Artificial Intelligence: Search Methods for Problem Solving

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Chapter – 11
A First Course in Artificial Intelligence
Lecture – 84
Deduction as Search
Implicit Quantifier Notation

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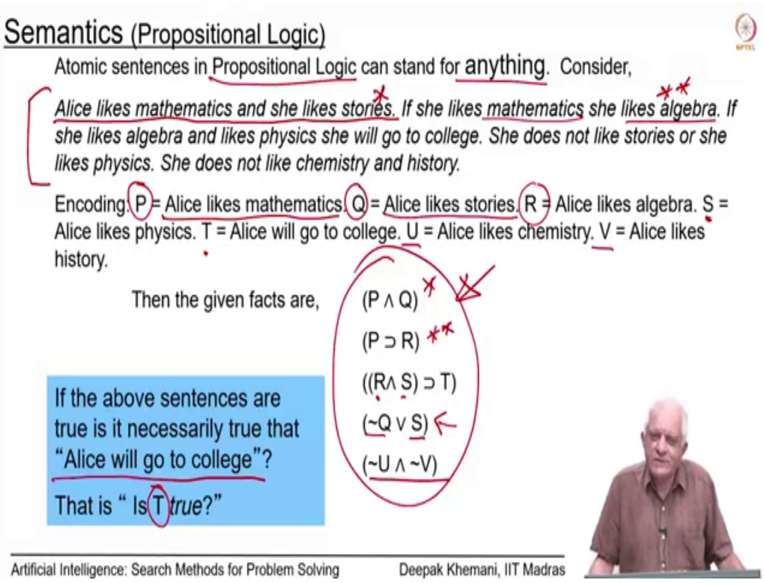

So let us get on with the proof process. In the last segment, we looked at the semantics of propositional logic emphasizing the fact. That you can use propositional symbols so stand for anything and we saw that P Q R S T all these stands for some statements which are there.

Once, we have done this encoding, what we are left with is a knowledge base which is what you can see this set of statements P and Q P implies R,R and S implies P and so on. And, we are saying that given this knowledge base can we find a proof for the statement T, that given this knowledge base is that a way of adding T to the knowledge base, that is the syntactic process of finding a proof. And, we will accept that to mean that the sentence is true because we have accepted our logic to be sound.

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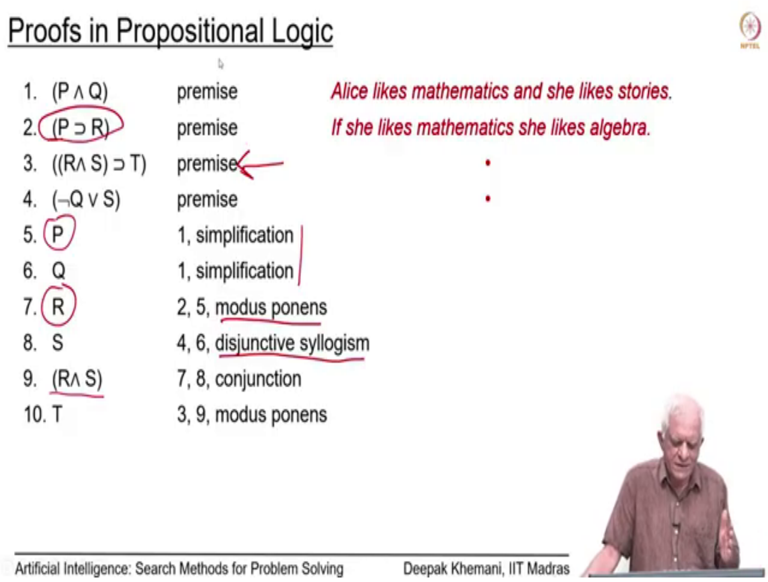

So, let us look at the notion of proofs in propositional logic, some statements are given to us. And, so, we accept them without questioning, because they are given to us they are the premises. These are the 4 statements that we have interested in here, Alice likes mathematics and she likes stories and so on and so forth.

And, these are the 4 statements and we are interested in showing that T is true. Now, the process of making inferences is to choose one of the rules of inference that we very briefly discussed and you can go back and revise them, you probably already know them to keep adding new statements.

