends on the accuracy with which uncertainty in earthquake size location recurrence and effects can be characterized.

If you can characterize uncertainty in very good way your model is going to be accurate. If accuracy is cannot be quantified in a proper way, then model is it is expected that it may not be like you know very accurate. All the models and processors for characterization of uncertainty of these parameters are available but they may be based on data which are collected over a period of time that geologically are very short. So, if you have a large span of like you know data which is collected from many decades or let us say hundreds of years then you it is chances are that that these models will be working well. So always as a result engineering judgment must be applied for the interpretation of PSHA results.

Now perhaps you may ask one question we talk about DSHA and PSHA so what we need to do. So, what NCSDP recommend particularly or insist rather for important project that both DSHA and PSHA should be carried out and once you carried out both DSHA and PSHA whichever is given you the you know the conservative result that should be considered for the design. But sometime what happens like we get more conservative values then it needs to be discussed again because otherwise it may be difficult to design the structures for your dam, they may be very uneconomical. So that is the how the committee does for that. So, this is one of the expert areas where you need to carry out both DSHA and PSHA.

If you get the results similar result from both the DSHA and PSHA fine. Now if you get like you know from the conservative approach, we will say we consider the worst case however then we need to see the economics also. So, with this thank you very much for your kind attention. Thank you.