Computer Graphics
Prof. Sukhendu Das
Dept. of Computer Science and Engineering
Indian Institute of Technology, Madras
Lecture - 9
Three Dimensional Graphics

Welcome back everybody to the lecture on computer graphics. We are into the second lecture of three dimensional transformations. In the first lecture we had seen the general form of the transformation matrix and also the significance of the different parameters in three dimensional space using homogeneous Cartesian coordinate form of representation of a point.

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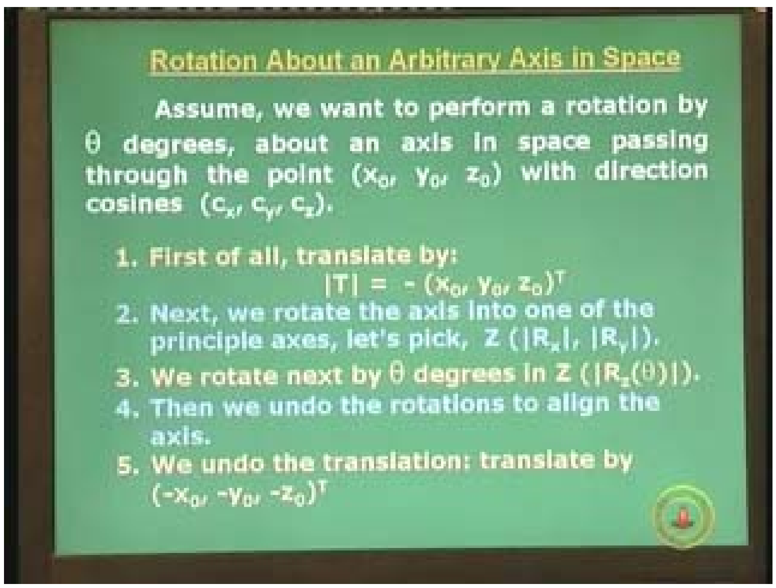

We basically have four elements forming a row or a column vector to represent a point in 3D and henceforth the transformation matrices is basically a 4 into 4 matrix. And we had seen in the Cartesian matrix that the top left 3 into 3 are responsible for all the affine transformations typically that is rotation, reflection, scale, shear and the two row matrices are responsible for perspective transformation and translation.

So we have seen the cases of reflection, shear as well as the translation in the last class. And mostly we have spent a lot of time discussing in depth about rotation matrices and Cartesian coordinate form and the generalized expression of rotation matrix about an arbitrary axis in space in 3D. So we will probably just touch upon that and will start from where we left in the last class just to maintain continuity and also since the concept about rotation about an arbitrary axis in space is very very important and that concept also is borrowed onto reflection about an arbitrary plane. We will also see that the same types of concepts are used.

We will probably look into the fact that how did we do a rotation about an arbitrary axis in space? Remember in three dimensional Cartesian coordinate form left and right hand system we have an arbitrary axis for which the direction cosines are known and we have to rotate by an angle theta. And a point or an object in 3D must be rotated by an amount theta and we look back into the slide and we find that the steps of this transformation are all given to you. So you assume that you want to perform a rotation by theta degrees as you can reach about an axis in space passing through a point. So the axis must be passing through a point x_0 y_0 z_0 and the direction cosines of that axis are Cx Cy and Cz respectively.

We need three parameters for direction cosine. Now we will see later on how direction cosines can be specified and if you are not clear about it you must go back and recall in geometry books about direction cosines. Let us go to the first step, the first step says translate by an amount T depending upon x_0 y_0 z_0.

The number 2 step is, next we rotate the axis onto the one of the principle axis and that was an important step which we discussed which means we need to rotate about x and then about y and that will help the axis coincide with the z axis. And once that is done we rotate in the third step by an amount theta degrees about the z axis for which the expression of the rotation matrix elements are all known to us. And then we undo the rotation that means in step number 4 and 5 we undo whatever we did in 2 and 1. Once 2 is known, 4 is known, expression for 3 is known and 1 and 5 are very straight forward because they are translation.

So you just revise what we did in step number 2 because that is the most important part. Once step 2 is solved all the others fall in place. So let us look at step number 2 the figure which describes and helps us to understand the rotation about the x and the y axis and will place the axis op along with the z axis. Hence if you look at the figure again the Cartesian coordinate system x y z, op is the axis about which we have to rotate by an amount theta and this axis op must coincide with the z axis and then we can rotate about z.

So the step number 2 involves two smaller steps in fact one is rotating by an amount alpha about the x axis and then by an amount beta about the y axis. And so two rotations of the axis op, one about x which will bring the axis op on the XZ plane, you have to visualize yourself, I will show you in the next figure that if you rotate by an amount alpha about the x axis the axis op will now fall on the zx axis and that is the first step of rotation for step number 2.

So after the first step of rotation of the axis op it has now fallen on the X_Z plane. So this figure shows that the first step of rotation is over and then we have to rotate by an amount beta about the y axis and that will place the axis op on the z axis.

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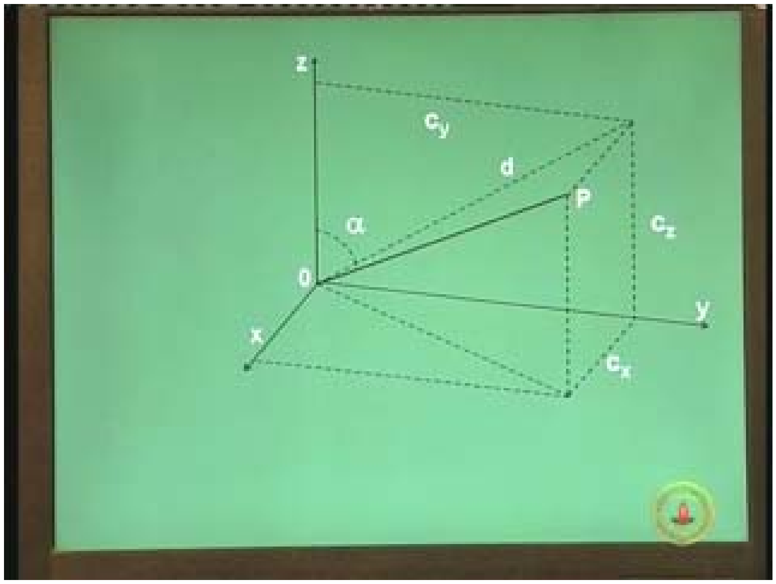

So I will roll this figure back for you to properly visualize how the axis op has coincided with the X_Z plane. This was the first scenario, so you have to visualize in 3D that if you rotate about the x axis by an amount alpha and of course the direction cosines C_x C_y C_z etc also shown in the figure. This small d is basically square root over of C_y square plus C_z square and so using those values of d and C_y we can find out the value of alpha.

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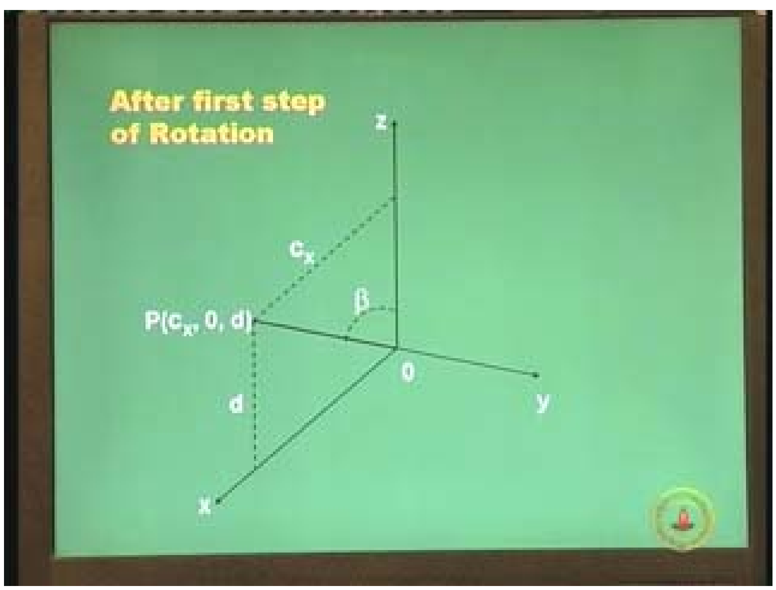

And once the rotation about an alpha takes place the next figure (Refer Slide Time: 00:05:47) which has been shown to you earlier is that axis op now coincides with the Z_X plane and then again you rotate by an amount of beta because beta is known because C_x

and d are all known. We have done this in the last class and the op will coincide with the z axis.

