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NPTEL
NATIONAL PROGRAMME ON TECHNOLOGY ENHANCED LEARNING

Pattern Recognition

Module 01

Lecture 07

Vector Spaces

Prof. Sukhendu Das
Department of CS&E, IIT Madras

So far we have discussed several properties of matrices from the concept of linear algebra and also we had seen methods to obtain Eigen values and eigen vectors of a matrix which will be used in many applications including some of our algorithms in pattern recognition. The other important concept which is related to matrices and linear algebra is vector spaces, we must understand this because we will seeing that most of the algorithms in pattern recognition they work in certain dimension in a certain space.

And they will be the concepts of projection on to certain low dimension subspecies and what are the corresponding side of vectors or numbers which represent or what we call as span that space we need to have some understanding in this course very elementary one before we proceed to algorithms in future lectures of pattern recognition. So what are vector spaces by definition.

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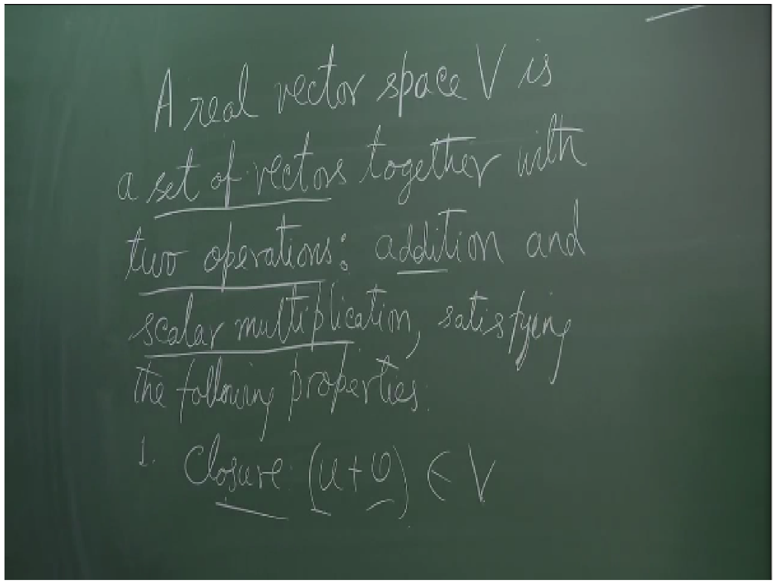

A real vector space V is a set of vectors together is basically a set of vectors or a collection of vectors together with two operations, and what are those two operations addition and the scalar multiplication. That means I should be able to add to vector multiple a vector with a scalar quantity and when I do these two operations with the set of vectors which belong to the vector space V it should satisfy a set of properties, okay.

That means this should work under the following conditions or properties which are also sometimes call actions this is very basic definition of vector spaces and we will give examples. So I will read it again real vector space V is a set of vectors together with two operations what are they addition and scalar multiplication, okay addition and scalar multiplication.

So these two operations on the side of vector should satisfy the following properties the each of this 10 properties will give the 10 names and the corresponding expression for that and we will try to go this as briefly as possible. The first of them is called the closure and that is a very causal word which is used, term which is basically used which says.

Then u+v if two vectors u and v are in the space v I did not write that okay, we will also define what are this scalars so we talked about set of vectors correct so assuming these two u and v are set of vectors okay, then this is in v when you add two vectors u and v it is in v, it is in this vector space v, correct. Some books will like that this is in v I am using this symbol to indicate that the vector addition of these two vectors u and v which lie within the space their addition will also lie within this, okay. You can ask me a question about what is a vector okay.

