Artificial Intelligence:

Representation: Reification and Abstract Entities

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Module – 04

Lecture - 04

We are looking at knowledge representation and we were looking at properties in particular and we introduced this notion of reification. So as I said, basically the idea of reification is to introduce abstract objects and by this I mean objects which are not in the domain but are introduced by us into the domain, kind of injected into the domain to help us reason about certain things. So in particular we were looking at properties and properties like height, weight, distance and say volume, temperature. These are properties we often associate with certain objects in the domain and how do we represent these properties essentially. So what we have said was that all these kind of properties are Types. It means that for example if we have a type of height which contains elements which correspond to height. And these elements are abstract elements in the sense that they are not concrete, tangible things that you can see. So if you say something like Height Mary then we say Height Mary is an object of type Height.

I am overloading the term height here, in the first instance it is a function which when applied to Mary gives us a new term. In the second instance its a type which means its a unary predicate which defines a certain type of object essentially. But its a little bit of overuse here and we can think of all these properties like height, weight, distance and temperature as types and any element of this type is basically a property of this type. Now so we can think of it in the domain D we add a new set called Height and then if we have Mary has object in the domain, then the function Height maps Mary into some element inside it which is a height of Mary.

so we can think of the height as the abstract object. Now unfortunately that does not help us enough to talk about height because when we talk about things like height, weight and distance we want to use units of measurement essentially. So we may want to measure height in feet or centimetres or meters. The height of Mount Everest is so much and so forth. So we want to come up with tangible quantities essentially. So what we can do is when we talk about units we can think of them as functions so for example units like inches and feet and so on. And one way to think of these functions is that these are functions so for example feet we can say is a function from numbers to objects of type height. So when we say 6 feet for example then essentially you are saying it corresponds to some element belongs to Height and then we saw that we can if you want to talk about the statement Mary is six feet tall. We can represent this as Height Mary which we have seen is a function which maps onto an object of type Height is equal to feet 6. So we have also seen that feet is a function that maps numbers to height so it takes 6 as an argument and tells us what is the height of Mary essentially.

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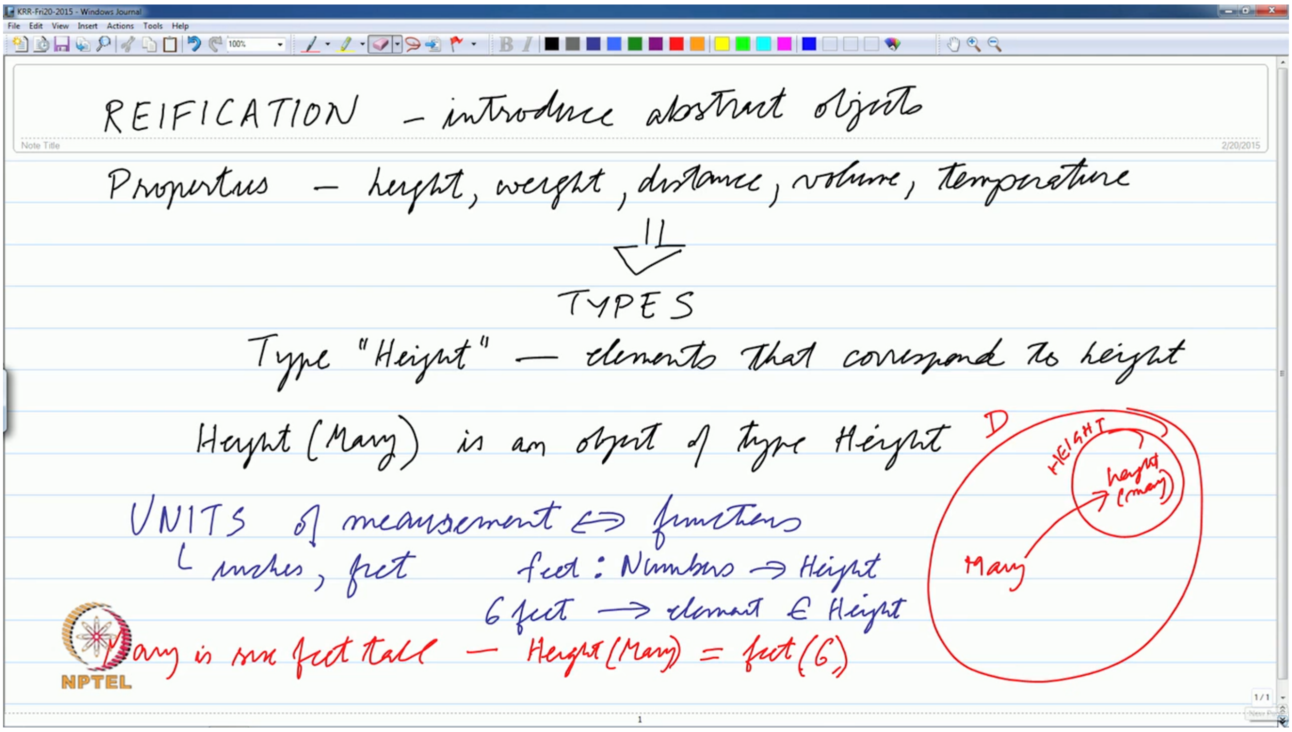

