Indian Institute of Technology Madras

NPTEL
National Programme on Technology Enhanced Learning

Pattern Recognition

Module 04

Lecture 04

Cauchy Schwartz Inequality

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So I will basically do some mathematical preliminaries in this first lecture okay, there is something called Cauchy Schwartz inequality I do not know whether you are aware of it or not.

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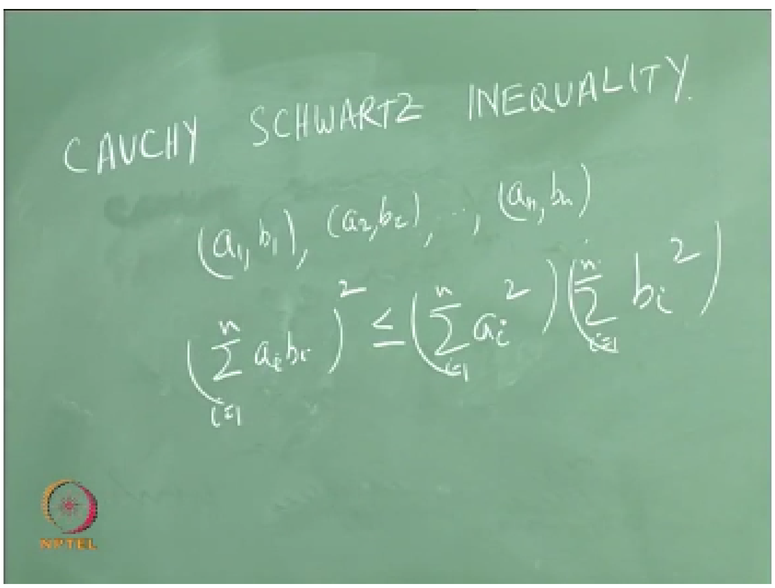

Cauchy Schwartz inequality suppose you have (a_1, b_1), (a_2,b_2), (a_n, b_n) in such ordered pairs okay, and what it basically says is that $(\Sigma a_i b_i)^2$, i=1 to n is less than or equal to $(\Sigma a_i^2)(\Sigma b_i^2)$. Now you look at the slide.

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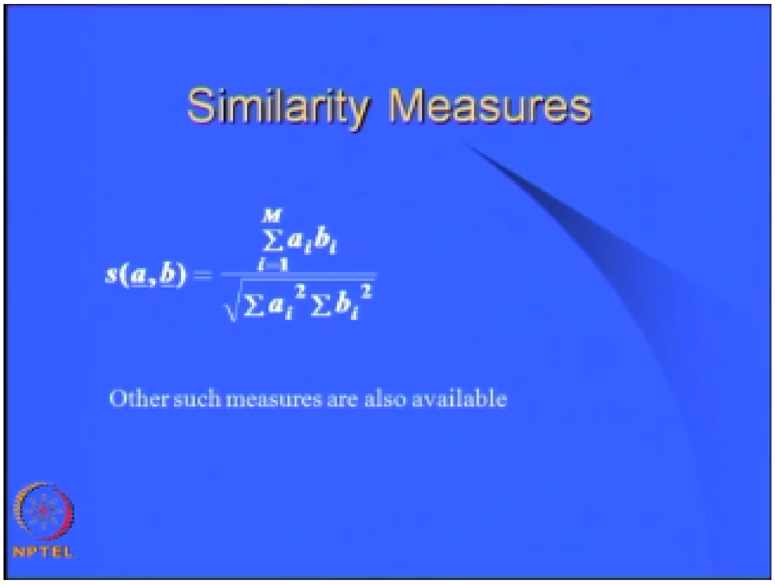

That means the expression in the slide the value lies between -1 and 1 right, okay now let me just prove it so this is the statement and then the proof is the following.

