Computer Graphics
Prof. Sukhendu Das
Dept. of Computer Science and Engineering
Indian Institute of Technology, Madras
Lecture - 20
Clipping: Lines and Polygons

Hello and welcome everybody to the lecture series on computer graphics. Today we start the concept of clipping lines and polygons and this is an important concept which is necessary in the areas of 2D viewing and 3D viewing. So we will discuss clipping lines and polygons.

We will first understand the concept of clipping lines because this was an important aspect which we saw in the case of 3D viewing when we needed to clip lines as well as polygons with respect to a two dimensional view port or a three dimensional Canonical View Volume or CVV. I hope you remember this when we studied three dimensional viewing. So that section was not covered during that period and we are going to understand the concept of clipping today. Let us see and understand using the picture in the next slide about what you mean by clipping.

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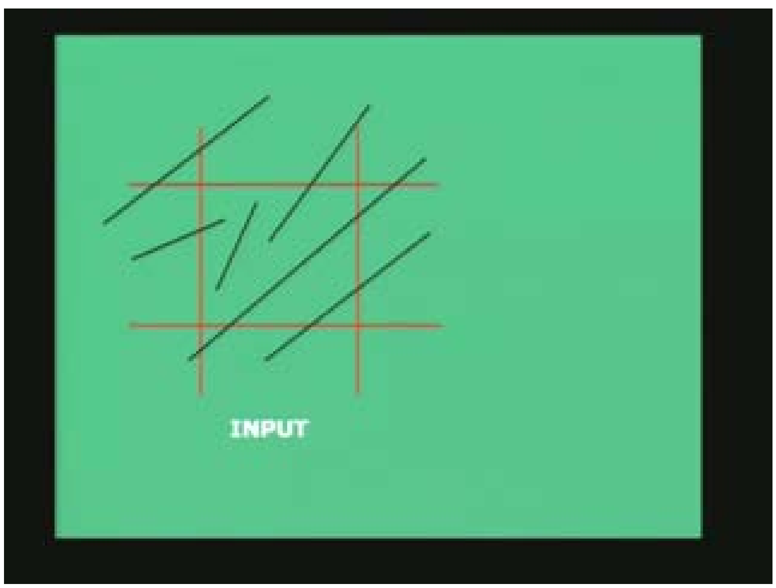

This is a two dimensional view port. There are set of lines in a two dimensional screen, the black lines which you can see and I would like to clip them in a respective rectangle which are defined by a pair of horizontal and vertical lines.

The pair of vertical and horizontal lines intercept at four vertices to give us a rectangle and that is our clipping rectangle. So the input to the algorithm are a set of arbitrary two dimensional lines could be drawn by Bresenham's line drawing algorithm and also an input clipping rectangle defined by a pair of X_{min} X_{max} Y_{min} Y_{max} coordinates which define this pair of vertical and horizontal lines defining the clipping rectangle for us. We can always define rectangle with the top right and bottom left coordinates, XY coordinates that gives us the rectangle. So we will use this rectangle to clip a set of lines in 2D and we expect this output to provide us a scenario like this.

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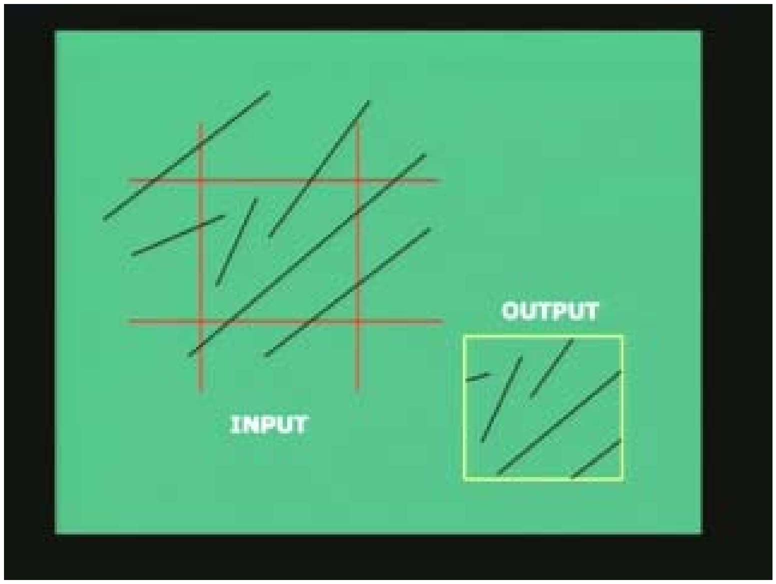

If you look at the screen now the output should be the portion of the lines or part of the lines which exist inside the clipping rectangle as well as the entire line if the case may be is inside then you should not clip it at all. Lines which stay outside completely of the clipping rectangle should be eliminated. If you look at the line to the top left of your screen as pointed by the pointer in the screen that line is completely left alone as it is outside the enclosed rectangle or the clipping rectangle to be very precise.

If you look at the line which is completely inside, there is one line which is completely inside as shown by this pointer, it is inside the clipping rectangle that should be left intact. If you see all other lines, part of the lines are clipped from the left, part to the right, part in the middle. So it depends upon what portion of the line which is inside the clipping rectangle should be kept intact. If the line is completely within the clipping rectangle it should be left untouched or unaltered. Whereas if the line is completely outside the clipping rectangle then of course you do not include any part of it, you do not have to clip that line at all.

We will discuss three different algorithms in this lecture on clipping and whatever algorithm we are going to study now will be involved in presentation of a line because you need to represent the lines in some form. Typically we will see that the parametric form is the one which is used.

We have already seen the case of a parametric representation of a line in the previous lectures and we will use that representation here today. And next we need to find out if the line is completely within the rectangle or outside the rectangle or if there is a part of the line which is inside and a part outside the rectangle. So there could be three different possibilities, three or four in fact one completely inside one completely outside, a part was inside which could be one end or it could be a certain middle section of a line which is inside the clipping rectangle.

So we have to find out the position of a line with respect to a clipped rectangle in two dimension and if it is completely inside you leave it untouched because all the points are passed on to the output.

If the line is outside the clipping rectangle never include it within your clipped rectangle and pass it out. Do not pass it to the output and the third and the fourth cases are, we have to find out which portion of the line is within the rectangle, is it a central middle part of the line or it is one end of the line, if you can see different cases of the lines in the picture we have seen, depending upon that we need to find out which portion of the line is viewable within the clipping rectangle and that is one we should show in the output. So come back and look into the figure where you see that you need to provide an output as given in the right bottom of your screen corresponding to an input like this.

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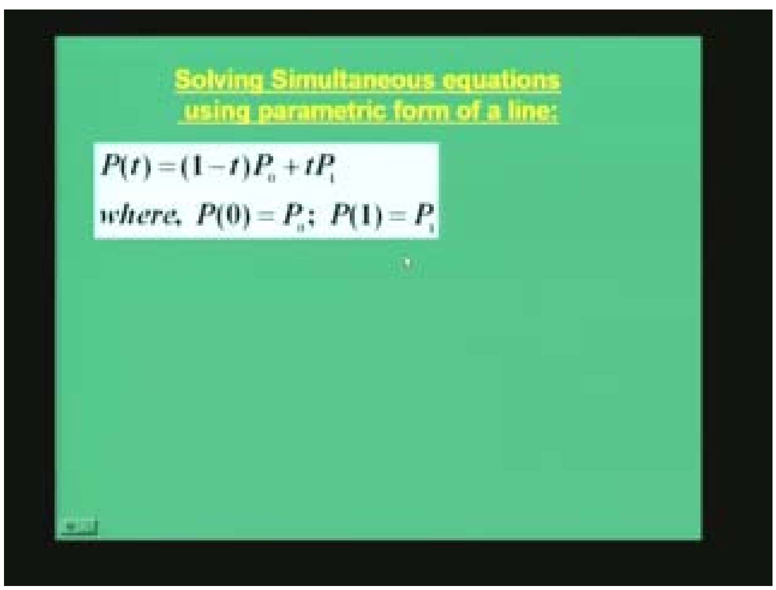

We will go through some theoretical foundations about parametric form of a line. Once again this is not new to you because you have used this in various forms in different lectures prior to this concept of clipping. You have used the parametric representation of a line earlier and now we look for a scenario where you try to solve simultaneous equations using a parametric form of a line. Now remember, when you talk of solving a set of simultaneous equations of a line. We have done this in the case of intersecting lines all the time in the course or in mathematics where you have a line Y equal to say m1 X plus c1.

You have another line Y is equal to m2 X plus c2. And when you solve this set of simultaneous equations representing this line you basically obtain the intersection point or the coordinates of the intersection point. That is what basically you do by solving simultaneous equations. Basically what you do is find out the intersection coordinates of a pair of lines. We do that using the representation of Y equals m X plus b the standard and very popular and common representation of a line.

We will try to do this using the parametric form of a line and we will see why this is useful here. So go back and let us look at the screen where you remember a point runs from P(0) to a point P_1. The two end points of the line are specified by P_0 and P_1. And at P_0 the parameter t is equal to 0, at the end point P_1. The parameter t is equal to 1 and the equation of the line can be given as this 1 minus t times P_0 plus t times P_1.

We know that, we use this equation that means as you vary t slowly from 0 to 1 when you vary t from 0 to 1 you are basically moving in the line from a point P_0 to the point P_1 by varying t. At t is equal to 0 you are at P_0 at t is equal to 1 you are at P_1. And for any value of t between 0 and 1 you are somewhere between P_0 and P_1 so we know that. That is the explanation again; you have seen it already earlier in previous lectures about parametric representation of a line. We will use it now again for solving simultaneous equations. Let us look at this picture now which is coming up on the right side of your screen.

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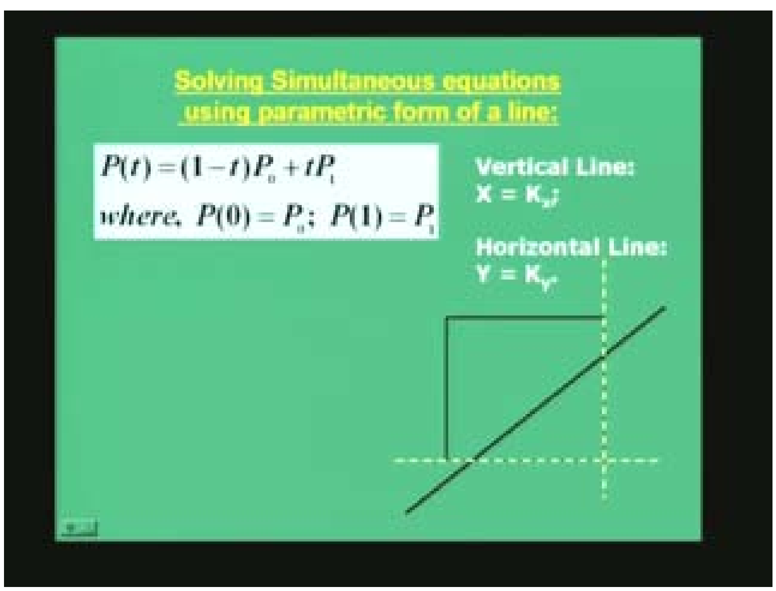

You are interested to clip a line represented by the parametric form with this rectangle. And what you are interested now is, let us say this line intercepts the bottom line of the rectangle and the right line of the rectangle.