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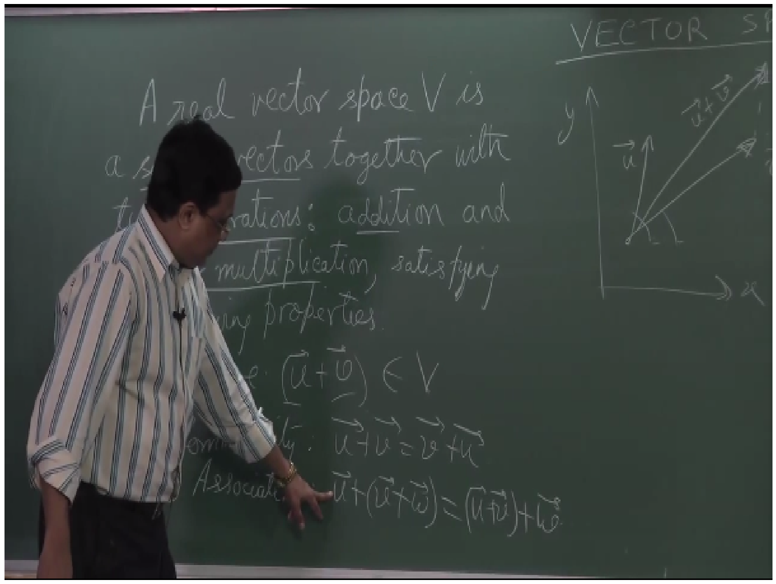

Usually in two dimensional or three dimensional space this is represented by a magnitude which indicates the length of a vector and some direction, in the two dimensional space x, y okay, if you take this x and y two dimensional space now of course it will probably also have a direction or identification of an angle okay. So this is the vector v I can have another vector u here, of a different direction and I am sorry I should not use the v here that is wrong.

Because v I am used for a vector space and I am using small letters to indicate the corresponding vectors, so I should write it as v okay, we have a different angle and a different magnitude. You can differences in 3D as well okay, so you can visualize as if this is one vector v and here one other vector u okay, so there are two vectors u and v in three dimensional space in this room of course in the field of pattern recognition will be dealing with by the large dimensional vectors, okay.

We will be able to visualize in two or three and imagine beyond three okay, and pattern recognition typically you have dimensions of hundreds of few hundreds often millions in certain cases, okay. if it is visualizing two or three dimension so if u and v are lying within the set of vectors in the vector space v in two dimension the corresponding vector this will be u+v, this also lies within the vector space v that is the idea which you have.

But let us look at the second property, commutivity or commutate okay, which says that u+v will give the same result if you add the vectors in the reverse order, so it does not matter. In which order you add the vectors, correct very close property with vector addition comes the associativity or it is also called associative property. Let us take three vectors u,v and w then the question is which one do you add first.

Well in this case let us say if you add this to first you get the same results as to be very correct, you should add the corresponding vector sign to indicate at this all vectors, some books might use a bold sign to indicate this vectors. In vector spaces some books on vector spaces we will actually might use at a different symbol or a bold sign to indicate this is a vector and using this arrow at the top to indicate this is a vector.

This is a vector space and okay, so there are few others such properties where axioms as they are called for a vector space I will quickly go through that, okay.

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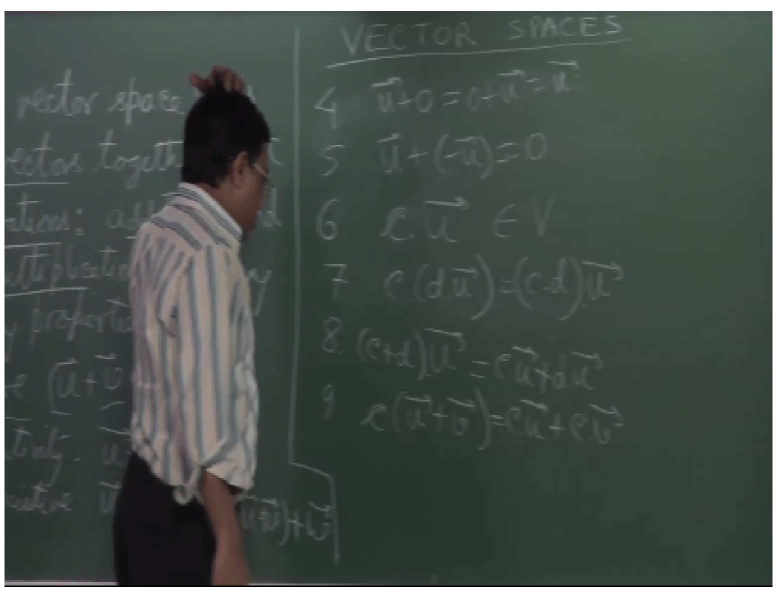