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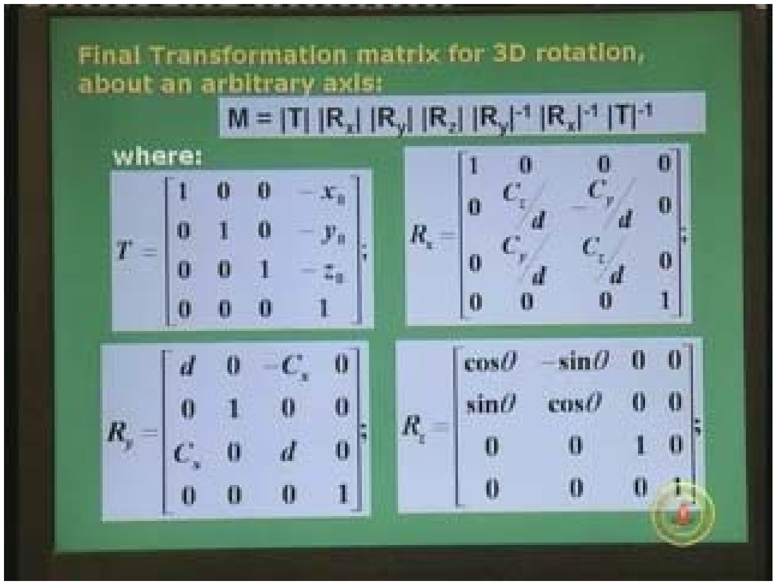

So coming back to the final transformation matrix for 3D rotation about an arbitrary axis, the first transformation is a translation followed by two rotations about x and y finally about z and the undo in the opposite sequence R_y inverse, R_x inverse and T inverse respectively where the corresponding expressions for the matrices T, R_x rotation about x is given here, rotation about y is given here. We already have discuss in this last class but just revisiting again and I also requested you to multiply all this matrices and get the final form for M it will be a 4 into 4 matrix and the final rotation about z is what we all know. So, to apply the z we actually have to use T, R_x and R_y. T will help the origin to move to the line or the line to move to the origin. R_x and R_y will help the axis op coincide with the z axis where we can use R_z to rotate about the z axis. So this is the final expression.

Actually we stopped here in the last class. And a small addition point here which may be noted is about the direction cosines of the axis.

So you may be given two end points of the axis or two arbitrary points on the axis about which the rotations has to take place. And from geometry actually you can find the direction cosines of a line from any two point coordinates on the line using the expression as given here on the screen.

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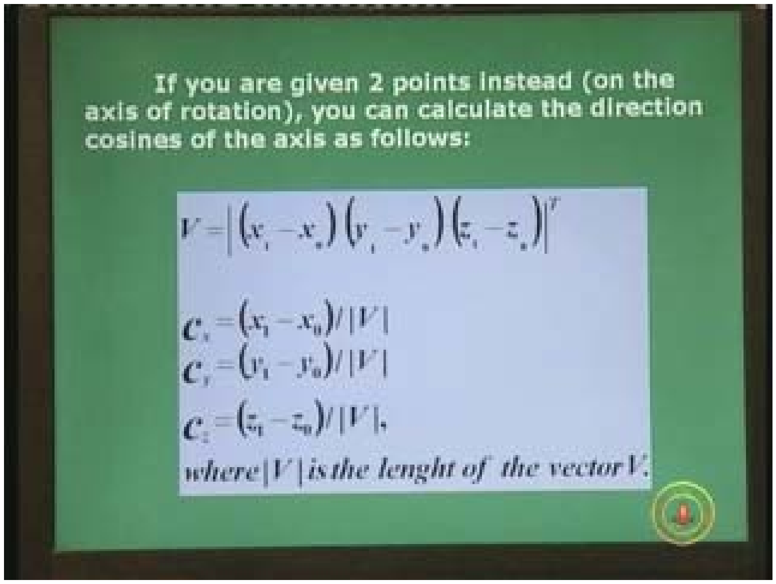

So if you are given two points instead of the direction cosines and these two points are lying on the axis of rotation as given in the bracket then you can calculate direction cosines of the axis using the expression as given below. You know you can compute V and then you compute C_x C_y C_z where V is the length of the vector V. So this how you compute C_x C_y and C_z respectively. Well, using the same concepts we move onto reflection through an arbitrary plane. And you remember as we had rotation about an arbitrary axis and the axis had to be coincided or basically brought in line with z axis.

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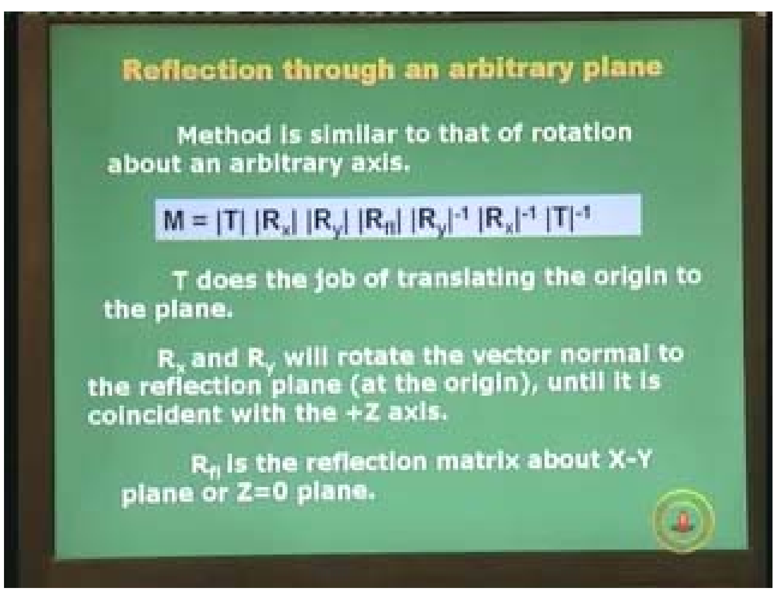

Similarly, now if we have an arbitrary plane and we need to provide a reflection we cannot do that straight away. What we have to now do is bring this plane and coincide with one of the orthogonal coordinate planes X_Y Z_X or X_Y Y_Z or Z_X and then we can apply reflection matrix. So the concept is similar and that is why we employ rotation with respect to z axis, to the concept of rotation about an arbitrary axis and here we actually apply in the middle or imply reflection.

The method is similar to that of rotation about an arbitrary axis and conceptually similar. I am I hope you can understand the sequence with respect to the expression as given here. And you can compare this expression with the expression given for a generalized rotation about an arbitrary axis. As we can see that the pattern of the expression is same in terms of sequence of matrix multiplications only the middle matrix R_z is replaced by a reflection matrix R_{fl}. R_{fl} is the reflection matrix about one of the X_Y Y_Z or Z_X planes and that is what is put instead of R_z in the previous transformation matrix when we talked about rotation about an arbitrary axis.

So coming back to reflection about an arbitrary plane we are talking of these different forms. So T does the job of translating the origin to the plane as like as in the previous case R_x and R_y will basically rotate the vector normal of the reflection plane at the origin and it will do that until it is coincide with the z axis. Therefore, when you are rotating the normal to the reflection plane using R_x and R_y you can assume that this vector normal is nothing but similar to the axis about which you were rotating. So you can visualize that op axis, you remember when we rotated about an arbitrary axis op that op can be visualized to be normal to your reflection plane. So when we are actually rotating a axis and coinciding it with the z plane the same thing is done here only what happens is we are not doing rotation but we are doing a reflection.