Rectangle is clipped by four pair of lines, we already know that. So we have Y_{max} Y_{min} X_{max} X_{min}, the four coordinates which are enough for you to specify the clipping rectangle. And in this case we are assuming that the line is getting clipped by the bottom line of the clipping rectangle and the right hand side vertical line of the clipping rectangle. So the vertical line equation is X equals K_x here the vertical on the right hand side and the horizontal bottom line is given by Y equals K_y that is the horizontal line.

So you are interested to basically find out what are the intersection points, you know you have a pair of intersection points with this parametric line with these two pairs of horizontal and vertical lines. You have this vertical line X equal to $K_{x,}$ you have the horizontal line Y is equal K_y and each of them will give an intersection with the line. So that is in it. When you find out this t you can calculate the two points where you have to clip the line. So what you basically do? Try to solve a a pair of equations using the parametric form basically is as simple as substituting this value of X equal to K_x and Y equal to K_y on to this parametric form.

Remember, P_0 and P_1 of the parametric form of a line are vectors in 2D with X and Y coordinates. So you need to substitute X equal to K_x and Y equals K_y on to the parametric form to get the values of t and you solve with respective pairs.

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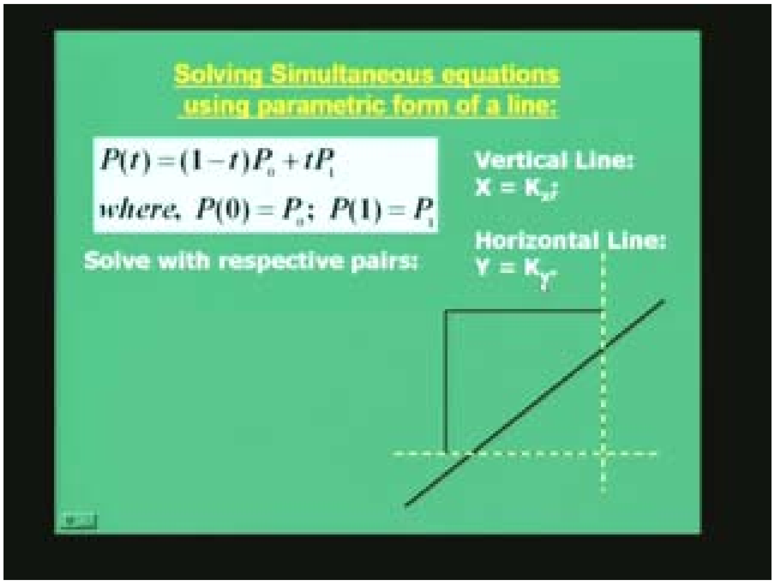

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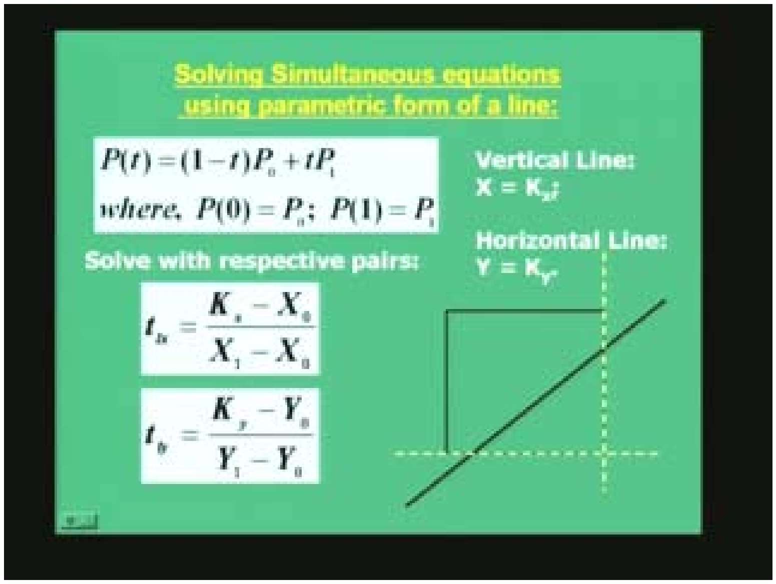

And the first solution between P(t) and X equal to K_x will give you a solution of l of x where l is the line and the parameter which we get for the intersection of this line with the vertical line X equals K_x will be given by this expression on the top of your screen that is K_x minus X_0/X_1 minus X_0 respectively and that the intersection between the line and the horizontal line Y equals K_y.

The line of interest which will be clipped and the horizontal line Y equals K_y is given by this $t_{ly.}$ So this is how you can easily solve for the value of the parameter t which will help to find out because now you can use this t and substitute back into the expression of P(t) for the line and find out the exact X and Y coordinates. Of course in one case the X is known and the other case the Y is known, anyhow because you are only clipping or finding out intersections with horizontal or vertical lines.

So, one of the parameters are anyway known to be used. Straight away you need to find out the other value. If it is the case of a vertical line which is X equals say K_x you need to find out the y coordinate of the intercept.

If it is a horizontal line which is Y equals K_y you need to find out the intersection of that line with the line to be clipped. And for that you need to find out the X coordinate because Y equals K_y is the equation of that horizontal line. So substitute these values of t on to P(t) and get those other X and Y coordinates and those will give you the coordinates of the intersection of the line with the horizontal and vertical axis.

I hope this concept is clear of how to obtain the t by solving simultaneous equations. Actually you can do it for any two arbitrary equations. If you have two lines given in the parametric form P_1(t) and another is in the case of P_2(t) you can actually use the same concept to obtain the t for both of these lines where it will intersect and obtain the corresponding coordinates, so that is also possible.

We will see that and if that is necessary we will use it later on. But right now this equation is very simple because when you are solving for simultaneous equations trying to solve for the intersection point using simultaneous pair of equations, one of the equation line is very simple because it is either a horizontal line or a vertical line which is nothing but the edge of the clipped rectangle.

So if it is the edge of the clipped rectangle one of the equations become very easy. Substitute back in to the P(t) and get the t value, substitute the t back and get your X and Y coordinates of intersection. So, if you look back this is one method by which we can obtain the t values and get the intersection of t. So keep this in mind. Whenever we say that obtain the intersection of the line with a horizontal or a vertical edge. In this case I have shown these two but it could be the left vertical or the top horizontal edge.

Also, if necessary we can do that but in this case the line intersects only with the bottom horizontal and the right vertical lines. And that is why we are interested to get the parameters of this line and that will in turn help us to obtain the coordinates of intersection. So keep in mind that whenever I say henceforth, you use the parametric form to solve the intersection of the edge of a rectangle with the line this is what I mean and this is how you obtain t using t substitute back into P(t) and get your X Y coordinates.

I hope this concept is very clear.

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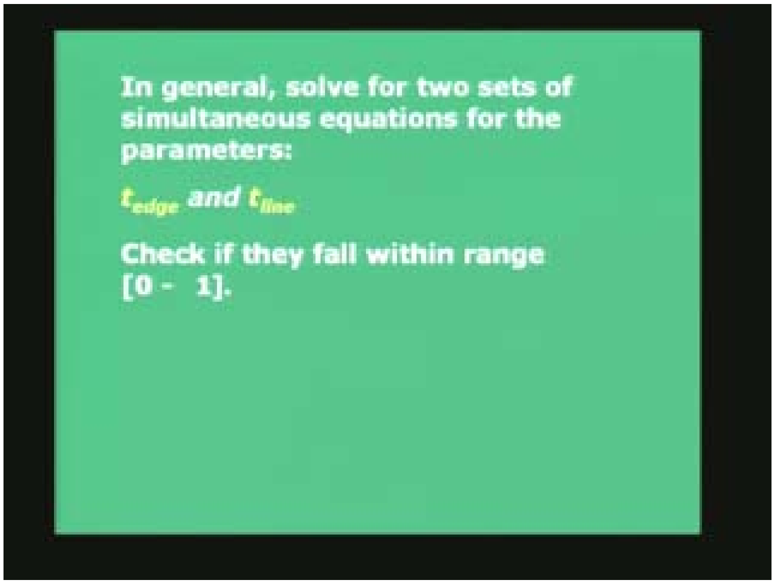

So let us move ahead to see a little bit more where we say in general we solve for two sets of simultaneous equations for the parameters t of edge and t of the line.

While I was talking about this but one of the line is anyway a very special case either it is a horizontal line or a vertical line. But each of the lines will have its own individual parameter t. So when I talk of t of edge I am talking of the parameter of the edge of the rectangle which is either the horizontal or the vertical line and the t line talks about the parameter of the line itself which is getting clipped.

So, in general you solve for two sets of simultaneous equations for the parameters t of edge on the edge of the rectangle and t of line for the line and once you can get this you basically have to check if they fall within the range 0 to 1. Why do you have to check if they fall within the range 0 to 1? Well the concept is very simple because if the value of t after solution can be anything but if it is less than 0 or more than 1 it typically means the following.

What does it mean? It means that the intersection point for that particular line is lying outside the range P_0 to P_1 and we are not interested if the intersection point is basically on the line which is extrapolated. It is possible that you know, you can have two finite lines and you can extrapolate them and they also intercept. But we are not interested in such an intersection where you have two sets of lines and the finite lines from P_0 to P_1 and P_2 to P_3, let us say you extrapolate them and they meet at a particular value of t. But if that value of t is more than 1 or less than 0 we are not interested to use that intersection point.

We are actually interested to find out if these two finite line segments actually intersect physically within the finite range of the values from P_0 to P_1 or in other words the t value which you get lies between 0 and 1 because you know that when t varies from 0 to 1, we are talking of points between P_0 and P_1 only. If it is less than 0 or more than 1 we are talking of an extrapolated line, line can be extrapolated to infinity, we all know that in two dimension or even in three dimension.

This parametric representation of a line can actually be extended to the concept of 3D where you actually clip a line with respect to a three dimensional surface. And of course the three dimensional surface if the equation is very simple in the form X equal to a constant or Y is equal to a constant you can visualize yourself that the intersection value can be obtained easily and also the coordinates very easily thereafter. So we are talking in 2D with intersection of two lines or in 3D where a line intersects a surface whatever the case may be in 2D or 3D we are actually intended to find out that in 2D there are two finite line segments and in 3D you have a finite surface basically a bounding rectangle again because it is a bounding volume.

In case of 2D you have 2D view port. And in the 3D what did you have? In 3D viewing when we use clipping we were clipping with respect to CVV or the Canonical View Volume. I hope you remember that.

So you have a finite volume there and you are interested to clip a line or with respect to that finite small canonical view volume and that is our main intention. So either 2D or 3D you have to visualize that you can use this method of solving simultaneous equations with the help of these parameters and that is not a problem. But once you get these values of t you need to find out whether the value of t actually lies between 0 and 1.

If it is outside that means less than 0 or more than 1, the finite line segments in 2D or the surface and the line segment in 3D are actually not physically intersecting they are extended because the plane in 3D and a line in 2D or 3D can be extended to infinity. Extrapolated segments of these lines do intersect that will always intersect unless the line is parallel, we know that.

Only two parallel lines will not intersect otherwise any non-parallel lines in 2D or in 3D a surface and a line as long as they are not in parallel they will always intersect. But that is not the key idea, we are interested only in finite line segments and finite surfaces and we are interested to find out whether those finite segments intercept surface or line in 3D and or two lines in 2D. And if those finite line segments do intercept, the values of the t which we will get will always lie between 0 and 1, you can visualize that very easily. So find out this t edge and t line check if both of them lie between 0 and 1 if not we do not take that intersection.