I will write the properties of the equations and just write the terminology associated with it, if this is 0 vector this is a property the 0 vector which basically says that if I add a 0 vector either way you get the same result, this is an element of addition I need an element of addition when you have a 0 vector within a v.

The fifth one is call the inverse elements of addition which says that if you add the inverse of a vector basically add the vector in the reverse direction then you will get a null vector that is 0 vector as it is called, sometimes call a null vector, but 0 vector is probably the base to of addressing that, the null word is use in some other space. Inverse elements of addition okay, these actually will be ones then it is called a scalar if you have a vector u again you must put these vector signs all over.

If you have a scalar value c it is not a vector so I am just putting the vector symbol okay, this is also in v that means we depth a vector u or v like the one which you have seen a w multiplied with a scalar quantity. So far we have used u, v, w as vectors and we are going to use c or some others scalar quantities like c or d or a or b, okay.

So this is also in v, and the number 7 if you take two scalar multiplications this is also equivalent to doing something like this, this is call commutivity powers scalar multiplications okay, this is a symbol scalar multiplication commuitivity of scalar multiplication you have a distributive property also of scalar multiplications with respect to additions so that means if you have a vector u and you multiply try to want to multiply that with a quantity which is an addition of two scalars c+d are scalar numbers like here.

See I am not put the vector sign then this can also be written as, you can also write similarly which is the distributive property over scalar multiplication with respect to addition of vectors this call the distributive property and here it is called distributive property of scalar both are distributive property of scalar multiplication with respect to additions actually the only difference is that we are distributing it over the vector addition here we are talking about multiplication with a vector that is the difference between the two, and the last one which should be also there if you multiplied with a scalar quantity call one and element of scalar multiplication you will get this.

So this ten properties are axioms combined with this definition that you have a set of vectors and there are two valid operations which we can do on any of this vectors in the vector space the fine layer vectors define your vector space you must keep this is mind when you do any of this operations, before going to discussions of more in detail about vector space is which deals with vector subspace let us say this is a concept of span what is the span of a basis vectors.

So subspace basis vectors in span by the basis vectors we will look at a couple of small examples of vector spaces, say examples okay we of course go with numerical examples but the vector space is can be span by even matrices so let an n 2 x 2 matrix given as a b c and d four elements it is all scalar quantities they are all real scalar quantities formed set of all possible matrices in fact possible infinite set of matricides then the all set of matrices put together can form also a vector space. That is also possible which also include the 0 vector 0 matrix this is also valid vector within the vector space.

Another example say pn of all polynomials of degree that must be given greater than less than equal to n then also form a vector space of course set a wall polynomial with degree less than equal to n and also form a vector space of course there are some constant which says that if the addition is define by adding coefficients of certain polynomials at powers and scalar multiplication is multiplying each coefficient by a scalar.

So there are certain condition which has to do with the coefficients of this polynomials and the corresponding power then they can also form but what it basically means is that set of polynomials also can form a vectors space.

