Algorithms for Big Data
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Lecture – 09
The Monty Hall Problem

In this segment, we are going to look at problem 1.2 from (Refer Time: 00:33).

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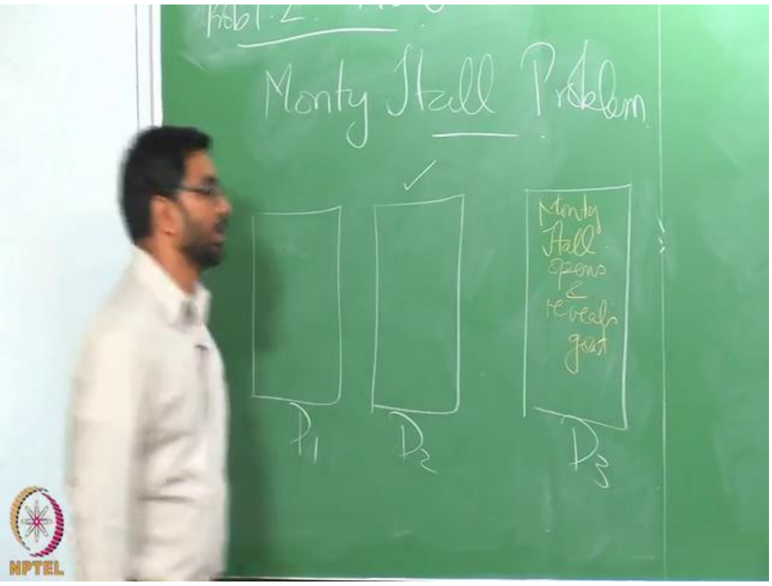

This problem is called the Monty Hall Problem. To give you bit of background Monty Hall was a game show host, and in that game he would play this the following random probabilistic game.

So, let us think of this, let me explain this context. So, you there are in this context there are 3 doors. What you know is that behind 1 of these doors, D1, D2, D3, behind 1 of these door is a special price, let say a car and behind other 2 doors there is something lockless desirable, let us just say according to the problem given here behind other 2 doors are goats. So; obviously, you prefer a car which is a lot more expensive and valuable. So, your interest, you are allowed to choose a door. Let us say we pick door D2 and then the game show host Monty Hall, he will he is aware of which door a holds the car.

So, we can look at your choice, if this choice may be the choice with the car it may the choice without the car. We do we do not know, but Monty Hall know that, but regardless 1 of these, other 2 doors is going to have a goat. So, what Monty Hall is going to do is, open the door which contains a goat. So, this and now he gives you the option, you have 2 options; 1 you can either retain this D2 or you can move to D1. So, there are 2 options; now you have the freedom to choose D2 or D2 ok.

And at first you may think this is not much value in choosing in moving over to D1 it may be you might be saying well why not I stick with D2, but as it turns out the you actually increase your chances of winning the car by moving over to D1. This is the Monty Hall Problem we need to prove with this is. In fact, the case, let us let us try to analyze these 2 scenarios; 1 scenario you choose you choose a door Monty Hall reveals another door where he has a goat, but you choose to stick to D2 that is scenario one.

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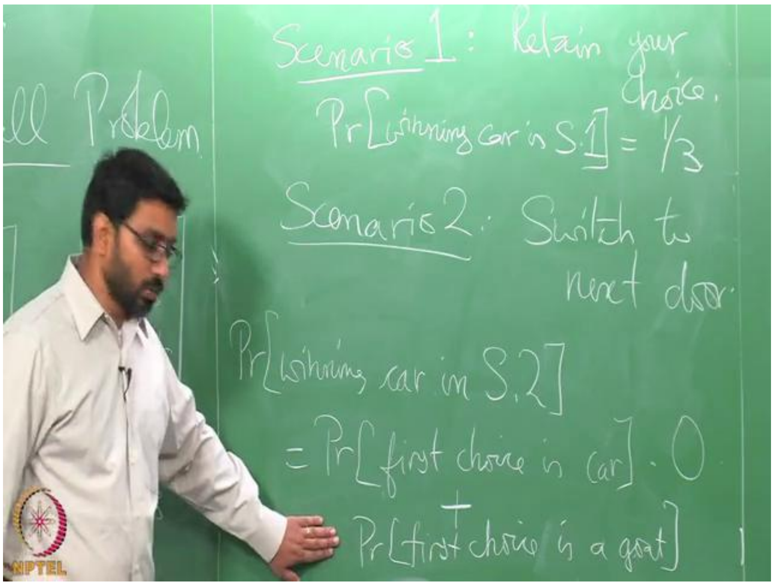