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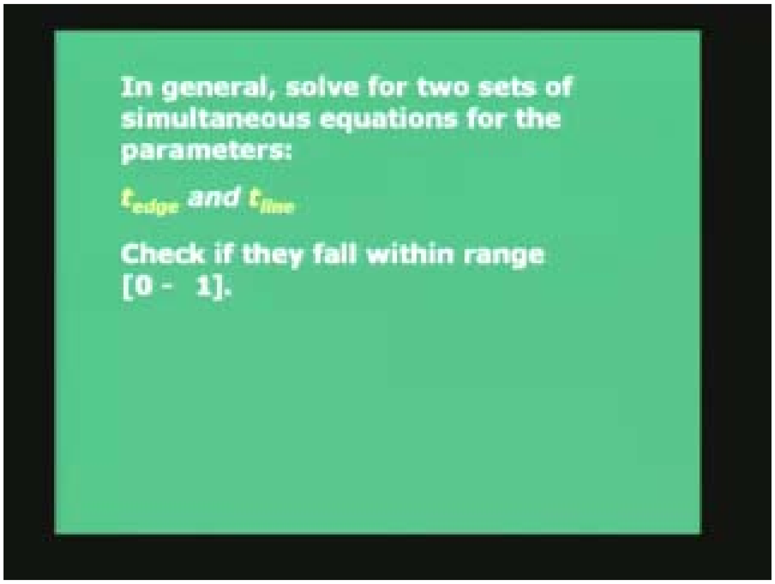

So we go back to that picture once again. In the slide we have you solved for two sets of simultaneous equations for the parameters t edge and line t line. We hope it is clear now, I hope that you have understood what t edge and t line is and the corresponding t values which you get by solving these simultaneous equations and then you have to check if they fall in the range 0 to 1.

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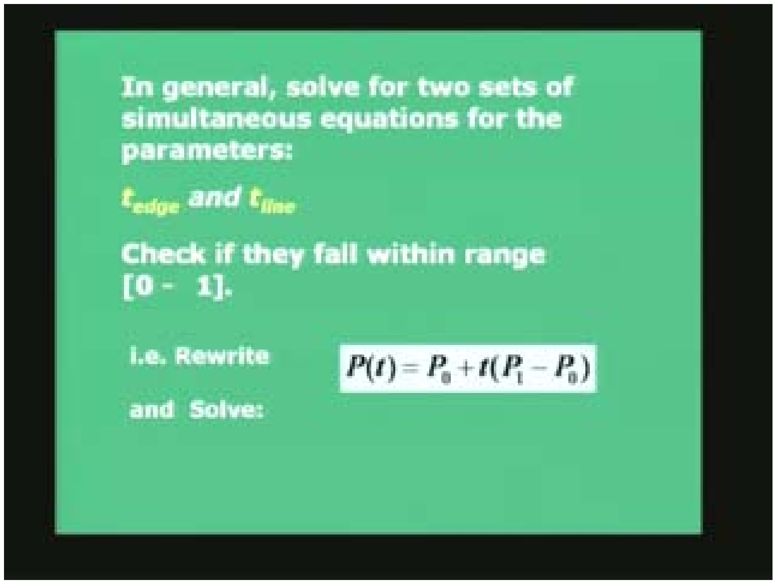

Look at this solution; we rewrite the equation of that P(t) the parametric form. I hope you have noted down the form which was given in the previous slide. It is almost similar, but rearrange the terms and write it in this form where you can see this form very easily, the t varying from 0 to 1 will make the P(t) run from P_0 to P_1.

We substitute t is equal to 0 here. We can see that P(t) is at P_0, if you substitute t is equal to 1 the P not cancels out and you are left with P_1. That is the same form so we are rewriting that same parametric form in this and you solve.

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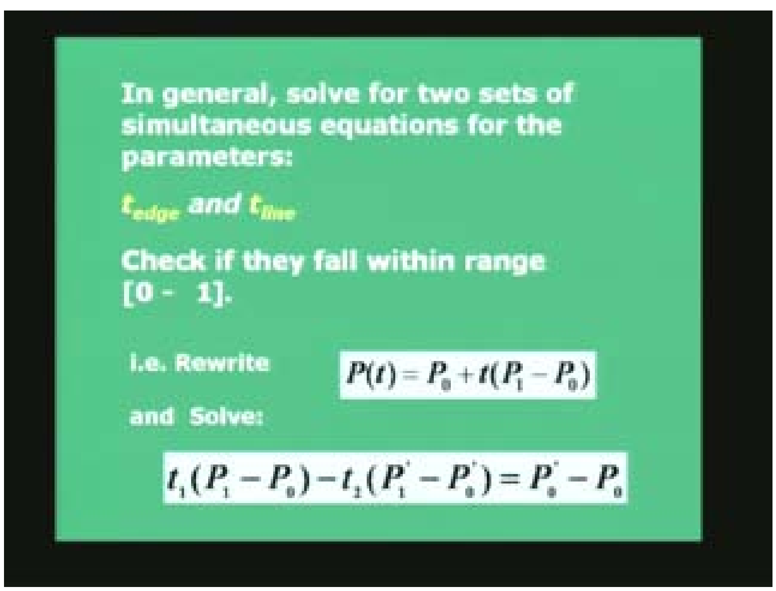

If you are having two lines where one runs from P_0 to P_1 and the other runs from P_0 prime to P_1 prime here we are talking of intersection of two lines. So there must be two pairs or four values of these parameters of the line which define the starting point and the ending point. So since there are two lines there should be two starting points and two ending points of a line.

So we start at P_0 end at P_1 is one line. The other line is at P_0 prime and it ends at P1 prime. So you can write the two equations for these pairs of lines and subtract one from the other you should be able to get the value of t. We are able to get the values of t and remember since this is a vector, now you actually get two equations from this set at the bottom of the screen. And using this pair of equations which you get you have two unknowns t_1 and t_2. You solve for this t_1 t_2, we basically can give t edge and t line if the lines under consideration are the edge of a clipped rectangle and the line typically.

That is the case. So you have P_0 P_1 for one line and P_0 prime and P_1 prime for the other one. So use this equation to solve for t_1 and t_2. That is the general form of the solutions to the simultaneous equations using the parametric form. This is something probably little bit new than what you have been using earlier as Y equal to mX plus b to solve for a line. But you can always use this.

Now let us go to the first algorithm for line clipping which is the Cyrus-Beck algorithm for line clipping.

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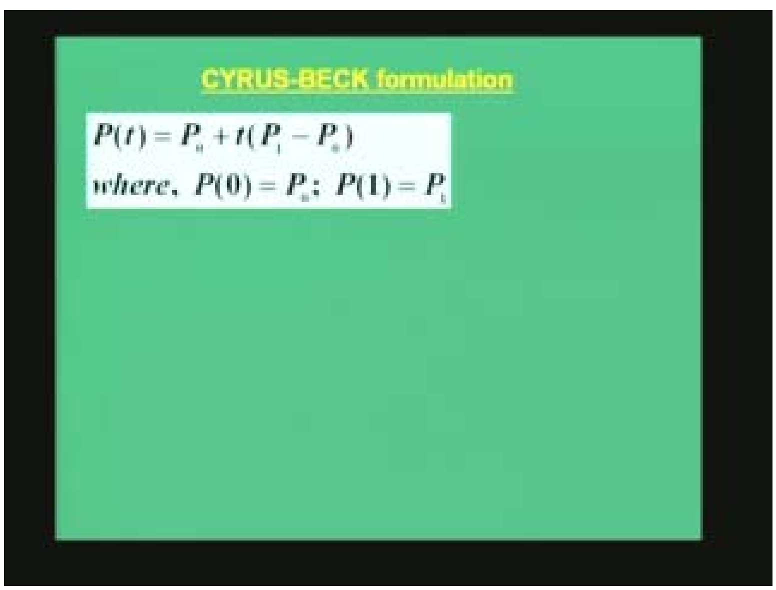

It uses the parametric form; most of them use the parametric form of line. So do not worry about that. So this was the representation, again it is repeated in the slide, parametric form of a line.

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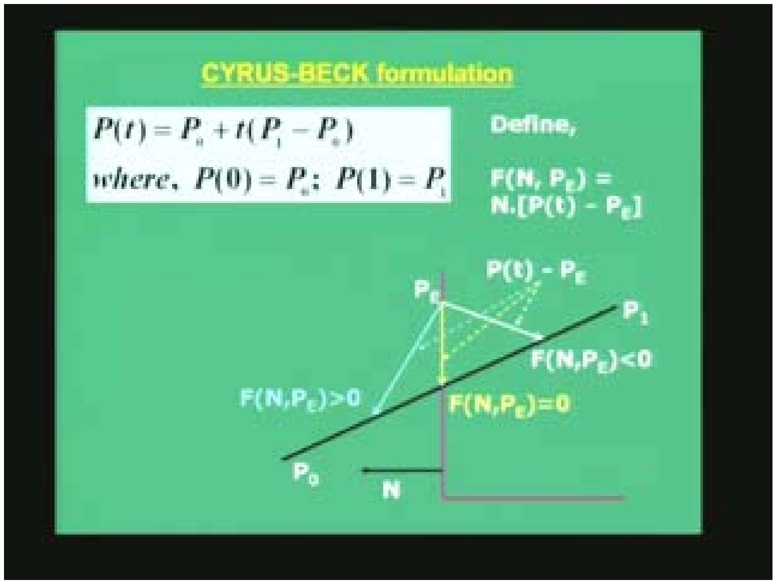

And let us look at this diagram to find out the mathematical foundation and concept of the Cyrus-Beck formulation for clipping a line.

Well, I am interested to keep this line which runs from point P_0 to this point P_1 on the top right which is P_1 and that is the end point and bottom left for P_0 that is the line which I am interested to clip. And we assume for the time being that the line intercepts only the left vertical edge of the clipping rectangle. It can intersect with any one of the four or in fact two of the four but let us say for the time being it intercepts only the left vertical, it can intercept any and for each you have to do this mathematical operation.

But let us start with any intersection edge. So we are talking about the left vertical edge of the clipped rectangle. In fact left and the bottom part of the clipped rectangle is shown in this pink colored line and the black line P_0 to P_1 is the one which is supposed to be clipped and it should be left only with the line from the vertical edge point here to the point 1. Well there are lots of notations here, first of all we know the parametric form of a line that is given in the top left. You define a mathematical relation a function of N and P_E where I will tell you what are these N and $P_{E.}$

N is the vector which is perpendicular to the edge being considered for clipping. Right now, since we are talking about the left vertical edge for clipping so the N will be pointing towards the left hand side. So basically in XY domain if X is going from left to right Y is going from top to bottom of your screen and the origin of the coordinate system is at the bottom left of your screen this vector N in 2D will be minus 1, 0. Just for those who understand we will be able to follow minus 1, 0 which is the vector N. So you are interested to find out, this function if you see N, P_E is defined as the dot product of this normal N with a vector P(t) minus $P_{E.}$

Now what is this P? That is of course one I should explain. But before that let us see this function F, it is the dot product of this N, now you know what this N is, it is the vector which is perpendicular to the edge which is used for clipping, which is clipping the line P_0 P_1 that is very clear I hope. And this is dot product of N with P(t) minus $P_{E.}$ First what is P_E subscript E because P(t) is known, that is nothing but the vector which points to any particular point on the line based on the value of t.