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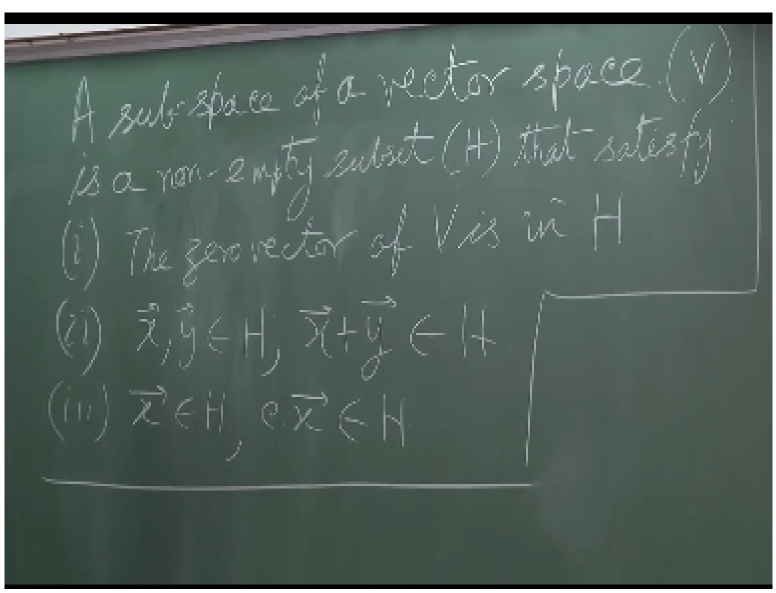

So after introducing the concept of vector spaces we will extend our concepts for that to sub spaces of vectors, so what is sub space is written here a sub space of a vector space v is another non empty sub set h okay it is a non empty sub set h that is satisfy the following property what are they first of all the 0 vector of v is in h so this is 0 vector in the should be part of h is well okay and number two point if x and y belong to the sub space a sub set h okay you are talking of a subset h which actually form the subspace of v okay then the addition of those two vectors also will be in h which is the subset a vectors form in the subspace and if x belongs to the subset h correct then a scalar multiplying c is a scalar constant if you multiply the vector x / s color that also shown in each.

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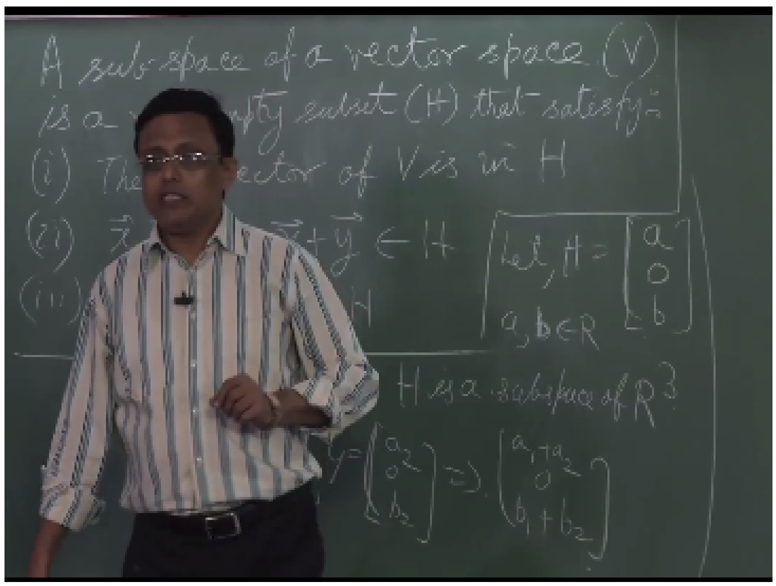

Let us take an example now to understand this so let h be a vector given as say a and b are real numbers okay we have to show that h is a subspace of in three dimension, so the first thing which we can do is verify this three properties with respect to this h so if we take a 0 vector in three dimension that is possible with h because you are just have to set a and b both equal to 0.

Take x and y belong in to h okay this also satisfy because if you just think of two vector say a 10 b1 as here x and let us say y is some other because they align with in h say a2 0 b2 simply what you do addition of this two vectors will give you very simply a1 + a2 0 b1 + b2 which is also within h.

That is very simple define as the second property also holds well this is regal to prove because you just multiplies this by scalars and simply means that. So this is a simple way to show that this forms a subspace of r^3 because in r3 that is in basically incidence and space which I am usually falling here the sub space is in two dimensions because the second dimension will be equal to 0 here that is the true of the component are acting.