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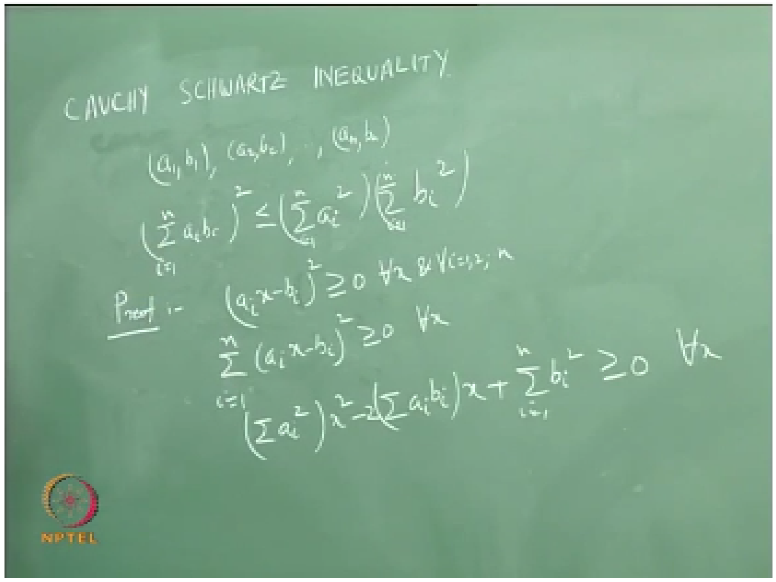

We know that $(aix-bi)^2$ this ≥ 0 for all x and for all i=1 to n is it correct, $(aix -bi)^2 \geq 0$ since this is a square term greater than or equal to 0 for all x and for all i=1 to n, I have n such things here. So naturally Σi=1to n, $(aix-ba)^2$ it ≥ 0 for all x right okay, that means $(\Sigma ai^2)x^2$- Σai bi)2x x+Σbi^2 i=1 to n$\geq$0 for all x am I right.

Now this is in the form of ax^2+bx+c right, this is in the form of ax^2+bx+c now suppose ax^2+bx+c $\geq$0 for all x then what can you say about a, b and c. What can you say about b^2-4ac term can you say anything. Now these all the all of them are real numbers so b^2-4ac it has to be real but can you say anything, look at this I am drawing a diagram here.

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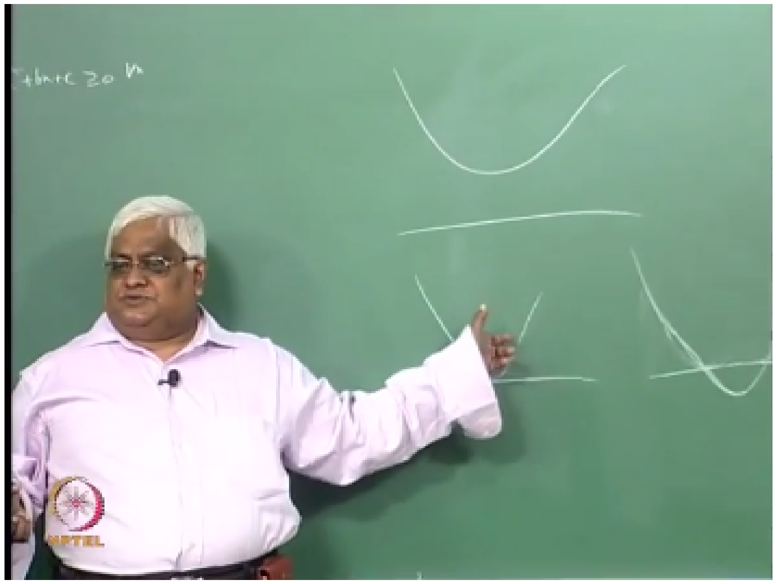

$ax^2+bx+c \geq 0$ that means for every x there are two possibilities for every x this is strictly greater than 0, that means the current may be something like this it never touches x-axis am I right, that is one possibility. And for some x it may be equal to 0 right, that means the curve may be for exactly one X it is equal to 0 or can it happen like this and it happened like this, for 2x it may be equal to 0, can it happen like this that cannot happen.

First this $ax^2+bx+c=0$ it can have two real roots right and if it has two distinct real roots then what happens is that in between those two real roots the function takes negative values it cannot be like this if you have two real roots that mean this is one root and this is one root then it is going to be look like this, this will be this will look like this okay, in between those two routes you should get negative values.