so we can talk about comparison now. Supposing we have a sentence Mary is taller than Peter. So one way we can do is define an ordering on elements of type Height. So once we define ordering we can define what do we mean by taller and things like that. So we can make a generic statement. We can say that if you want to say Mary is taller than this thing then you want to say that there exists a number n and there exists a number m such that Height Mary is equal to feet n . Remember this a a statement in FOL, you can equate two terms and say that they are equal and Height Peter is equal to feet m. And n is greater then m.

notice that we have expressed the statement that Mary is taller than Peter without really talking about their actual height. Essentially we are saying that there exists some number n which is the height of Mary and there is another number m which is the height of Peter and n is greater than m. So we can express statements like this in FOL. Now we have this problem, supposing we say that lets say John walked for 3 kilometres and then ran for 900 meters. Then the question is how much did he travel?

So these kinds of problems occur all the time in kids books when they are learning maths and we should be able to devise mechanisms in which we can represent such problems and solve them essentially. So one of the questions we have to answer is how does one add 3 kilometres to 900 meters. That means we have to somehow find a conversion from one type to another type. Notice that these 3kms and 900 meters we will assume they are of the same type Height or I should have called it type Length but basically it doesn't matter, basically its a one dimensional measure and it represents objects of one dimension. So how do we add 3 kilometres to 900 meters. Obviously when we talk about addition in this domain then we have to talk about saying that you must always convert them to the same units and only then you can add. How can we convert 3kms to meters? We can say that anything in kilometres is equal to something in meters x, lets say this dot represents multiplication, 1000. so if we had represented somewhere that he walked for 3 kilometres then we can inference using this statement which says that so this is basically for all x.

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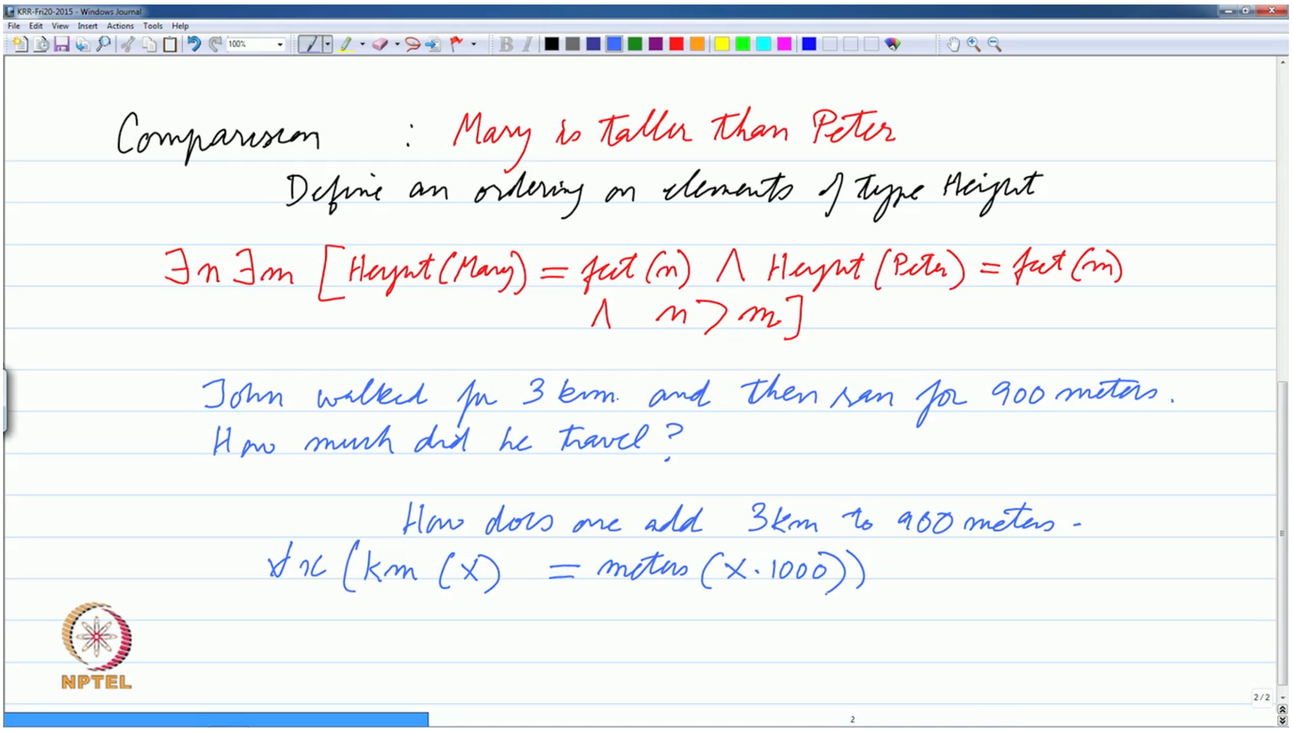