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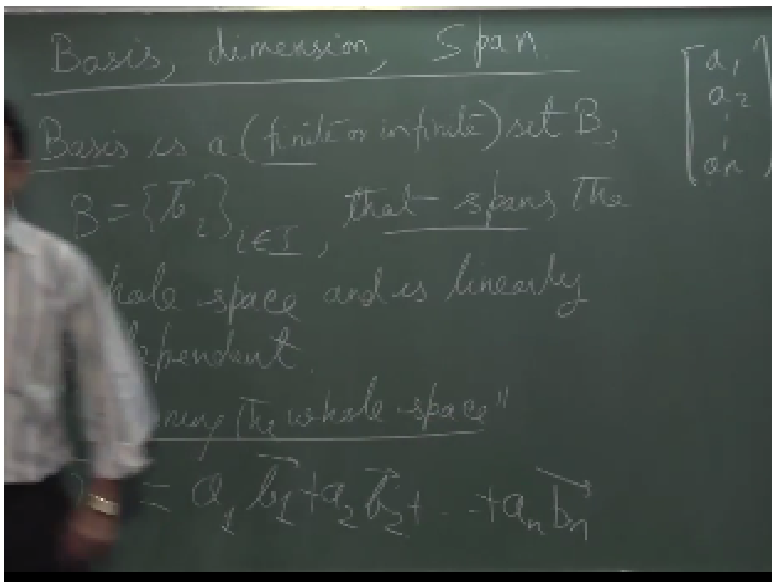

So bashed on this the last concept which we are going to talk about is what you mean by basis vectors and what is its relation with the dimension of a vector space and what you mean by a basis vectors spanning a space or a subspace the relationship between basis dimension and span. Let us define the word basis, basis is a set finite or infinite although we will be mostly worried about or dealing with finite basis set b where b consist of elements no sorry not this way bi where bi are vectors.

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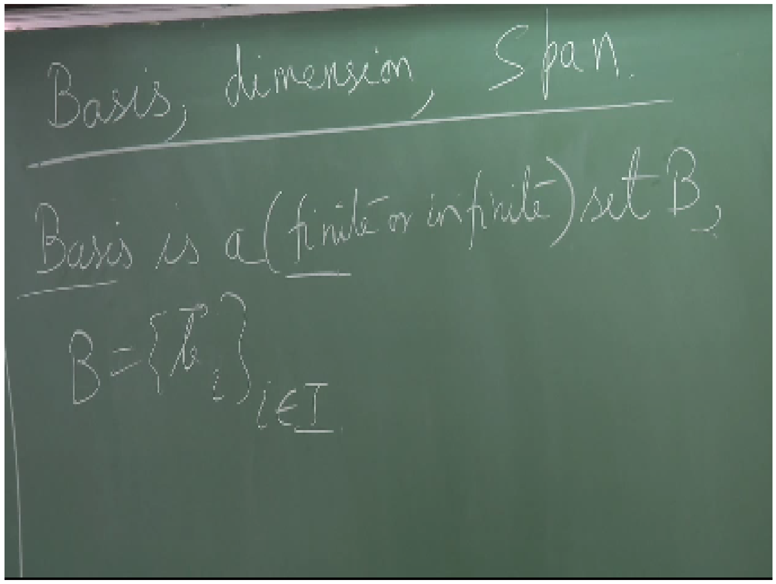

So basis is a finite or infinite set b consisting of the set of vectors with some indices I okay typically taken from a set of integers Bi and typically often indexed by set I sometimes you will just refer that is the first vector second vector by the set of integers here that this that this spans the whole set the whole space that means the set of vectors which form the set b it span the whole space and is linearly independent.