So that means either this case has to happen our this case has to happen this case means there is no real route, there is no real route means what can you say about b^2-4ac, if b^2 -4ac is greater than or equal to 0 is greater than or equal to 0 if it is greater than 0 then you are going to have two distinct real roots. So b^2-4ac cannot be greater than 0 are you understanding, if it is equal to 0 then you are going to get exactly one root –b/2a.
So b^2-4ac has to be less than or equal to 0 are you understanding b^2-4ac it has to be less than or equal to 0 right.

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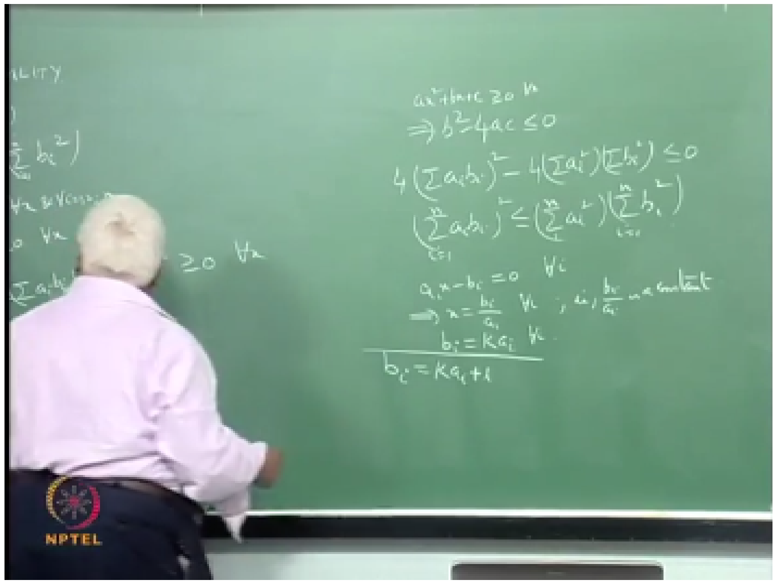

This implies b^2-4ac should be ≤ 0, so b^2 means the square of this that is $4(\Sigma\ a\ bi)^2$-$4(\Sigma ai^2)\Sigma(bi^2)\leq 0$ right, that means $\Sigma(ai\ bi)^2\leq\Sigma(ai^2)$ and $\Sigma(bi^2)$ it is clear. Now the next question is when is this equal to 0 this is equal to 0 when all these are equal to 0 right, all these are equal to 0 means aix- bi should be equal to 0 for all aix, I mean aix - bi should be equal to 0 okay, that means aix-bi=0 for all i that means what x=bi/ain for all i.

So that means this has to be a constant x must be same for all i, that means bi/ai has to be a constant that is bi/ai is a constant okay. So equality will hold when bi/ai is a constant that means what there is a linear relationship between bi's and ai, bi= constant ai, bi is equal to some Kai for all i, that means if you look at this you can look at that expression that is there on the board if ai, if bi =some Kai then what will happen to the count the quantity SAB it is equal to 1, am I correct it is equal to 1 or – 1, can it be equal to -1.

Suppose K is the negative quantity then that minus will appear in the numerator, but it does not appear in the denominator it is equal to +1 or- 1 and bi=Kai that means what, you have a vector a1 to am okay, and you are multiplying it by a constant K then you will get the other vector b1 to bm, that means what actually they are the same vectors the direction is actually the same the magnitude is different than the angle between them is either 0 or 180˚ then cosine 0 is +1 cosine of 180˚ is -1.

Ad this is the this is linear relationship but what it says is that the linear relationship you are making it go through the origin if we does not go through the origin then you will get a sum constant plus some constant L right, since the constant there is no other constant here this goes through the origin alright. Now if you add that constant that is bi= Kai + l, then what will happen to $\bar{b}$ I think I will write here now.