When t is equal to 0 it is P_0 when t is equal to 1 it is P_1 we know that. That is P(t). P_E is any arbitrary point, actually you can choose any arbitrary point on the vertical edge which is being used for clipping or which is under consideration because the line can intersect more than one edge but we will consider one edge after another. So in this case we are talking of the vertical edge, the left vertical edge so when we are considering that a point P is an arbitrary point on the vertical edge.

So let us go back to the slide, this is $P_{E,}$ this is the point which is marked as $P_{E,}$ you can take any other arbitrary point. So I have chosen this point here on the left vertical edge and since the equation of this vertical edge is some X equal to kX. So it is absolutely no problem, you just choose an arbitrary Y and that is the coordinate of N.

So, if that is P you should not be able to visualize what this vector Pt minus P_E is. Remember Pt is a vector which is pointing to any point on the line, P_E is a vector which is a point on the vertical edge and the subtraction of the two Pt minus $P_{E,}$ I have shown three different vectors with three different colors if this is Pt pointing here then the Pt minus P_E is this light blue color line. If Pt is lying on the vertical edge then this downward pointing yellow colored vector is your Pt minus $P_{E.}$ You see this dashed line shows what this vector is.

If the point is somewhere here you have Pt minus P_E as your white line. So this example shows what is going to be your Pt minus P_E vector for three different values of Pt. Pt is a vector which points to any point on the line and P_E is another point on the vertical edge. So Pt minus P_E vector will be the difference between these two positions. And for three different t values, for three different points on the line P_0 P_1 I have picked up three arbitrary points in fact one of them is on the vertical edge.

I have shown these three different cases of Pt minus P_E for you to understand. And I have used three different colors to show three different variants or examples of Pt minus P_E for three different points. I hope the point is very clear. There are three different examples of Pt minus P_E and you are interested to find out or given an arbitrary point P_E and the N is also clear the function of N dot P_E is the dot product of this N with a Pt minus $P_{E.}$ Of course it is also a function of t in something. It is a function of N, function of P_E and of course also a function of t which is not given inside the function but in literature it is called of as a function of N dot $P_{E.}$

Now, if you see for these three different cases of these points on the lines one point is outside the clipping rectangle, one is actually on the vertical edge of the rectangle which is used for clipping and another is completely inside.

We assume that one point is inside which is pointed by the white line and for each of these three points the function value is shown. One is negative because you see it is a dot product, so dot product of Pt minus P_E with N is positive with the point when it is outside, it is 0 if the point is on the edge of the clip rectangle because you can see this yellow vector which is going down Pt minus P_E for us

If this is your Pt then this vertical edge is your Pt minus P_E and it is orthogonal to N. And you know that the dot product of two orthogonal vectors is equal to 0 you know that dot product. So Pt minus P_E and Pn orthogonal vectors and when you have that the dot product is equal to 0. That is why the functional value is equal to 0 for the point here. And in this case if you see, the dot product is negative, why? It is because they are pointing away from one another. N is looking towards the left Pt minus P_E is looking to the right, the dot product when you take will be a negative point.

I hope the concept is now clear with the help of this figure. Three different cases of this functional value which we have defined are F N, P_E is a function of the normal N and the normal to the edge which is used for clipping. Then of course the $P_{E,}$ now remember we can actually change $P_{E.}$ I advise you to change the value for this figure P_E because remember P_E is at any point on the vertical edge so change its position that is the Y coordinate.

Change that position and try to draw the vectors yourself, what you will find, when I am looking at the sign of this functional value F N, P_E whether it is positive or negative or 0 that will not change. The dot product value might change depending upon the P_E value which I am asking you to change and draw these vectors once again for these three cases. But remember the sign will never change.

The dot product of two vectors looking towards the same direction is positive. If they are orthogonal it is 0 and if they are pointing towards the other side it is negative. So we know these are the dot products. So we are looking at the sign of the dot product in fact is what we will try to look and these are the cases. These negative 0 and positive signs will not change for this functional value when you change your P of E.

You can look into the picture once again and see that this is what is going to happen and so what we say is solve for t. Now solve for t using this particular expression which is F of N minus P_E because it is going to be the dot product and when the dot product is equal to 0 that is N of P, we are talking of the intersection of this line P_0 P_1 with the vertical edge and that will give us the point of intersection and that is given in two lines and we actually know how to get the intersections. But in this case you can use this dot product itself and equate it to 0 and at the point of intersection this dot product always will be equal to 0.

We have already seen this yellow color vector and the yellow color expression F of N P_E equal to 0, so substitute N P_E which is nothing but the same expression here and the value of the dot product is 0. Again extending this concept here we were looking into the same figure, three different vectors, the dot product is negative for a point, if it is outside the clipped rectangle.

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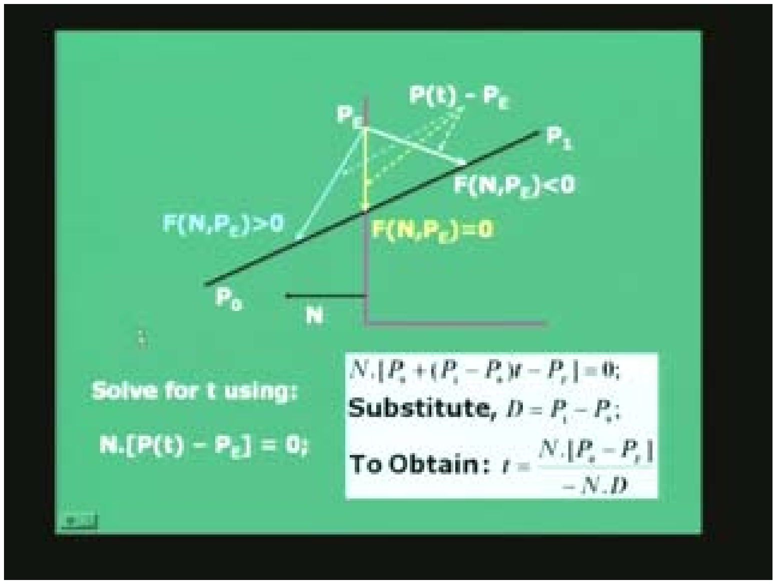

If the point is on the edge of the clipped rectangle the functional value is equal to 0 and if the point is within the clipped rectangle the value is negative.

So, if you remember these three criteria, something similar to what we had for a decision variable in the case of Bresenham's we had the D sign changing from positive, negative and at the midpoint was equal to 0, so almost conceptually the same but only the functional value is different. That was integer arithmetic in Bresenham's but this is a dot product of two vectors but again the sign is important.

We are talking of positive outside, 0 at the point of intersection and negative inside. So you remember that, we are saying that use the value of the function when the dot product is equal to 0 to solve for t.

So if you substitute N dot, Pt is this, you remember the expression of Pt? Please go ahead and substitute and you will find that this is the expression which you get, Pt minus P_E is equal to 0, that is what you get for this one N dot and what we do is, you introduce a variable D it is P_1 minus P_0 and you obtain the expression of the parameter t as this, that is very easy for you to substitute and get. You can easily get this t from substituting D on to the expression given here which is N dot Pt minus P_E equal to 0.

So this is your t value. This is at the t value which gives the t for that line to be clipped. What is the value of t at which the vertical edge of the clipped rectangle exactly intercepts that line? So exactly at that point where it clips the vertical edge and we have a P_0 P_1 line exactly at the point where it clips we have the parameter t obtained using this. So that is the expression of t you get, just remember this expression which will come back again for us N dot P_0 minus P_E N dot D. Let us ensure that all the parameters are known to us. N is known because the vertical edge so the vector normal to that edge is also known to us.

In this case the example is minus 1, 0, D is also known because P_0 and P_1 the endpoints of the line are known. So D is known, N dot D can be obtained, P_0 is known. P, you choose any arbitrary point on the vertical edge so that is also known. Since the vertical edge expression is known you just choose an arbitrary Y coordinate and P can be chosen very arbitrarily. It is interesting to note, that is what I asked you sometime back when I was talking with respect to the previous slide, please vary P and draw these three vectors yourself to have a feeling about this dot product being negative, positive and also at 0 at the exact point of intersection. So P is an arbitrary value, P_0 P_1 is given, hence D is given, N is given and you should be able to obtain the value of t exactly. So let us move ahead.

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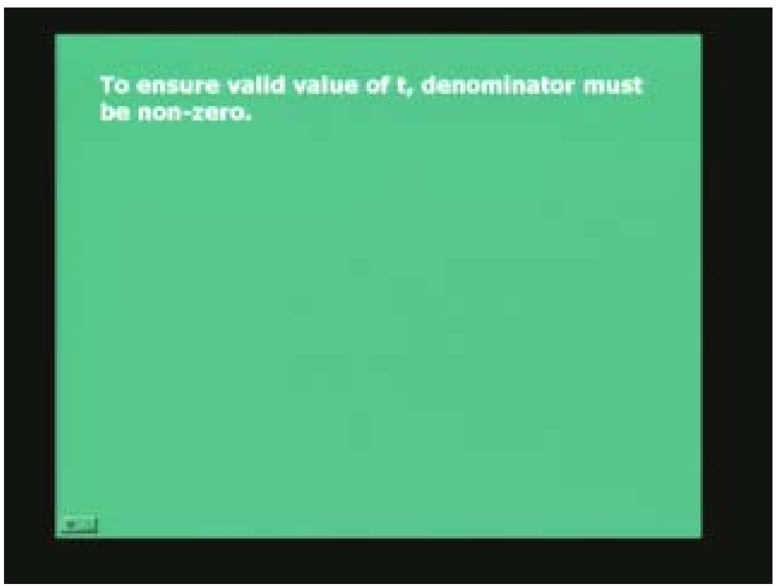

To have a valid value of t the denominator must be a non 0 value because if you previously have looked at the expression of t, if you roll back a little bit the denominator of D is the dot product of N dot D. dot product is a scalar quantity we all know that and if the scalar quantity at the denominator of the expression of t is equal to 0 then you have a problem. So the denominator of the expression of t must be a non 0 value. So you must have a non 0 value of N dot D there is no doubt about it. When do you get a 0 value? We will also check it out but assuming it is non 0 assuming that D and N both are not equal to 0 you first check whether N dot D dot product is not equal to 0.

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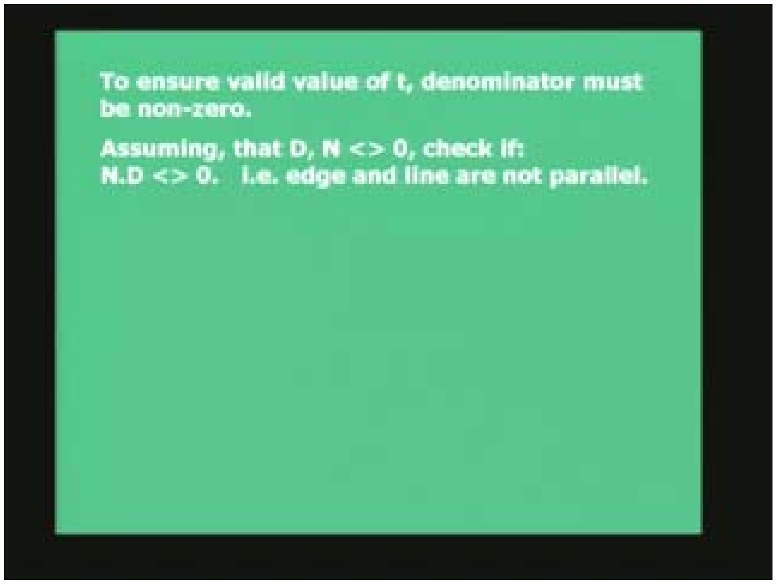

Now, if either D or N or 0 vectors, but of course N cannot be a 0 vector because it has to be orthogonal so that P_0 P_1 are two different points so that also we know. We will actually check whether the dot product of N dot D is equal to 0. And if it is equal to 0 then you have a problem otherwise if it is not equal to 0 the clipping edge of the rectangle and the line to be clipped, remember clipping edge of the rectangle, there are four such edges, the edge being considered and the line which will be clipped they are not parallel if the dot product is 0 otherwise they could be parallel.