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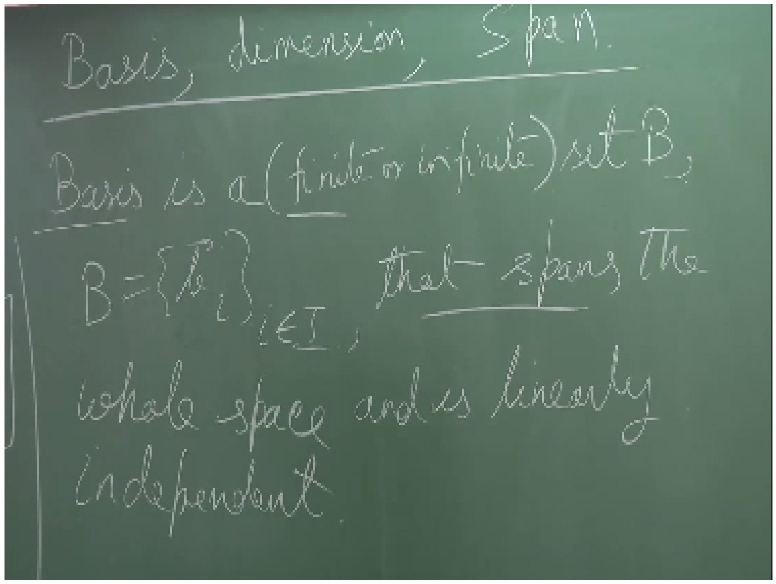

The spanning the whole space this is a term which is casually used spanning the whole space means the falling what does it mean spanning the whole space you must understand this and then this will probably becomes spanning the whole space means that any vector v in the space which we are considering can be represented by ai b1.

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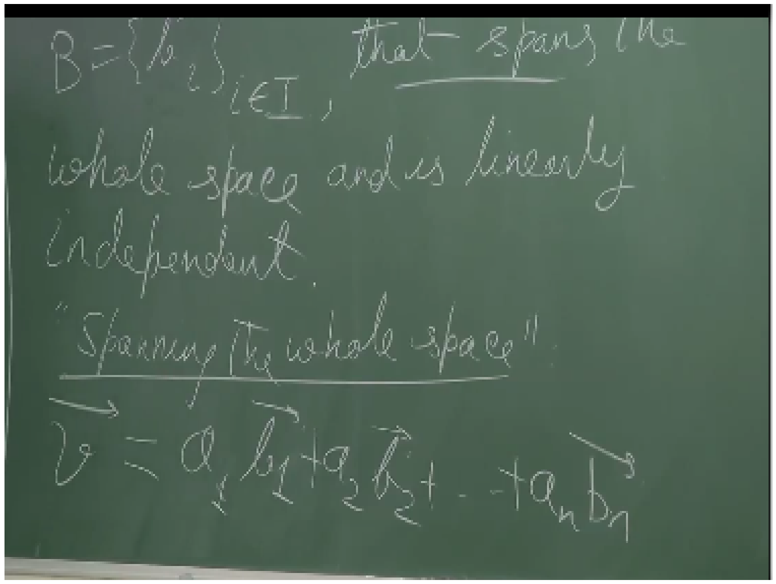

Spanning the whole space means writing a vector v in this space represents the linear weight at some of the basis vectors b1 b2 up to bn each of them multiplied by the corresponding scalar a1 a2 and so on up to an and this is the simply scalars of course there is a constant of linear depends and independency here which will come to that between the basis vectors bi okay.

But this spanning of the whole set means that this combination represents any arbitrary vector v and the given a vector v as long as a unique combination of this ai. We can actually form another vector like this with the coefficients a1 a2 up to an.