So, we can add P, and Q, and R, and S in this order then R and S and then T. So, it is not a very long proof as you can see. Let us look at this in a little bit detailed, why can we add P and Q? We are given that P and Q is true. So, therefore, P is true, there was a rule called simplification.

This again emphasizes the fact that it is a symbolic process, it is we are not looking at meaning, you can not say that if you have said that P and Q is true P is; obviously, true. Yes of course, it is; obviously, true, but to be able to say that in a proof you must have a rule which allows you to say that. And, the rule is simplification and it says that if there is a conjunct you can take any one of them and add it to the knowledge base how do we get R?

We get R, because we know that, P is true and we know that P implies R is true and now we can use modus ponens to generate R. How do we get S? There is another rule, which is called disjunctive syllogism, which is that if not Q or S is true and not Q is false, then S must be true, because this is an or and one of them at least has to be true.

So, we have got S. Then, we get R and S why, because we have R and we also have S and there is a rule called addition or conjunction sorry not addition, it is called conjunction, which allows you to make a compound sentence given to simple sentences. Once, we have R and S we can use our statement, that R and S implies T and we can add T to the statement essentially.

This is the process that I am sure you are familiar with all the proofs that you wrote in your geometry class or algebra class, followed this particular pattern. This is also called natural reduction, because this is the kind of thing that humans always do.

We are calling it forward chaining here or forward reasoning here essentially.

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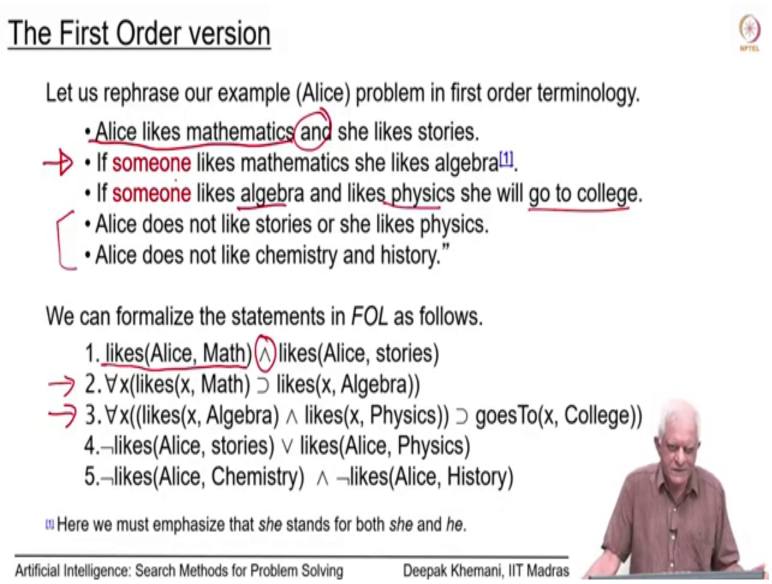

Let us move on to first order version of this story. We stated the relation between mathematics and algebra, saying that if Alice likes Math's, then she likes algebra. Now, let us say that this statement is universally true which means, that if anyone likes Math's, then they like algebra and the universal statement in English is written like that.

If someone likes Math's, then she likes algebra yeah. And, the other statement that, if you like algebra, and if you like physics, then you will go to college. So, remember I am saying if you like, which means in some sense in English we are saying that if anyone likes algebra and if anyone likes physics, they will go to college. Or in other words for all X, if you like algebra and if you like physics you will go to college.

So, our representation has become richer now, as you can see instead of using statements like PQR in propositional logic, we are using predicates. So, one of the predicates we are using is likes. So, this says that likes Alice Math's.

That is the statement, that Alice likes Math's. And, of course, and has come here there is an and, and the second part she likes stories has come as another statement. Statements 2 and 3 are the universally quantified statements. Which is at anyone who likes Math's like's algebra, and anyone who likes algebra and who likes physics will go to college.

And, statements 4 and 5 are simply the translation of these 2 into first order logic here. So, 5 statements in English and 5 statements in first order logic the corresponding statements.

It is a same story that we talked about in propositional case except for the fact that we have made these rules about connections between Math's and algebra and between algebra physics and college, universal in the sense they apply to everyone essentially. And, therefore, we can prove this for Alice, but we could have also proved it for Suresh or John or Peter or whatever the case may be. So, let us look at the proof in the first order version.