And now after the R_x and R_y operations of successive rotations the reflection plane which was arbitrarily in space will now coincide with the X-Y plane or because the normal s is coincided with the plus z, so this is a plus z so and this is the X_Y so the plane has become X_Y and we can employ a reflection across or about the X-Y plane which is the expression for which it is known. So R_{fl} is the reflection matrix about X-Y plane or z equal to 0 plane and so that is understood.

So we move onto different concepts of three dimensional transformations now. Basically we have understood the different affine transformation of rotation, reflection, scale and shear. In the last class we have studied in greater depth about arbitrary rotations, arbitrary reflection, translation and also we know the most. In fact the only transformation which is left now is the projective transformation or perspective transformations. And before we learn or understand more about it we will just go through a few terminologies which are necessary to understand about projective geometry and with its perspective orthographic.

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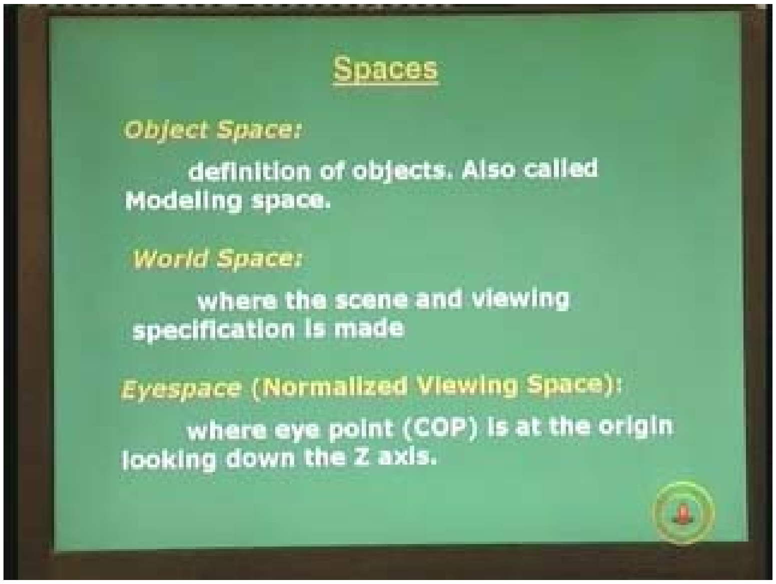

And we need to know about what are called the spaces. There are various types of spaces existing in both coordinate geometry and computer graphics literature object space.

We talked about object modeling not in details in any previous lecture but in the introduction where we say that you have to basically define the geometry and then define your object, its location, its size, its parameters, in attributes and they are defined with respect to an eyespace three dimensional space or two dimensional space mostly 3D in this case since we are talking about three dimensional transformation.

So let us visualize everything in 3D. You have to define an object in 3D and it is defined in the object space. It is also called the modeling space where you can model a sphere, a cylinder, simple objects or any complex structures made up of building blocks etc. We will talk about 3D modeling more later on but you can visualize small objects can be joined together and be made a complex object. So those objects are all defined in the object space or modeling space.

We also have the world space where you have to position the objects somewhere in the world. So you have made your objects and you need to position them, orient them and scale them if necessary and position and them basically somewhere with respect to the world coordinate system and that is what is called the world space where the scene and the viewing specification is made.

And of course now we have the normalized viewing space or eyespace where we defined a COP term called the center projection. We will find out whether it is called the eye point where it is the origin the eye point or the center projection the COP will define that later on in the next couple of slides z the origin and we are basically looking down the z axis so that is the eyespace or the normalized viewing space.

Sometimes this space and world space are considered to be identical but they could be different where nobody stops you from having two different spaces. We need the 3D image space for the projection space. So we are talking of a 3D projective space with the dimensions that are all normalized. The dimensions are in the range minus 1 to plus 1 in x and y axis and it is in 0 to 1 in the z axis. So basically it is something like three dimensional rectangular parallel pipette not exactly a cube because the z axis dimension is less than the x and y rectangular parallel pipette.

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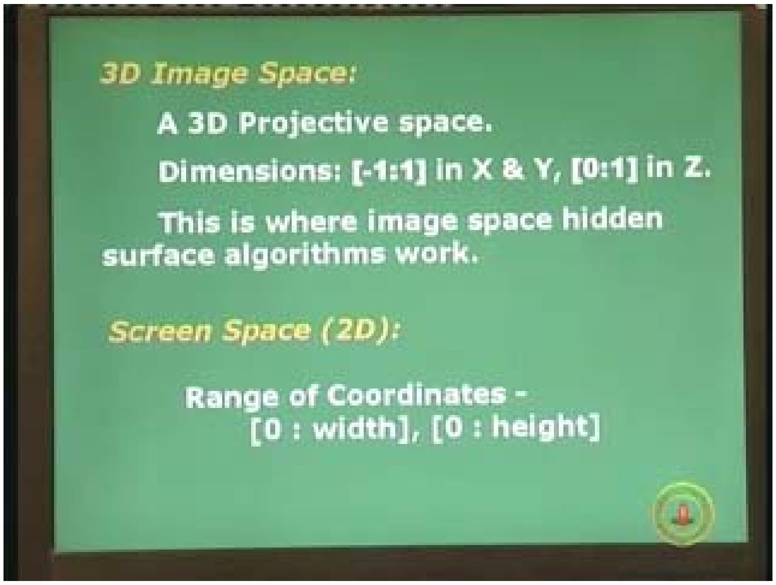

Basically we will see this projective normalized 3D image space later on if not in this lecture when we talk about different algorithms for projective geometry and trying to draw three dimensional pictures, this is very very important and it is where we will say that the image space hidden surface algorithms work.

Hidden surface algorithms work in the image space to help you when rendering on this picture facing the viewer. We will understand all those. But you just have to know now that the 3D image space runs from minus 1 to plus 1 in the x and y domain and from 0 to 1 in the z axis. And the last of them is the screen space which you are basically viewing. A screen space is a 2D space on which the entire world or the entire 3D pictures will be projected and its range runs from 0 to a certain value depending upon the width and height of the screen.

Typically for a normal region it could be 644 for a T or for a higher resolution we have seen that it could draw to even 1024 pixels. It depends upon the pixels and the window size you choose on the screen, the screen space is defined from 0 to a particular value. So we move onto the projection geometry now. That is the one which is leftover and we will look at several planar geometric 3D to 2D projections. After of course we will describe at length a few fundamental ones depending upon the time limits we have which are very very useful. Of course but there are other type of projection geometries which are used in engineering designs and drawings especially in the field of mechanical engineering or civil engineering which we will just mention but we will talk about those which are used extensively in the field of computer graphics and a little literature.

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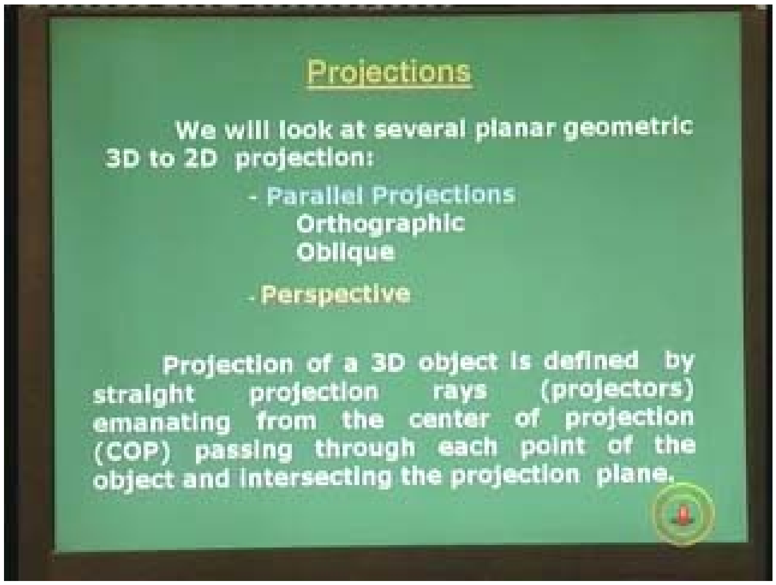

Essentially we will say that there are two different types of projection geometries. Let us be very specific here; one is called the parallel projection or the parallel projection geometry and the other one is called the perspective geometry. There are two types and again parallel we will say that there were essentially again could be two types one is orthographic or oblique. Of course their other types of parallel projections also we will name them,

We will look at a chart but typically under parallel projection we look into orthographic projections and oblique projections of parallel projections under that category. And in the perspective projection also there are two or three different categories and we will touch upon them as we proceed.