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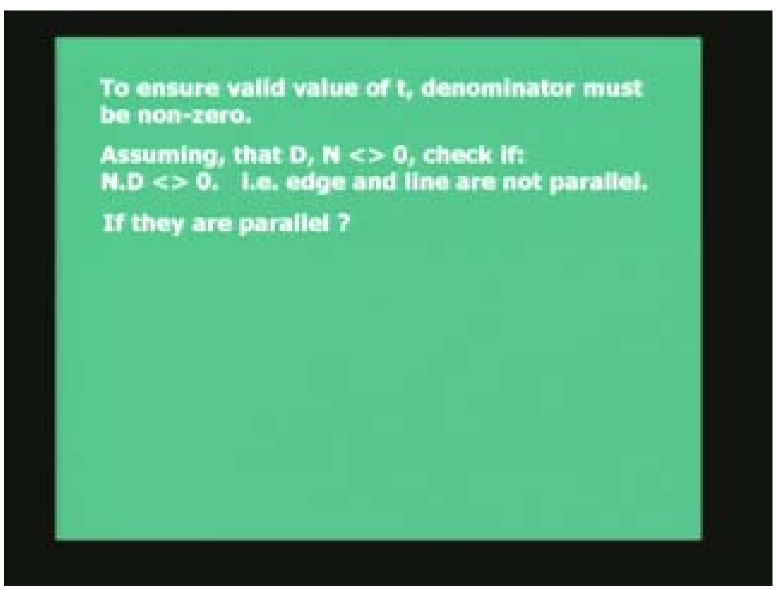

When the dot product is equal to 0 we know that the two vectors are perpendicular N dot D is equal to 0 ensure that they are perpendicular to each other.

Only when the two vectors are orthogonal N and D then only you will have the dot product equal to 0. When will it happen? It will happen when N is the normal to the line being clipped. So if the line and the edge they themselves both are parallel either vertically or horizontally whatever the case may be.

Then only you will have in the case of N dot D the dot product is equal to 0. In this case you will not be able to obtain an intersection that is why basically t goes to infinity so mathematically it is correct. But please do not try to compute the case when the edge and line are parallel. But how to test it? Just check the dot product if it is 0 you do not need to compute the t because these two lines will never intersect and you do not need an intersection point at all.

So you only try to obtain an intersection point and obtain the value of t only in the case when the lines are not parallel. Whenever the lines are not parallel N dot D that dot product is guaranteed to be non 0, it will never be 0. So if they are parallel, do not go for intersection. If they are not parallel N dot D the dot product is not equal to 0 and then you move ahead and try to calculate the value of t using the expression given in the previous slide.

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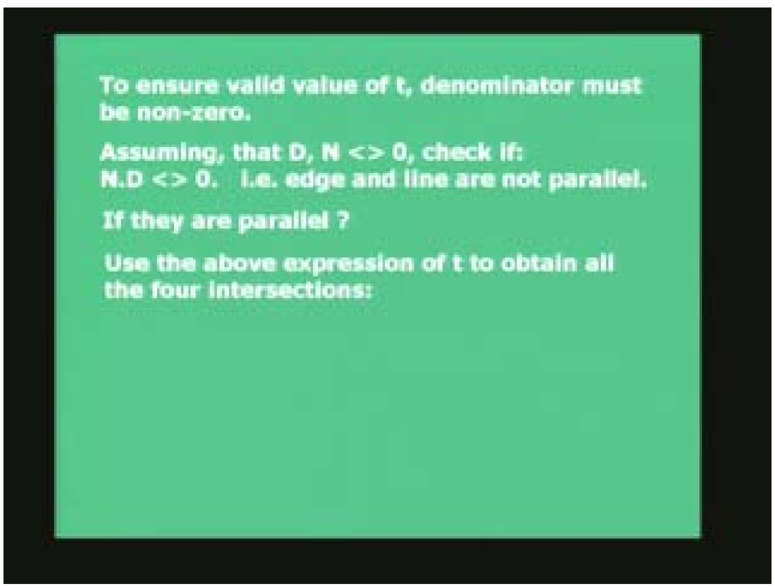

So use the expression of t which has been given in the previous slide to obtain all the four intersections. Why do we talk about four intersections? One intersection one value of t, there are four lines, there are four edges of the clip rectangle, two vertical, two horizontal ones if you remember, two vertical edges and two horizontal edges of the clip rectangle.
So you should be able to obtain in general four intersections because you keep taking the line with one edge, second edge, third edge and fourth and so on and solve these pairs

one after another you will be having four values of t. So, four values of t for the four edges of the clip rectangle and those four edges of the clipping rectangle will give you four values of t for the line to be clipped. So it is obvious that there are four intersections.

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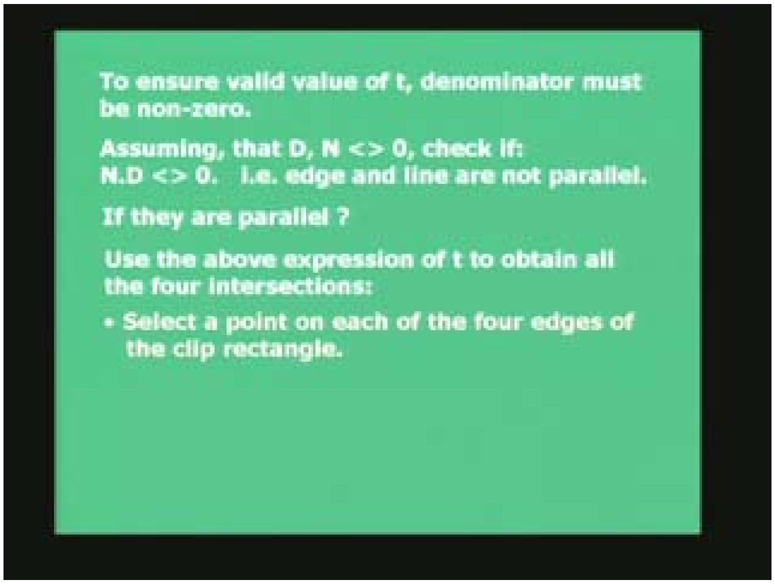

Select the point on each of the four edges of the clip rectangle. This is the method, that means basically you obtain the t, the selection of a point is basically choosing an arbitrary value of P_E and then use that to obtain four values of t provided that of course N dot D is not equal to 0 in one of these cases.

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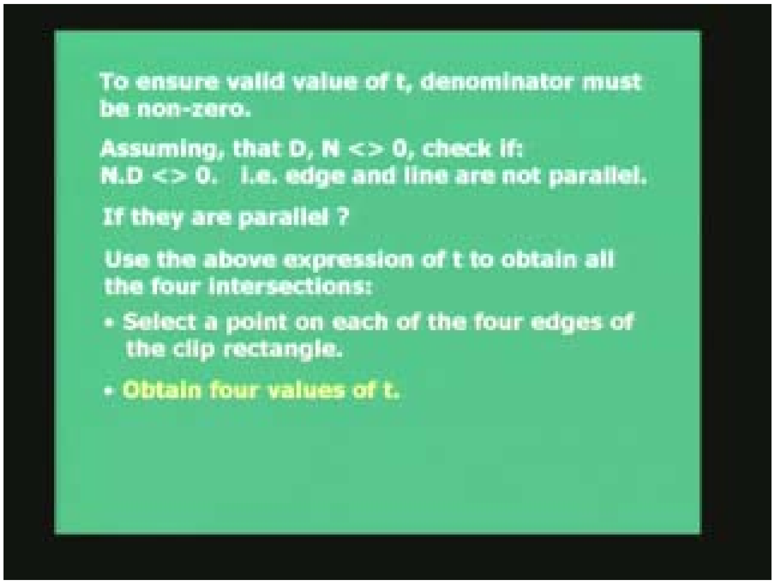

In these cases you will probably have two intersections out of four that is the possibility. But in the general case you will have four intersections with the four edges of a clipping rectangle and hence you will have four values of t for four values of intersections. So obtain the four values of t that is your next point. It is something like the steps of the algorithm is what we are looking for the Cyrus-Beck formulation because the mathematical formulation we have understood now.

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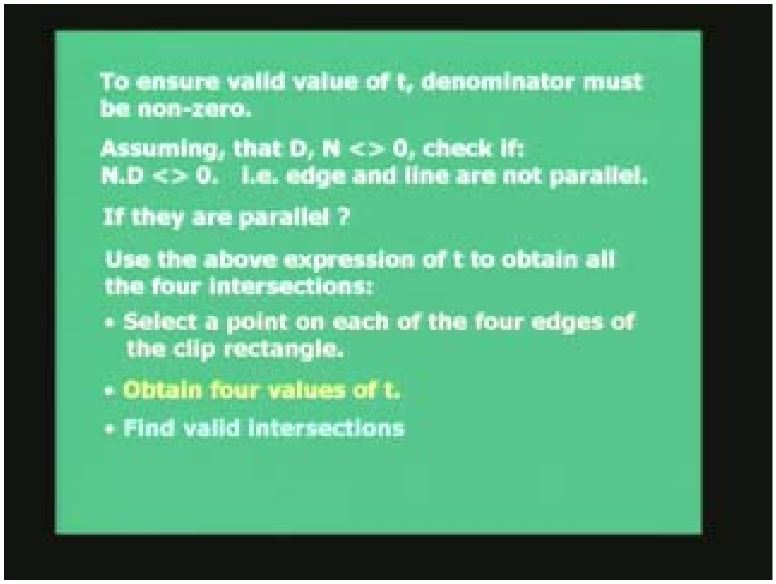

And find the valid intersections now this is the most important point.