You retain your choice, and scenario 2; you switch. So, in scenario 2 you would say well I choose D2 earlier on, but let me give up on D2 and switch to D1. At the end that whichever door you have chosen is going to be opened up and you going to get the prize that is held behind that door and; obviously, your interest is to get the car.

So, now you have to analyze these 2 scenarios and figure out what is the probability of getting a car in either of these 2 scenarios. So, what is the probability of getting a car on the scenario 1? Well, if you think about it in scenario 1, you are going to choose a door uniformly at random between say D1, D2 and D3 and you are going to stick to that.

So, probability of car in scenario 1 equals 1 over 2. what about scenario 2 scenario 2 is a little bit more tricky. But let us think about what happens in scenario 2, there are two possibilities that could have happen, 1 is your first choice in this particular instance D2, your first choice could have been a car and if there had been in the case you have you lose. But on the other hand the other scenario is the first choice that you made was not a car, and remember and if this was a goat and Monty Hall has to reveal the other door that also hides at goat. Then the third door that you have going to choose. In fact, has the car. So, if you started out with choosing a door that is not hiding a goat rather choosing a door that is hiding a goat then you are going to necessarily win the car. So, these are the 2 cases that in case with each other. So, let us analyze them a bit carefully.

So, let us write out the probability that we are interested in, it if this is the probability of winning a car in scenario 2, do that this is our interest. Let us see the 2 cases. So, there are 2 possibilities, first choice is car and under this first choice being a car, you the probability of winning a car becomes 0. The second scenario is, then probability their first choice is a goat and if your first choice is a goat. Let us see what happens if your first choice is a goat, this you move gone moving to the other door which is necessarily a car. So, this is going to be hence point 1 and what is this work out to be, probability the first choice is a car is one-third.

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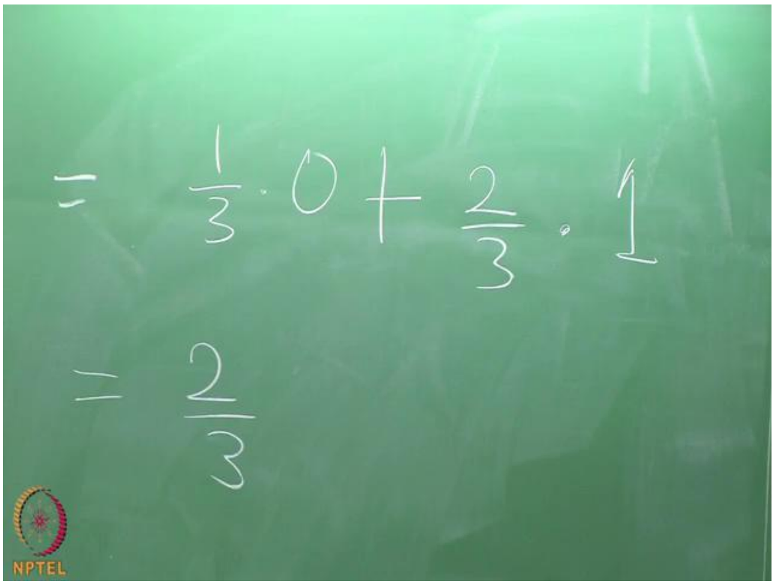

But its times to 0, so that is essentially does not matter then the second option is at is at the first choice is a goat and that is going to happen with probability two-thirds and if that is the case you necessarily win the win the car, and that is the probability one. So, this works out to be 2 over 3, and notice 2 over 3 is significantly better than 1 over 3. So, what this tells you and might be bit counter interactive and you mainly to think about this a little bit. What this tells you a scenario 2 is, significantly improving your chances of winning the car and.

So, this is 1 more example, where careful analysis of this sort of a randomized situation reveals something that could be little bit counter interactive certainly was counter interactive for me. So, that is it.