What is a projection? We need to define what projection is. Here we are talking of a projection of a 3D object which is defined by straight projection rays called the projectors. Light rays you can visualize them as coming out of the object and getting inside the screen. So there is something called projector rays or projectors or projection rays which are emanating out from the center of projection or COP.

We use a diagram to define what COP is or center projection is and that passing through each point of the object and intersects the projection plane. So it is a ray, it is a straight line. You can traverse any intersection on the ray but typically you can say it starts from a center of projection and passes through the object and hits the projection plane or hits the screen. That is where you have this concept of projector rays coming out of the center of projection. Also, the reverse could be considered where rays come out of the screen pass through the object and hit in the move to and center of projection. But typically we motionally visualize that the rays come out of the center of projection. So that is the projection space and I think we will look into a figure to understand what you mean by projections here. Let us look into the next slide.

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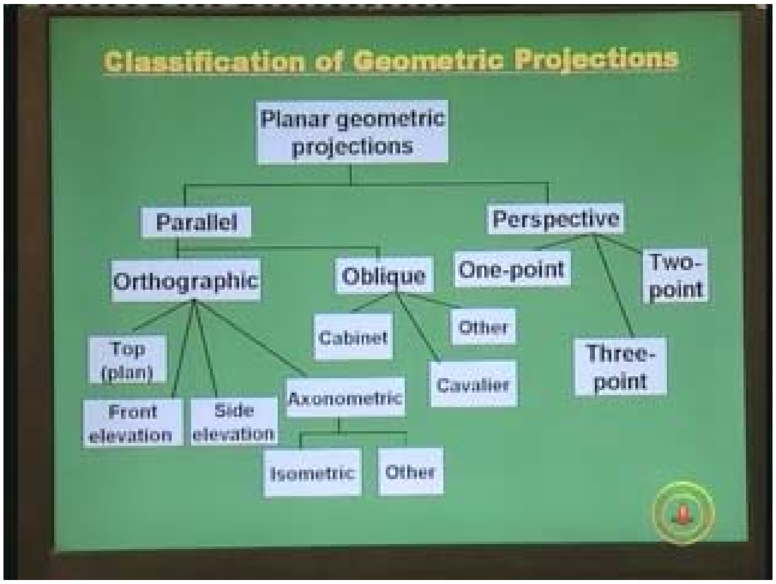

Classifications of different geometrical projections: here all types of projections are given in this layout. But as I said before we will classify them under two categories called the parallel and perspective as given here. Thus, planar geometric projections fall under two categories namely parallel and perspective. Under parallel you have orthographic and oblique which we will discuss here and under perspective we will have one point, two point and three point perspective projections. We will understand and see in this lecture about what are these one point, two point and three point projection systems under the category of perspective projections and under parallel what are orthographic and oblique projections.

Again, we will not discuss in detail about the sub classes under orthographic or oblique. As I said before most of these are very useful for CAD designs in mechanical, civil and architectural designs. But under orthographic you have the top plane view, the front elevation view, the side elevation view or what is called the axonometric view.

We will see the isometric category under axonometric which will give you an idea of what is orthographic projection. And also under oblique projection we will have a cabinet cavalier and other type of oblique projection also. We will see under orthographic projection what is axonometric or isometric projections and what are oblique projection as well. These are the different categories and classes and sub classes of geometric projections. We do not need to get into depth of all different sub categories. We will definitely touch upon the main and important ones.

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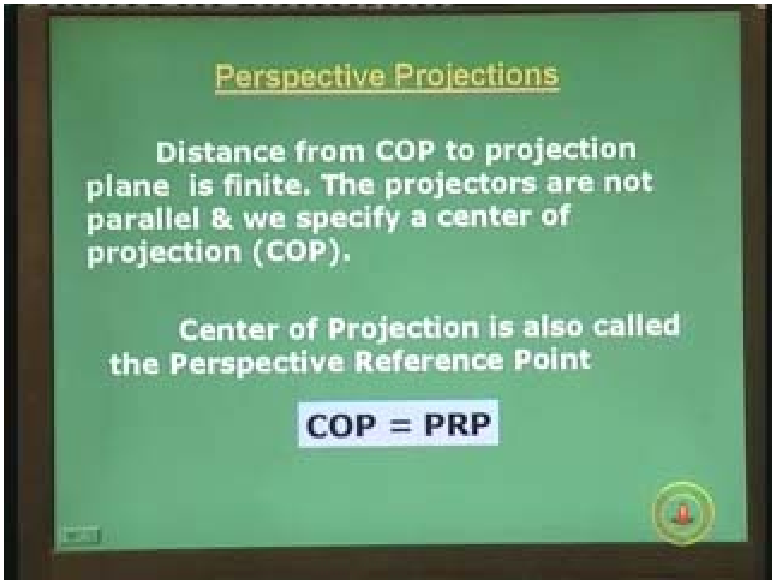

We will first move onto perspective projections which is very natural to the human eye. Any camera system which you see digital cameras, analog cameras the cameras which set on the satellite typically follow the nature or rule of perspective geometry or perspective projections, our eyes also follow the same emotion of perspective projections and we will see what are the laws in terms of both optics and geometrical relations which relates the three dimensional quantities in space to the two dimensional projections or which will be available on the screen in case of a perspective projection.

We define what is called in case of perspective projection that the distance from the center of projection or COP. Now we are probably using this term quiet often. The COP is a very common term. We must get to a figure very soon to understand what is COP in terms of projection rays. It will come very soon where right now we assume that COP is a point in 3D space in the world space. It is a point and the rays are coming out of the center of projection passing through the object and hitting the screen. And now we are talking of the distance of the center of projection distance of the center of projection to the projection plane.

We say that from center projection the rays are coming out and going to the projection plane. So the physical distance between the projection plane and the COP is finite. Of course you can immediately guess that if it is infinite it becomes orthographic projection. In the case of perspective projection this distance is finite, the projectors are not hence parallel and we have to specify a center of projection or there is another term called Projective Reference Point PRP or COP, they are used interchangeably. We will see what this particular point is. But this is the definition of perspective projection where the distance is considered to be finite.

So, center of projection is also called a Perspective Reference Point or Projective Reference Point as we talked about. So COP and PRP will probably be used interchangeably and you should not get confused. Basically we assume that both of these mean the same thing. The Center Of Projection COP or PRP is called the Projective Reference Point both these mean the same. Let us get through a few more term analogies before getting into the figure and the equations which derive the relationship of projective geometry. One is perspective foreshortening. Now this is a very important concept here which comes in perspective geometry.

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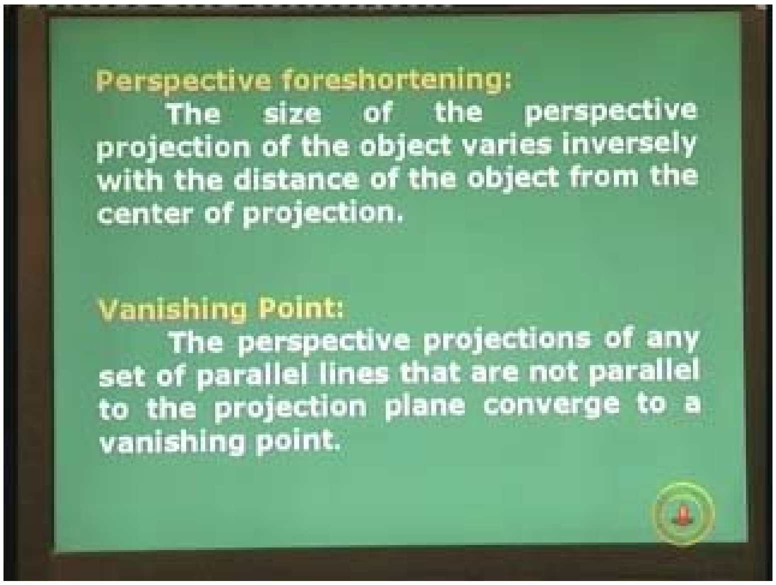

It says that the size of the perspective projection of the object that means after the object has been projected on the screen then that size varies inversely with the distance of the object from the COP or center of projection. That means the size is inversely proportional to the distance of the object from the COP that means closer the distance is or shorter the distance is larger is the object, larger is the distance and smaller is the object. This is very easy to understand. If you take any object it could be a ball or any other object and you bring it closer to your eye or closer to your camera it will appear bigger or if you take it away from the viewer that is yourself or from the camera typically it will tend to appear very small. That means perspective projection does not give you the actual idea of what is the size of the object because a small object closer to you could appear bigger and a large object which is quiet far away from you may appear smaller.