If x is in kilometres then you multiply x by 1000 then thats it meters so it gives us a mechanism for this thing. And then we can have rules which say that you can add numbers in the same unit essentially. So i will leave this as a small exercise to figure our how to do it.

We can say that feet, instead of being a function from number to objects of type Height, we can say it is the other way round, Height to Numbers. And then if you want to still say that Mary is 6 feet tall then we can say that, same as before. We have Mary and we apply this function Height to Mary so we get an object of Height type. But now we have a function called feet which we apply to this object of type Height. And what this gives us is a number and this number is 6. so this is an alternate way of looking at it essentially.

Now we can define an ordering amongst feet, just as we defined Mary is taller than Peter, we can define an ordering on the number returned by feet, say that feet x is greater than feet y if x is greater than y.

now if I want to talk about statements such as the following that Mary was 4 feet tall in 2006. so may be Mary is a young girl or she was a young girl in 2006 and she was 4 feet tall. How do i represent these kinds of statements? Any idea? Which of the two alternatives that we discussed about representing height is better. Because now I want to do something like the following. I want to say that remember we were talking about adhoc predicates, so I will choose a predicate called height which is still overloading the name height and then I would say for this, when I talk about predicate Height, I will talk about a person or a thing and its height and may be a date followed by that essentially. So my schema becomes as follows:

Height Subject, height, date. Now which one do you prefer essentially. Do you prefer to talk in terms of abstract objects or do you prefer to talk in terms of numbers. The second schema that we are seeing right now essentially brings everything down to numbers which ofcourse makes things like addition simpler. If you say that he walked for 3 meters and he waked for 6 meters, then both 3 and 6 are numbers and you can add them up provided ofcourse we are talking about the same units.

So may be in this mechanism it kind of makes sense to say that since I am saying that Mary is 4 feet tall, this kind of naturally corresponds to this representation so i could use this representation as a term.