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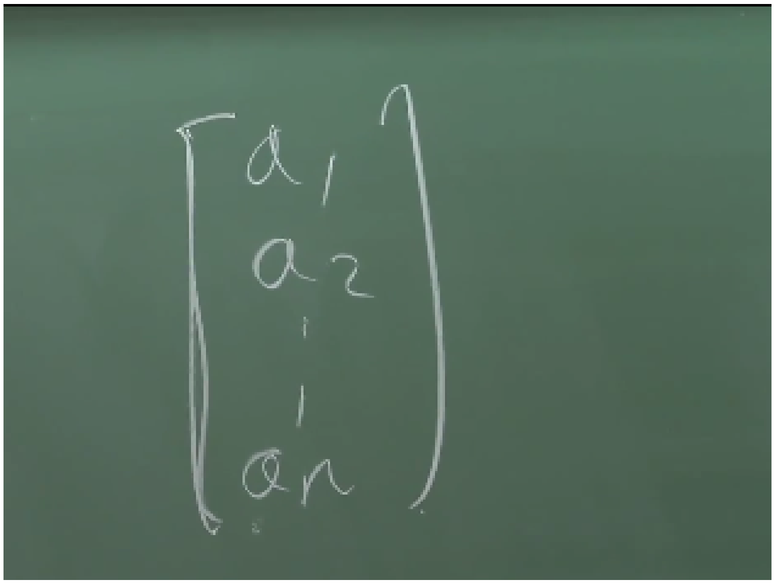

Which is form by the coefficients of these basis vectors as long a this combination of the coefficients are unique then you have linear independents of the basis vectors that means the basis vector can be linearly dependent or independent I will give examples of that using a figure but if there linear independent then I can find a unique set of ai to represent any arbitrary vector v however if the basis vectors are dependent then I can have several combinations a ,non unique set of combinations of this coefficients here to represent the arbitrary vector v here okay.

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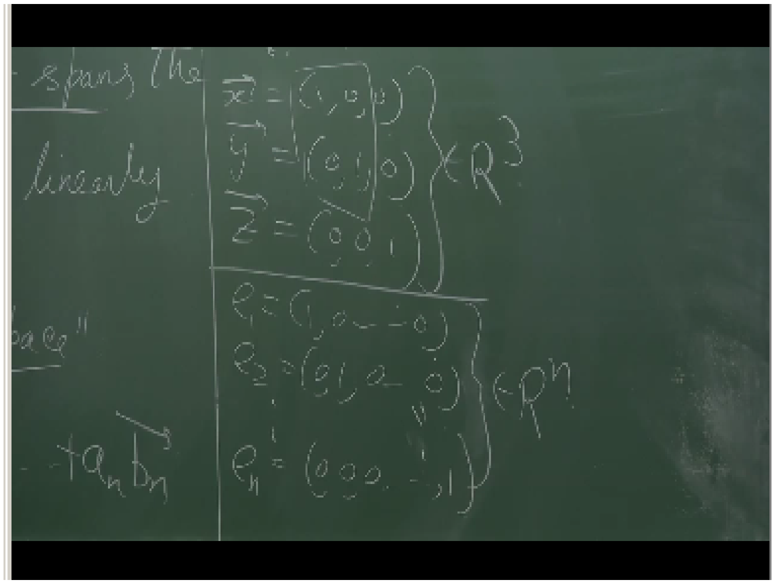

Let us take an example the most simplest case of basic vectors which we have in three dimensional space could be our xyz very simply in three dimension this is corresponding to 3 dimension basic vectors this can be established to n dimensional also saying that each consists of limits a is that elements in each in elements in each.

And look at the if you look at this is matrix then you can identify matrix the diagonal place of rest of all zero these are examples classic examples of linearity independent bases vectors I repeat linearly independent bases vectors which expands the corresponding dimension in this case.

This spans the space of dimension N okay if you just take this combination then you taking of two dimensional space x and y okay and if you want me to write this I will just rub of this example and in this particular form this particular form will indicate that as if I have something like the components of the as v1, v2 and we want v2 as shown up to vn with a component of n dimensional vector and I can write this equal to say A1 and 100 last A2.

So on which is wrong b1, b2, bn are replaced by this corresponding vectors e1,e2,en which I just wrote and the same this are linear if I take arbitrary vector v with corresponding components I need to have unique set of Ai s which will represent this vector however in many of case where if I do not have linear independent in the bases vectors of Bi s or as given.