A typical example, when we look at the sky and try to observe the sun and the moon typically of course it is very difficult to visualize both of them or see both of them together on the sky. Sometimes they are visible but typically in the day time if you try to understand the size of the sun and in the full moon nights you try to observe the size of the moon. Typically in most cases you will find that the size of the moon which is clearly visible on the clear sky and it appears much larger and bigger than the size of the sun.

Now we know all from basic knowledge of science and geography that the size of the sun is many many times larger than the size of the moon, why moon? It is much larger than the size of the earth and definitely much more than the size of the moon. But in spite of being such a large object, the sun is so far away when compared to the distance of the earth and the moon that the sun typically appears almost of the same size or even smaller than the moon when we look through our eye or through any camera and this is because of the nature of the perspective geometry or perspective projection.

And this concept which you can see for yourself, take an object, a large object a ball, it could be a football or a tennis ball or a cricket ball whatever the case may be and bring it closer to your eye or closer to a camera and you will find that the size of the object goes bigger and when you take it away from the viewer or take it away from the camera the size of the object starts to shrink.

This is an effect of perspective foreshortening because the sentence that states on the screen that the size of the projection of the object varies inversely with the distance. So when the distance is large the size is going down and if the distance is becoming shorter the size goes up. Assign the moon was an example I talked about. You can try this experiment yourself right now with any particular object.

Take an object far away from you, it appears too small and when you bring it closer it appears bigger. So this is called as the concept of perspective foreshortening. We will see with examples of equation as to how we can model this. The basic concept of a vanishing point in perspective geometry: Well let us read the definition as given in this screen and then I will try to explain what is vanishing point. We also have figures describing vanishing point.

The definition says that the perspective projections of any set of parallel lines that are not parallel to the projection plane converge to a vanishing point. Now this is a very interesting and a confusing definition if you have not heard. See this concept of a converging and parallel, now we know all from geometry that parallel lines, lines which are parallel never intersect even at infinity or basically intersect at infinity. But we are saying that we are talking of a finite point called a vanishing point where these parallel lines tend to converge or meet in the projection geometry or in the projection plane.

I will take give a simple example of how to visualize this vanishing point. Well if you stand on a high road, a long straight road preferably linear and straight. You can stand on a railway line of course you have to be very carefully that you do not take up the hazards of outstanding when there is traffic on either the road or on the rail track.

But assuming those things are taken care of and you are able to stand on a rail track or in the center of a road and look along the direction of the road sitting or standing on the center of the road or on the rail track and assuming that the road and the rail is straight, whatever the case may be and what do you find? Do you know that the end of the two roads or the rail tracks is parallel? That is how the railway line moves. But as you keep looking longer and longer further away from you along the rail track you can start to visualize. It seems now as you go towards the horizon or towards the end of the rail track, when you start looking you are not traveling but standing basically. Those two lines tend to meet as they seem to converge at some point which is at a very far distance, the distance you probably cannot see where they are probably meeting but you know that at some point they are meeting because you are seeing a perspective projection of a point in infinity.

And perspective projection of a point in infinity can be finite. This vanishing point is a point which is accurate infinity why? Because this is the point where two parallel lines meet. Two tracks of the railway line, you know since they are parallel they only meet in infinity. So that point is at infinity but the projection is a finite point it is called the vanishing point because that projection when we take onto the projection plane which is you eye when your seeing that railway track it is called the vanishing point, it is a finite point in a two dimensional plane.

We will see with an example what this vanishing point is and it is also related in some sense with perspective foreshortening not directly but in an indirect manner.

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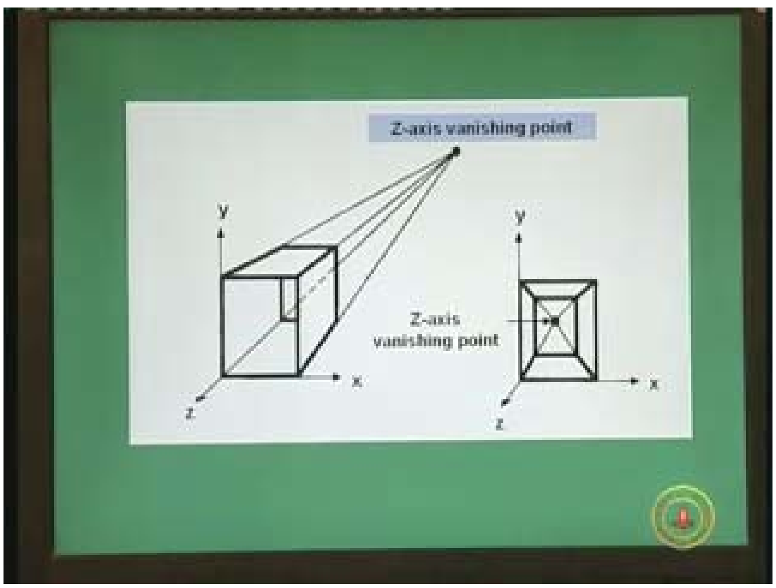

And here is the example of perspective projection geometry and a z axis vanishing point. Here there are two figures as you see on the screen, they are actually same. Let me explain taking the left hand side first because it is a tricky diagram whereas the right hand side is a two-dimensional diagram. I must say this is given in the book by Foley Van Dam and from there we have borrowed it. And this is a basically a queue sitting where the x y axis is coinciding with one plane of the rectangular parallel pipette and z axis is the viewing direction.

And typically if you see for yourself that if I take the four lines which are supposed to be parallel lines because it is a cube and those are parallel lines or parallel to all z axis but they are not parallel to the to the viewing plane or the projection plane. And if that is so those lines which actually never meet in 3D appear to meet at a point as you see on the left hand side figure as drawn. So you typically tend to have a trapezoidal figure. You can extrapolate those lines or extend those lines yourself by scale and they will meet at a point called the z axis vanishing point.

Why it is called the z axis vanishing point? Because that vanishing point is obtained by extrapolating or extending lines, any two lines you can take, in this case I have taken four lines but you can take any two lines which are parallel to z axis. And since the z axis is not parallel in this case to your viewing plane or the screen the way the 3D diagram shows, henceforth the projections of those two lines will not be parallel and if they are not parallel on the screen they will meet at a point called the vanishing point which is a finite point on the screen. That point actually is an infinite point in 3D but it is a finite point in two dimensional screen and is called z axis vanishing point. Again I repeat it is called z axis vanishing point because the lines the parallel lines which meet on the projection plane or the screen to form the vanishing point are all parallel to z axis.

Remember, if you take lines which are parallel to x and y they will not form a vanishing point in this case because typically we assume that it is especially to do with the y axis because the y axis is parallel to the projection plane.

Now we look at the same diagram and we will change it slightly such that now we make the screen exactly parallel to the x y plane. Now the viewing geometry in the left hand side diagram was such that the viewing axis was neither parallel to x y z. It was only assumed parallel to be y axis but now we are putting the viewing plane or the projection plane on the x y plane. And we are looking through the rectangular parallel pipette at z axis vanishing point that can be obtained in the similar manner as at the left hand side diagram by extending or extrapolating the four parallel lines which are parallel to z axis.

And then our form, in fact both the points which are pointed by the z axis vanishing point are basically at the same point. But they are from two different perspective geometry reference points. That means the projection plane has shifted and that is why the z axis vanishing point has shifted. But you will always get a vanishing point whenever you are taking parallel lines and those parallel lines are not parallel to your projection geometry or plane.