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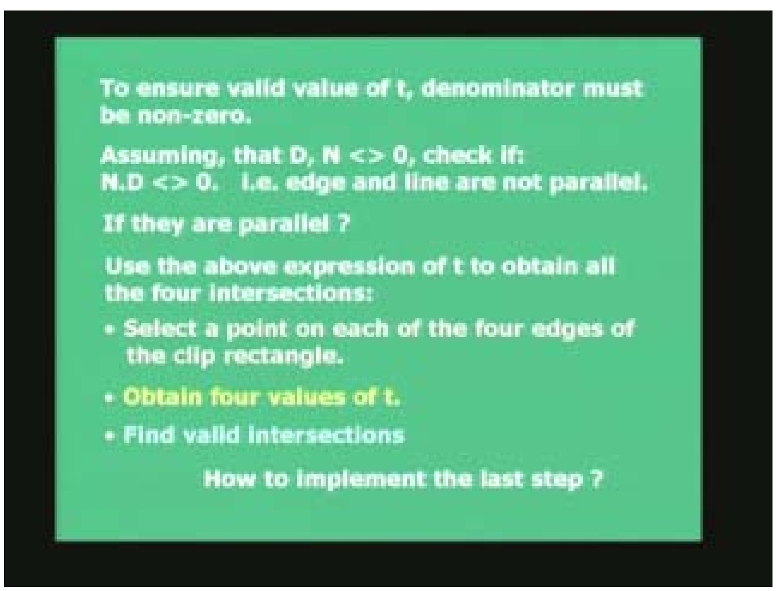

This is the most important point and we must have to look into the last part of today's lecture, how to implement this last step because the mathematical foundation tells you how to select an arbitrary point on the four edges of the clip rectangle that is basically select the four of the P_E's necessary for the expression.

Then use the expression of the t to obtain the four values of t and find if they are valid, what do you mean by valid? First of all the t values must be within 0 and 1. Any two or four values of intersection does not guarantee always that the line is within the clip rectangle. We will see that.

So the implementation of the last step is the key now after the mathematical foundation of the Cyrus-Berg formulation to find out what do you mean by valid intersections.

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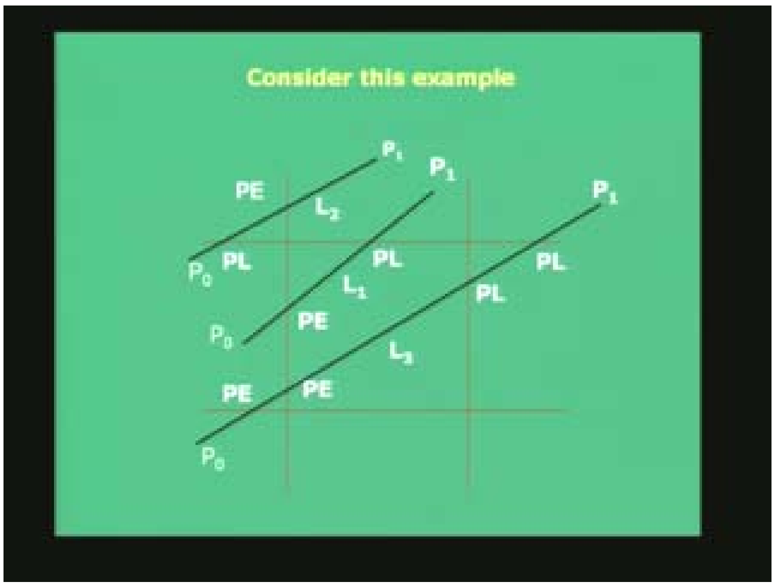

Let u look at this particular example. Let us consider this example where you will find that there are three lines I have considered. What are these three lines? L_1 in the middle, L_2 and L_3, I hope you can see these three lines L_1 L_2 and L_3 and L_1 runs from point P_0 to P_1, L_2 runs from a point P_0 to P_1 do not worry about these two different similar Ps which are used here, I should have used probably P_2 P_3 P_4 P_5. Well P_0 to P_1 let us consider line L_2 runs from P_2 to P_3 and line L_3 runs from P3 to P_4. It is a small mistake although I have used the same P_0 P_1 in all these cases but usually use different symbols, the three different lines.

What is more interesting is I have labeled the intersections of the line L_1 which is running from the middle one P_0 to P_1 and intercepts with the two left vertical and top horizontal edges at point PE and PL. This PE and PL are points which have important labels, important functions and properties, we will see that. But in this case of line L_1 there is absolutely no problem because what you have to do is basically find out these PE and PL and once you know that they are between 0 to 1 for your P_0 and P_1 then you basically retain the portion of the line L_1 from PE to PL.

If you see the line L_3 where the line is much bigger and in fact physically crosses all the intersection points, you need to find out the part of the intersections. You will actually have four intersection points physically lying amid 0 to 1 that exists for this particular case and what you have to do is find out some minimum values of this t, minimum and maximum values or suitable values which will clip the line between this PE of L_3 to PL of L_3. So you need to keep the part of the line which is inside or the outside.

The interesting part is line L_2 which also has two intersections and those intersections are lying between 0 and 1 but the problem is those intersection points are lying outside the clipped rectangle. So, these are three different cases which we have seen where in one case of course you can have a case that the line is completely inside, I have not considered that but you will have two valid intersections which you have to clip and you may have four valid intersections for which you need to find out the two valid ones in the middle and clip and retain the part which is actually physically inside.

You may have two intersections where the values of t are amid 0 to 1 but the entire line is outside the clip rectangle although you will have the values of t amid 0 to 1. That is the case of the line L_2 which we were discussing in the last one. You need to throw off that line, you should not retain that line although the values of t are between 0 and 1.

So the last step of this algorithm after selecting t is after getting four values of t, the last one is the most important part where we need to find valid intersections. So how to implement the last step is critical and we had seen an example, the problem of selecting valid intersections with the help of this example. So now with the remaining time we will get into it to find out how to get valid t's.

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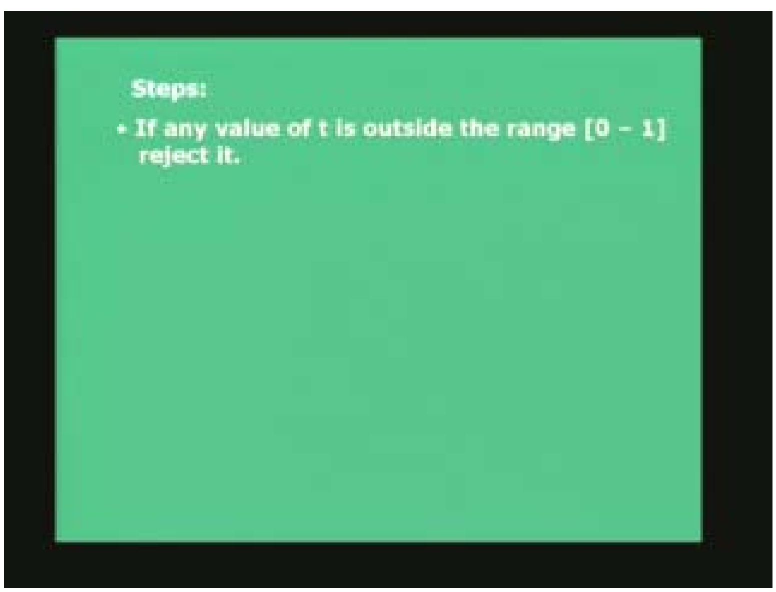

Steps are the following of the Cyrus-Beck algorithm after the mathematical formulation.

The steps are; if any value of t is outside the range 0 to 1 reject it, this we all know, this is very straight forward and a simple case that if any value of t is outside this range you do not even consider it.

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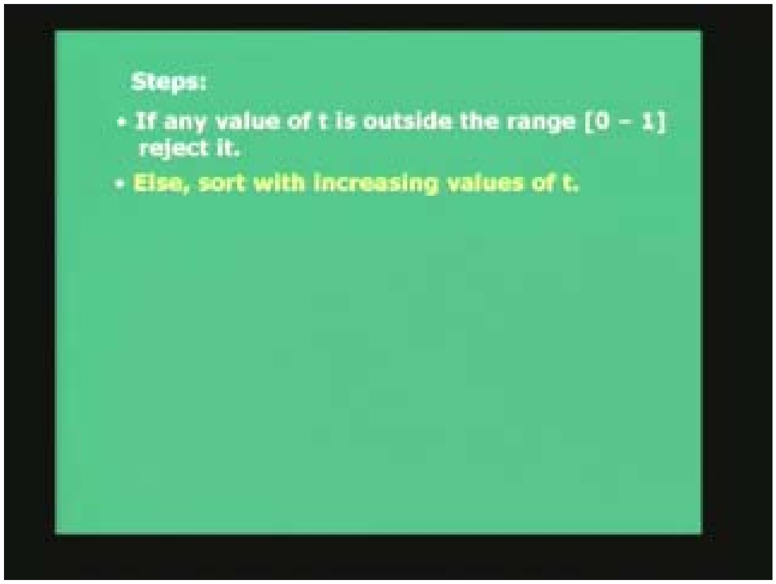

Else if you get the values of t which lie within the range 0 to 1 you sort them with increasing values of t that is from the minimum value towards the maximum value of t is from it is something like moving from left to right within P_0 to P_1 of the line. So that is what you do by sorting with increasing values of t.

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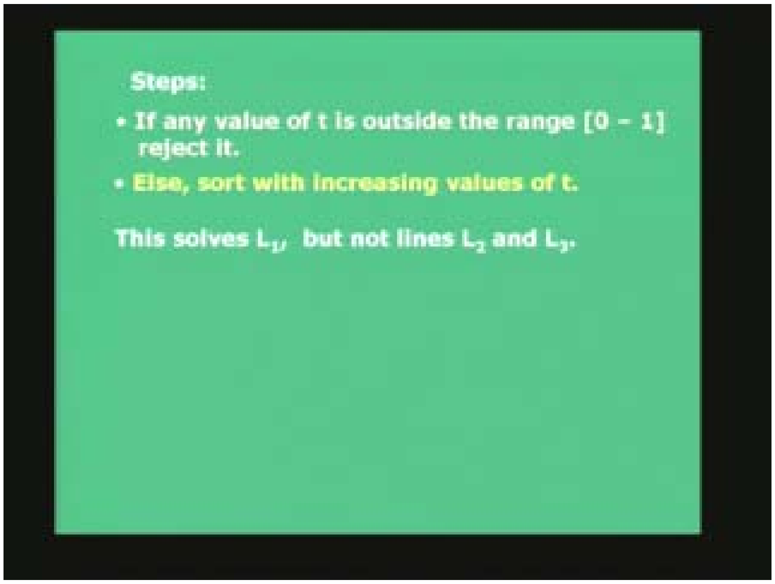

Now these two steps to solve the problem of trying to find valid intersections solves line L_1.

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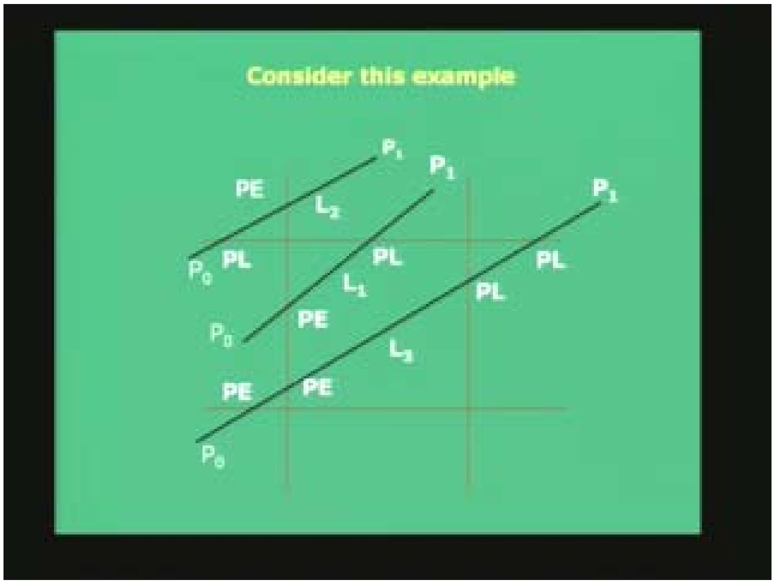

How does it solve L_1? If you go back to this figure and you find only two valid intersections is which are lying between 0 to 1 you know with this line L_1 running from P_0 to P_1 you will actually get four values of t, absolutely no problem. But you will find only a pair, two values of t which lies amid 0 to 1. So after you select these two which is lying between 0 to 1 you arrange them in the increasing order and that will give you these values of t, sort them in increasing order and that is the range from P_1 to P_3 is what you will get as valid.