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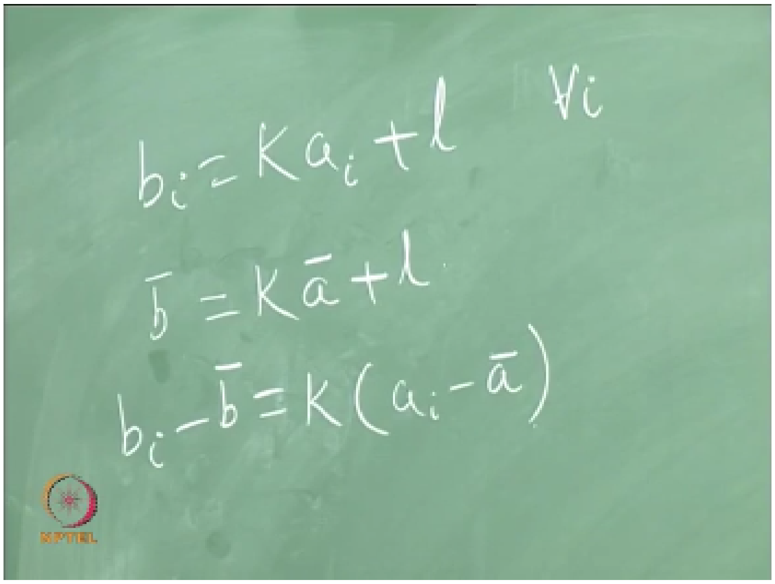

bi =Kai+l then what will happen to $\bar{b}$ this is the mean of b this is actually equal to Kā+l the mean of b, that is $\bar{b}$, $\bar{b}$=Kā+l then bi-$\bar{b}$=Kai-ā I am just subtracting this from this then bi- $\bar{b}$=Kai-ā this l and l is getting constant they are getting cancelled. Now you have this form sum bi=some constant times ai only thing is that here we are subtracting the mean and we are subtracting the mean here. So if there is a linear relationship between bi's and ai's with K and L then the correlation coefficient is +1 or -1 it is clear then the correlation coefficient is +1 or -1. Please doubts. Sir, probably we take there will be l divided by something now bi=Kā+l divided by number of examples here second, second sir, L divided by something when we will take $\bar{b}$. Okay we will just see so this step is not clear to you right yes.

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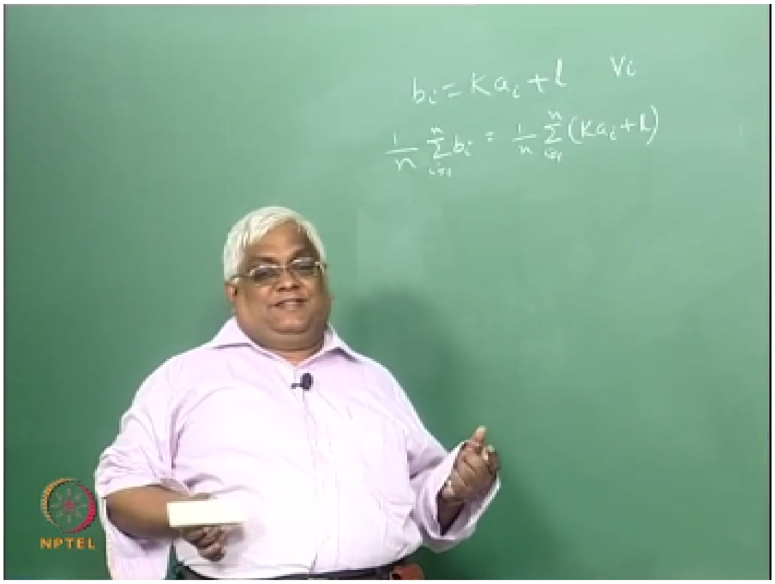

$1/n\Sigma b_i$, $i=1$ to N this is $\bar{b}=1/n\Sigma$ $i=1$ to n, Ka_i+l right, you got it take it off okay, so when there is a linear relationship between the two variables then the correlation coefficient is +1 or -1, but the converse is not true wait, wait when there is a linear relationship between the variables then the correlation coefficient is +1 or -1. Suppose the correlation coefficient is +1 or -1 then also you can say that there is a linear relationship.

But for 0s it is not true when the random variables are independent and the correlation coefficient is 0, but when the correlation coefficient is 0 you cannot say that the random variables are independent okay. I will also tell something that we will be using it in the coming lectures Dehlia fact okay.