Remember, if you take the horizontal or vertical lines of your cube as we can see especially on the right hand side diagram and those form a parallel line they will never meet in the projection plane or in the screen because as they are parallel lines in 3D as well as in 2D.

When you take parallel lines which are not parallel to the projection screen they will intersect the projection plane on the screen and they will actually meet. Those projection lines which are parallel to z axis in this case intersect the projection plane and henceforth they also are not parallel to each other and they will meet at a point called the vanishing point. So I hope that ideally you have to draw these diagrams by yourself to visualize what it is. Please take a pencil and a paper and scale and draw this diagram and try to visualize and see what we mean by the z axis vanishing point.

We will go to the equations of perspective geometry. I will leave it as an exercise for you to find out the equation of the vanishing point because it is a finite point. You should be able to find out the expression or the value of the vanishing point given a set of parallel lines and the projection geometry.

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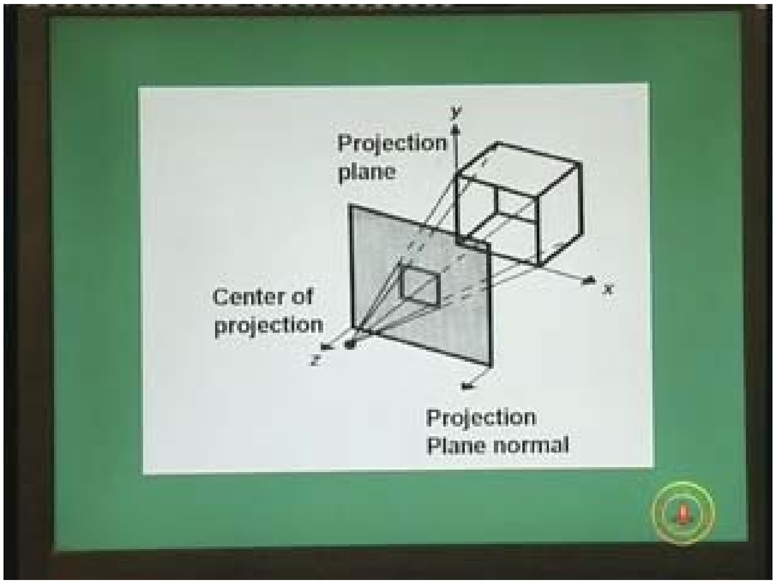

Now this is the typical example of a projection plane or the projective geometry where as you can see you have an x y z axis the three dimensional scenario and you have the same cube or a rectangular parallel pipette. And now you have for the first time the center of projection which is marked as a large dot here in the center of the screen. This is the COP or the PRP perspective reference point or center of projection COP or PRP and rays come out of the COP or PRP cross the projection plane and intersect the object.

Actually if the projection plane is kept behind the object and that is also not a problem as the rays will actually pass through the object hypothetically and may 0 be actually passing through but since they are all virtual rays in computer graphics, virtual reality plays a very big role and it has contributed to many aspects in virtual reality. So virtual rays can pass through the screen and hit the object or pass through the object and hit the screen if the screen is behind the object.

So right now what I have done is, in this picture I have taken four arbitrary vertices of the three dimensional structure and join the center of projection to the corresponding vertices of the object. And wherever those projection rays or projector as you call it intersect the projection plane as given here as the grey colored plane here we get the projection of that point on the projection plane. And the projection plane normal is also given here. So that projection plane is normal in this case and we can assume it to be along the z axis. So center of projection is on the positive z point and looking towards negative z or you looking towards origin and rays intersect the plane and then strike the object and you get the projection of that three dimensional structure on the projection.

So this is the diagram which illustrates what is my center of projection which is at the finite distance from the projection plane and the object actually could lie between the projection plane and the COP or it could lie behind projection plane. Nobody stops you from visualizing and considering any sort of scenario where the projection plane could be almost anywhere with respect to this COP and the corresponding object.

So you can now try to visualize the diagrams of the vanishing point with respect to this figure. This was the diagram which was illustrating the vanishing point. Remember there is no COP here in the true sense. We are not able to see the COP and do not confuse the z axis vanishing point with the COP. z axis vanishing point or any vanishing point comes due to the intersection of parallel lines, projections of parallel lines in 3D or their projections are taken and they are not parallel any more in the projection plane and if they are not parallel they will meet. In 3D they are parallel but in the projection plane they are not parallel and hence they will intersect to form a finite point in 2D called the z axis vanishing point.

The z axis vanishing point or any vanishing point is a finite point in 2D and it is basically projection of an infinite, point at infinity where parallel lines in 3D will meet. So that is very interesting and in this case center of projection is again virtual. But it is easy to visualize this is more to do with projector rays. To get a projection of an object on the projection plane you need the center of projection to be defined with respect to the axis and the projection plane and it is normal otherwise you can not draw the rays. Once the COP is defined you draw the rays from the COP or PRP to the object. Intersection of those rays with the projection plane will give you the projection points and you can construct the projection of a 3D object onto 2D.

We have discussed about z axis vanishing point. This is an example of x axis vanishing point and z axis vanishing point together to describe again. And as you see here that the center of projection is defined, we have x y z axis here. I hope you can see it and we again have a cube defined here as x y z. So, one of the vertices of the cube is at the origin of the coordinate system.

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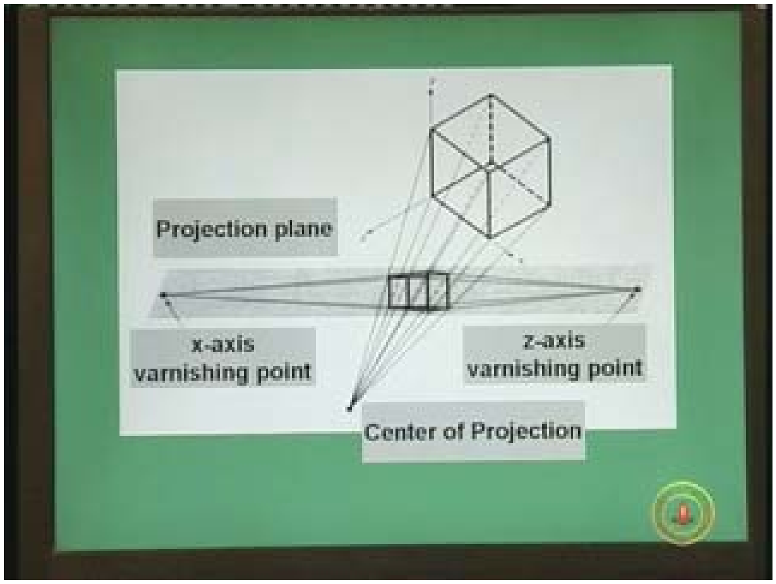

We have a projection plane, interestingly this projection plane, we assume that it is parallel to the y axis. The z and the x axis intersect that plane but y axis does not intersect that plane because we can assume the y axis is parallel to the projection plane. And as long as the y axis does not intersect the projection plane, any lines which are parallel to y axis will also be parallel in the projection plane after they are projected.

So now as you can see here, since x axis and z axis will intersect the projection plane and what I have done basically is taken all the 4 plus 4 about exactly basically eight vertices of the cube and have tried to project all of them on the projection plane. The process is very simple as I told you, you have to take all the eight vertices and draw projector lines or rays from the vertices to the COP. Or take lines, join COP to the vertices and wherever these lines intersect the projection plane you mark those points. Those are the projections of the vertices in 3D onto the projection plane. And you can join the respective vertices in 2D and get the projection of the three dimensional cube on the two dimensional projection plane.

Now this you can do for any object which has vertices or even curves. And then you can take point by point and keep projecting and drawing lines in the center of projection COP and get the projection points on the projection plane and reconstruct the 2D diagram. All this is possible.

But here we are talking of more than one vanishing point on the projection plane. It is more than one vanishing point because as you can see here lines which are parallel to the z axis can be projected onto the projection plane but they will not be parallel any more. They will not be parallel any more because the z axis intersects the projection plane and hence the projections of parallel lines, lines which are parallel to z axis will now meet at one point called the z axis vanishing points.