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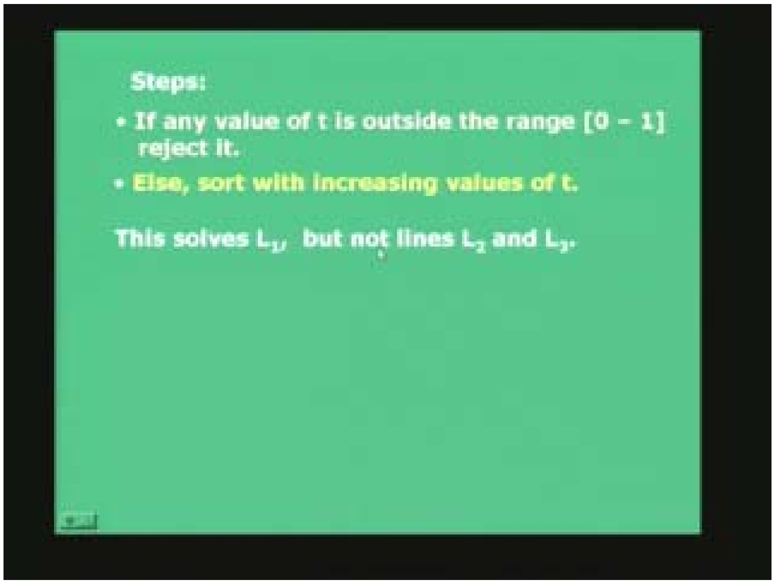

So, you can easily solve that for L_1 line but not for lines L_2 and L_3 which needs some special treatment.

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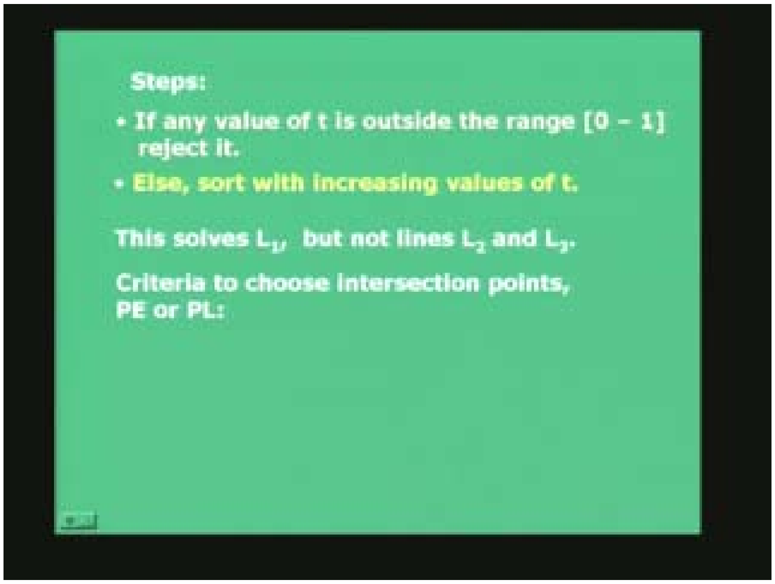

So criteria to choose intersection points P(E) or P(L) which was labeled in the previous figure. I will bring that figure once again but let us look at the criteria for choosing this.

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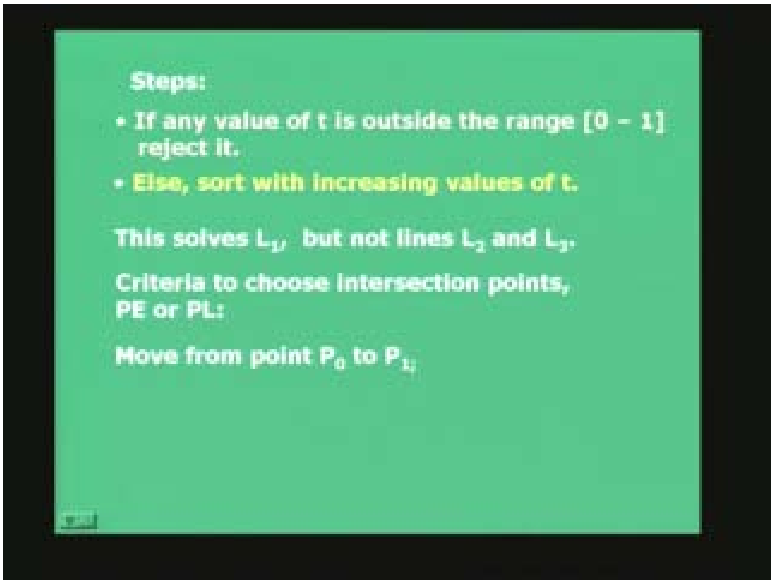

You basically move from P_0 to P_1.

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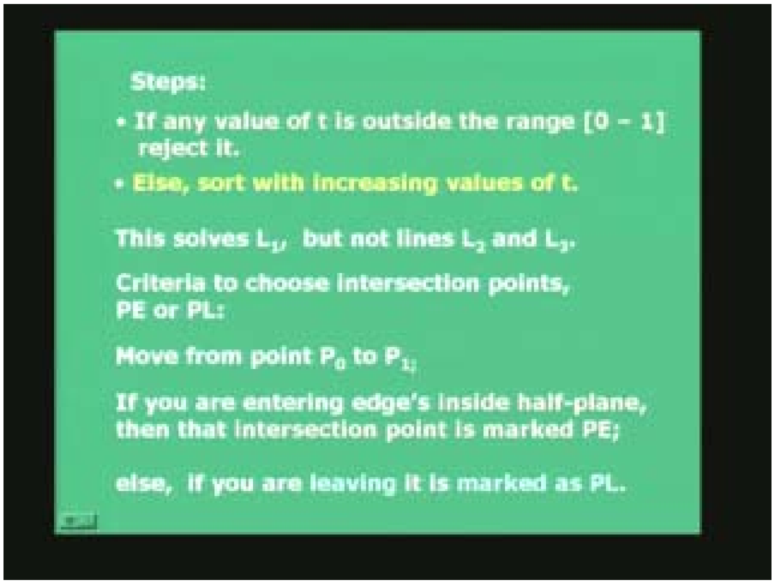

And as you move which is nothing but how you do move from P_0 to P_1? You keep increasing the values of t and move from point P_0 to P_1. As you move from P_0 to P_1, if you are entering the edges inside half-plane, this is an interesting concept, a new concept I am introducing. Entering the edges inside half-plane then that intersection point is marked as PE else if you are leaving it is marked as PL.

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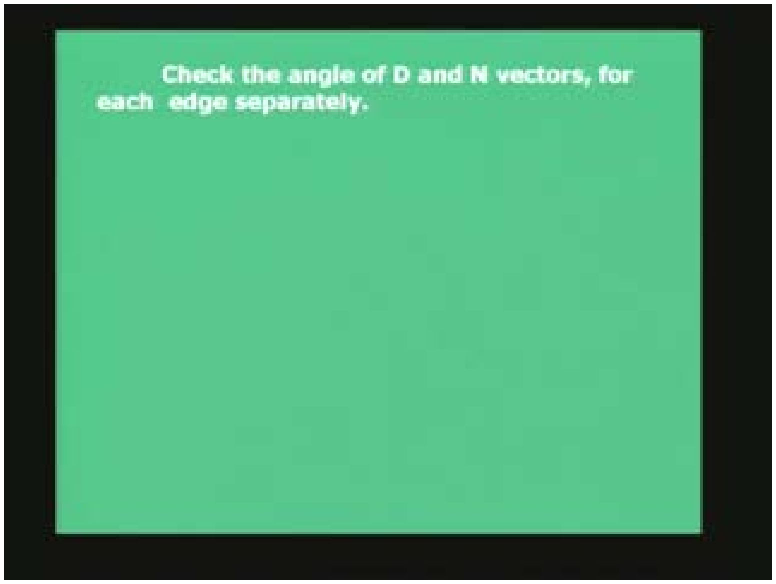

Well, you can visualize with respect to the figure which is given in the previous diagram. I am bringing that figure here, if you look at the line L_1 as you are increasing t you are moving from P_0 towards P_1 at one point you are entering into the clip rectangle from the outside world. And that clip rectangle which is to the right of this left vertical edge is what I will call as the inside half-plane. Any vertical line in 2D space divides my two entire D space into what I will call as half-planes.

I consider a two dimensional plane, if you put a line anywhere, any arbitrary line, vertical or horizontal or inclined one, it will split that plane into two parts, it is true for a surface as well in 3D. In 3D when you have a surface, the surface splits the volume or the space into three half worlds or three half spaces. But let us understand only 2D because we are considering examples in 2D it is easy to visualize and that is given in the slides and the screens as well.

So if you take a vertical line to be very precise in this case because clip rectangle will have horizontal and vertical lines only. So let us take this vertical line which we are considering. A vertical line splits the world into two parts, the half-plane which is outside the clip rectangle another half-plane which is inside. So when you are moving from P_0 to P_1 at some point you will be entering a half-plane. When you are crossing that vertical edge you will be entering into the half-plane which is inside the clipping rectangle. And then again keep on extending, keep on moving towards P_1 but increasing values of t we do not increment t do not worry about that for the time being.

But logically if you do that or visually if you can perceive it, as you keep increasing more at some point you will be leaving the half-plane because if you look at the line L1 it enters the half, it enters the rectangle and leaves it. So at some point you will be entering the half-plane to the right of the clip line entering the clip rectangle and also leaving it and going to the other world where it will be outside the range.

So we are talking of entering and leaving that is why the subscripts E and L indicate entering into the clip rectangle and leaving this clip rectangle or entering into the half-plane of the world, half-plane which is inside the clip rectangle and leaving the half-plane which is also inside. This PE and PL have special significance. The subscripts indicate that you have to enter into the clip rectangle and leaving it. That is what you are doing when you move from P_0 to P_1 for the line L_1. So that is very straight forward and that is what you mean by it.

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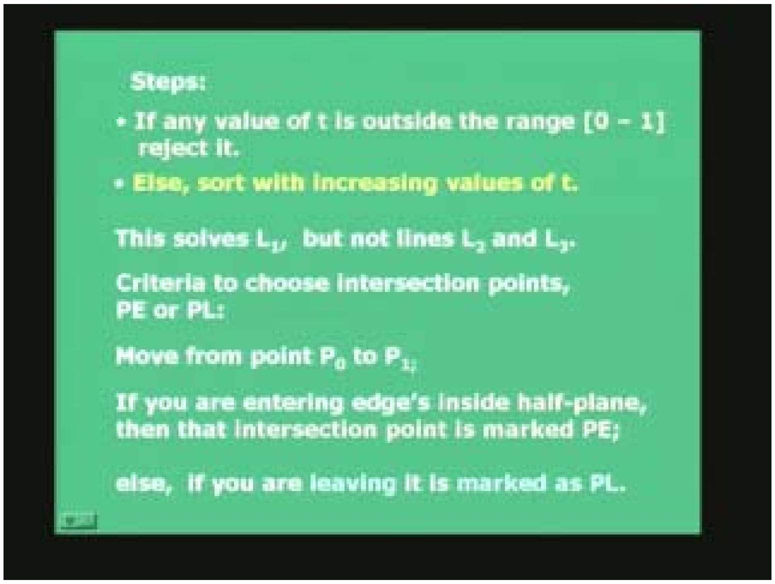

If you are entering edges inside half-plane then that intersection point is marked as PE else when you are leaving is marked as PL. I hope with the example of this previous figure this concept is very clear how you have to mark PE and PL.