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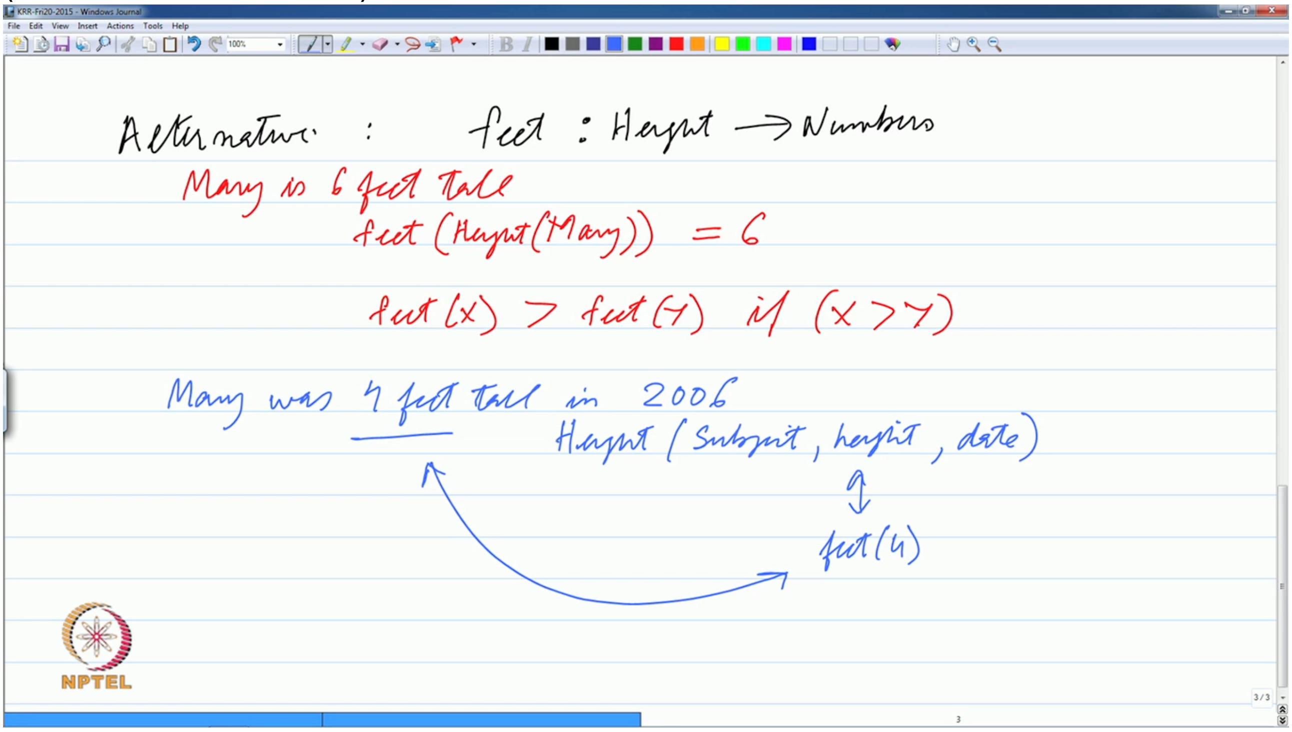

we are so used to talking in terms of numbers that we have not thought about representing numbers essentially. How can we represent numbers. I told you a story last time about Mc..... and he told somebody that I want to spend my life studying what is a number that a man may know it and what is a man who may know the number and he was told that its a job of a lifetime. So if i say Mary has 6 apples, what am I really saying? We can boil it down to a concrete representation. Whenever I use a number what am I trying to say? What is a number? What is 6 essentially. When i say Mary has 6 apples what am I really saying or is it a shortcut for essentially.

If you want to explain to somebody, lets say somebody comes from Mars or somewhere and you say that i will give you 3 apples and the creature says what do you mean you will give me 3 apples. So you need to explain what do you mean by 3 apples. If i want to define numbers, lets say i want to define natural numbers. I want them to be discrete objects in my domain. I may have apples and oranges in my domain but now i want to add numbers also. In the sense that i want to be able to count and add, subtract, do all that kind of stuff. But what is this number, how do i add numbers to my domain in a logical sense. So without spending too much time on this, anyway this is not our main goal. We can see that numbers correspond to set, so the first thing that we are talking about is somehow numbers are associated with sizes of sets.

A number is a cardinality of some set so we define a sequence but we define a sequence of sets and essentially we talk about cardinalities of sets and when we talk about Mary has 6 apples we are saying that the set of apples that Mary possesses is in one to one correspondence with a set which I will call 6. 6 is a name of a set and essentially when i say that Mary has 6 apples, what I am fundamentally saying is that the set of apples that Mary has is in one to one correspondence with a set which I will call 6. and essentially so this notion of quantity becomes a notion of a definition in set theoretic terms. We are saying that numbers correspond to sets of certain sizes which is also an abstract notion. So some people have said that 6 corresponds to all those sets which have the cardinality 6. now its a little bit of recursive definition but in some sense they all have the same cardinality. The set of 6 apples and the set of 6 oranges what they have in common is that the cardinality of two sets is 6 essentially. And 6 essentially corresponds to abstract concept of cardinality of a set.