Similarly, lines which are parallel to x axis will also not be parallel anymore to the projection plane and they will meet and form x axis vanishing point.

Why we do not have a y axis vanishing point? I did tell that we are assuming in this figure which is again borrowed from the book by Foley Van Dam for authors on computer graphics principles and practice where this diagram shows that these four lines of the cube which are all parallel to y axis form parallel lines as projections on the projection plane. That means if you take these four lines on the cube which are all parallel to y axis since the projection plane is also parallel to the y axis, these parallel lines in 3D will also form parallel lines in the projection plane. You can try this out using the COP and it is also almost evident in this figure.

Thus vertical lines which are all four vertical lines you can see in 3D of this cube, three continuous lines and one dash line at the back there are three plus one that is four such vertical lines of the cube and they have formed four vertical lines on the projection planes. And these four lines remain parallel. They remain parallel because the y axis does not intersect the projection plane. If you tilt the projection plane and make the y axis intersect the projection plane. Then these vertical lines will not remain parallel in the 2D after they have been projected.

But in this case I kept repeating that the projection plane is parallel to y axis and all vertical lines which are parallel to y axis also remain parallel in 2D. And so they will not meet in 2D and hence we do not have a y axis vanishing point. Since the x and z axis meet the projection plane, all four lines which are parallel to z axis will not remain parallel on the projection plane and they will intersect and form the z axis vanishing point. And all lines which are parallel to x axis will not remain parallel on the projection plane. Once they are projected they will form x axis vanishing point.

This is interesting that you can have one vanishing point, one point, two points or three points projection system. And typically for simplicity one axis vanishing point is usually chosen but you can have a two axis x and z vanishing point. You can also have x y and z axis vanishing point where you take the projection plane in such a manner that all the axis intersect. None of the axis is parallel to the projection plane, they will intersect at one point and that is the case where the entire three axes meet at some point with the projection plane and none of them are parallel. Then you also have a three axis vanishing point x y and z respectively.

So you remember the diagram which we talked about where we had three different types of projective geometry. One was one point, two points and a three point, they are nothing but an x y or z in case of one point. This is an example we have just seen of a two axis vanishing point or two point projection geometry. It is of no problem as you can also similarly have a three point perspective point, it is nothing but three vanishing points or perspective geometry.

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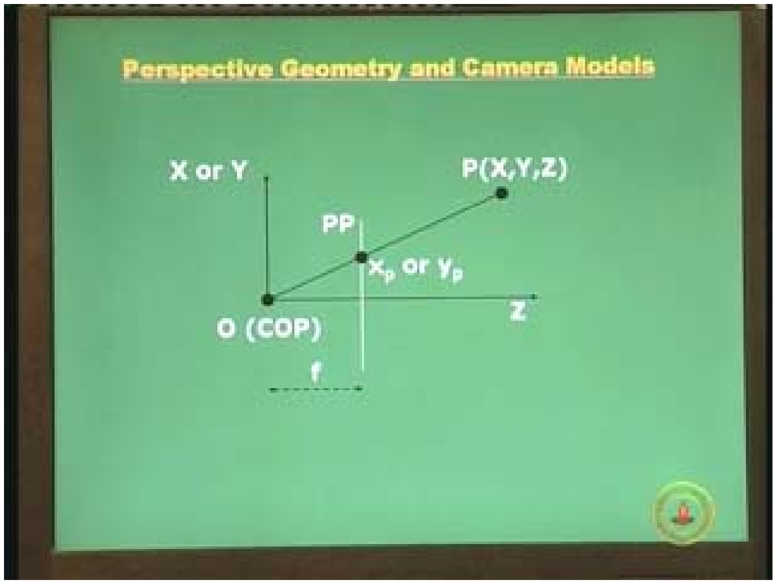

We will go through the equations of perspective geometry and camera models to understand before we move onto orthographic projection. As you go along here I have given a two dimensional projection of the three dimensional geometry which we have just seen in a few slides before the vanishing point or somewhere in-between we discussed about COP and points coming out of the COP intersecting the projection plane and then going on and touching the point on the 3D.

So here if you see in the screen you assume your screen to be X and Z or Y and Z. So if it is x then y is coming out towards you as you see on the screen. If it is a Y-Z plane, your screen is a Y-Z plane then X is coming out of you, it does not matter. We have taken the origin of the coordinate system to be for our convenience for the time being. This is a special case it is not general that the COP or the center of projection always has to be on the origin of the coordinate system.

But in this case let us assume that, and the PP are the projection plane which at the distance f from the origin, this f is something similar to a focal length parameter or a perspective geometry parameter. And we will see that how it is related to the focal length of the lens as well, not directly of course but let the PP or the projection plane is at the distance f from the origin or the COP in this case is the same. And a point in 3D is given by this point P(X, Y, Z).

So the projection of this point P(X, Y, Z) on the projection plane PP can be easily obtained by the concept we talked about. That is, take a ray or join a line between P and O or COP and P. In this case since O and COP are same, take a ray which starts from COP, intersects the projection plane and goes and touches the point P or take a simple straight line which joins P and the COP on the origin in this case.

Intersection of that ray, intersection of that ray with the projection plane gives the projection of a point P in 3D onto the 2D plane PP. So I will mark it as X_p. So if the vertical axis is x, we have for corresponding x the X_p or the X projection point. And similarly if this is Y instead then we should have Y_p, the subscript indicates that it is a 2D geometrical value whereas X Y Z capital letters indicate 3D coordinates of the point P.

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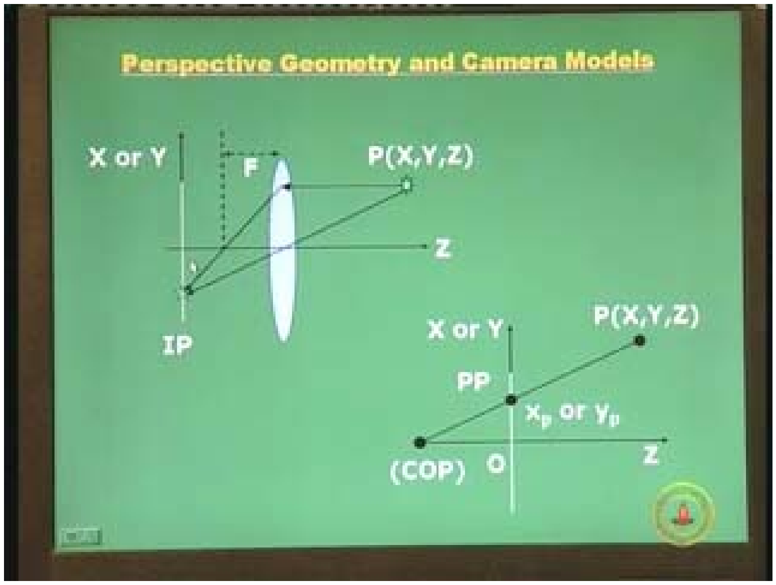

We will look at this model from a different angle. Now I will try to give an analogy with that diagram with respect to the concepts of optics which typically happen in a camera. This is to do with the imaging models talked about more in classical areas such as image processing and viewing geometry and all that.

So typically if you look into the concept of optics, you have a point in space at X Y and Z and you are viewing that with a lens. This is typically in the case of your eye or in the camera, you have lens in front and retina at the back side or your analog and digital camera where you have lens in the front which is used to focus the nearest object and you have a screen or an array of sensors in the back side if it is a digital camera. If it is an analog camera you have a photosensitive film on which the object is focused. And that concept of optic says that rays which come out of the point p pass through the optical center of the lens, pass through undeviated and will fall on the screen which we call as the image plane, in this case it can be considered to be a projection plane.

In this case we will take as an image plane IP and rays which are running parallel to what we call as the optical axis of the lens, in this case horizontal axis, z is the optical axis of the lens, they will get deviated, they will deviate and then pass through a point which is at the distance capital F from the lens. This is very interesting, capital F from the lens and these two rays after they bend and once they are undeviated, if the object is actually focus properly, this will go and meet at a particular point on the image plane x and y.