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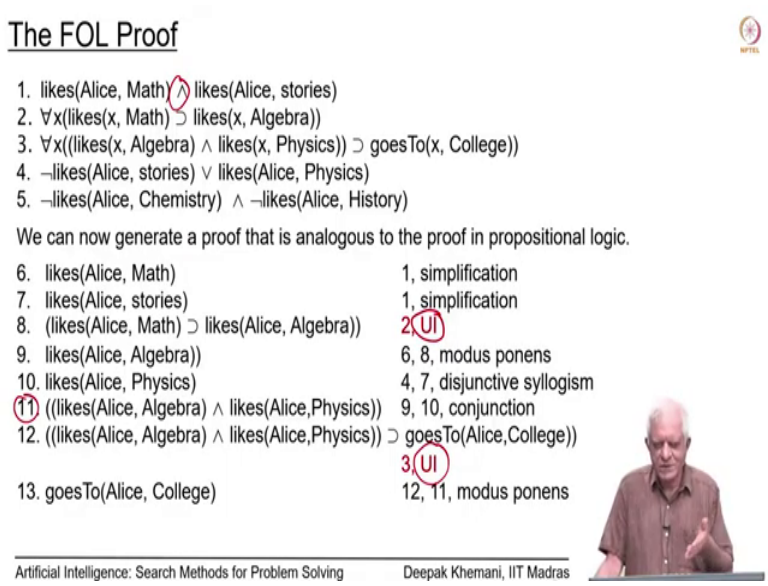

So, this is a knowledge base that we started with the 5 sentences that we saw in first order logic. And, we can generate a proof which is very analogous to the proof in propositional logic.

So, the first statement is simply simplification and because we have something and something here, we can take one of them and add it to the this thing, we did that in propositional logic. So, there is no difference only the representation has changed from proportional symbol to a predicate in first order logic. So, stories also in the same way.

This is new, that you can say that if Alice likes Math’s then she likes algebra. This is something we had said in the propositionals story specifically. In the first order story we said that if anyone likes Math’s they will like algebra.

So; obviously, using this rule of universal instantiation if you remember, we can say that these are universal statement that applies to everyone. So, it must apply to Alice as well. And, therefore, we can make this inference that if she likes Math's she likes algebra.

Now, we are on our way, because we can use modus ponens as before to conclude that she likes algebra. We can also conclude that she likes, physics by the same the disjunctive syllogism that we did in propositional case, but now we can produce another statement, which is the conjunction that we did in the first case also.

But, another application of the universal instantiation rule, which says that the general rule that if anyone likes algebra and they like physics they will go to college applies to Alice also. So, we can explicitly say, that if she likes algebra and she likes physics then she will go to college. Having said this we have already said this in statement 11 here, that she likes algebra and she likes physics.

So, we can take 11 and 12 and apply modus ponens and there we are with this proof of the statement that goes to Alice College essentially. So, when we moved from propositional logic to first order logic, what we did of course, we changed the story we made it more general that anyone who likes Math's likes algebra and so on. But, we saw that given that we could still prove that Alice will go to college essentially.

It is a little bit like the Socratic argument that we started with which say that all men are mortal Socrates is a man and therefore, Socrates is mortal. So, here we are saying that all people who like Math's they like algebra and so on and so forth. And, Alice likes Math's and she likes algebra and therefore, she will go to college.

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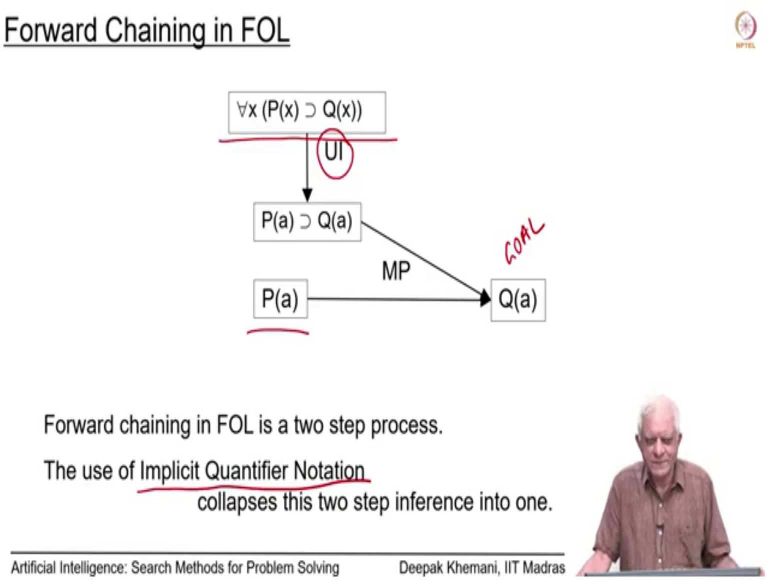