Now it was due to John von Neumann that we have a very nice definition of numbers. So this definition works like this essentially that the number 0 I give a name to the empty set phi which you can write it as an empty set. So 0 corresponds to the empty set essentially. If we have nothing then you have as many objects as the empty set essentially. So that makes kind of sense. Number 1 corresponds to the set which contains 0 and the contents on 0 which is an empty set. So number 1 is a set which I make by taking the set 0 and adding it to a set and add contents of the set 0. so you must remember that numbers now are sets essentially.

so how do I get that number? So i got 1 by saying its a union of a set containing 0 union 0. If I look at the number 2 then I am saying that take the set containing 2 and take the union with the set 2. This makes any sense? It will make sense provided you remember that numbers are union themselves essentially. Sorry not 2 it should be 1 the previous set.

and what is the set 1, 1 is the set that we have just drawn here which is phi phi which is the number1. So i put this, I want to take the union with the same set. So the first object is a set containing 1 and the second object is 1 itself. So 1, the set containing 1 union with 1 essentially which ofcourse gives me a set of these three objects. The set containing phi and phi. Sorry I am taking the union so I wont have this set.

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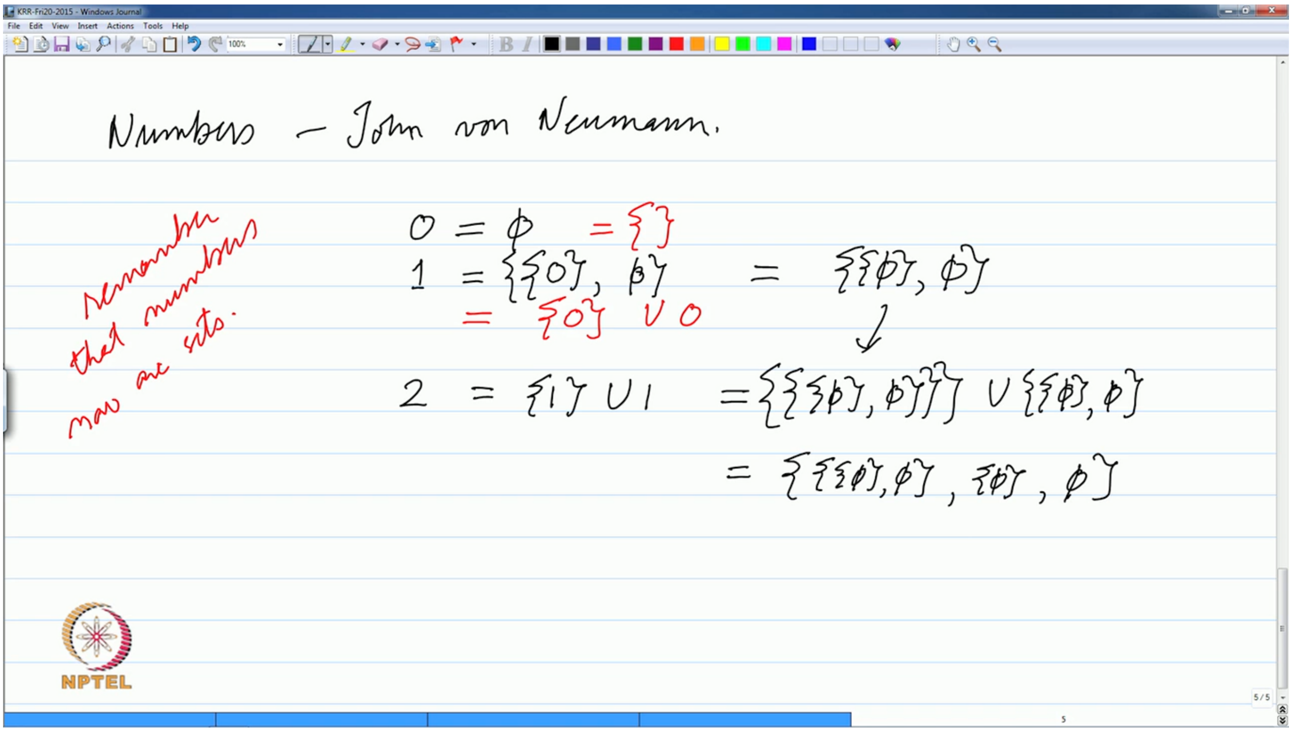

Okay I think I messed it up so lets start all over again. So a nice idea of what numbers are was given by John von Neumann. And what he essentially says is that you start out by defining 0 as an empty set which you can write like this. Then we define the number 1 as set containing 0 union 0. so if you substitute what 0 is which is set phi which is set containing phi union phi which is the set of phi because if you take a union of anything with an empty set you get that number essentially.

Now if we define a number 2 as a set containing 1 union of 1. the set containing 1 is this set which contains 1 and we are taking a union with 1 which is this set itself. So notice that when i say 1 here I have got 1 here. And when I say this 1 here I have got this 1 here.