Well if I remove the lens, now I can tell you that well I remove the lens and I will also remove the ray which is bent. I will just take the center line and replace the same figure with this model. So this is done in most image processing concepts of perspective geometry where I will say that line passes through the projection plane at the lens and COP somewhere where this point meets and basically what you can visualize is you can take the line and deviate which passes through the optical center of the lens. And take the projection screen in front of the lens between the object and the lens and when you have that then this point X_p or Y_p comes here, the origin becomes COP, the origin could be taken here and this is the projection plane X or Y and Z.

So there is almost a similarity or analogy which you have to visualize between this model considered for perspective geometry and the lens model which is coming out of optics theory. And we continue with the perspective geometry. So this is the scenario with which we were just talking about. And this perspective projection plane PP has a distance f from the origin or the center of projection. As it is called about P(X, Y, Z) is the point in 3D and this ray which joins the COP and the point in 3D intersect the projection plane at a point which has coordinate X_p or Y_p depending upon whether the vertical axis is x or y respectively which we are viewing along the z axis. This is one model.

The other model is where the origin is considered to be on the projection plane and the COP is on the left side at the distance of minus f on the negative z axis. And on the ray we join the COP and the point in 3D intersects the point PP again. But in this case the origin is taken to be at the intersection of the projection plane and the z axis, instead of taking it at the COP or COP z or minus z.

In this first figure the projection plane is at the distance of plus f from the origin. In this case the projection plane is at the origin COP is at the distance minus f in the negative z axis. Now these are the two different projective models which are quiet simplistic. These models give us very simplistic equations of course for perspective geometry. These are used in many image processing books as well. It is used in many computer graphics literatures as well. And with respect to these two figures we will see what are the corresponding equations of these two figures. They are almost similar, as you can see structurally the figure is very much the same.

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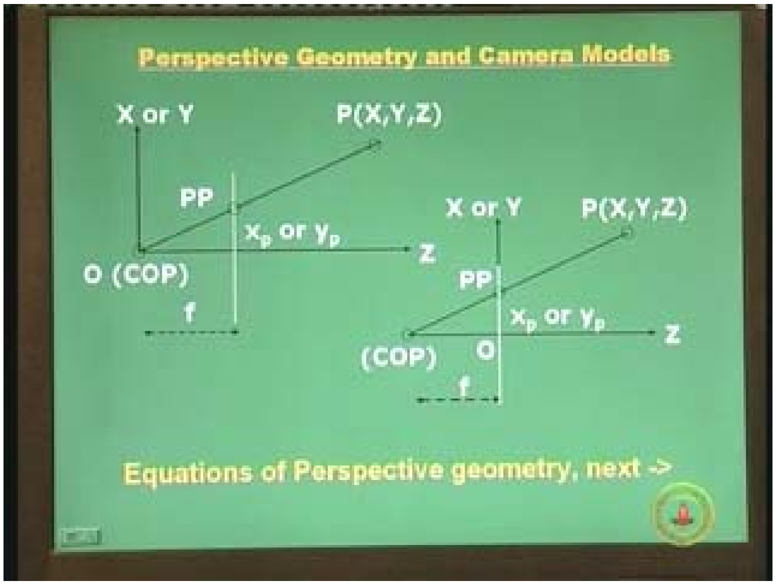

Just as a question of whether we move the origin, take it at the COP or take it the origin at the projection plane itself is the question. Of course the question which you should ask is where is the lens gone as we talked about in the couple of slides back. Well the lens is there but we have derived this simple structure from the optics of the lens itself and we do not worry about the lens anymore.

We are worried about the 3D object, the origin of the system and the projection plane where the projector ray joins the COP and the point P intersects. So that is the point where we have the projection of the 3D onto the 2D projection plane and that is all which is interesting for us and that gives us very easy equations which you can write yourself now using similar triangles and derive equations and derive the expressions of X_p or Y_p in terms of x y z and the f the parameter for projection. And we can change that distance between O and COP in this case or between in the projection plane. And O and that parameter f actually controls what is going to be the value of your projection point X_p or Y_p on the projection plane. Remember, the projection plane is in 2D so it is a small x y domain and in the 3D the point is in x y and z domain.

So let us look at the equations of perspective geometry next corresponding to these two figures. I will give you two expressions and you can see they are very similar. A small difference exists as like the small difference in these figures itself and here are the equations of perspective geometry.

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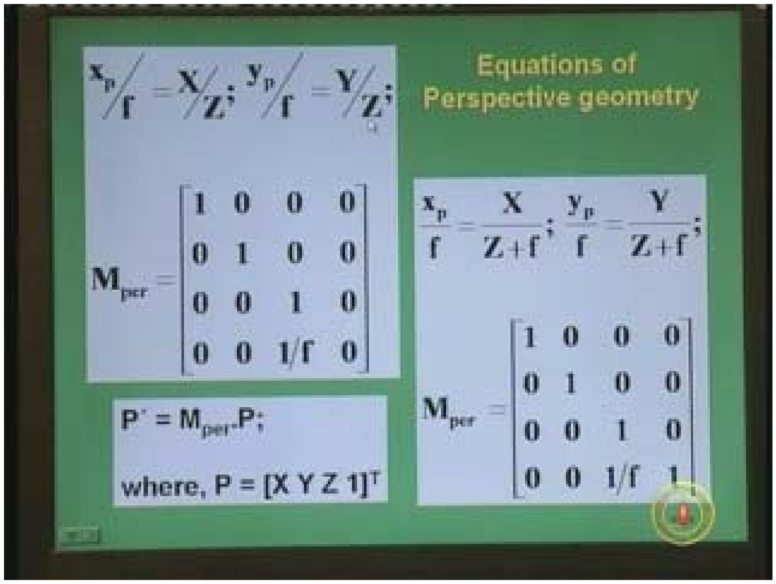

From the first figure you can use similarity of triangles and write expressions as given on the top. Linear equations X_p by f is equal to x by z and Y_p by f is equal to y by z. I go back to that figure (Refer Slide Time: 00:49:20) which we just discussed. As you see it from the similarity of triangles here, X_p by f is equal to x by z or Y_p by f is equal to y by z. You can draw a vertical line here from the point p and drop the projection onto the z axis. That will be your x or y and that divided by z will be your X_p by the f, the distance between projection plane and f, so that is from the similarity of triangles. You can write those two equations as given here on the top and that basically gives us the relationship of the perspective geometry transformation matrix 4/4 as given here

P prime the projection, obtained the projection of a point on a 2D axis is M per multiplied by the point P where M per is the projection matrix as given here. It is almost an identity matrix except that the last row last column is not unity, it is 0 and in the last row you have to put 1 by f factor.

If you use this M per and then use P as the [X Y Z 1] in a homogeneous form. Try to use this, you will give get a P prime as your transform coordinates and you will get the expression as given in the top row. These are the equations with respect to the first figure. Let us just also look at expression as given in the second figure. And this is the expression as given here X_p by f or x by z plus f. This is with respect to the second figure. You can write this equation yourself again using similarity of triangles and these expressions when they are expressed in matrix form under perspective geometry will give M per or the perspective transformation matrix as given here.

You see that the expressions of both these matrices, as I promised you earlier, were same. This is actually an identity matrix with just the one factor 1 by f given here. In this case of course the difference between the two matrices is the last row, the last column element is 0. So the two matrices have exactly similar nature except at one location where the last row last column is different 0 and 1 respectively. And you have the 1 by f parameter in the last row which basically takes care of the projection transformation parameter. And you can use this M per for both the two models in the two expressions. Use P as a homogeneous coordinate system P[X Y Z 1], use M per post multiplied by P or premultiply by the M perspective transformation matrix and you get P prime which will be basically in 2D homogeneous form and you can get your expressions which are expressed in a linear or non-linear form as given in the two models on the top.

So we stop here with the discussion on perspective geometry which we have discussed so far. And we continue in our next class with a few discussions on perspective geometry and also on the orthographic transformation matrices or orthographic projections which we did not touch in this class. We will start with few discussions or extensions of perspective geometry and then finally the orthographic geometry in the next class. Thank you.