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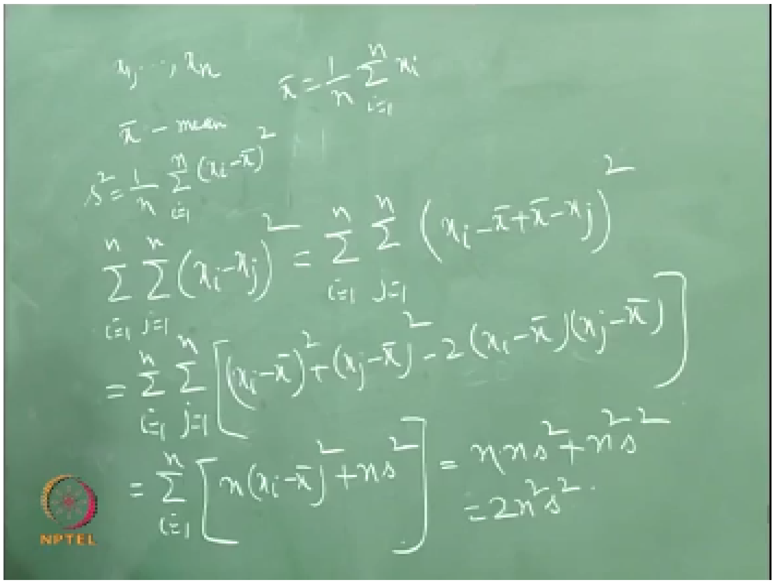

Suppose you have n such real numbers x1 to xn and mean is $\bar{x}$ this is the mean we all know the formula of variance let us just say the formula of variance=1/n Σi =1 to n $(xi-\bar{x})^2$ okay. Now variance is supposed to give you what variance is supposed to give you some sort of a scatter of the data how much of variation you have in the data, you might be wondering that why in order to measure the variation in the data.

We are taking the difference between the values and the mean we want to measure the variation in the data set right, then why are we taking the difference between the values of the mean, how is mean entering into the picture have not you got this doubt, have you understood what I wanted to say why are we taking the distance or the difference from the mean, when we want to measure the variation within the data set how is mean coming into the picture okay.

Now let us see suppose let us not take the mean then what we will do we can always do something like this are you understanding what I am trying to write we take all possible such differences and then you take the squares and then we will divide it by some quantity here that is a different thing, okay. Now let us just see what this is going to give us this is equal to what I will do is $(xi-\bar{x}+\bar{x}-xj)^2$ okay.

So now this is equal to Σi= 1 to n, Σj=1 to n what do you have here $(xi-\bar{x})^2+(xj-\bar{x})^2$- okay, now what is this let us see Σi=1 to n let us take this Σj=1 to n inside this is the term which is independent of j, so what we are going to get this is $n(xi-\bar{x})^2$ right, now Σj=1 to n $(xj-\bar{x})^2$ this is n times x^2 okay, now what about the this one Σj=1 to n of this here $2(xj-\bar{x})^2$ it is something independent of j so we can take it out.

$\Sigma j=1$ to n $(xj-\bar{x})^2$ what is the value of that it is equal to 0 are you sure it is equal to 0 $\Sigma j=1$ to n, $(xj-\bar{x})^2 = \Sigma j=1$ to N, $xj-\Sigma j=1$ to N $\bar{x}$ right, this is equal to $n\bar{x}$ - this is $n\bar{x}$, what is $\bar{x}$ is i or j it does not matter exciting right, so that is equal to 0 so this is equal to 0 right. Now what will happen to this now we will take this $\Sigma i= 1$ to n inside this will be $n\Sigma i=1$ to n $xi -\bar{x}2ns^2+$, so this is basically nothing but $2n^2s^2$.

So basically you are going to get a quantity which is a function of s^2 so you need not take this you can be satisfied with this after all this is a constant multiple of this right, see even if you look at this $2n^2s^2$ look at $(xi–xj)^2$ it occurs 2 times 1 as $(xi–xj)^2$ another as $(xj–xi)^2$ right. Now so that is occurring two times so in fact totally how many terms are there here n^2 terms and there is something that is occurring twice so that is why $1/2n^2$.

So you need not we need not have to go through our look at this we can actually look at this by looking at this we are actually looking at this by looking at this we are actually looking at this, I hope this is clear to you because this is a constant multiple of this, we will stop here.

End of
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