So, this process as I have been saying is forward reasoning or forward chaining, it is a 2 step process for us for the moment. What is the inference we are doing? We are given two statements P of a some random predicate that we are talking about. And, we are given a universal statement, which says that for all x P x implies Q x.

And, our goal is to show that Q of a is true essentially. As we saw in the Alice example this is a two step process.In the first step we apply the rule of universal instantiation, which is here to generate a specific statement to a, P a implies Q a and then we use our Godel modus ponens to arrive at Q a essentially.

Now, we will try to make life simple for us, because eventually our goal is to write programs you know not just to do mathematics.We will use something called the implicit quantifier notation, which will collapse this two step process into one step essentially.

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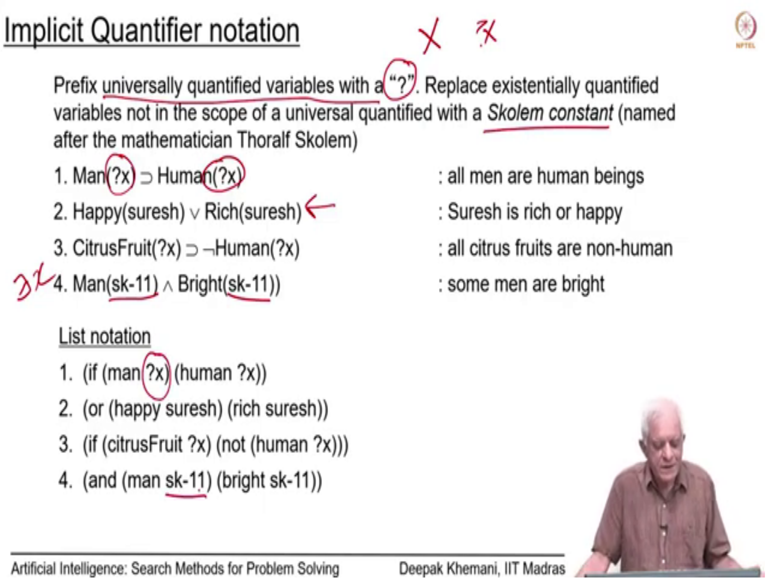

So, the implicit quantifier notation, if you have a universally quantified variable we prefix it with the question mark. So, instead of X we will write it by question mark X. And, we will drop the universal quantifier, because now it is understood between us it is implicit, that if there is a question mark before a symbol or a variable it means it is universally quantified. We will also handle existentially quantified variables.

But, that is a little bit thing that we will not get into right now, because we have only one week to do all of logic. Ah. But, there is a process calls skolemization, which was given to us by thoralf skolem, where we have skolem constants and skolem functions, which take care of the existential quantifier. For our simple study here we will not even deal with existentially quantifiers.

So, the 2 4 statements that we talked about in first order logic are now expressed in a implicit quantifier form. So, what you can see is that whatever was universal has been converted into implicit form essentially. Whatever was proportional remains as before and whatever was existential, there remember that there was a there exists x here. There exists x who is a man and x is bright, we replace it by a special constant which is called as skolem constant.

As I have mentioned here this sk stands for Skolem essentially. So, this is the implicit quantifier form in which we can represent our first order logic statements. Essentially, we are going to throw away those two symbols for all x and there exist x and somehow have an understanding as to which variable is implicit and which variable is universal and which variable is existential in nature.

Existential variables are a little bit more involved, but we will not get into that. This is a simple existential statement in which you can replace it with the skolem constant, but there are others which have to be replaced by skolem functions, but we will not look at them in this week. In list notation the same thing happens, the variable is replaced by either a question mark variable or a skolem constant in this simple case.