Here these are the cases where there independent means I cannot represent any bases vector a linear combination of the others anyone cannot be represent by the others then this will be linked and there will be cases when they are not linearly independent that means they are linearity dependent rather than being orthogonal like this then I will not have unique I have A is to represent this vector I will show stop with one example in 2 d.

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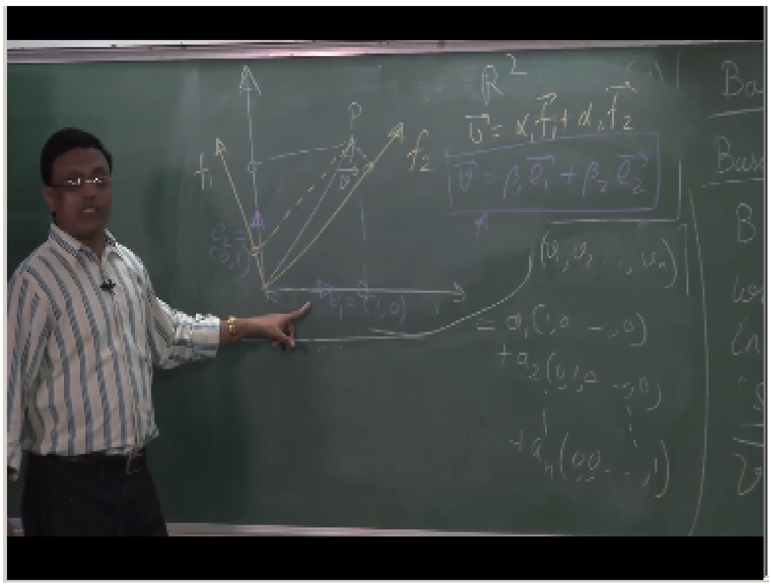

In 2 dimensional space x, y these are my e1,e2 so this is a 1,0 two dimensional spacing then I have e2=0,1 now I take another set let us say so with this what it basically says if I have a vector v and I want to represent this vector v with two components v1,v2 as a linear combination of e1 and e2 are to projected along both the dimensions and find out the length of the components along e1 and e2 and that will give me my corresponding coefficients which will be unique.

This is that I will not have any 2 components and plugged into where I represent that vector that is not possible if I do not have linear independents that means the test are linear not independent of another that means it is something like a drawing let us say here is one circuit here and there is another vector here okay which I write as well I write this f1 and this is f2 like you had e1 and e2 you have f1 and f2 okay.

And you follow the same principle basically what we need to do is this is the point p which represented vector and as you projected along e1 and e2 you need to do the same thing so the project on left you have to travel along orthogonal to this so I will use the same color and to project along this you have to travel along this direction here this is also represented here okay but is only a unique representation why we will not be a unique representation because the two components f1 and f2 are not orthogonal to each other that is number 1 that means you can represent f1 in terms of f2 actually or wise versa.

And so you will have many combinations possible actually what you do now is if you can represent f1 is the function of f2 they are able to represent v as a function of only f1, f2 by itself you do not need both okay and that shows the reason of the linear indecencies here of x and y if you take this to be two axis e1 and e2 with respect to f1 and f2 so this v is some $\alpha 1f1+\alpha 2f2$ versus if you write the other one which is $v=\beta 1e1+\beta 2e2$ this one is a unique representation with $\beta 1,\beta 2$.

Because the e1, e2 actually are linear independent bases vectors spanning the two dimensional space right this is not but you can represent them because represent by non orthogonal set f1, f2 which are not linearly independent so that is also possible in most representations top of this support and this bases and in that particular case we will say that e1 and e2 spans the entire sub spaces into dimensions.

That means all vectors v in two dimensional space can be represented by the linear combination like this on the two bases vector e1 and e2 okay so that is what is the mean so this two dimensional sub space can be visualized is a sub space in three dimension and three dimension for certain higher dimensions so we will stop here with this and hop this knowledge helps you to understand principles of pattern classification and technician.

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