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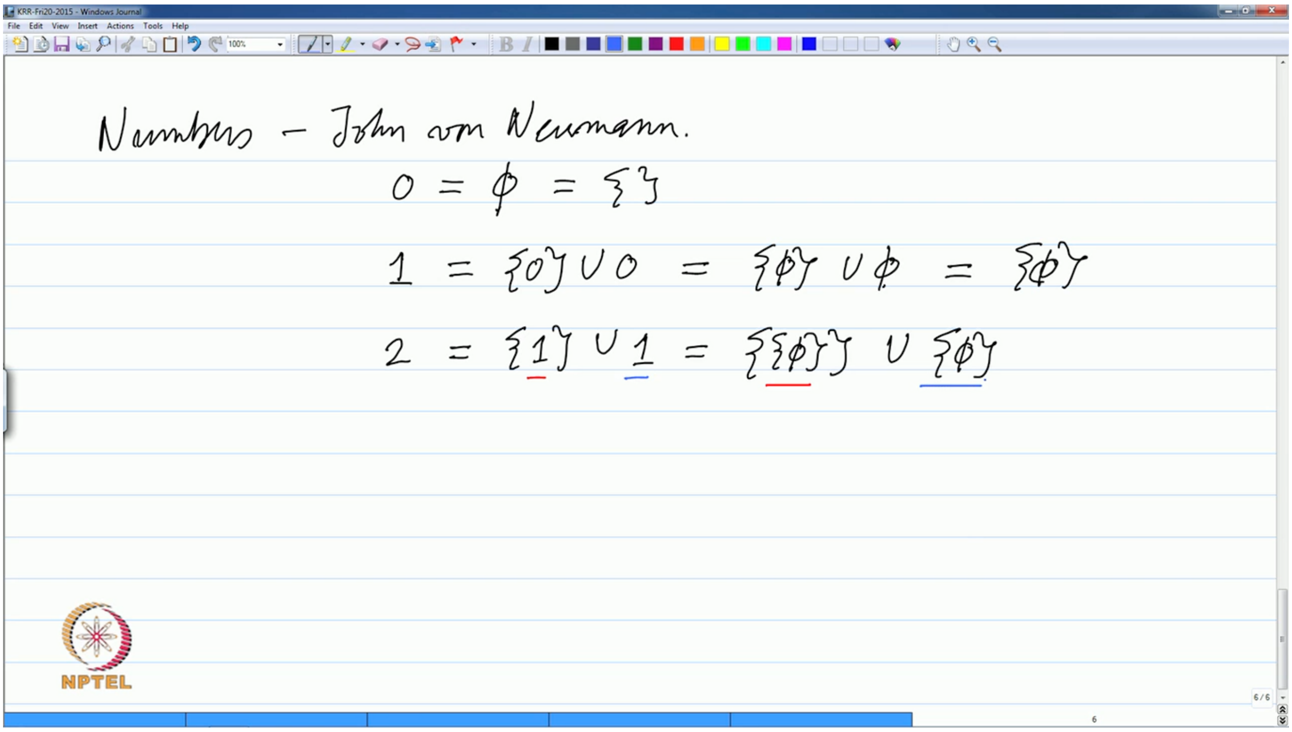

This is equal to the set which contains phi and empty set. The basic idea here is that a successor of a natural number N is defined as a set containing that number N union with the set N. So we can now talk about the number 3. so its a set containing 2 union with 2. so we just wrote the set containing 2 so if we take the set containing 2 so I will just do a shortcut here. It will be this whole thing then the elements of that set which is the set containing phi and then phy. So you can see here that the number 3 corresponds to a set which has three elements essentially. So in this way we can define the set of natural numbers starting with the empty set by saying that the successor function defines a set by taking the previous number and taking the union of previous number with the set containing the previous number. But when we talk about natural numbers for the rest of this course we will essentially talk of the sequence.

We will say that 0 which is the constant that we think of, then successor of 0, then successor of successor of 0 and successor of successor of successor of 0 and so on. And we will end up defining the sequence of elements in which the successor function is implicitly defined and we have a definition. So if you say what is the number 2000, you can say that it is 1999 union with elements of 1999 and in some sense we have a logical definition for every set essentially. So we can construct a set of that many elements and the definition of the set of that particular cardinality is the definition of the number itself. So in the next class we will come back to Reification and we will try to see where are the other places we need to do reification.