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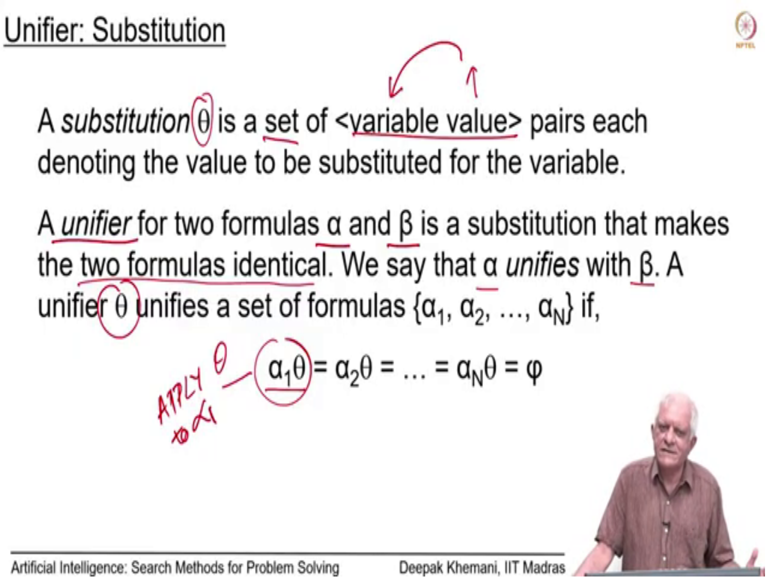

Now, there is something called as substitution or a unifier it is a core part of logical reasoning, we will just have a very quick look at this. A substitution and we use Greek symbols like theta is a set of variable and value pairs. Where each pair tells you, what value can you put in place of the variable or what value, what values can replace variables in some formulas?

And, that happens, if you substitute, if you apply the substitution. Then, we have a notion of a unifier and we will very quickly go over this. We say that two formulas alpha and beta.

There is a unifier for them, if there is a substitution such that the two formulas become identical. We say that alpha unifies with beta and again the unifier is a substitution, if we apply the substitution to a set of formula the alpha 1, alpha 2, alpha n, then they get unified, if when you apply the substitution.

So, when you write this, this stands for applying it essentially. So, we are applying it to alpha 1 here and we have apply to alpha 2, we apply to alpha n and so on. And, we find that all the formulas have become the same then we say that they have been unified.

So, it there is a whole algorithm for unification, again we will not go into that it can deal with all kinds of complex formulas not just simple ones that we are looking at. But, our notion is to explore how is reduction is done in search? So, we will keep it simple as you see.

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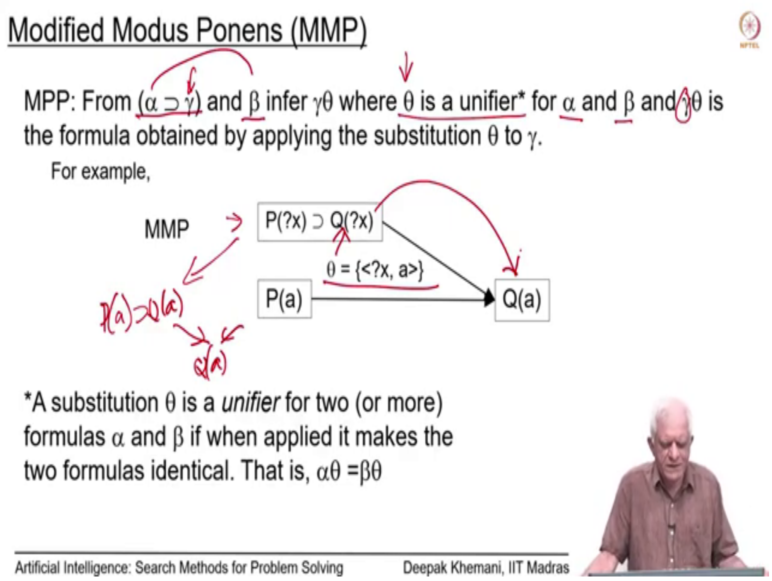

So, now we have a modified modus ponens rule. And, it uses this implicit quantifier formulas and it uses a notion of a substitution. And, it says that if you are given alpha implies gamma and you are given beta. If, somehow you can make alpha and beta look same, that is if there is a unifier theta for alpha and beta.

Then, you take the unifier theta and apply the theta to gamma. What is gamma? Gamma is the consequent. We are still doing modus ponens we are just doing it when the two formulas alpha and the two formulas in the modus ponens do not look identical. One of them may have a variable, which is implicitly quantified and so on.