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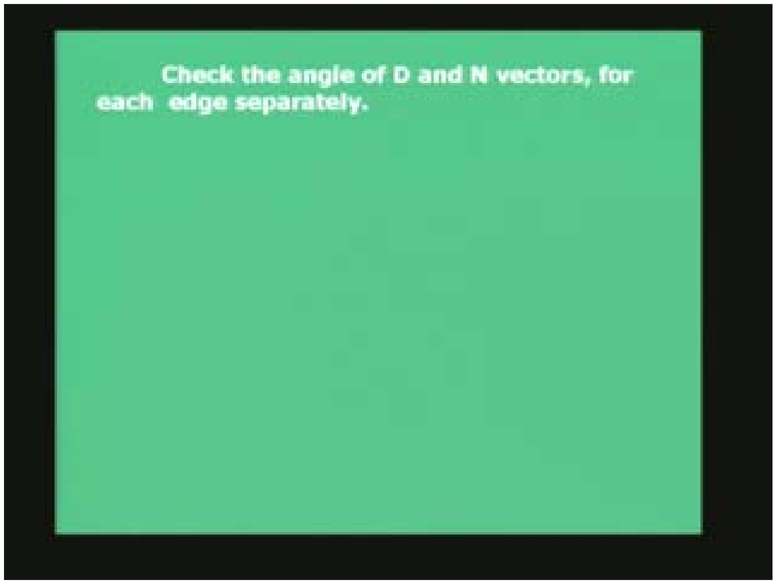

Well, check if the angle of D and N vectors for each edge separately at that point of intersection.

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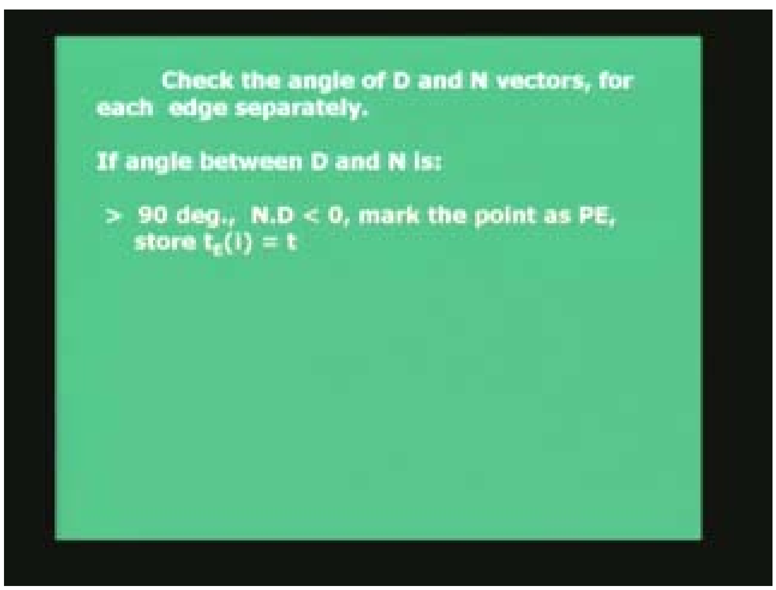

And if the angle between the D and the N is 90 degree and is actually more than 90 degree, greater than 90 degree then the dot product is negative and marked at point as P of E. Remember, if you are entering, if the value is more than 90 degree the dot product will be negative. Mark that point as PE and store it as an one dimensional array form t of E that E indicates the entering not exiting, entering the half-plane inside, half-plane of the clip rectangle and for that value of I it is t, that is the first point I is equal to 1.

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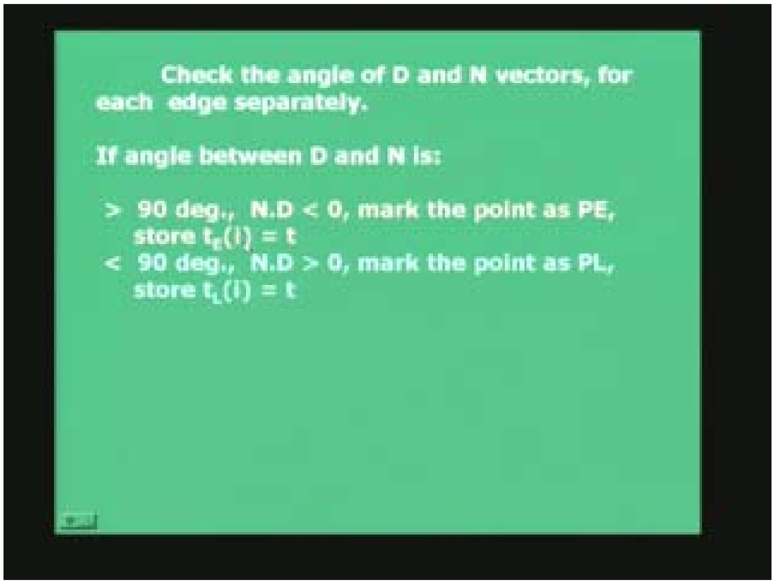

And if it is less than 90 degree then the dot product you can check this with the line L1. You can check these statements with respect to the line L_1 given in the previous figure PN PL are marked at the entry and exit part of the clip rectangle. If the dot product is positive then that angle between D and N is less than 90 degree, the dot product is positive, mark the point as PL and store it as another array t_L L of I is equal to t. I will increment if you have more number of intersection points, I will increment from 1, 2 and so on. So that will take long to happen.

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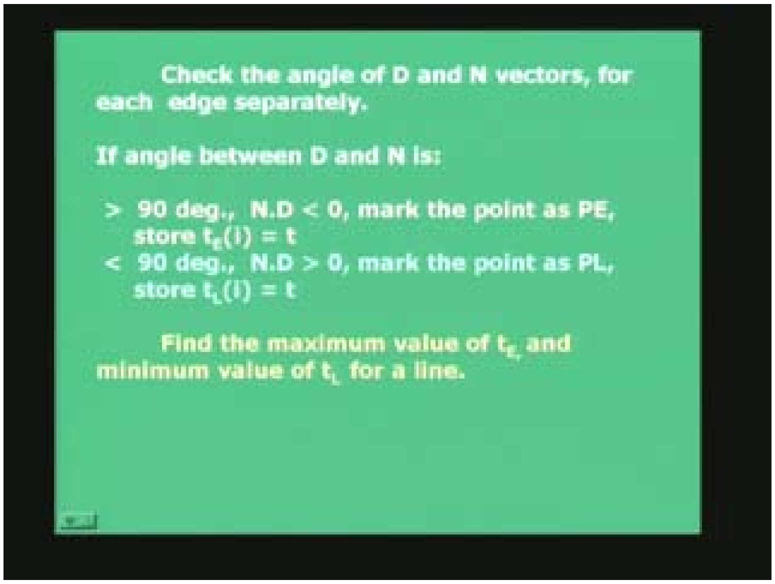

Find the maximum values of t_E, look at the next line find the maximum value of t_E and minimum value of t_L for a line.

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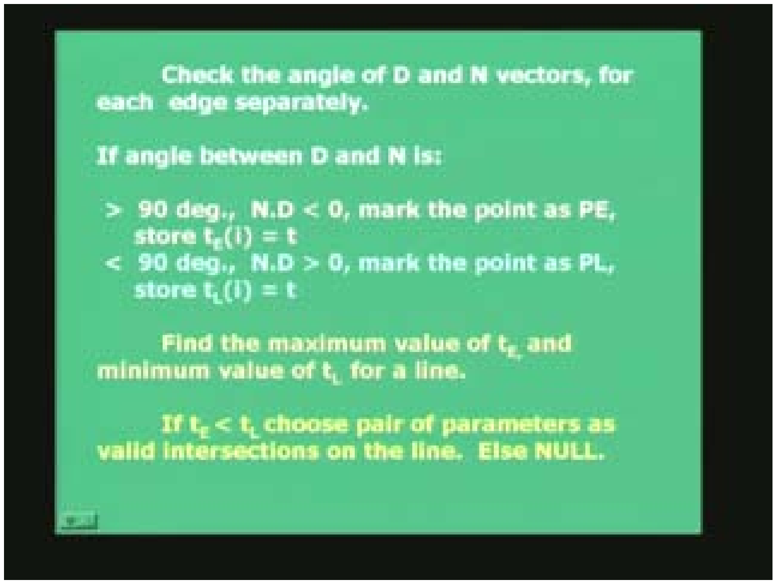

And if t_E is less than t_L choose this pair of parameter as the valid intersection on the line Else NULL.

So what you do? As you increase the values of t you check for this dot product being positive and negative. Now this is that Cyrus-Berg formulation which we have done. Basically look at that dot product, if that dot product is negative or positive based on that you decide whether the point is a PE entering or a PL which is leaving the clip rectangle. That is number one. And solve for the value of t you know the expression of t that numerator divided by N.t.

We have seen that expression how to solve for simultaneous pair of equations using parametric form of a line. So solve for those values of t and whether that point is of PE category or PL category, store that in two separate arrays is the t_E of I and the t_L of I and store these corresponding values of t in both arrays. Then each array you sort and find the maximum value of t_E and the minimum value of tL maximum value of t where it is actually entering and the minimum value of tL where it is actually leaving.

If you find that the t_E for the t remember all of these are between 0 to 1 there is no doubt you are only traveling from 0 to 1. If t_E is less than t_L then you choose the pair of parameters as valid intersections of the lines. Else you do not do. And this we will sort out to help you lines L_1 L_2 and L_3 on the screen. If you look back, this is what we will wind up. You see for the line L_1 you have PE and PL just two pairs. That is the t for the point PL is more than the t for the point PE and this is a valid intersection.

This is not true for the line L_2. If you look at L_2, the t value for PE will be more than the t value for PL. You will first get a PL and then a PE. You will be first in to leave and then enter, that is something wrong. So t value for PE intersection with the vertical edge will be more than the t for the top horizontal line.

So the t for PE will be more than the t for PL. So that is not a valid intersection, throw it out, what happens, so that is L_2 which we will not consider at all to be clipped. L_1 is clipped, what about L_3?

Well, there are two PE's and two PL's, sort those find the maximum PE which is this point, find minimum value of P for this PL which is this and check if this t is more than P and take those two as the valid intersection pair. This is how we throw out lines or take valid intersections and clip it. And out of more than two you select maximum minimum of PE and PL category within the t_E and t_L list and then find out if the t_L is always more than t_E then it is a valid intersection. This is the basic concept and theory of Cyrus-Beck formulation to find out valid intersection points. We will continue in this lecture to give the entire algorithm and complete the discussion and then move on to the next algorithm for clipping.

Thank you very much.