So, essentially what we are saying is if you want to infer from alpha implies gamma if you want to infer beta look for a unifier theta here, which unifies alpha and beta. Alpha is the antecedent in the in the implication and beta is the given fact essentially.

So, if you can make alpha and beta the same by applying some theta, then apply the same theta to the consequent in the implication statement, which is gamma and then you have your implication. So, the figure I hope makes it looks simple. So, the first sentence here is a Universalist quantifier statement. Remember that there is an implicit quantifier all for all x P x implies Q x. And, what is given to us is P of a essentially?

So, again if you look at if P is men and Q is mortal and the first statement says all men are mortal and if a stand for Socrates, then this is a Socratic argument essentially. But, now we are saying that to make this inference do not go through the process of universal in sensation, earlier we did that remember we went from this. And, we went and said that this must be true and then from this and this we said Q of a must be true.

So, this two step process we are now collapsing into one step and we are saying, that given a universally quantified formula like this. If you can find the substitution which will make them the same, then you can go ahead and directly infer Q of a.

Because, the substitution theta when you apply to this formula Q of x gives us this formula Q of a and that is our goal in any case. Notice also that this makes life simple for programming, you do not have to keep guessing as to what instance of universal instantiation you should apply, that comes from the data itself essentially. So, this is a modified modus ponens rules.

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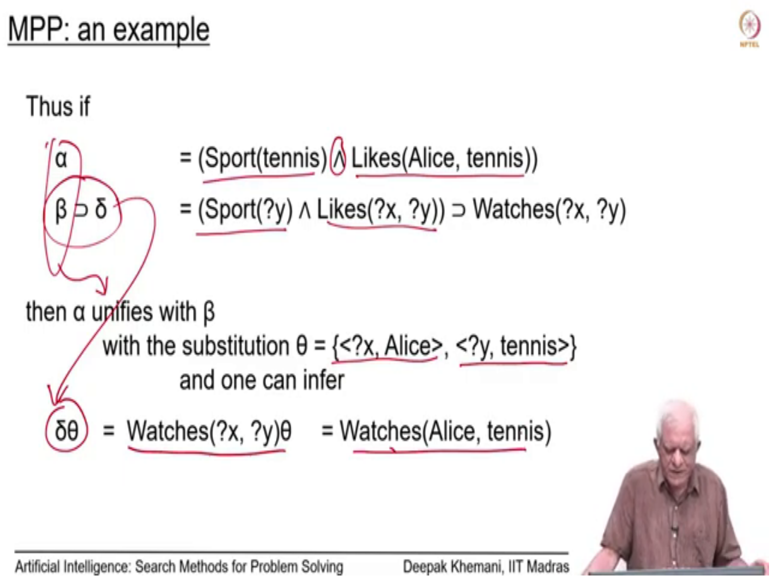

So, here is an example, if alpha stands for the statement, it is a compound statement, it uses and here.It says, alpha says that tennis is a sport and Alice likes tennis. This beta and thetas beta and delta say anyone who likes the sport will watch that sport essentially.

So, you can see that this sport y can match with tennis being a sport and likes x y can match with likes and Alice tennis. So, we can find a substitution to do that. So, we should be able to infer that, Alice will watch tennis. So, that is what it says here. You have to unify alpha with beta here and the theta that you get this x is replaced by Alice, y is replaced by tennis, then we apply this theta to delta, delta is a consequent there remember.

And, so, we substitute this apply this theta to this formula and we get this formula, which is what we want to show that Alice will watch tennis. Now, just use the compound formula, now let us try to take care of that as well.

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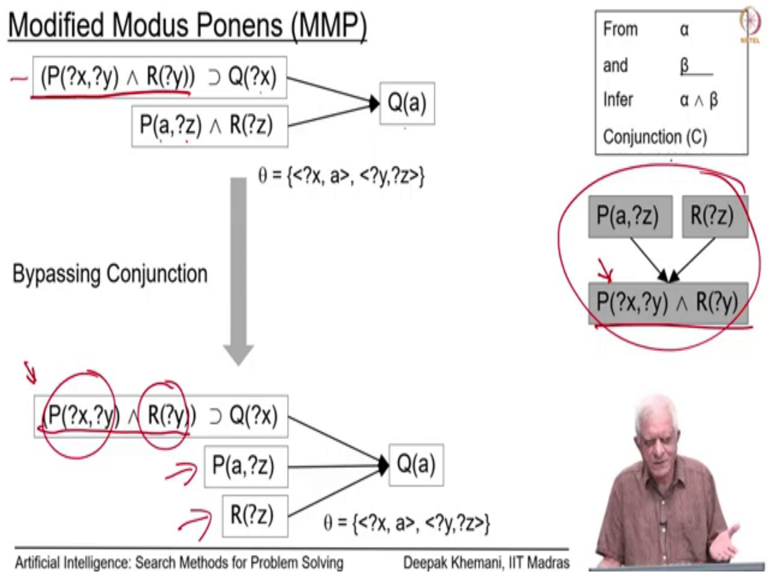

This is a rule that we are talking about. So, you can map it to tennis and Alice and that kind of stuff, but essentially what it says is that, if you are for all x, for all y, this is the first statement here.

P x y and R of y implies Q of x. So, this x and this x are the same. And, if you are given P of a and some variables that we do not even know what it is, we can still go ahead and infer Q of a. So, this is an application of the modified modus ponens rule, but it needs the conjunct as a antecedent in the rule. Very often that is not the case and very often you will get those two formulas separately in your knowledge base.

So, we may know something about P, we may know something about R, and we may have to pull together those two formulas to form a conjunct. Now of course, logic allows us to do that, we can use the rule of conjunction, which will say given alpha and given beta, you can infer alpha or beta, you can add alpha and beta to the formula.

So, given these two things which are given to us separately, you can now combine it to make a compound formula, which is nice because now we can apply modus ponens as you can see here.This conjunct matches this conjunct and then we can make the inference. So, between this formula and this formula, we can infer Q of a.

Essentially,, but we can sidestep applying this conjunction process as well by simply saying that, modify the modus ponens still further to say, that if the antecedent has two formulas here. And, if they are available independently and separately, you can still apply the rule and make the inference that you wanted to make essentially.

So, we having compacting the proof process. First we said do over with universal instantiation. Now, we are saying do over with conjunction as well essentially. So, of course, you can use conjunction and use this process to get a compound formula and use that directly, but why not simply implement your system, so, that this will happen implicitly directly essentially.

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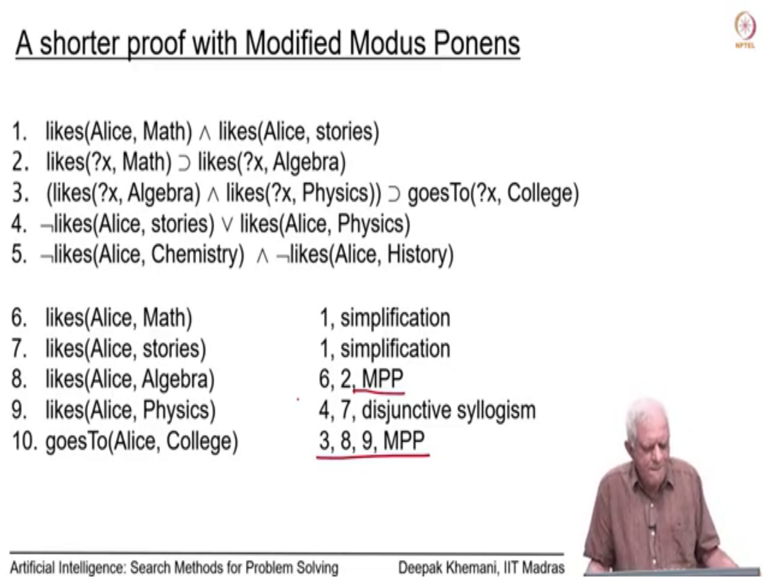

So, now back to our earlier story, you can see that we can find a much shorter proof here, we have used a modified modus ponens. So, the universal instantiation step has vanished. And, here we have used even the further modified modus ponens, where we took three arguments and directly concluded that Alice will go to college essentially ok.

So, this gives us a flavor of forward chaining or forward reasoning in first order logic. And, we get some ideas as to how we can generate proofs for formulas? And, this process is called theorem proving. We have looked at one aspect forward reasoning we will go back and look at backward reasoning or backward chaining, in the next session. So, see you then.